Research Article
The Phase Transition of Higher Dimensional Charged Black Holes

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We have studied phase transitions of higher dimensional charge black hole with spherical symmetry. We calculated the local energy and local temperature and find that these state parameters satisfy the first law of thermodynamics. We analyze the critical behavior of black hole thermodynamic system by taking state parameters (Q, Φ) of black hole thermodynamic system, in accordance with considering the state parameters (P, V) of van der Waals system, respectively. We obtain the critical point of black hole thermodynamic system and find that the critical point is independent of the dual independent variables we selected. This result for asymptotically flat space is consistent with that for AdS spacetime and is intrinsic property of black hole thermodynamic system.

1. Introduction

In recent years, the study of critical behavior of black holes has still received a lot of attention. Particularly, the idea of including the variation of the cosmological constant Λ in the first law of black hole thermodynamics has attained increasing attention. Matching the thermodynamic quantities with the ones in usual thermodynamic system, the critical behavior of black holes can be investigated and the phase diagram like the van der Waals vapor-liquid system, critical exponents, and Clapeyron equations can be obtained [1–14]. For the de Sitter spacetime, the thermodynamic properties and phase transition of black hole with the method of the equivalence quantities have been investigated, and it is shown that the result is similar to AdS black hole [15–20]. This helps to further understand black hole entropy, temperature, heat capacity, and so on, and it is also very important to improve the self-consistent geometric theory of black hole thermodynamics.

As is well known, there are Hawking radiations for black holes in asymptotically flat spacetime and nonasymptotically flat ones. Because the heat capacity of black hole in

asymptotically flat spacetime is negative, this black hole in asymptotically flat space is thermodynamically unstable. For studying the thermodynamics properties and phase transition of black hole, we should firstly verify that the black hole is thermodynamically stable.

In order to restore thermodynamic stability so that equilibrium thermodynamics and the phase structure can be studied, we must consider the whole systems that include not only the black hole under consideration but also its environment [21, 22], as self-gravitating systems are spatially inhomogeneous, which is different from the usual thermodynamic system. Any specification of such system requires not only thermodynamic quantities of interest but also the place at which they take the specified values. In this paper, we have studied the phase transition of higher dimensional charged black hole with spherical symmetry. To solve this problem we can place the black hole inside a finite concentric spherical cavity, whose radii are fixed and larger than the one of black hole. The temperature is fixed on the surface of the cavity, which could be physically realized by placing a heat bath around the cavity. We will keep the charge inside the cavity also fixed. This will define a canonical ensemble [23–28].

We will study the phase structure and the thermodynamic properties of the various dimensional charged black hole in this ensemble.

References [29–35] have investigated the critical behavior of various black holes in AdS spacetime with Ehrenfest scheme. They found that the phase transition of black hole is the continuous one in AdS spacetime, and the thermodynamic quantities of the critical point satisfy the Ehrenfest equation. References [36–38] have obtained the same conclusion by studying the thermodynamics and state space geometry of black hole in AdS space. Moreover, it is interesting to study the thermodynamics behavior of the ensemble, which we construct for asymptotical flat space, and identify whether the ensemble meets the Ehrenfest equation. We obtain the phase diagram for the thermodynamic quantities (Q, Φ) at the critical point and the result similar to that in AdS black hole. The thermodynamic quantities satisfy the Ehrenfest equation, so the corresponding phase transition is a continuous phase transition.

The paper is arranged as follows: In Section 2 we first review the higher dimensional charged black hole in asymptotically flat space and give the quasilocal thermodynamic quantities of the canonical ensemble. In Section 3 the critical behavior in different dimensional charged black hole is investigated. Finally, the paper ends with a brief conclusion (we use the units $G_d = \hbar = k_B = c = 1$).

2. The Thermodynamical Quantity of the Charged Black Hole with the Ensemble Theory in Higher Dimension

The solution for charged black hole in spacetime dimensions with $d > 3$ reads

$$ds^2 = -Vdt^2 + \frac{dr^2}{V} + r^2 d\Omega_{d-2}^2, \tag{1}$$

where the $V(r)$ functions entering the metric are given by

$$V(r) = 1 - \frac{m}{r^{d-3}} + \frac{q^2}{r^{2(d-3)}}. \tag{2}$$

Here the parameter m is related to the ADM mass (M) of black holes:

$$M = \frac{(d-2)\,\omega_{d-2}}{16\pi}m; \quad \omega_{d-2} = \frac{2\pi^{(d-1)/2}}{\Gamma\left((d-1)/2\right)}, \tag{3}$$

where ω_{d-2} is the volume of unit $(d-2)$ sphere. The parameter q is related to the electric charged Q as

$$Q = \frac{\sqrt{2(d-2)(d-3)}}{8\pi}\omega_{d-2}q. \tag{4}$$

The entropy of the system is given by

$$S = \frac{\omega_{d-2}}{4}r_+^{d-2}, \tag{5}$$

where r_+ is the radius of the outer event horizon defined by the condition $V(r_+) = 0$. Let us first consider the

cavity as a boundary with a radius r_B to study quasilocal thermodynamics along the line of the procedure in [22]. Then, the local temperature measured at the boundary is given by [22–28]

$$T_{\text{loc}} = \frac{T}{\sqrt{V(r_B)}} = \frac{(d-3)\left(r_+^{2(d-3)} - q^2\right)}{4\pi\sqrt{V(r_B)}r_+^{2d-5}}. \tag{6}$$

For fixed charge Q, the entropy calculated from the first law of thermodynamics is

$$S = \int \frac{dM}{T}. \tag{7}$$

Applying the first law of thermodynamics, the total thermodynamic internal energy within the boundary r_B is obtained as

$$E_{\text{loc}} = \int_{M_0}^{M} T_{\text{loc}}dS = \int_{M_0}^{M} \frac{T_{\text{loc}}}{T}dM$$
$$= \frac{(d-2)\,\omega_{d-2}}{8\pi}r_B^{d-3}\left(\sqrt{V(B_0)} - \sqrt{V(r_B)}\right). \tag{8}$$

Taking $V(B_0) = 1$ at the boundary of cavity [26], we can obtain

$$E_{\text{loc}} = \frac{(d-2)\,\omega_{d-2}}{8\pi}r_B^{d-3}\left(1 - \sqrt{V(r_B)}\right), \tag{9}$$

where

$$V(r_B) = \left(1 - \frac{r_+^{d-3}}{r_B^{d-3}}\right)\left(1 - \frac{q^2}{r_+^{d-3}r_B^{d-3}}\right). \tag{10}$$

So we can define the reduction quantities for corresponding

$$x = \frac{r_+^{d-3}}{r_B^{d-3}},$$

$$\tilde{q} = \frac{q}{r_B^{d-3}},$$

$$b_q(x) = \frac{(d-2)\,\beta_B}{8\pi r_B}, \tag{11}$$

$$h = \frac{(d-2)\,\omega_{d-2}}{8\pi}r_B^{d-3}\left(1 - \sqrt{(1-x)\left(1 - \frac{\tilde{q}^2}{x}\right)}\right),$$

$$S = \frac{\omega_{d-2}}{4}r_B^{d-2}x^{(d-2)/(d-3)}.$$

For $r_+ > q$ and $r_B > r_+$, we get $q < x < 1$. The states functions in the cavity satisfy the first law of thermodynamic system [22, 39]:

$$dE_{\text{loc}} = T_{\text{loc}}dS + \Phi dQ + \sigma dA, \tag{12}$$

where

$$S = \frac{\omega_{d-2}}{4}r_+^{d-2},$$
$$A = \omega_{d-2}r_B^{d-2}. \tag{13}$$
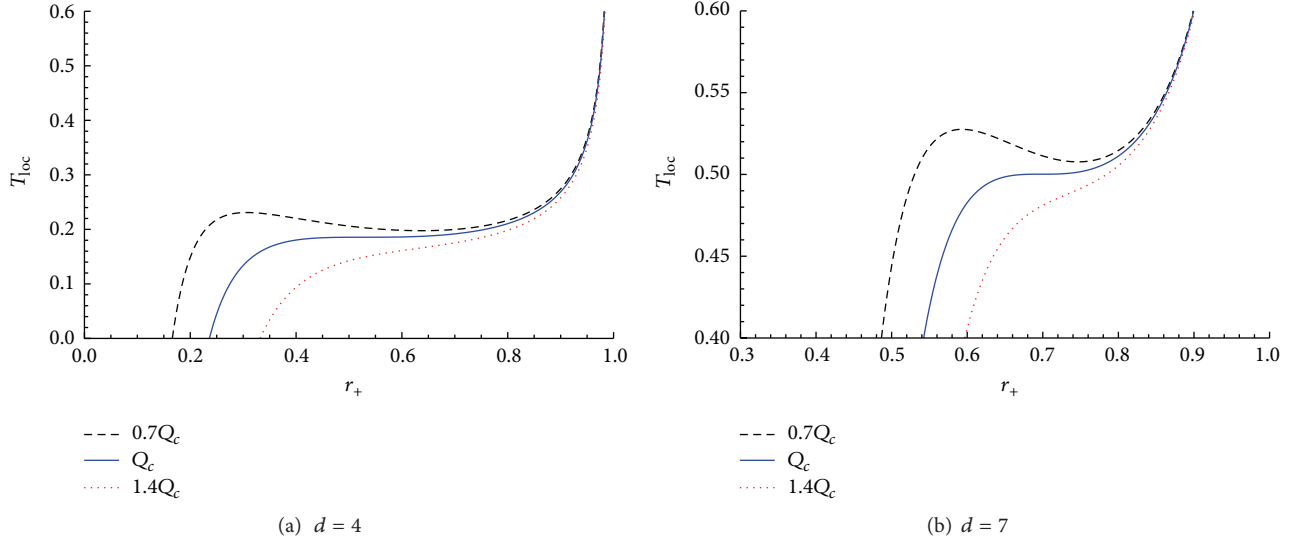
(a) $d = 4$

(b) $d = 7$

Figure 1: The T_{loc}-r_+ curves for $d = 4, 7$, respectively. From top to bottom the curves correspond to the charges $0.7Q_c$, Q_c, and $1.4Q_c$.

From (9) and (12), we can obtain

$$
\left(\frac{\partial E_{\text{loc}}}{\partial S} \right)_{Q,A} = T_{\text{loc}}
$$
$$
= \frac{(d-3)\left(1 - q^2/r_+^{2(d-3)}\right)}{4\pi r_+ \left(1 - r_+^{d-3}/r_B^{d-3}\right)^{1/2}\left(1 - q^2/r_+^{d-3}r_B^{d-3}\right)^{1/2}},
$$
$$
\left(\frac{\partial E_{\text{loc}}}{\partial Q} \right)_{S,A} = \Phi
$$
$$
= \frac{(d-2)q\left(1 - r_+^{d-3}/r_B^{d-3}\right)}{\sqrt{2(d-2)(d-3)}r_+^{d-3}\left(1 - r_+^{d-3}/r_B^{d-3}\right)^{1/2}\left(1 - q^2/r_+^{d-3}r_B^{d-3}\right)^{1/2}},
$$
$$
\left(\frac{\partial E_{\text{loc}}}{\partial A} \right)_{r_+,Q} = \sigma
$$
$$
= \frac{(d-3)}{8\pi r_B}\left[1 - \frac{1 - r_+^{d-3}/2r_B^{d-3} - q^2/2r_+^{d-3}r_B^{d-3}}{\left(1 - r_+^{d-3}/r_B^{d-3}\right)^{1/2}\left(1 - q^2/r_+^{d-3}r_B^{d-3}\right)^{1/2}} \right].
$$

(14)

3. The Critical Effect in Charged Black Hole in Cavity

For the thermodynamic ensemble, we take the state parameters T_{loc}, S as the independent variables to study the critical behaviors of thermodynamic ensemble. When these state parameters Q and r_B are invariable quantities, the critical values of black hole radii r_B, black hole event horizon r_+, and the black hole temperature are decided by

$$
\left(\frac{\partial T_{\text{loc}}}{\partial r_+} \right)_{r_B,Q} = 0,
$$
$$
\left(\frac{\partial^2 T_{\text{loc}}}{\partial r_+^2} \right)_{r_B,Q} = 0.
$$

(15)

Table 1: Numerical solutions for r_+^c, Q^c, T_{loc}^c, and Φ^c for given values of $d = 4, 5, 6, 7$, respectively.

d	r_+^c	Q^c	T_{loc}^c	Φ^c
4	0.527864	0.236068	0.185589	0.324920
5	0.610537	0.320704	0.301913	0.221054
6	0.661764	0.378834	0.404099	0.177014
7	0.698081	0.404347	0.500148	0.151505

We can calculate the position of the critical points in different dimensional spacetime. The results are shown in Table 1 (we take $r_B = 1$).

Table 1 shows that the critical values of r_+^c, Q^c, and T_{loc}^c increase as the spacetime dimension d increases, and Φ^c decreases as the spacetime dimension d increases. Figure 1 shows T_{loc} curves with black hole radii for different d. we can find that there is a phase transition near critical point with the charge $Q \leq Q_c$. The critical temperature T_{loc} increases as the spacetime dimension d increases. From the heat capacity of system

$$
C_Q = T_{\text{loc}}\left(\frac{\partial S}{\partial T_{\text{loc}}} \right)
$$

(16)

we can find that the instability state will appear with the charge of spacetime less than the critical charge. By using Maxwell's equal area law we discover the possible two-phase coexistence curves in the process of phase transition [39, 40].

Next, let us calculate the free energy of the black hole in order to study phase transition between the black holes and the hot flat space [24, 28]
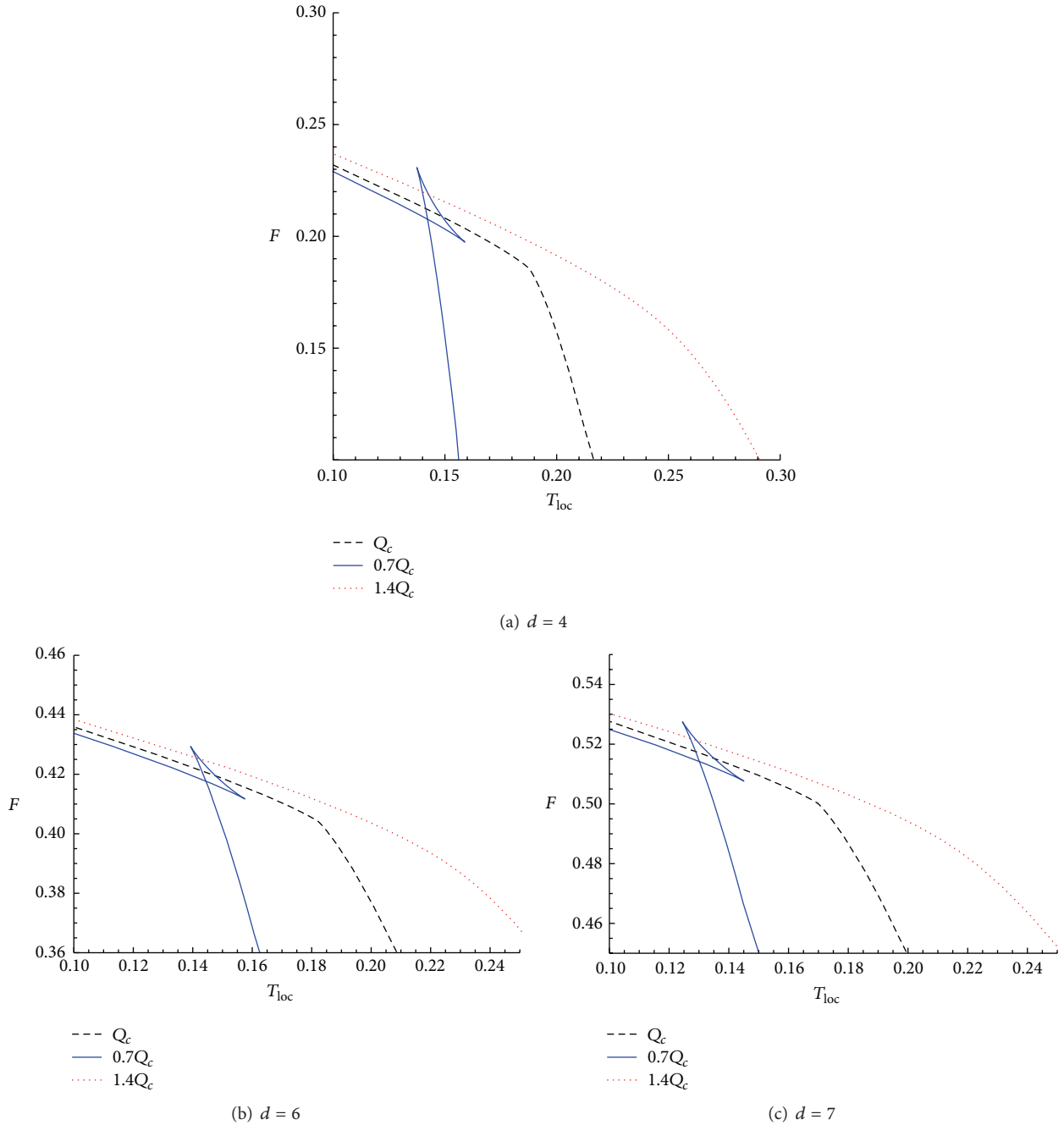
$$
F = E_{\text{loc}} - T_{\text{loc}}S.
$$

(17)

(a) $d = 4$

(b) $d = 6$

(c) $d = 7$

FIGURE 2: The F-T_{loc} curves for $d = 4, 6, 7$, respectively. From top to bottom the curves correspond to the effective temperatures $0.7Q^c$, Q^c, and $1.4Q^c$.

We can plot the relations curve for the free energy F and the local temperature T_{loc}, with $r_B = 1$, and Q takes values near critical charge. Figure 2 shows that the system is the two-phase coexistence state, when the charge of black hole is smaller than the critical charge Q_c. This result is consistent with Figure 1. From Table 1, it is found that the critical temperature and critical charge increase as the spacetime dimension increases. This result is consistent with the conclusion in [3] for higher dimensional AdS black hole.

Further, We can plot the curve for the free energy F and the local temperature Q, with $r_B = 1$, and T_{loc} take values near critical temperature in Figure 3. From Figure 3, we can see that there is a phase transition at the quasilocal temperature less than the critical quasilocal temperature in different dimensions. The results are consistent with the profile of F-T_{loc} in Figure 2.

From (12), we know that we can select the independent variables (Q, Φ) or (σ, A) for the black hole thermodynamic
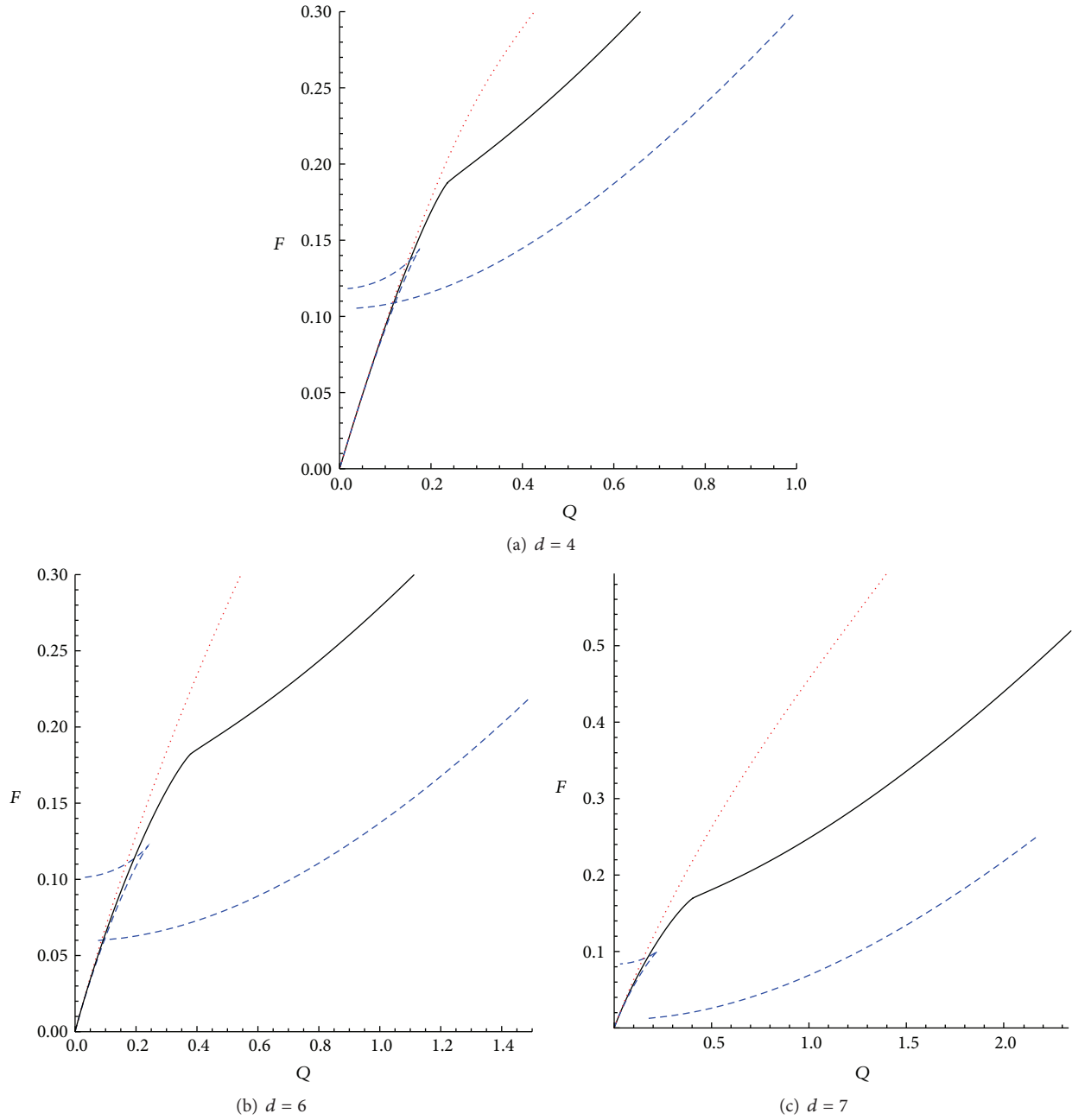
(a) $d = 4$

(b) $d = 6$

(c) $d = 7$

FIGURE 3: The F-Q curves for $d = 4, 6, 7$, respectively. From top to bottom the curves correspond to the effective temperatures $0.7T_{\text{loc}}$, T_{loc}, and $1.4T_{\text{loc}}$.

ensemble. When taking r_B and T_{loc} as constants, we select the dual independent variables (Φ, Q), and the critical position is given by the following conditions:

$$
\left(\frac{\partial Q}{\partial \Phi}\right)_{T_{\text{loc}}, r_B}
$$

$$
= \frac{(\partial T_{\text{loc}}/\partial r_+)_Q}{(\partial \Phi/\partial Q)_{r_+} (\partial T_{\text{loc}}/\partial r_+)_Q - (\partial T_{\text{loc}}/\partial Q)_{r_+} (\partial \Phi/\partial r_+)_Q}
$$

$$
= \frac{(\partial T_{\text{loc}}/\partial r_+)_Q}{\partial (\Phi, T_{\text{loc}})/\partial (Q, r_+)} = f(r_+, Q) = 0,
$$

$$
\left(\frac{\partial^2 \Phi}{\partial Q^2}\right)_{T_{\text{loc}}, r_B} = \left(\frac{\partial f(r_+, Q)}{\partial Q}\right)_{T_{\text{loc}}, r_B} = 0.
$$

(18)

We can plot the curve for the free energy Φ and the local temperature Q in Figure 4, with $r_B = 1$, and T_{loc} take values near critical temperature.
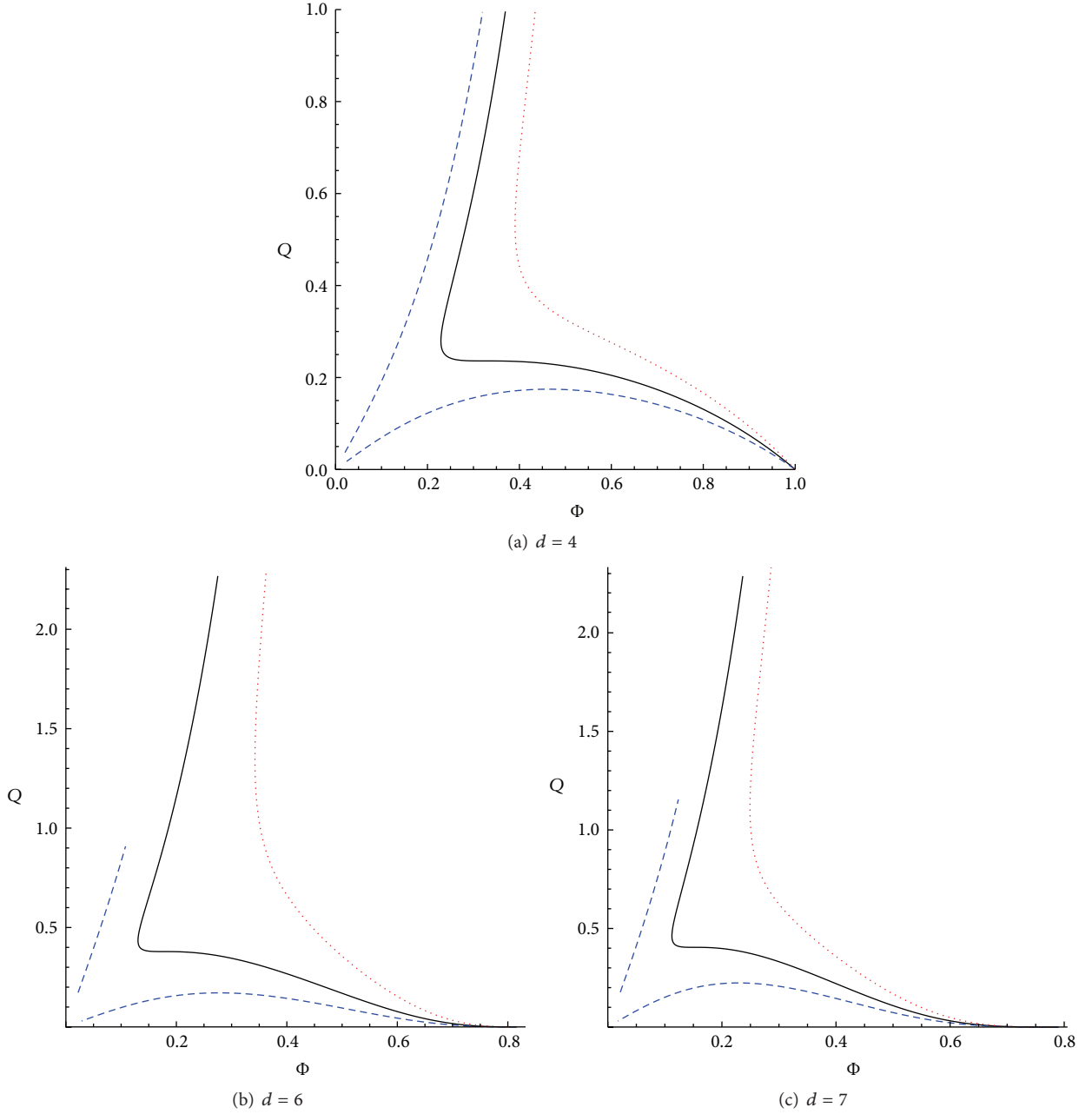
(a) $d = 4$

(b) $d = 6$

(c) $d = 7$

FIGURE 4: The Q-Φ curves for $d = 4, 6, 7$, respectively. From top to bottom the curves correspond to the effective temperatures $0.7T_{\text{loc}}$, T_{loc}, and $1.4T_{\text{loc}}$.

When taking r_B and T_{loc} as constants, we select the dual independent variables (σ, A), and the critical position is given by the following conditions:

$$\left(\frac{\partial A}{\partial \sigma}\right)_{T_{\text{loc}}, Q} = \frac{1}{8\pi r_B}$$

$$\cdot \frac{(\partial T_{\text{loc}}/\partial r_+)_{r_B}}{(\partial \sigma/\partial r_B)_{r_+} (\partial T_{\text{loc}}/\partial r_+)_{r_B} - (\partial T_{\text{loc}}/\partial r_B)_{r_+} (\partial \sigma/\partial r_+)_{r_B}}$$

$$= \frac{(\partial T_{\text{loc}}/\partial r_+)_{r_B}}{\partial (\sigma, T_{\text{loc}}) / \partial (r_B, r_+)} = f(r_+, r_B) = 0,$$

$$\left(\frac{\partial^2 A}{\partial \sigma^2}\right)_{T_{\text{loc}}, Q} = \left(\frac{\partial f(r_+, r_B)}{\partial \sigma}\right)_{T_{\text{loc}}, Q} = 0.$$

$$(19)$$

Comparing (15), (18), and (19), we can find that the equation of critical point is the same. So we can obtain the same critical point with different independent dual variable in the
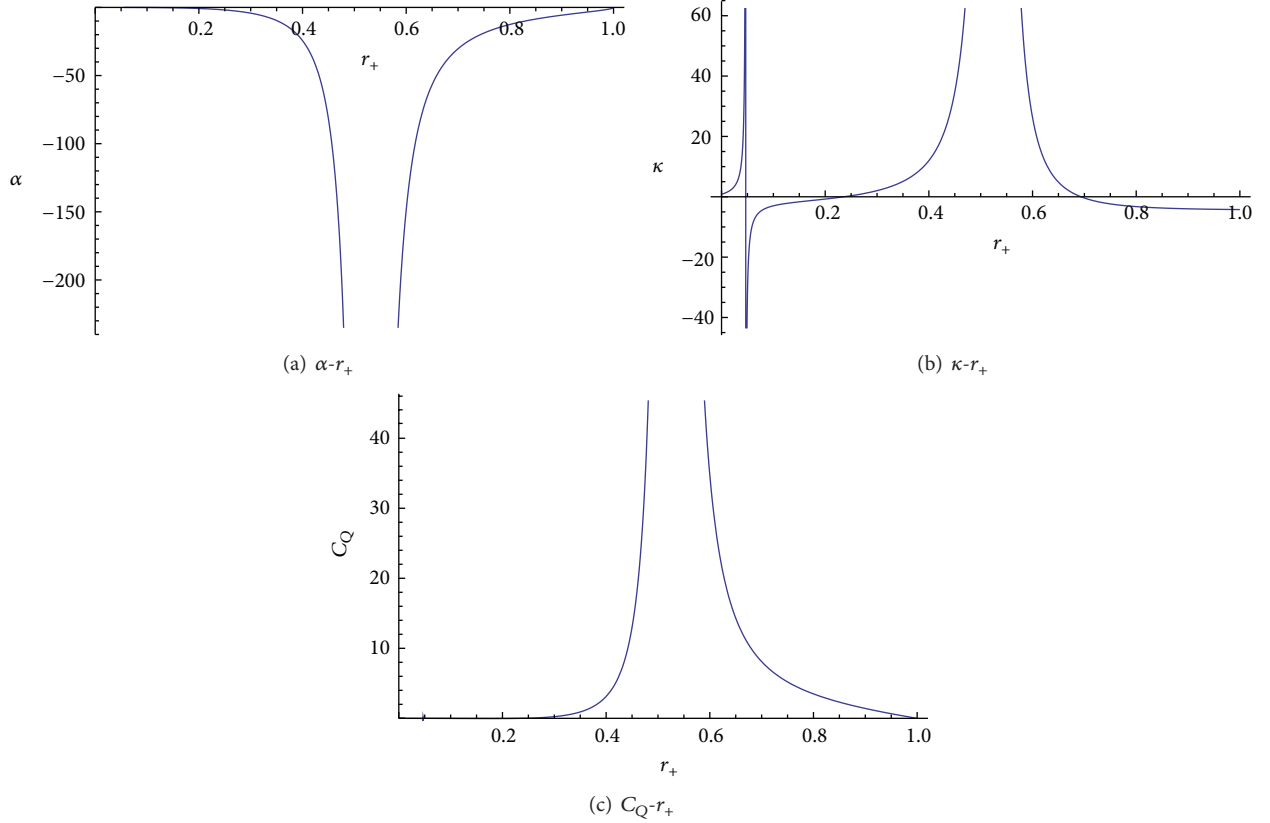
(a) α-r_+

(b) κ-r_+

(c) C_Q-r_+

FIGURE 5: α-r_+, κ-r_+, and C_Q-r_+ curves for higher dimensional charged black hole with $d = 4$.

thermodynamic system. The critical point is unaltered with the choice of independent dual variables.

According to Ehrenfest's classification, when the chemical potential and its first derivative are continuous, whereas the second derivative of chemical potential is discontinuous, this kind of phase transition is called the second-order phase transition. For van der Waals system there is no latent heat and the liquid-gas structure does not change suddenly at the critical point. Therefore this kind of phase transition belongs to the second-order phase transition and continuous phase transition. To discuss the critical behaviors of system near the phase transition point with r_B unaltered, we will evaluate the second-order partial derivation of chemical potential:

$$\alpha = \frac{1}{\Phi}\left(\frac{\partial \Phi}{\partial T_{\text{loc}}}\right)_Q = \frac{1}{\Phi}\left(\frac{\partial \Phi}{\partial r_+}\right)_Q \left(\frac{\partial T_{\text{loc}}}{\partial r_+}\right)_Q^{-1},$$

$$k_T = -\frac{1}{\Phi}\left(\frac{\partial \Phi}{\partial Q}\right)_{T_{\text{loc}}} = -\frac{1}{\Phi}\left[\left(\frac{\partial \Phi}{\partial Q}\right)_{r_+}\left(\frac{\partial T_{\text{loc}}}{\partial r_+}\right)_Q\right.$$

$$\left. - \left(\frac{\partial \Phi}{\partial r_+}\right)_Q \left(\frac{\partial T_{\text{loc}}}{\partial Q}\right)_{r_+}\right]\left(\frac{\partial T_{\text{loc}}}{\partial r_+}\right)_Q^{-1},$$

$$C_Q = T_{\text{loc}}\left(\frac{\partial S}{\partial T_{\text{loc}}}\right)_Q = T_{\text{loc}}\left(\frac{\partial S}{\partial r_+}\right)\left(\frac{\partial T_{\text{loc}}}{\partial r_+}\right)_Q^{-1}. \qquad (20)$$

From (15), (18), and (19), we know that these quantities, α, k_T, and C_Q, obtained in (20), approach infinity at the critical point. So the phase transition, which happens in the black hole thermodynamic ensemble, is a continuous phase transition. The Gibbs free energy and the first-order partial derivation for these two phases are continuous at the critical point of continuous phase. We also depict the curves of α-r_+, κ-r_+, and C_Q-r_+ in Figure 5, respectively, at the constant pressure. From these curves, we find that the specific heat of black hole thermodynamic system at constant charge C_Q, the expansion coefficient α, and the compressibility κ have infinite peak.

For usual thermodynamic system, the entropy change and volume change are equivalent, respectively, as $dv^1 = dv^2$ and $dS^1 = dS^2$, near critical point; Ehrenfest had calculated the Ehrenfest equation:

$$\frac{dP}{dT} = \frac{\alpha^1 - \alpha^2}{k_T^1 - k_T^2},$$

$$\frac{dP}{dT} = \frac{C_P^1 - C_P^2}{Tv(\alpha^1 - \alpha^2)}, \qquad (21)$$

in which superscripts 1 and 2 represent phases 1 and 2, respectively. Recently, the Ehrenfest equation for AdS black

hole thermodynamic system [29–35, 41] has been studied and the Prigogine-Defay (PD) relation is obtained:

$$\Pi = \frac{\Delta C_P \Delta k_T}{TV (\Delta \alpha)^2} = 1. \tag{22}$$

For the black hole thermodynamic system, we can rewrite the equation of Ehrenfest as

$$\left(\frac{\partial Q}{\partial T_{\text{loc}}}\right)_\Phi = \frac{\alpha_2 - \alpha_1}{k_{T_2} - k_{T_1}} = \frac{\Delta \alpha}{\Delta k_T},$$
$$\left(\frac{\partial Q}{\partial T_{\text{loc}}}\right)_S = \frac{C_{Q_2} - C_{Q_1}}{T_{\text{loc}}\Phi (\alpha_2 - \alpha_1)} = \frac{\Delta C_Q}{T_{\text{loc}}\Phi \Delta \alpha}. \tag{23}$$

Using

$$\left(\frac{\partial T_{\text{loc}}}{\partial Q}\right)_\Phi = \left(\frac{\partial T_{\text{loc}}}{\partial Q}\right)_S$$
$$- \left(\frac{\partial T_{\text{loc}}}{\partial S}\right)_Q \left(\frac{\partial \Phi}{\partial Q}\right)_S \left(\frac{\partial \Phi}{\partial S}\right)_Q^{-1}, \tag{24}$$

according to (18), the critical points satisfy

$$\left(\frac{\partial T_{\text{loc}}}{\partial r_+}\right)_Q = \left(\frac{\partial T_{\text{loc}}}{\partial S}\right)_Q \left(\frac{\partial S}{\partial r_+}\right)_Q = 0. \tag{25}$$

From (5), we know

$$\left(\frac{\partial S}{\partial r_+}\right)_Q = \frac{\omega_{d-2} (d - 2) r_+^{d-3}}{4} \neq 0. \tag{26}$$

So, (24) can be written as

$$\left(\frac{\partial T_{\text{loc}}}{\partial Q}\right)_\Phi^c = \left(\frac{\partial T_{\text{loc}}}{\partial Q}\right)_S^c. \tag{27}$$

Substituting (27) into (23), the Prigogine-Defay (PD) ratio (Π) can be calculated as

$$\Pi = \frac{\Delta C_Q \Delta \kappa_T}{T_{\text{loc}}\Phi (\Delta \alpha)^2} = 1. \tag{28}$$

Hence, when Q is constant, the phase transition occurring at $T_{\text{loc}} = T_{\text{loc}}^c$ is a second-order equilibrium transition. This is true in spite of the fact that the phase transition curves are smeared and divergent near the critical point. This result is in agreement with the result of the AdS black holes.

4. Discussion and Conclusions

In this work, to study the properties of black hole in asymptotically flat spacetime, we built a stable black hole thermodynamic ensemble in the view of thermodynamics. we calculated the local energy and local temperature and find that these state parameters satisfy the first law of thermodynamic equation (12). Based on this condition, we analyze the critical behavior of black hole thermodynamic ensemble through taking the state parameters (Q, Φ) of black

hole thermodynamic ensemble corresponding to the state parameters (P, V) of van der Waals system, respectively. We obtain the critical point of black hole thermodynamic system and find that the critical point is independent of the dual independent variables we selected. This result for asymptotically flat space is consistent with the conclusion for AdS spacetime [1] and is an intrinsic property of black hole thermodynamic ensemble. Firstly, from Table 1 we can find that the critical temperature and critical charge of black hole ensemble increase with the spacetime dimensional increase, which is similar to the critical behavior of AdS black hole. Secondly, the second partial derivatives α, κ_T, and C_Q of Gibbs free energy of black hole thermodynamic ensemble are divergent at the critical point; the result is the same as the result of AdS spacetime. Finally, the Prigogine-Defay (PD) relation at critical point in black hole thermodynamic ensemble is consistent with the AdS black hole. The PD ratio satisfies (28) and is independent of spacetime dimension.

In the black hole thermodynamic ensemble, we built a concentric spherical cavity, whose radii are fixed and larger than the ones of black hole, to achieve the condition of thermodynamic stability for black hole ensemble. The reason, for which AdS black hole can achieve the thermodynamic stability, is that there is a cosmology constant Λ. Through comparing the critical behavior of thermodynamic ensemble with the ones of AdS spacetime, we can argue that the radii r_B of concentric spherical cavity are related to the cosmology constant Λ in the thermodynamic view.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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