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Research Article

Thermodynamic Product Relations for Generalized Regular Black Hole

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We derive thermodynamic product relations for four-parametric regular black hole (BH) solutions of the Einstein equations coupled with a nonlinear electrodynamics source. The four parameters can be described by the mass (m), charge (q), dipole moment (α), and quadrupole moment (β), respectively. We study its complete thermodynamics. We compute different thermodynamic products, that is, area product, BH temperature product, specific heat product, and Komar energy product, respectively. Furthermore, we show some complicated function of horizon areas that is indeed *mass-independent* and could turn out to be *universal*.

1. Introduction

The thermodynamic product for Reissner Nordstrøm (RN) BH, Kerr BH, and Kerr-Newman (KN) BH [1] has been examined; a simple area product is sufficient to draw a conclusion that the product of area (or entropy) is a *universal* quantity. The universal term is used here to describe when the product of any thermodynamic quantities is simply *mass-independent*. Alternatively mass-dependent thermodynamic quantities imply that they are *not* universal quantity. This is strictly followed throughout the work.

In case of RN-AdS [2], Kerr-AdS, and KN-AdS BH [3] one can not find a simple area product relation; instead one could find a complicated function of event horizon ($\mathscr{H}^+$) area and Cauchy horizon area ($\mathscr{H}^-$) that might be universal. Very recently, we derived for a regular Ayón-Beato and García BH (ABG) [4] the function of $\mathscr{H}^+$ area and $\mathscr{H}^-$ area is

$$f\left(\mathscr{A}_+, \mathscr{A}_-\right) = 384\pi^3 q^6, \tag{1}$$

where the function should read

$$\begin{aligned} f\left(\mathscr{A}_+, \mathscr{A}_-\right) &= \mathscr{A}_+\mathscr{A}_-\left(\mathscr{A}_+ + \mathscr{A}_-\right) + 24\pi q^2 \mathscr{A}_+\mathscr{A}_- \\ &\quad - 256\pi^4 q^8 \left(\frac{\mathscr{A}_+ + \mathscr{A}_-}{\mathscr{A}_+\mathscr{A}_-}\right) \\ &\quad - \frac{\mathscr{A}_+\mathscr{A}_+}{\mathscr{A}_+ + \mathscr{A}_+ + 4\pi q^2}\left[\left(\mathscr{A}_+ + \mathscr{A}_-\right)^2\right. \\ &\quad + 24\pi q^2\left(\mathscr{A}_+ + \mathscr{A}_-\right) - \mathscr{A}_+\mathscr{A}_- - \frac{256\pi^4 q^8}{\mathscr{A}_+\mathscr{A}_-} \\ &\quad \left. + 176\pi^2 q^4\right]. \end{aligned} \tag{2}$$

It indicates a very complicated function of horizons area that turns out to be universal. But it is not a simple area product of horizon radii as in RN BH, Kerr BH, and KN BH. This has been very popular topic in recent years in the GR (General Relativity) community [1] as well as in the String community [5] (see also [6–10]). These universal relations are particularly interesting because they could hold in more general situations like when a BH space-time is perturbed by surrounding matter. For example, KN BH surrounded by a ring of matter the universal relation does indeed holds [1].

Recently, Page and Shoom [8] have given a heuristic argument for the universal area product relation of a four-dimensional adiabatically distorted charged rotating BH. They in fact showed that the product of outer horizon area

and inner horizon area could be expressed in terms of a polynomial function of its charge, angular momenta, and inverse square root of cosmological constant. It has been argued by Cvetič et al. [5] that if the cosmological parameter is quantized, the product of $\mathscr{H}^+$ area and $\mathscr{H}^-$ area could provide a "looking glass for probing the microscopy of general BHs".

However, in this work, we would like to evaluate the thermodynamic product formula for a generalized regular (singularity-free) BH described by the four parameters, namely, m, q, α, and β [11]. This class of BH is a solution of Einstein equations coupled with a nonlinear electrodynamics source. We examine complete thermodynamic properties of this BH. We show some complicated function of physical horizon areas that is indeed mass-independent but it is not a simple area product as in Kerr BH or KN BH. We also compute the specific heat to examine the thermodynamic stability of the BH. Finally we compute the Komar energy of this BH.

It should be noted that Smarr's mass formula and Bose-Dadhich identity do not hold for $\alpha = 3$ and $\beta = 3$ when one has taken into account the nonlinear electrodynamics [12–15]. It also should be mentioned that, for some regular BHs [16], once the entropy is taken to be the Bekenstein-Hawking entropy [17, 18] the first law of BH thermodynamics is no longer established because there is an inconsistency between the conventional 1st law of BH mechanics and Bekenstein-Hawking area law. The authors [16] also showed the corrected form of the first law of BH thermodynamics for this class of regular BHs. We should expect that this is also true for regular ABG BH and it should be valid for arbitrary values of α and β.

The plan of the paper is as follows. In Section 2, we have described the basic properties of the generalized regular BH and computed various thermodynamic properties. Finally, we conclude our discussions in Section 3.

2. Generalized Regular BH Solution

The gravitational field around the four-parametric regular BH solution is described by the metric

$$ds^2 = -\mathscr{B}(r)\,dt^2 + \frac{dr^2}{\mathscr{B}(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{3}$$

where the function $\mathscr{B}(r)$ is defined by

$$\mathscr{B}(r) = 1 - \frac{2mr^{\alpha-1}}{\left(r^2+q^2\right)^{\alpha/2}} + \frac{q^2 r^{\beta-2}}{\left(r^2+q^2\right)^{\beta/2}}. \tag{4}$$

And the electric field is given by

$$\begin{aligned}\mathscr{E}(r) = q\Bigg[&\frac{m\alpha\left\{5r^2-(\alpha-3)q^2\right\}r^{\alpha-1}}{\left(r^2+q^2\right)^{\alpha/2+2}}\\ &+\frac{\left\{4r^4-(7\beta-8)q^2r^2+(\beta-1)(\beta-4)^4\right\}r^{\beta/2}}{4\left(r^2+q^2\right)^{\beta/2+2}}\Bigg],\end{aligned} \tag{5}$$

where the parameters are described previously. This is a class of regular (curvature free) BH solution in GR. The source is nonlinear electrodynamics. In the weak field limits the nonlinear electrodynamics becomes Maxwell field.

In the asymptotic limit, the above solution behaves as

$$\begin{aligned}-g_{tt} &= 1 - \frac{2m}{r} + \frac{q^2}{r^2} + \alpha\frac{mq^2}{r^3} - \beta\frac{q^4}{2r^4} + \mathscr{O}\left(\frac{1}{r^5}\right),\\ \mathscr{E}(r) &= \frac{q}{r^2} + \alpha\frac{5m}{2r^3} - \beta\frac{9q^3}{4r^4} + \mathscr{O}\left(\frac{1}{r^5}\right).\end{aligned} \tag{6}$$

It may be noted that, up to $\mathscr{O}(1/r^3)$, we recover the charged BH behavior and the parameters m and q are related to the mass and charges, respectively. From the electric field behavior we can say that α and β are associated with the electric dipole moment and electric quadrupole moments, respectively. It also should be noted that, in the limit $\alpha = \beta = 0$, we obtain the RN BH and, in the limits $\alpha = 3$ and $\beta = 4$, we recover the ABG BH. This BH solution can be treated as a generalization of ABG BH [19]. The above metric as well as the scalar invariants $R, R_{ab}R^{ab}, R_{abcd}R^{abcd}$ and the electric field are regular everywhere in the space-time. Hence, in this sense it is called a regular BH in Einstein-Maxwell gravity. The first regular model is discovered by Bardeen [20] in 1968.

The BH horizons can be obtained by setting $\mathscr{B}(r) = 0$; that is,

$$\begin{aligned}&\left(r^2+q^2\right)^{(\alpha+\beta)/2} - 2mr^{\alpha-1}\left(r^2+q^2\right)^{\beta/2}\\ &\quad + q^2r^{\beta-2}\left(r^2+q^2\right)^{\alpha/2} = 0.\end{aligned} \tag{7}$$

In this work we restrict our case $\alpha \geq 3$ and $\beta \geq 4$.

Case 1. We have set $\alpha = 4$ and $\beta = 5$. In this case the horizon equation is found to be

$$\begin{aligned}&r^{10} - 2mr^9 + \left(4m^2+5q^2\right)r^8 - 6mq^2r^7\\ &\quad + \left(4m^2q^2+9q^4\right)r^6 - 6mq^4r^5 + 10q^6r^4\\ &\quad - 2mq^6r^3 + 5q^8r^2 + q^{10} = 0.\end{aligned} \tag{8}$$

To find the roots we apply Vieta's theorem; we find

$$\begin{aligned}\sum_{i=1}^{10} r_i &= 2m,\\ \sum_{1\leq i<j\leq 10} r_ir_j &= 4m^2+5q^2,\\ \sum_{1\leq i<j<k\leq 10} r_ir_jr_k &= 6mq^2,\end{aligned}$$

$$\sum_{1\le i<j<k<l\le 10} r_i r_j r_k r_l = 4m^2q^2 + 9q^4,$$
$$\sum_{1\le i<j<k<l<p\le 10} r_i r_j r_k r_l r_p = 6mq^4,$$
$$\sum_{1\le i<j<k<l<p<s\le 10} r_i r_j r_k r_l r_p r_s = 10q^6,$$
$$\sum_{1\le i<j<k<l<p<s<t\le 10} r_i r_j r_k r_l r_p r_s r_t = 2mq^6,$$
$$\sum_{1\le i<j<k<l<p<s<t<u\le 10} r_i r_j r_k r_l r_p r_s r_t r_u = 5q^8,$$
$$\prod_{i=1}^{10} r_i = q^{10}. \tag{9}$$

Eliminating mass parameter we find the following set of equations:

$$\sum_{1\le i<j\le 10} r_i r_j - \left(\sum_{i=1}^{10} r_i\right)^2 = 5q^2,$$
$$\sum_{1\le i<j<k\le 10} r_i r_j r_k = 3q^2\sum_{i=1}^{10} r_i,$$
$$\sum_{1\le i<j<k<l\le 10} r_i r_j r_k r_l - q^2\left(\sum_{i=1}^{10} r_i\right)^2 = 9q^4, \tag{10}$$
$$\sum_{1\le i<j<k<l<p\le 10} r_i r_j r_k r_l r_p = 3q^4\sum_{i=1}^{10} r_i,$$
$$\sum_{1\le i<j<k<l<p<s<t\le 10} r_i r_j r_k r_l r_p r_s r_t = q^6\sum_{i=1}^{10} r_i.$$

By further elimination in (10), finally we find the following mass-independent relations:

$$\sum_{1\le i<j<k<l\le 10} r_i r_j r_k r_l - q^2\sum_{1\le i<j\le 10} r_i r_j = 4q^4,$$
$$3\sum_{1\le i<j<k<l<p<s<t\le 10} r_i r_j r_k r_l r_p r_s r_t = q^2\sum_{1\le i<j<k<l<p\le 10} r_i r_j r_k r_l r_p,$$
$$\sum_{1\le i<j<k<l<p<s\le 10} r_i r_j r_k r_l r_p r_s = 10q^6, \tag{11}$$
$$\sum_{1\le i<j<k<l<p<s<t<u\le 10} r_i r_j r_k r_l r_p r_s r_t r_u = 5q^8,$$
$$\prod_{i=1}^{10} r_i = q^{10}.$$

In terms of area $\mathcal{A}_i = 4\pi r_i^2$, the mass-independent relations are

$$\sum_{1\le i<j<k<l\le 10}\sqrt{\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k\mathcal{A}_l} - 4\pi q^2\sum_{1\le i<j\le 10}\sqrt{\mathcal{A}_i\mathcal{A}_j} = \left(8\pi q^2\right)^2,$$
$$3\sum_{1\le i<j<k<l<p<s<t\le 10}\sqrt{\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k\mathcal{A}_l\mathcal{A}_p\mathcal{A}_s\mathcal{A}_t} = 4\pi q^2\sum_{1\le i<j<k<l<p\le 10}\sqrt{\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k\mathcal{A}_l\mathcal{A}_p},$$
$$\sum_{1\le i<j<k<l<p<s\le 10}\sqrt{\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k\mathcal{A}_l\mathcal{A}_p\mathcal{A}_s} = 640\pi^3q^6, \tag{12}$$
$$\sum_{1\le i<j<k<l<p<s<t<u\le 10}\sqrt{\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k\mathcal{A}_l\mathcal{A}_p\mathcal{A}_s\mathcal{A}_t\mathcal{A}_u} = 1280\pi^4q^8,$$
$$\prod_{i=1}^{10}\sqrt{\mathcal{A}_i} = \left(4\pi q^2\right)^5.$$

From this relation one can obtain the mass-independent entropy relation by substituting $\mathcal{A}_i = 4\mathcal{S}_i$. Thus the mass-independent complicated function of horizon areas that we have found could turn out to be a *universal* quantity.

Case 2. Now we have set $\alpha = 5$ and $\beta = 6$. In this case the horizon equation is given by

$$r^{12} - \left(4m^2 - 8q^2\right)r^{10} - \left(4m^2q^2 - 22q^4\right)r^8 + 26q^6r^6 + 17q^8r^4 + 6q^{10}r^2 + q^{12} = 0. \tag{13}$$

Let us put $r^2 = x$; then the equation reduces to sixth-order polynomial equation:

$$x^6 - \left(4m^2 - 8q^2\right)x^5 - \left(4m^2q^2 - 22q^4\right)x^4 + 26q^6x^3 + 17q^8x^2 + 6q^{10}x + q^{12} = 0. \tag{14}$$

Again we apply Vieta's theorem; we get

$$\sum_{i=1}^{6} x_i = 4m^2 - 8q^2,$$
$$\sum_{1\le i<j\le 6} x_i x_j = 22q^4 - 4m^2q^2,$$

$$\sum_{1\le i<j<k\le 6} x_i x_j x_k = -26q^6,$$
$$\sum_{1\le i<j<k<l\le 6} x_i x_j x_k x_l = 17q^8,$$
$$\sum_{1\le i<j<k<l<p\le 6} x_i x_j x_k x_l x_p = -6q^{10},$$
$$\prod_{i=1}^{6} x_i = q^{12}. \tag{15}$$

Eliminating mass parameter, we obtain the mass-independent relation:

$$\begin{aligned}
\sum_{1\le i<j\le 6} x_i x_j + q^2\sum_{i=1}^{6} x_i &= 14q^4,\\
\sum_{1\le i<j<k\le 6} x_i x_j x_k &= -26q^6,\\
\sum_{1\le i<j<k<l\le 6} x_i x_j x_k x_l &= 17q^8,\\
\sum_{1\le i<j<k<l<p\le 6} x_i x_j x_k x_l x_p &= -6q^{10},\\
\prod_{i=1}^{6} x_i &= q^{12}.
\end{aligned} \tag{16}$$

In terms of area $\mathcal{A}_i = 4\pi r_i^2 = 4\pi x_i$ the above mass-independent relation could be written as

$$\begin{aligned}
\sum_{1\le i<j\le 6} \mathcal{A}_i\mathcal{A}_j + 4\pi q^2\sum_{i=1}^{6}\mathcal{A}_i &= 14\left(4\pi q^2\right)^2,\\
\sum_{1\le i<j<k\le 6} \mathcal{A}_i\mathcal{A}_j\mathcal{A}_k &= -26\left(4\pi q^2\right)^3,\\
\sum_{1\le i<j<k<l\le 6} \mathcal{A}_i\mathcal{A}_j\mathcal{A}_k\mathcal{A}_l &= 17\left(4\pi q^2\right)^4,\\
\sum_{1\le i<j<k<l<p\le 6} \mathcal{A}_i\mathcal{A}_j\mathcal{A}_k\mathcal{A}_l\mathcal{A}_p &= -6\left(4\pi q^2\right)^5,\\
\prod_{i=1}^{6}\mathcal{A}_i &= \left(4\pi q^2\right)^6.
\end{aligned} \tag{17}$$

Again we have found mass-independent complicated function of horizon areas that could turn out to be universal in nature.

Case 3. Now we have set $\alpha = 4$ and $\beta = 6$. In this case the horizon equation is

$$\begin{aligned}
r^{10} - 2mr^9 + 6q^2r^8 - 6mq^2r^7 + 12q^4r^6 - 6mq^4r^5\\
+ 11q^6r^4 - 2mq^6r^3 + 5q^8r^2 + q^{10} = 0.
\end{aligned} \tag{18}$$

Again we apply Vieta's theorem; we get

$$\begin{aligned}
\sum_{i=1}^{10} r_i &= 2m,\\
\sum_{1\le i<j\le 10} r_i r_j &= 6q^2,\\
\sum_{1\le i<j<k\le 10} r_i r_j r_k &= 6mq^2,\\
\sum_{1\le i<j<k<l\le 10} r_i r_j r_k r_l &= 12q^4,\\
\sum_{1\le i<j<k<l<p\le 10} r_i r_j r_k r_l r_p &= 6mq^4,\\
\sum_{1\le i<j<k<l<p<s\le 10} r_i r_j r_k r_l r_p r_s &= 11q^6,\\
\sum_{1\le i<j<k<l<p<s<t\le 10} r_i r_j r_k r_l r_p r_s r_t &= 2mq^6,\\
\sum_{1\le i<j<k<l<p<s<t<u\le 10} r_i r_j r_k r_l r_p r_s r_t r_u &= 5q^8,\\
\prod_{i=1}^{10} r_i &= q^{10}.
\end{aligned} \tag{19}$$

Eliminating mass parameter we have found the following set of mass-independent relation:

$$\begin{aligned}
3\sum_{1\le i<j<k<l<p<s<t\le 10} r_i r_j r_k r_l r_p r_s r_t\\
= q^2\sum_{1\le i<j<k<l<p\le 10} r_i r_j r_k r_l r_p,\\
\sum_{1\le i<j\le 10} r_i r_j = 6q^2,\\
\sum_{1\le i<j<k<l\le 10} r_i r_j r_k r_l = 12q^4,\\
\sum_{1\le i<j<k<l<p<s\le 10} r_i r_j r_k r_l r_p r_s = 11q^6,\\
\sum_{1\le i<j<k<l<p<s<t<u\le 10} r_i r_j r_k r_l r_p r_s r_t r_u = 5q^8,\\
\prod_{i=1}^{10} r_i = q^{10}.
\end{aligned} \tag{20}$$

If we are working in terms of area then the above mass-independent relation could be written as

$$3\sum_{1\le i<j<k<l<p<s<t\le 10}\sqrt{\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k\mathcal{A}_l\mathcal{A}_p\mathcal{A}_s\mathcal{A}_t} = 4\pi q^2\sum_{1\le i<j<k<l<p\le 10}\sqrt{\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k\mathcal{A}_l\mathcal{A}_p}, \tag{21}$$

$$\sum_{1\le i<j\le 10}\sqrt{\mathcal{A}_i\mathcal{A}_j} = 24\pi q^2,$$

$$\sum_{1\le i<j<k<l\le 10}\sqrt{\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k\mathcal{A}_l} = 3\left(8\pi q^2\right)^2,$$

$$\sum_{1\le i<j<k<l<p<s\le 10}\sqrt{\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k\mathcal{A}_l\mathcal{A}_p\mathcal{A}_s} = 11\left(4\pi q^2\right)^3,$$

$$\sum_{1\le i<j<k<l<p<s<t<u\le 10}\sqrt{\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k\mathcal{A}_l\mathcal{A}_p\mathcal{A}_s\mathcal{A}_t\mathcal{A}_u} = 5\left(4\pi q^2\right)^4, \tag{22}$$

$$\prod_{i=1}^{10}\sqrt{\mathcal{A}_i} = \left(4\pi q^2\right)^5.$$

Once again these are mass-independent relations that could turn out to be universal. So, we can compute different thermodynamic product relations for different values of α and β which are mass-independent.

The Hawking [18] temperature on $\mathcal{H}^i$ reads off

$$T_i = \frac{\kappa_i}{2\pi} = \frac{1}{4\pi}\left[\alpha\frac{r_i}{\left(r_i^2+q^2\right)} - \frac{(\alpha-1)}{r_i} + \frac{q^2(\alpha-\beta)r_i^{\beta-1}}{\left(r_i^2+q^2\right)^{\beta/2+1}} + \frac{q^2(\beta-\alpha-1)r_i^{\beta-2}}{\left(r_i^2+q^2\right)^{\beta/2}}\right]. \tag{23}$$

It should be noted that the Hawking temperature product depends on the mass parameter and thus it is not a universal quantity.

Another important parameter in BH thermodynamics that determines the thermodynamic stability of BH is defined by

$$C_i = \frac{\partial m}{\partial T_i}. \tag{24}$$

For generalized regular BH, it is found to be

$$C_i = \frac{(\partial m/\partial r_i)}{(\partial T_i/\partial r_i)}, \tag{25}$$

where

$$\frac{\partial m}{\partial r_i} = \frac{\left[\alpha r_i^{\alpha}\left(r_i^2+q^2\right)^{\alpha/2-1} - (\alpha-1)r_i^{\alpha-2}\left(r_i^2+q^2\right)^{\alpha/2}\right]}{2r_i^{2(\alpha-1)}} + \frac{q^2\left[(\beta-\alpha-1)r_i^{\beta-\alpha-2}\left(r_i^2+q^2\right)^{(\beta-\alpha)/2} - (\beta-\alpha)r_i^{\beta-\alpha}\left(r_i^2+q^2\right)^{(\beta-\alpha-2)/2}\right]}{\left(r_i^2+q^2\right)^{\beta-\alpha}},$$

$$\frac{\partial T_i}{\partial r_i} = \frac{1}{4\pi}\left[\frac{\alpha-1}{r_i^2} - \frac{\alpha\left(r_i^2-q^2\right)}{\left(r_i^2+q^2\right)^2} + q^2(\alpha-\beta)\frac{\left\{(\beta-1)r_i^{\beta-2}\left(r_i^2+q^2\right)^{\beta/2+1} - (\beta+2)r_i^{\beta}\left(r_i^2+q^2\right)^{\beta/2}\right\}}{\left(r_i^2+q^2\right)^{\beta+2}}\right] + \frac{1}{4\pi}\left[q^2(\beta-\alpha-1)\frac{\left\{(\beta-2)r_i^{\beta-3}\left(r_i^2+q^2\right)^{\beta/2} - \beta r_i^{\beta-1}\left(r_i^2+q^2\right)^{\beta/2-1}\right\}}{\left(r_i^2+q^2\right)^{\beta}}\right]. \tag{26}$$

The BH undergoes a second-order phase transition when $\partial T_i/\partial r_i = 0$. In this case the specific heat diverges.

Finally, Komar [21] energy computed at $\mathscr{H}^i$ is given by

$$E_i = \frac{1}{2}\left[\alpha\frac{r_i^3}{(r_i^2+q^2)} - (\alpha-1)\,r_i + \frac{q^2(\alpha-\beta)\,r_i^{\beta+1}}{(r_i^2+q^2)^{\beta/2+1}} + \frac{q^2(\beta-\alpha-1)\,r_i^{\beta}}{(r_i^2+q^2)^{\beta/2}}\right]. \tag{27}$$

In the limits $\alpha = 3$ and $\beta = 4$, one obtains the result of ABG BH.

3. Discussion

In this work, we examined thermodynamic product relations for generalized regular (curvature-free) BH. Generalized BH means the BHs represented by the four parameters, that is, m, q, α, and β. We determined different thermodynamic product particularly area products for different values of α and β. We showed that there is some complicated function of horizon areas indeed mass-independent that could turn out to be universal. We also derived the specific heat to determine the thermodynamic stability of the BH. Finally, we computed Komar energy for this generalized BH.

Competing Interests

The author declares that they have no competing interests.

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