

Research Article

A Thermodynamic Approach to Holographic Dark Energy

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Received 31 December 2016; Revised 9 March 2017; Accepted 20 April 2017; Published 30 July 2017

Academic Editor: Burak Bilki

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We propose a method to relate the holographic minimal information density to de Broglie’s wavelength at a given universe temperature T . To figure this out, we assume that the thermal length of massive and massless constituents represents the cut-off scale of the holographic principle. To perform our analysis, we suppose two plausible universe volumes, that is, the adiabatic and the horizon volumes, that is, $V \propto a^3$ and $V \propto H^{-3}$, respectively, assuming zero spatial curvature. With these choices in mind, we evaluate the thermal lengths for massive and massless particles and we thus find two cosmological models associated with late and early cosmological epochs. We demonstrate that both models depend upon a free term β which enters the temperature parametrization in terms of the redshift z . For the two treatments, we show evolving dark energy terms which can be compared with the ω CDM quintessence paradigm when the barotropic factor takes the formal values $\omega_0 = -(1/3)(2 + \beta)$ and $\omega_0 = -(1/3)(1 + 2\beta)$, respectively, for late and early eras. From our analyses, we nominate the two models as viable alternatives to dark energy determined from thermodynamics in the field of the holographic principle.

1. Introduction

Almost two decades ago, the universe acceleration has been observed [1–3] and gradually confirmed [4–9], showing that our universe seems to be dominated by an exotic antigravitational component, dubbed dark energy. Even though the universe speed-up is today well-consolidate [10, 11], its physical origin has not been clarified. If the cosmological principle holds, one needs to introduce within Einstein’s gravity an additional effective fluid which exhibits a negative pressure, dominating over matter today. This is the philosophy of the concordance paradigm, named the Λ CDM model. In this approach, one makes use of a vacuum energy cosmological constant (for the sake of clearness, the cosmological constant approach is physically different from dark energy. In the second case, there is no need to introduce the effects due to quantum vacuum energy into Einstein’s equations, employing an evolving barotropic factor) [12–22], providing a constant

equation of state [23–27] which counterbalances the action of gravity after the transition time (which represents the onset of acceleration [28], i.e., when the cosmological constant starts dominating over pressureless matter). In this picture, the universe energy budget is essentially composed of cold dark matter for about 23%, of 4% of baryons as visible matter, and of 73% of cosmological constant [29–34]. The Λ CDM model shows an excellent guidance with current data [35–44] but it is severely plagued by the coincidence and fine tuning problems [45, 46]. These issues do not enable concluding that the Λ CDM model represents the final paradigm to describe the universe dynamics (in particular, the measures of cosmological constant and matter magnitudes are extremely close to each other, i.e., $\Omega_\Lambda/\Omega_m \sim 2.7$, at our time [38, 47–49], whereas the predicted and observed cosmological constant values differ for about 123 orders of magnitude [50]). As a natural consequence, a wide amount of cosmological models has been introduced to extend the concordance model in

order to get possible alternatives to dark energy derived from prime principles (see, e.g., [51–59] and references therein).

Among all, an enticing prospect is to postulate the validity of the holographic principle [60, 61]. The principle is supported by the fact that the maximum entropy inside a physical region may be not extensive, growing as the surface area. In this landscape, a cut-off scale is naturally introduced, being responsible for the cosmic acceleration today. In cosmology, one can therefore assume the existence of a minimal information region, consistent with current cosmological constant's value, but physically different from vacuum energy [62–64]. Thereby, the understanding of dark energy's nature is shifted to find how the aforementioned cut-off scale evolves in terms as the universe expands. Several different possibilities have been investigated during the last years [65, 66]. In particular, a remarkable class of models is likely represented by choosing cut-off scales which reproduce the Λ CDM behavior at low- and high-redshift regimes [67]. In such a way, the corresponding holographic dark energy naturally reduces to the Λ CDM paradigm at late and early times, respectively, in agreement with observations and without concordance and fine tuning issues. We here present how to obtain the holographic cut-off assuming as size of minimal information the universe's thermal lengths, hereafter λ_{Th} , associated with massive and massless particles. We propose that the holographic dark energy behaves like a gas, whose cut-off length corresponds to the standard de Broglie's wavelength at a given temperature T [68, 69]. For massive and massless particles we have $\lambda_{\text{Th},1} = h(2\pi mk_B T)^{-0.5}$ and $\lambda_{\text{Th},2} = h(2\pi^{1/3} k_B T)^{-1}$, respectively [70, 71]. In particular, as λ_{Th} becomes much smaller than the distances among particles, the dark energy gas obeys the Maxwell distribution, showing instead quantum effects in the opposite case. This implies the existence of two regimes in which one can choose, at small scales, quantum effects associated with photon, whereas one can choose classical behaviors at large scales, associated with matter. The importance of our approach lies in the fact that, considering the validity of pure thermodynamics and that the holographic principle holds at cosmic scales, as two intrinsic elements of Einstein's theory, it is possible to recover effective dark energy scenarios, instead of postulating the dark energy form by hand in Einstein's equations and without modifying general relativity. To guarantee that the holographic cut-off is due to the thermal lengths, we assume the temperature parameterized by the scale factor as simple as possible, by departing from the standard temperature of the Cosmic Microwave Background (CMB) in terms of a simple power law. The consequences are here described at both scales. We investigate how the cosmological models evolve with respect to the redshift z and we fix theoretical limits over the free parameters of our approach. Moreover, we check the goodness of our holographic dark energy by means of first-order perturbation equations and we compare our treatment with the Λ CDM and with the quintessence models. The paper is structured as follows: in Section 2, we briefly report how to relate and motivate the holographic principle to cosmology. In Section 3, we discuss the method to build up the holographic cut-off in terms of the thermal length at

both late and early regimes. In Sections 3.1 and 3.2, we discuss some consequences of our choices at the two investigated stages of universe's evolution. Finally, Section 4 is devoted to conclusions and perspectives of our work.

2. The Approach of Holographic Dark Energy

A consolidate conjecture states that general relativity may break down as it is applied to small distances in the regime of very high energies. The level of small distances and/or high energies may cause the breakdown of geometrical continuum as well. Examples are offered by lattice or grids in which the interactions are modified at either classical or quantum levels [72–79]. Assuming quantum effects, in analogy to what happens in quantum mechanics, there would exist indivisible units with corresponding minimal lengths associated with certain physical domains. If a minimal length exists, namely, \mathcal{L} , one then expects a corresponding minimal area, naively proportional to \mathcal{L}^2 . In this picture, a standard hypothesis is to imagine that thermodynamics do not break down. In doing so, the associated entropy is modified by the existence of the aforementioned minimal surface and the Bekenstein-Hawking surface law is still valid. This scheme has been severely supported even by other frameworks which are in favor of the existence of minimal lengths, among which are the generalized uncertainty principle, string scenarios, doubly special relativity, and so forth [80]. Another approach which predicts a minimal length is based on the holographic principle. The principle has been firstly discussed by 't Hooft and extended to other formalisms, such as string theory by Susskind [81, 82]. It states that the degrees of freedom of a spatial region reside not in the bulk but in the boundary. The number of boundary degrees of freedom per Planck's area should not be larger than one, so that the basic demand lies in postulating that the maximum entropy inside a physical region is not extensive and, in particular, that it grows as the surface area. Exceptional emphasis has been devoted to applying the principle in the context of homogeneous and isotropic universe [60]. In this landscape, the byproduct of extending the holographic postulate to cosmology imposes the minimal information density as the source of dark energy. Indicating dark energy's density with ρ_X , we find that it is proportional to an infrared cut-off scale, equivalent to \mathcal{L} . From the above perspective, the effective dark energy becomes

$$\rho = \frac{\mathcal{A}}{\mathcal{L}^2}, \quad (1)$$

with \mathcal{A} a constant which is plausibly close to the present critical density (hereafter $8\pi G = 1$), $\rho_{\text{cr}} = 3H_0^2$. This can be viewed as the existence of an information density entering the Einstein's energy momentum tensor, so that, for a precise choice of \mathcal{L} , the universe can accelerate in agreement with recent observations. Thereby, the problem of understanding the dark energy nature is practically shifted to defining somehow \mathcal{L} , although the physical nature is however completely reinterpreted with respect to the cosmological constant. To define \mathcal{L} , probably the most appealing and suggestive

frameworks take the size \mathcal{L} inside the class of functions able to describe unified dark energy models, in which dark energy emerges as an effect of dark matter. Although reasonable, this procedure generated a wide number of models which have been built up ad hoc [65, 66, 83–88]. A simple and consolidate strategy is to assume L proportional to the Ricci scalar or to root mean square of second-order geometrical invariants [67]. This puts in correlation \mathcal{L} with invariant quantities leading to a geometrical infrared cut-off scale, satisfying the problem of causality, portrayed in [89]. Dark energy is thus interpreted as a geometrical acceleration, in strict analogy with the classes of modified gravities which make use of higher order curvature invariants [90]. These approaches differ from choosing cosmological distances as \mathcal{L} cut-off scales and candidate to be invariant. Even though they are invariant, the above picture does not however take into account the universe thermodynamics in terms of geometry. Indeed, more recently the idea that thermodynamics may be obtained by means of corresponding spaces of equilibrium in which the states identify the properties of the system in terms of geometric equilibrium has been investigated [91, 92]. Thus, motivated by the idea that there would exist a correspondence between thermodynamics and geometry, one would expect that the cut-off could be represented by thermal lengths. The thermal length provides a direct measure of the thermodynamic uncertainty for a thermodynamic system. In other words, supported by the fact that universe constituents show a thermal scale due to their equilibrium temperatures, it is licit to presume that the cut-off size is imposed by the temperature itself and by its evolution. The main consequences of treating the cut-off as a thermal minimal size may give relevant information on dark energy's evolution. In the next section, we describe the two possible thermal lengths, associated with massive (dust) and nonmassive (photons) universe constituents. We show the simplest case in which dark energy is modeled as perfect gas, in which the holographic cut-off is proportional to the CMB temperature and we suppose a power law dependence on the redshift z , parameterized by an additional free constant β .

3. Thermal Cut-Off Scales and Consequences in Cosmology

As stated above, the holographic principle certifies the existence of a further energy density, hereafter ρ_X , which is associated with the minimal information density of a given physical region. In particular, the term ρ_X turns out to be different from the standard pressureless matter density, ρ_m , and may manifest under certain conditions a negative pressure. We here assume that ρ_X naturally enters the energy momentum tensor of Einstein's gravity, without adding quantum vacuum energy associated with the cosmological constant. To do so, we employ the validity of the cosmological principle, so that the large scale geometric and physical properties can be accounted adopting the Friedman-Robertson-Walker (FRW) line element:

$$ds^2 = dt^2 - a(t)^2 [dr^2 + r^2 (\sin^2 \theta d\phi^2 + d\theta^2)], \quad (2)$$

with zero spatial curvature (the generalization to $\Omega_k \neq 0$ is straightforward), $\Omega_k = 0$. For a source given by a perfect fluid, the whole universe dynamics are expressed in terms of the Friedman equations:

$$H^2 = \frac{\rho}{3}, \quad (3a)$$

$$\dot{H} + H^2 = -\frac{1}{6} (\rho + 3P_X), \quad (3b)$$

which determine the universe evolution at all stages. The dots represent the derivative with respect to the cosmic time t and matter is supposed to be pressureless; that is, $P_m = 0$. In this scheme, only perfect fluids are source of gravitational field and the net total density is essentially composed of $\rho = \rho_m + \rho_X + \dots$ where at first approximation one can neglect radiation, neutrinos, relics, and so forth. Our main idea is to model the whole universe evolution as a thermodynamic system [93, 94]. Under this hypothesis, the laws of thermodynamics appear to be mathematically consistent with an isotropic and homogeneous geometric background [95]. In particular, thermodynamic laws in which arbitrary functions of time are present in spite of the lack of symmetry are compatible with equations of state which have to be imposed on the resulting solution. For instance, it is possible to use the first law of thermodynamics to get expressions for the universe temperature which may furnish numerical results in agreement with recent CMB observations [23, 96–98]. By relating the holographic principle to the Friedmann equations (3a) and (3b) and by (1), one has

$$\Omega_{\text{tot}} \rho_{\text{cr}} = \rho_{m,0} a^{-3} + (1 - \rho_{m,0}) \mathcal{L}(a)^{-2}, \quad (4)$$

where we guarantee that $H(z = 0) = H_0$ today invoking that $\mathcal{A}/\rho_{\text{cr}} = 1 - \rho_{m,0}$. The problem of determining the dark energy evolution is shifted to reconstructing $\mathcal{L}(a)$ at different stages in terms of the scale factor, or equivalently by using the redshift z . In other words, since $\mathcal{L}(a)$ is not known a priori, one passes from postulating a dark energy evolution to understanding which cosmological size can be related to the holographic principle. The most general holographic dark energy model, that is, the one in which no specific cut-offs have been proposed, has been severely investigated and (4) effectively represents a precise class of models inside the most general ones. The choice of (4), indeed, enters the set of models in which there exists a split between matter and holographic dark energy with no interactions between them. Moreover, the cut-off scale is supposed to be analytical with respect to the redshift z . These naive conditions have been imposed to be consistent with current knowledge on dark energy's evolution and to guarantee that at the very small redshift regime the holographic models may reduce to the effective Λ CDM paradigm. All holographic dark energy models, such as Ricci dark energy models and second-order invariants, are contained in the above classes of generalized holographic frameworks. In particular, to be consistent with observations, we suppose that dark energy departs smoothly from a pure cosmological constant. This is mostly supported by observations that suggest that at either late or early times

dark energy slightly departs from a pure constant. In other words, whatever form of dark energy density one considers, the prescription at late and early times may guarantee that

$$\begin{aligned}\rho_{X,0} &\approx 1 - \rho_{m,0} && \text{at late times,} \\ \rho_{X,\infty} &\approx 1 - \rho_{m,0} && \text{at early times.}\end{aligned}\quad (5)$$

For example, in the case of Chevallier-Polarski-Linder parametrization [99, 100], the equation of state for dark energy becomes $\omega = \omega_0 + \omega_1(1 - a(t))$, which turns out to be a constant at late and early times, giving, respectively, $\omega_{\text{late}} = \omega_0$ and $\omega = \omega_0 + \omega_1$. The second imposition may be stringent for models in which dark energy evolves faster than a^{-3} at high redshift. Hence, one can imagine overcoming this problem, assuming that the reconstructed dark energy term satisfies at least

$$\frac{\Omega_X(z)}{(1+z)^3} \sim \Omega_{m,0}, \quad (6)$$

with $\Omega_X \equiv \rho_X/\rho_{\text{cr}}$, $\forall z$.

Together with the requirements of (5), one needs that standard thermodynamics are not violated due to fast transitions or anomalous evolutions of ρ_X during intermediate phases; that is, $0 < z < \infty$. This request excludes possible dark energy oscillation or anomalous behavior and in particular provides that dark energy smoothly evolves, suggesting that the CMB temperature would be a simple power law in a FRW universe. Moreover, we suppose that dark energy behaves like a perfect gas, having that it does not interact with any other fluid constituents. All these requirements are essentially certified by the great goodness in comparing cosmic data coming from the last scattering surface with respect to the temperature observed today. Assuming that the universe is composed of massive and massless particles, such as dust and photons, respectively, and considering a local thermal equilibrium, it is natural to expect a thermal length associated with each constituent. In particular, the thermal length is built up by using de Broglie's wavelength in a fluid configuration at a given temperature. For an ideal gas, the constituents are disposed with average distances among them. If such a distance, namely, λ_{space} , is due to the length determined from the volume occupied by the gas, one thus obtains

$$\lambda_{\text{space}} \propto V^{1/3}. \quad (7)$$

Clearly, we need the correct universe's volume in order to characterize λ_{space} . We notice that $V = V_0 a^3$ fulfills the weak energy condition and represents the simplest choice to model the volume evolution in terms of the scale factor. Here, one can imagine that the volume evolution has been built up to the first-order approximation with V_0 proportional to the universe's radius, that is, $1/H_0$. However, other approaches have recently suggested that the volume scales by means of the apparent horizon [101–103], leading to a volume $V \propto H^{-3}$. The former approach defines plausible causal regions with entropy proportional to H^{-2} . Analogously, at domains of large or small redshifts one can first approximate the volume with $V \propto a^3$, without contradicting the causality

condition [104, 105]. To match thermodynamic properties within the holographic principle, one can suppose that the cut-off scale \mathcal{L} is defined by taking the thermal length of a given massive or massless gas. As discussed above, this is physically supported by the thermal length definition itself. In particular, the thermal length is essential in thermodynamic systems, since it represents a direct measure of the thermodynamic uncertainty, leading to a localization of a given particle set, of known mass, with the average thermal momentum. Hence, from the definition of thermal length it follows that the holographic gas behaves classically only if de Broglie's wavelength is much smaller than particle distances. For the two cases, the particle distances in the FRW universe are

$$\lambda_{\text{space}} \propto a, \quad (8a)$$

$$\lambda_{\text{space}} \propto H^{-1}, \quad (8b)$$

respectively, for the adiabatic volume $V \propto a^3$ and for the horizon volume $V \propto H^{-3}$, so that, using the approximation of perfect gas, one has

$$\lambda_{\text{Th},1} = \frac{\alpha_1}{\sqrt{T}}, \quad (9)$$

$$\lambda_{\text{Th},1} = \frac{\alpha_2}{T}, \quad (10)$$

which are valid for massive particles and massless gases, respectively, with T the observable temperature of the universe, that is, the temperature we observe from the CMB experiments [68–71]. The two constants $\alpha_{1,2}$ have been explicitly reported in Section 1. As confirmed by observations, a suitable landscape is to assume that at late times the fluid characterizing dark energy is classical, whereas possible quantum effects may be revealed only at early times, with higher energy scales. At a first glance, it behooves us that at $z \gg 1$ the presence of photons suggests using (10), as thermal length, whereas the opposite case is expected at $z \ll 1$, by making use of (9).

This implies that (9) might be used at small redshift to guarantee a classical regime, while (10) might be used at early times to enable possible quantum effects. In other words, two cases are in principle possible, at different redshift regimes, leading to different scenarios. We summarize them as follows (with the convention $V_0 = 1$):

$$\lambda_{\text{Th},1} \leq \lambda_{\text{space}}, \quad (11)$$

$$\lambda_{\text{Th},2} \geq \lambda_{\text{space}}. \quad (12)$$

Since, as stated, these choices are valid for classical and quantum regimes, respectively, we have

$$\alpha_1^{-2} T_{z \ll 1} \geq a^{-2}, \quad (13a)$$

$$\alpha_2^{-1} T_{z \gg 1} \leq a^{-1}, \quad (13b)$$

in the case of adiabatic volume, and

$$\alpha_1^{-2} T_{z \ll 1} \geq H^2, \quad (14a)$$

$$\alpha_2^{-1} T_{z \gg 1} \leq H, \quad (14b)$$

in the case of apparent volume. In all the aforementioned frameworks one can use the well-consolidate temperature parametrization [106]:

$$\frac{T}{T_0} = a^{\beta-1}, \quad (15)$$

which fulfills the requirements that we mentioned above. In principle, modifying the temperature definition with the parametrization (15) would modify the use of the cosmic distances, increasing the incidence of the duality problem [107] between the luminosity and angular distances. The request that the duality problem does not significantly modify observations can be accounted by assuming that β is close to $\beta \sim 1$ and may be parameterized by

$$\beta = 1 + \delta, \quad (16)$$

with $\delta \geq 0$ defined as a small positive number. With the above recipe, we can now consider some consequences of our approaches to late and early times in the next subsections.

3.1. Late-Time Evolution. The two thermal lengths lead to approaches at different stages of the universe evolution. The first analysis can be performed by using the adiabatic volume at the small redshift regime. This epoch corresponds to our time, in which, making use of the thermal length for massive particles, one finds

$$\mathcal{V}_{\text{late}}^{\text{adiabatic}} = \Omega_{m,0} (1+z)^3 + (1 - \Omega_{m,0}) (1+z)^{1-\beta}, \quad (17)$$

where $\mathcal{E} \equiv H/H_0$ is the normalized Hubble rate. Using the temperature parametrization of (15) and requiring $z \approx 0$, one finds

$$\alpha_1 \geq \sqrt{T_0}, \quad (18)$$

which certifies that α_1 gives rise to the lowest limit corresponding to the value $\alpha_1 \approx 1.65$ K. Combining (1) with (15), assuming $\alpha_1 = \sqrt{T_0}$, we have

$$\Omega_{X,0} \equiv 1 - \Omega_{m,0} = \frac{\mathcal{A}}{\rho_{\text{cr}} \alpha_1^2} T_0, \quad (19)$$

and, using $\Omega_{m,0} = 0.27$ and $\alpha_1 \sim T_0$, we simply get $\mathcal{A} > 0$, as requested by modeling the dark energy density in (1), and moreover

$$\mathcal{A} = (1 - \Omega_{m,0}) \rho_{\text{cr}}, \quad (20)$$

which confirms that \mathcal{A} is close to the critical density. The cosmological model depends upon two parameters, that is, $\Omega_{m,0}$ and β , and mimics the Λ CDM behavior at the level of background cosmology, providing a dark energy term which can be approximated at first order as

$$\Omega_X \approx (1 - \Omega_{m,0}) [1 + (1 - \beta) z], \quad (21)$$

reproducing at $z = 0$ the exact value of $\Omega_\Lambda = 1 - \Omega_{m,0}$, in perfect agreement with observations. In particular, the first correction to the cosmological constant case, that is,

$(1 - \Omega_{m,0})(1 - \beta)z$, turns out to be small if compared to $1 - \Omega_{m,0}$, when $\beta < 1$. At higher redshifts, the model mimics the ω CDM paradigm, which corresponds to a quintessence field, whose dark energy contributes scales as $(1+z)^{3(1+\omega)}$. In this picture, one finds that the two models are formally equivalent if one imposes

$$\omega = -\frac{1}{3} (2 + \beta). \quad (22)$$

The ω CDM model is thought as the simplest approach, generalizing the concordance paradigm, but it is derived from postulating a quintessence term, built up in terms of a slowly rolling dynamical scalar field. In this scheme, one invokes a noncanonical kinetic term in the Lagrangian. Our treatment predicts the same dynamics, without invoking a quintessence scalar field, that is, without introducing by hand another ingredient within Einstein's equations. Hence, although the model here presented can be formally compared with the ω CDM approach, the physics associated with it turns out to be completely different. The need of a different origin of quintessence, indeed, enables one to avoid the introduction of scalar fields in Einstein's energy momentum tensor. The need of a precise cut-off scale is essentially enough to extend the concordance paradigm by means of a running dark energy term in which the barotropic factor is constant. From those considerations, one does not need to expect that the propagation of perturbations is equivalent to the ω CDM approach, as we discuss later in the work. On the contrary, the pressure, energy density, and equation of state of our model may be formulated in terms of β only, which effectively works as a thermodynamical scale. The principal thermodynamic quantities of our model are summarized as

$$\rho_X = \rho_{\text{cr}} (1 - \Omega_{m,0}) a^{-1+\beta}, \quad (23a)$$

$$P_X = -\frac{\rho_{\text{cr}}}{3} (1 - \Omega_{m,0}) (2 + \beta) a^{-1+\beta}, \quad (23b)$$

$$S \propto \frac{\rho_X \mathcal{V}}{T} = V_0 \rho_{\text{cr}} (1 - \Omega_{m,0}) a^3, \quad (23c)$$

where the last relation suggests that the entropy, S , scales as the volume, giving rise to the important fact that the entropy density is constant, as one expects in the concordance model, but without invoking a cosmological constant a priori. Moreover, since observations show that $\omega \approx -1$, it turns out that $\beta \sim 1$ and ~ 0 , confirming what we previously discussed in (16). The same procedure can be performed in the case of the apparent volume, leading to the same Hubble evolution of (17). However, in this case one finds

$$\alpha_1 \leq \frac{T_0}{H_0^2}, \quad (24)$$

which corresponds to an upper limit for α_1 which is positive in order to guarantee that $\lambda_{\text{Th}} > 0$. Particularly, for $\alpha_1 = T_0/H_0$, one gets

$$\mathcal{A} = H_0^{-1} (1 - \Omega_{m,0}) \rho_{\text{cr}} = 3 (1 - \Omega_{m,0}). \quad (25)$$

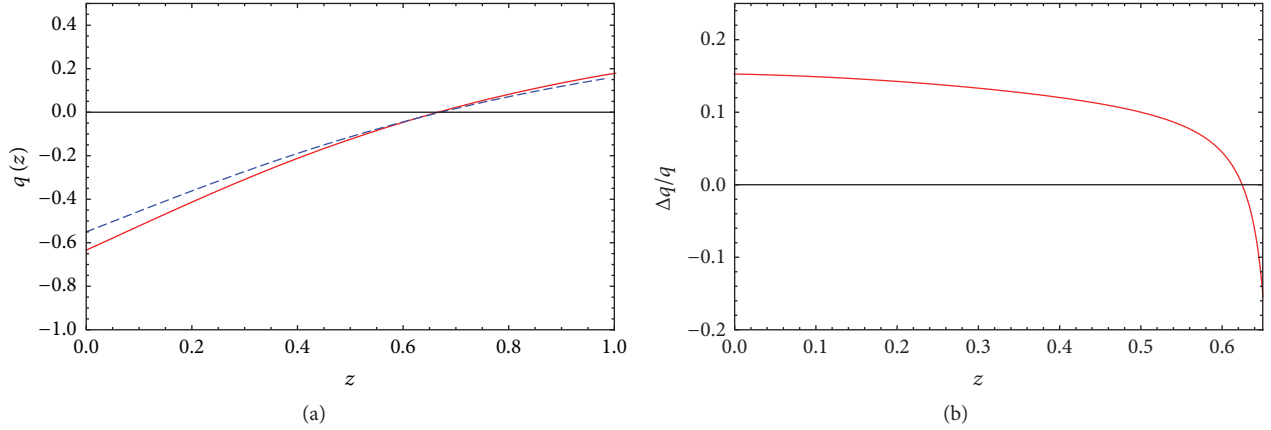


FIGURE 1: (a) Behaviors of the deceleration parameters for our model (red line) and for the concordance paradigm (dashed line), with indicative values $\Omega_{m,0} = 0, 3$ and β as reported in (29). (b) Variations between our model and the Λ CDM approach.

In both cases, (20) and (27), \mathcal{A} is of the same order of $1 - \Omega_{m,0}$, but in the second case it is much closer to ρ_{cr} , as requested by the holographic principle when building up (1). Thus, the universe accelerates when the quantity \dot{H}/H^2 determines a deceleration parameter's evolution inside the interval $-1 \leq q_0 \leq 0$. For the model $\mathcal{E}_{late}^{adiabatic}$, for both the adiabatic and horizon volumes, using $a \equiv (1+z)^{-1}$ and

$$\frac{dz}{dt} = -H(z)(1+z), \quad (26)$$

we get

$$\frac{\dot{H}}{H^2} = \frac{3\Omega_{m,0}(1+z)^{2+\beta} + (1-\Omega_{m,0})(1-\beta)}{2[1 + \Omega_{m,0}(-1 + (1+z)^{2+\beta})]}, \quad (27)$$

which enters the present value of the deceleration parameter as

$$q_0 = \Omega_{m,0} - \frac{1}{2} \left[1 + \frac{\beta}{2} (1 - \Omega_{m,0}) \right]. \quad (28)$$

Inside the observational domains, predicted by model independent measurements of the deceleration parameter, that is, $q_0 = 0.6361^{+0.3720}_{-0.3645}$ [108], we obtain

$$\beta = 1.2460^{+1.2947}_{-1.2733}, \quad (29)$$

where we considered for the matter density an indicative value, $\Omega_{m,0} = 0.30^{+0.05}_{-0.05}$, with errors in (29) propagating by the standard logarithmic formula, as follows:

$$\delta\beta = \left| \partial_{q_0} \beta \right| \delta q_0 + \left| \partial_{\Omega_{m,0}} \beta \right| \delta \Omega_{m,0}, \quad (30)$$

where ∂_{q_0} and $\partial_{\Omega_{m,0}}$ indicate the partial derivatives with respect to q_0 and $\Omega_{m,0}$, respectively. Those results are compatible with the Λ CDM paradigm [109–116] and correspond to a barotropic factor inside the interval:

$$\omega = -1.0820^{+0.4316}_{-0.4244}, \quad (31)$$

which testifies the goodness of our approach, lying in the expected intervals capable of accelerating the universe today. For the sake of completeness, in Figures 1(a) and 1(b), we plot the behaviors of the deceleration parameter of our model versus the deceleration parameter in the Λ CDM framework. They evolve similarly in the regime of small redshifts, as one can notice from (a). In (b), we report the percentage of variation, which has been defined as

$$\frac{\Delta q}{q} \equiv \frac{q - q_{\Lambda\text{CDM}}}{q_{\Lambda\text{CDM}}}. \quad (32)$$

It indicates discrepancies smaller than 20% between the two models, in the plotted redshift domain. This is confirmed by Figure 2, in which we compare the behaviors of our model versus the concordance paradigm. In particular, assuming $\Omega_{m,0} = 1/3$, for three redshifts appropriately chosen within the interval $z \leq 2$, one has

$$\begin{aligned} \frac{\Delta q}{q} &= -\frac{2}{3}(1-\beta), \\ \frac{\Delta q}{q} &= -\frac{3}{2} - \frac{5(2+\beta)}{2+2^{2+\beta}}, \\ \frac{\Delta q}{q} &= \frac{3}{2} \left(3 - \frac{29(2+\beta)}{2+3^{2+\beta}} \right), \end{aligned} \quad (33)$$

evaluated at $z = 0$, $z = 1$, and $z = 2$. The percentage function $\Delta q/q$ increases as β increases. In particular, for the indicative value $\beta = 1.2490$, it is easy to get that $\Delta q/q = \{16.6\%; 8.83\%; 4.24\%\}$, for the above set of redshift. At fixed β , the two deceleration parameters are much more compatible as the redshift increases. However, at our time, corresponding to $z = 0$, the deviation increases significantly if one passes from $\beta = 1.2490$ to $\beta = 1.5$. This indicates that the favorite values are inside $\delta \simeq 0$ with $\beta > 1$. In other words, the approach indicates slightly departing from $\beta = 1$ and values larger than it.

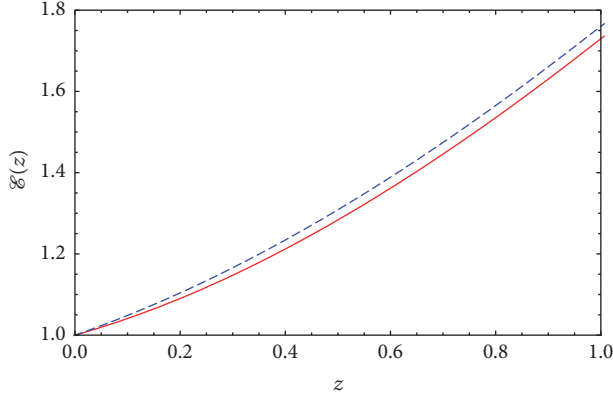


FIGURE 2: Behaviors of our model and Λ CDM normalized Hubble rates \mathcal{H} , with indicative values $\Omega_{m,0} = 0.3$ and β as reported in (29).

3.2. Early Time Cosmology. Analogous considerations may be carried out by investigating the consequences of our recipe at higher redshift domains, so that, at early times, following (11) and (13a) and (13b) we require that $\lambda_{\text{Th}} > \lambda_{\text{space}}$. From this, it follows that the normalized Hubble rate is

$$\mathcal{H}_{\text{early}}^{\text{adiabatic}} = \Omega_{m,0} (1+z)^3 + (1 - \Omega_{m,0}) (1+z)^{2(1-\beta)}. \quad (34)$$

In this case, using again the temperature parametrization but at the regime in which $z \gg 1$, one obtains again (20), which confirms that \mathcal{A} is close to the critical density. For the horizon volume, the two regimes provide the same result of (27). To be compatible with the concordance paradigm, but differently from the ω CDM case, one needs that the sound speed

$$c_s^2 = \frac{2(1 - \Omega_{m,0})(\beta - 1)(1 + 2\beta)}{9\Omega_{m,0}(1+z)^{1+2\beta} - 6(1 - \Omega_{m,0})(\beta - 1)} \quad (35)$$

is negligibly small. Imposing the former property on c_s , one gets that the sound speed tends to zero when $z \rightarrow \infty$ as $\beta > -1$, which is supported by theoretical considerations, since $\beta > 0$. Analogously, c_s cannot exceed the stiff matter limit; that is, $c_s = 1$. To figure this out, we summarize the following cases:

$$\begin{aligned} \beta &= 1, \\ c_s &= 0; \\ 0 &< \beta < 1, \\ c_s &\longrightarrow 0, \\ \beta &> 1, \\ c_s &\longrightarrow 0, \end{aligned} \quad (36)$$

which are associated with different intervals of $\beta > 0$. Notice that the case $\beta > 1$ implies a sound speed which goes faster to zero than $0 < \beta < 1$. Moreover, in analogy to the late-time

case, one can write down the thermodynamic quantities of particular interest for such a model, given by

$$\rho_X = \rho_{\text{cr}} (1 - \Omega_{m,0}) a^{2(\beta-1)}, \quad (37a)$$

$$P_X = -\frac{\rho_{\text{cr}}}{3} (1 - \Omega_{m,0}) (2 + \beta) a^{2(\beta-1)}, \quad (37b)$$

$$S \propto \frac{\rho_X V}{T} = V_0 T_0^{-1} \rho_{\text{cr}} (1 - \Omega_{m,0}) H^{-3}. \quad (37c)$$

With those considerations in mind, at a first approximation one can write the first-order perturbation equation approximating the sound speed with $c_s = 0$. This gives

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m\delta = 0. \quad (38)$$

When the redshift increases, the matter density influences the whole dynamics and so one can suppose checking how much the growth factor deviates from the concordance model. To do so, we can define the deviation parameter Δ as

$$\Delta = 1 - D(a) D_{\Lambda}^{-1}(a), \quad (39)$$

implying that the deviations of our model are smaller than 5%, with $D(a) = \delta/a$ for our model and $D_{\Lambda}(a) = \delta_{\Lambda\text{CDM}}/a$ for the concordance model. We can therefore define in perturbation theory the following growth index definition:

$$f = \frac{d \ln \delta}{d \ln a}, \quad (40)$$

whose evolution can be obtained by rewriting (42), with boundary conditions made by using the last scattering redshift $z_{\text{LSS}} \approx 1089$ and $f(a_{\text{LSS}}) = 1$. In particular, we have the growth index equation as

$$f' + \frac{f^2}{a} + \left[\frac{2}{a} + \frac{1}{\mathcal{E}} \frac{d\mathcal{E}}{da} \right] f - \frac{3\Omega_m}{2\mathcal{E}^2 a^4} = 0. \quad (41)$$

In Figures 3(a) and 3(b), we report the behaviors of Δ and f for our model. The discrepancies as reported in the above plots are again nonsignificant, showing a great compatibility between the standard cosmological model and our paradigm. A further remark is based on noticing that our approach is nominated to predict dark energy at late stages and even a dark term at earlier epochs of universe evolution (the model, indeed, does not provide a pure pressureless contribution at higher redshifts and so one cannot conclude that dark energy is negligibly small when the universe size decreases). Naively, any early phase of dark energy would imply the relation

$$\frac{d}{dt} \left(\frac{1}{aH} \right) < 0, \quad (42)$$

which guarantees the existence of inflation at $z \rightarrow \infty$. Thus, we may argue whether our approach satisfies the above requirement, giving rise to $\ddot{a} > 0$ at small scales. From the corresponding equation of state, one immediately concludes that our model works fairly well in predicting a phase in which $\omega < -1/3$, which turns out to represent the minimal condition to guarantee that inflationary time exists in the past. In other

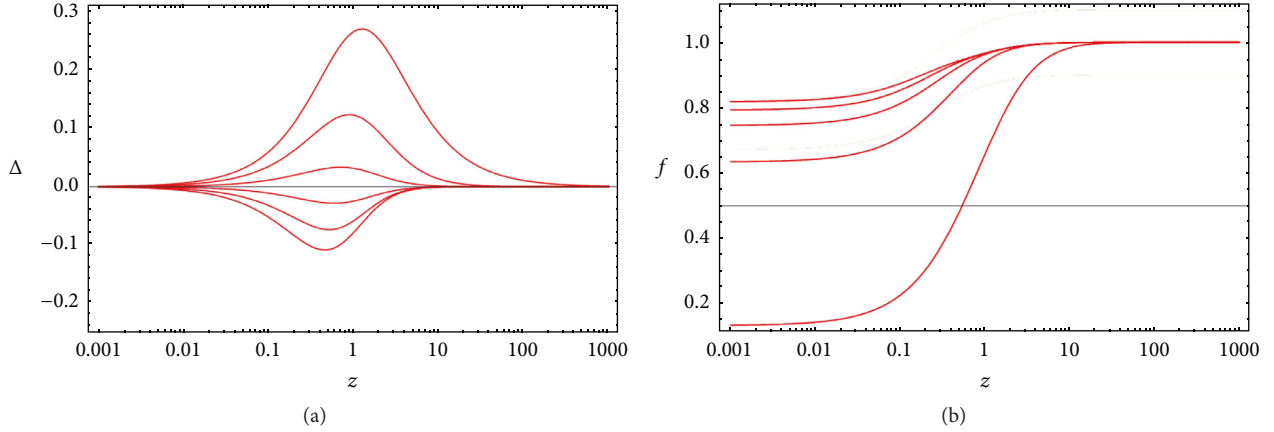


FIGURE 3: In (a) we obtain our model with an equidistant grid spacing, split by five steps with the same distance. (a) shows slight departures from our framework and the concordance model. This is even confirmed in (b). There, we report the behavior of the growth factor $f(a)$ in terms of the redshift z . Both redshift domains span from $z = 0$ to $z \approx 1000$.

words, the above relation is satisfied and the accelerated behavior of our model manifests at either late and early phase, providing, under certain conditions, that inflation may be predicted as a byproduct of our thermal lengths, unifying the dark energy description at late times with the corresponding inflationary epoch at early times.

4. Final Outlooks and Perspectives

In this paper, we assumed the validity of the holographic principle to characterize the dark energy evolution. To do so, we supposed that the cut-off scale, entering the minimal information density, is given by the well-consolidate de Broglie's wavelength associated with massive and massless particles, that is, matter and photons, respectively. The corresponding cut-off scales become the thermal lengths which can be used at late and early times, respectively, for classical and quantum regimes. We built up two cosmological models parameterizing the temperature with respect to the redshift z , using a standard power law, depending upon a constant β which enters our approach as a free parameter. To do so, we chose the adiabatic and the horizon volumes, that is, $V \propto a^3$ and $V \propto H^{-3}$, respectively. We found that, in both regimes of small and high redshifts, our models mimic the Λ CDM behavior by means of an evolving dark energy term. The former dark energy evolution seems to be compatible with the ω CDM quintessence model if the barotropic factor reads $\omega_0 = -(1/3)(2 + \beta)$ and $\omega_0 = -(1/3)(1 + 2\beta)$ at late and early times. We found a fairly good agreement of our models with the concordance paradigm. In particular, we found that the deceleration parameter evolution at small redshift spans within the observational interval predicted by observations and agrees with the Λ CDM predictions. Moreover, we found that the shift between our framework and the concordance model turns out to be small at high redshift. Further, even the growth index evolution slightly departs from the predictions of the standard cosmological model. This nominates the approach of thermal length within the holographic principle as a viable alternative to dark

energy. In future works, we will better understand whether more complicated models of holographic dark energy may be obtained, considering thermal length parameterized by specific temperature evolutions. Moreover, we will clarify the correct ranges of available β employing the use of cosmic data, to better understand if $\beta > 1$ or $\beta < 1$. Further, we will check the goodness of our model at early times with respect to more complete small perturbation analyses.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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