

Research Article

Black Holes and Information: A New Take on an Old Paradox

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Interest in the black hole information paradox has recently been catalyzed by the newer “firewall” argument. The crux of the updated argument is that previous solutions which relied on observer complementarity are in violation of the quantum condition of monogamy of entanglement, with the prescribed remedy being to discard the equivalence principle in favor of an energy barrier (or firewall) at the black hole horizon. Differing points of view have been put forward, including the “ER = EPR” counterargument and the final-state solution, both of which can be viewed as potential resolutions to the apparent conflict between quantum monogamy and Einstein equivalence. After reviewing these recent developments, this paper argues that the ER = EPR and final-state solutions can—thanks to observer complementarity—be seen as the same resolution of the paradox but from two different perspectives: inside and outside the black hole.

1. Introduction

Black holes have provided an alluring yet confusing arena for the study of physics. One suddenly encounters paradoxes when standard concepts, which are taken for granted in other physical environments, are applied to a black hole and its surroundings. A particularly notorious paradox is the apparent destruction of information when matter falls through a black hole horizon [1]. This loss of information presents a direct challenge to the principles of quantum theory and, although it has been the subject of intense scrutiny, the puzzle continues to persist.

There once was a commonly held viewpoint that the information paradox could be resolved by virtue of a framework that is known—in the spirit of Bohr—as horizon or observer complementarity [2]. However, a recent addition to this debate suggests that the information in question never does make it past the horizon. Rather, the black hole horizon is surrounded by a high-energy barrier—or “firewall”—which thermalizes infalling matter on impact [3]. The firewall argument is based on an apparent violation of the quantum condition of “monogamy of entanglement” but is by no means generally accepted. This is because the loss of quantum

monogamy would be no less costly than giving up the long-cherished equivalence principle of Einstein relativity. The latter principle dictates that one encounters an approximately flat spacetime when falling through the horizon.

One viable resolution of the firewall puzzle is the “ER = EPR” counterargument as put forth by Maldacena and Susskind [4]. Those authors adopt the same basic argument as the firewall proponents but draw a different conclusion. In order to preserve a natural state of approximately flat space at the horizon, Maldacena and Susskind suggest that wormholes (*i.e.*, Einstein-Rosen or ER bridges) allow for disturbances to travel between the Hawking-radiated matter [5, 6] and the environment within the black hole interior.

Another possible resolution is the so-called final-state solution, which was first put forward as a solution to the information paradox by Horowitz and Maldacena [7] but latter updated for the firewall scenario by Lloyd and Preskill [8]. The proposed procedure allows information to escape the black hole through the postselection of a specific final state at the black hole singularity. In effect, a process of quantum teleportation is used to transfer the state of the infalling matter to that of the exterior radiation. But, as post-state teleportation is formally no different than the propagation of

quantum information through wormholes [9], the two discussed solutions—ER = EPR and final state—would appear to share some similarities on at least a superficial level. Here, we will make a much stronger claim.

The conclusion in this paper is that the ER = EPR and final-state solutions are simply two sides of the same argument. Each is reached separately on the basis of the position of the observer whose perspective is in question. In both cases, a wormhole serves as the conduit for information transfer but, in one case, the information travels out of the black hole and, in the other, the information is rather transmitted inward. The key to this identification lies within the auspices of observer complementarity, which then remains central to the information paradox in spite of its recent detractors.

Although wormholes are suggestive of their own special brand of grand-paternal-like paradoxes and associated violations of causality, these conflicts are, as discussed later, only apparent and resolved within the relevant theories.

2. Black Holes and Information

Black holes were originally thought to consume all matter which fell past their horizons, without any hope for recovery. Essentially, they acted as cosmic drains in spacetime, permanently removing matter and energy from the Universe. This point of view was, however, challenged by Hawking, who showed that black holes evaporate over time and eventually disappear after being converted to radiation [5, 6]. This evaporation process meant that black holes could, after all, restore matter and energy to the Universe. The information content that was carried by the infalling matter was, however, quite another story.

Black holes came to be viewed, rather than drains, as scrambling machines. Indeed, even an initially pure state of collapsing matter would apparently be converted to a mixed state when it reemerged in the outgoing radiation [1]. The problem of black holes then took on a different perspective: It appeared that energy was conserved but information was completely scrambled. This viewpoint is especially problematic when confronted with the quantum-mechanical requirement of unitary evolution. A pure state that has collapsed into a black hole must, by the tenets of quantum mechanics, be recoverable (in principle) from the radiation which eventually replaces the evaporating black hole. That the information about this state (or that of any infalling matter) appears to be lost is what constitutes the black hole information paradox.

The problem is often recast in terms of the “nice-slice” point of view (see Figure 1). Here, one considers a collection of nonintersecting and (mostly) space-like surfaces, each of which crosses the horizon such that, once inside, it slowly curves so as to avoid the singularity for as long as possible. The implication for infalling matter is that there will always be at least one nice slice which is intersected by both the matter in its original form and its reincarnation as emitted radiation. If this radiation retains any quantum information about the infalling matter, then the information exists twice on the same space-like slice, which directly violates the so-called no-cloning principle of quantum mechanics [10]. For a

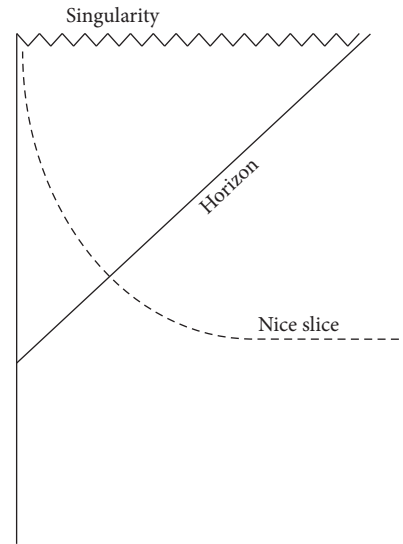


FIGURE 1: Schematic of a single “nice slice” crossing a black hole horizon.

more detailed explanation of the no-cloning theorem, please see Appendix A. And so we are left with the paradox of either the cloning of information or else its destruction, with both options being in violation of sacred quantum principles.

2.1. Complementarity at the Horizon. A solution was offered to this problem by Susskind and collaborators, who employed a principle that was originally known as horizon complementarity [2]. This principle stated that two observers—on either side of a black hole horizon—may disagree on an event inasmuch as they would never be in a position to compare their respective experiences. The concept was later expanded and renamed as observer complementarity, which follows the same principle but now applying to all causally separated observers and not just those separated by black hole horizons [11].

Regarding the black hole information paradox, observer complementarity allowed infalling information to be cloned at the horizon without any violation of quantum mechanics. The argument considers two observers: one named Alice, who falls into the black hole, and another named Bob, who remains outside and witnesses Alice’s descent. As per Einstein’s equivalence principle, the details of which are expanded on in Appendix B, Alice must experience approximately flat space as she enters the black hole (assuming a large enough black hole, as we always do). However, from Bob’s perspective, Alice must be thermalized near the horizon, with her information being carried away by the emitted radiation. According to horizon complementarity, both events can happen. This is because Alice and Bob can never disagree with one another’s experience once Alice has crossed the horizon. Suppose that Bob collects Alice’s radiation and then follows her into the black hole in an attempt to produce a paradoxical situation. Then he can never meet up with nor receive a signal from Alice before his destruction at the singularity; as such a meeting or signal would require Alice to have access to more

energy than that contained by the black hole. In this way, horizon complementarity allows cloning at the horizon and, therefore, the preservation of any information that had passed into the black hole.

3. Horizon Complementarity up in Flames

Although horizon complementarity was long considered a suitable resolution to the information paradox, the debate has been sparked anew following the “firewall” argument of Almheiri et al. who are commonly known as AMPS [3]. (Similar concerns had been expressed, but with much less fanfare, in earlier articles [12–15].)

The firewall argument assumes a relatively old black hole (here, “relatively old” means that, for the black hole-radiation system, the radiation subsystem should be the dominant one, if only by an infinitesimal amount) and begins with the acceptance of the same postulates as presented in the original horizon-complementarity framework. Those relevant to the current discussion include the expectation of (approximately) flat space for someone crossing the horizon, any information held within the black hole can eventually be retrieved from the Hawking radiation, and anyone remaining outside the black hole should see no violation of conventional physics. With these restrictions in mind, AMPS considered three isolated matter systems, as depicted in Figure 2, that are present during the evaporation process: one system, *A*, which lies within the black hole horizon, another system, *B*, consisting of Hawking radiation that is emitted late in the evaporation process, and a third system, *C*, which is composed of some much-earlier-emitted Hawking radiation. To make the argument fully come to life, one assumes that the collapsed matter was initially in a pure state and that *A* is chosen, without loss of generality, so that its state is the purifier of *B*’s (and *vice versa*).

The problem for horizon complementarity arises when the entanglement between the systems is considered with reference to the aforementioned postulates. Importantly, the requirement of flat space at the horizon entails a high degree of entanglement between the interior and the modes just outside the horizon (such an entangled state for the near-horizon region is referred to as the Unruh vacuum state [16]). As a consequence, *B* must be highly entangled with its purifier *A*. However, in order for an external observer to be able to consistently recover information from inside the black hole and—at the same time—see nothing paradoxical, there must typically be a high degree of entanglement between samples of early and late radiation. That is, *B* must also be highly entangled with *C*. (The condition of an “old” black hole, as per fn. 1, rules out the possibility that this conclusion can be avoided by both *B* and *C* being sufficiently entangled with systems behind the horizon.) This presents a situation where System *B* violates the principle of monogamy of entanglement, which restricts a system to be strongly entangled with only one other system at a time (this follows directly from the strong subadditivity of entropy [17] and the argument is outlined in Appendix C). This violation would not necessarily be a problem for horizon complementarity, except that AMPS

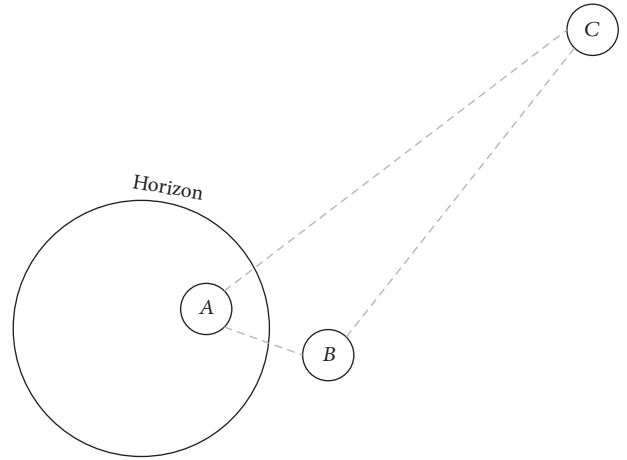


FIGURE 2: Systems involved in the firewall argument, with dotted lines showing the entanglement required by horizon complementarity.

confirm the existence of a frame which allows a single observer to witness both the late-early entanglement (*B*-*C*) as well as the trans-horizon entanglement (*A*-*B*). This was the spanner in the works for observer complementarity, which only applies if no violation is ever witnessed.

The resolution of the problem, as prescribed by AMPS, is to do away with the trans-horizon entanglement and accept a sea of high-energy particles in the vicinity of the horizon. In other words, discard the equivalence principle in order to preserve the unitarity of the evaporation process. However, it may not be necessary to throw away any sacred principles in order to resolve the black hole information paradox. All that might be needed is a wormhole.

4. The “ER = EPR” Counterproposal

Although AMPS concluded that the horizon of a black hole is a place of fiery death, Maldacena and Susskind proposed a different view of their argument, which might just save the equivalence principle [4]. The crux of Susskind and Maldacena’s counterargument is the recognition of what was a hidden assumption in the AMPS presentation. This assumption, to be elaborated on below, was related to the ability of the three relevant systems, *A*, *B*, and *C*, to transmit the effects of disturbances to one another.

Maldacena and Susskind argued that *A*, the system lying inside the black hole, must be in some sense “identified” with *C*, the distant system of early Hawking radiation, in order for both *A* and *C* to be highly entangled with System *B* near the horizon. However, as Susskind and Maldacena also point out, this identification cannot work unless a disturbance at *C* directly affects *A* (and *vice versa*). In particular, given this identification, a disturbance at *C* can be expected to create particles at *A*, which an infalling observer would view as part of a firewall. However, since System *C* was supposed to be emitted early in the evaporation process, it should be too far away from the black hole for such a disturbance to affect the journey of most infalling observers. To this end, AMPS claim

that there must always be a firewall at the horizon irrespective of any interference effects at C .

The assumption that Susskind and Maldacena took issue with is this inability for the distant radiation to rapidly transmit an effect to the interior of the black hole. If a disturbance of the distant radiation (or C) could somehow be felt by System A , then the effect would be to create a firewall on the horizon for any observer in the vicinity. On the other hand, if an observer fell in without any interaction occurring on the distant radiation, he or she would indeed witness approximately flat space at the horizon. But in neither case would there be an observed violation of monogamy of entanglement because a distant observer choosing to act (or not) on C could never be sure about the entanglement between A and B , whereas an infalling observer could never be sure about any relation between B and C .

What is then required to bypass the AMPS argument is a mechanism that would allow the far-away System C to transmit, almost instantaneously, an effect to the interior System A . For this purpose, Susskind and Maldacena introduced the notion of Einstein-Rosen (ER) bridges or, as they are more commonly known, wormholes. The presence of a wormhole connecting System C to System A would provide the necessary “shortcut” for any effect at C to influence A . In this way, it can be ensured that any firewall would arise as the result of interference on a near-horizon matter system (in this case, A) rather than as a preordained requirement of the black hole environment. On the other hand, such a shortcut would manifest itself as an instantaneous action at a distance, leading to an apparent violation of the principles of special relativity. But it is, indeed, only an apparent violation. A disturbance at C is transmitted to A via a legitimate pathway through spacetime given that a wormhole is a direct consequence of two systems being entangled. This argument—that wormholes are part and parcel with entanglement—is known as $ER = EPR$, (it should be emphasized that this identification is conjectural and not supported by direct evidence) where EPR refers to the Einstein-Podolsky-Rosen brand of entanglement as per the famous thought experiment [18]. Further illustration of the interpretation that ER bridges link entangled systems is provided in Appendix D, where the EPR thought experiment is used to demonstrate the concept further. This counterargument may or may not put the matter to rest, depending on one’s taste. However, it is by no means the only workable solution in the literature. Let us turn to another.

5. The Final-State Solution

Well before firewalls and $ER = EPR$, there was the final-state solution, as first proposed by Horowitz and Maldacena [7] and later updated by Lloyd and Preskill [8]. This proposal addressed the black hole information paradox by employing the quantum-mechanical notions of teleportation [19] and post-state selection [20] as a means for transferring information from the black hole interior to particles outside the horizon. As in standard quantum teleportation, the final-state solution relies on an entangled pair of particles, namely,

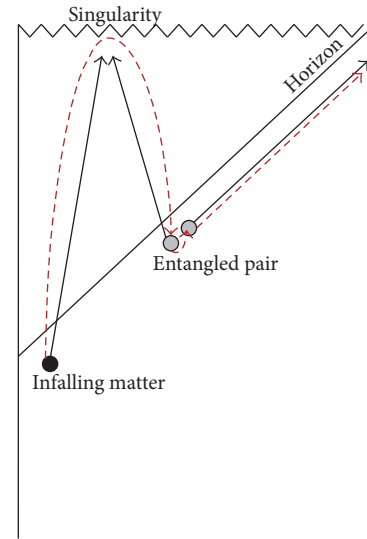


FIGURE 3: Schematic description of the final-state solution, with the red dotted line showing the path taken by the information contained in the infallen matter as it is teleported back in time.

positive- and negative-energy pair that initially straddles the horizon. (In the Hawking picture of black hole evaporation [1], entangled pairs are produced at the horizon whereby the positive-energy particle moves outward to become a quantum of Hawking radiation, while the negative-energy partner falls in and eventually lowers the mass of the black hole.)

Let us, for current purposes, denote the positive-energy particle as System 2 and its negative-energy partner as System 3, with System 1 reserved for denoting a suitable particle in the original infalling matter system. (This setup is illustrated in Figure 3.) More to the point, System 1 will be the particle that is responsible for the annihilation of System 3. In their proposal, Horowitz and Maldacena regard this annihilation event as a “measurement” of the two particles for which a specific final state is specified (*i.e.*, postselected). As System 3 is entangled with System 2, the outcome of this measurement must also affect the latter system. In fact, through the combination of post-state selection and quantum teleportation, it can be ensured that System 2, the incipient Hawking particle, is in the exact same state as that of System 1, the infalling bit of matter.

The required protocol is essentially a special case of quantum teleportation that adopts post-state selection as a means for negating the need for any classical communication. The information is still transferred by using quantum entanglement to generate a communication channel, but there is a notable difference from the standard case: The teleported information appears to be available at System 2 before the measurement of Systems 1 and 3 is actually carried out. The interpretation is that the information follows a channel which moves backwards through time in order to be teleported outward from the black hole.

Information flowing backwards in time may sound far-fetched. However, this protocol is remarkably similar to

the procedure of post-state teleportation which has been described by Lloyd et al. and shown to avoid all of the usual paradoxes that are associated with time travel [9]. Indeed, post-state teleportation was developed to provide a self-consistent quantum description of time travel, inasmuch as general relativity allows for this possibility through the existence of closed time-like curves and wormholes. It was further argued by Lloyd and Preskill that any issues of causality and unitarity violation in the Horowitz-Maldacena protocol would be small enough to be corrected by considerations from quantum gravity [8].

Lloyd and Preskill also addressed the preservation of monogamy of entanglement. For instance, the negative-energy particle, System 3, may appear to be entangled with both System 1 and System 2—but, from System 3's perspective, it is only ever entangled with a single system and can only ever be sure of which system is acting as its purifier when they are in causal contact.

Given that the resolution of the black hole information paradox (and firewall problem) does ultimately depend on wormholes, one might wonder which of the two discussed solutions—ER = EPR and final state—is the correct one. After all, both solutions employ similar procedures of transferring information across the black hole horizon via wormholes, but two distinct solutions is typically one too many. To this end, we will employ observer complementarity to shed some light on the situation.

6. Observer Complementarity Revisited

Adopting the concept of observer complementarity, we will now argue that ER = EPR and the final-state solution can be viewed as two sides of the same coin rather than two separate solutions of the black hole information paradox.

Let us start by considering two observers, Alice and Bob, who are interacting with some black hole. Alice falls into the black hole while Bob stays behind to collect radiation which is emitted early in the evaporation process. Alice knows nothing about the state of the Hawking radiation that Bob is collecting, and so she expects to experience flat space while falling through the horizon. The trans-horizon entanglement, which is necessary to ensure a drama-free passage for Alice, does not amount to a violation of quantum monogamy because it is the only entanglement that she ever sees. Once inside, Alice happens to observe a negative-energy particle (System 3) along with a bit of the original matter (System 1). Assuming that Alice is (somehow) protected from the tidal forces that are acting deep within the black hole, she will see the two infalling particles heading towards annihilation as they approach the singularity. Alice grows concerned that the information held by the infallen matter will be forever lost inside the black hole. However, using her knowledge of quantum entanglement and post-state selection, Alice soon realizes that, as long as the annihilation event acts as a measurement, the information can be teleported backwards in time to the positive-energy partner (System 2) which is now moving away from the horizon. Alice then concludes (before her own violent destruction) that the final-state

solution allows information to be retrieved from the black hole via a quantum channel of communication and that it does so without violating the condition of monogamy of entanglement.

But, now, what about Bob's perspective? Bob has been collecting early radiation and so is quite aware of the entanglement between this and the late radiation (respectively, C and B), which must be present to ensure that the infallen information can be retrieved. However, Bob also knows that the equivalence principle should hold at the horizon and, as such, the late radiation must be entangled with matter across the horizon (System A). To resolve this apparent conflict, Bob concludes that his measurement of the Hawking radiation must have influenced the state of the particles at the horizon—perhaps even producing a firewall, which would then thermalize any infalling matter (including Alice). In order for the influence of these measurements to reach the horizon in time, Bob deduces that a wormhole must connect the interior of the black hole to the Hawking radiation. In this way, Bob comes to the realization that the ER = EPR conjecture is needed to explain the overall procedure. It can also be noticed that Bob need not account for Alice's experience within the black hole because, as far as he is concerned, Alice is thermalized upon entry. Similarly, Alice need not account for Bob's actions, which take place outside of her region of causal contact.

Even though both Alice and Bob interact with the same black hole, their locations on either side of the horizon result in much different experiences. However, they can never compare notes, as Bob cannot reach Alice after she crosses the horizon and Alice cannot send a signal that would reach Bob in time if he decides to jump in after her (as discussed in Section 2.1). (Note that Alice could not use post-state teleportation to signal Bob, as this would be in violation of the so-called unproved-theorem paradox [9].) This situation may be problematic for our usual notion of classical physics but is quite acceptable in the framework of observer complementarity.

One might argue that Alice, as a part of System A , plays an essential role in the EPR = ER protocol and thus her perspective cannot be discounted when interpreting this proposal as a resolution of the information paradox. Nonetheless, such an argument is overlooking the potential of observer complementarity, given that this is indeed a true principle of the fundamental theory (for current purposes, we are assuming that it is). For ER = EPR and the final-state solution alike, the role of the interior is to enable information about the initial state of the black hole to eventually reach the external radiation (System C) without endangering the entanglement between the pairs (Systems A and B). Alice and Bob are never in causal contact, and so the best that either can do is to observe what is happening on their respective side of the horizon and then try to infer what is happening on the opposite side. Given that the underlying process is quantum teleportation, the only question left is if the conduit of the teleported information should be viewed as an Einstein-Rosen bridge or rather as a postselected measurement. Our claim is that it will always be viewed as the former from Bob's perspective and the latter from that of Alice. If this appears

implausible, it is no more or less implausible than Susskind's original scenario: Whereas Alice is happily alive (until the tidal forces set in), Bob is sure that she has already suffered a fiery death.

The previous results can be summarized as follows: From inside the black hole, information is teleported out and preserved via the final-state protocol whereas, outside the black hole, information about any disturbance is teleported inward so that quantum monogamy is preserved via the ER = EPR mechanism. In both cases, there is a quantum communication channel that enables the information in question to propagate. Any difference of opinion lies only in the position of the observer, inside or outside the horizon. However, observer complementarity makes it clear that such differing opinions are par for the course.

7. Conclusion

The black hole information paradox and its recent “firewall” development can be resolved by using the notion of wormholes as quantum channels of communication. As reviewed here, two procedures that describe just such resolution solutions are the ER = EPR argument and the final-state solution. Although these are understood as two distinct resolutions, we have argued here that the ER = EPR and final-state solutions can be viewed as precisely the same proposal, only from two different perspectives. The key to our argument is the quantum-gravity inspired principle of observer complementarity, namely, that two observers can disagree on events provided that they remain out of causal contact. Ironically, the same basic principle (in the guise of horizon complementarity) was long thought to provide the answer to the information paradox, until it was recently shot down by the proponents of the firewall. With apologies to Mark Twain, the demise of observer complementarity may have been greatly exaggerated.

Appendix

A. The No-Cloning Theorem

The no-cloning theorem is essentially a restriction on the possibility of producing two identical quantum states from a starting point of one state. The theorem was outlined in 1982 in [10, 21], and it applies to any general quantum state. The proof involves looking at two quantum states which share a Hilbert space. The argument that follows then focuses on the question of what operations could be performed on a system which combines the two states into a tensor product without specifying either state. The application to general states relies on leaving the state that we wish to copy as an arbitrary unknown state.

This use of an unknown state is similar to the procedure used in quantum teleportation, as in [19], but there is a fundamental difference to keep in mind. At the end of the teleportation procedure, the unknown state has been transferred to the secondary particle while being “destroyed”

at the original particle. So, when the teleportation procedure is complete, only one particle holds the unknown state.

In the no-cloning theorem, however, the question under scrutiny is the possibility of producing two copies of an unknown state which exist simultaneously in two particles. With that aim in mind, possible operations which might result in such a cloned state are considered. The use of a measurement operation is ruled out as it will result in a changed state after the procedure. This leaves the possibility of using unitary operator on the tensor product which might clone the unknown state. What is seen in [10, 21] is that there is no unitary operator which can clone a general unknown state from one particle to another in order to end up with two copies of the state in question.

This conclusion effectively ensures that no operation performed on a system of two particles can produce the same state in both particles. In terms of the situation described in the black hole scenario, this restricts what happens to the information being held by the state that crosses the horizon. By the no-cloning theorem, this state could appear inside the horizon or outside, but it cannot be in both places on the same space-like slice as this would be an example of a cloned state.

B. The Equivalence Principle

The original equivalence principle is a reference to an idea which was first derived in [22]. The concept behind the principle is the matching of the effects found in gravitational fields with effects produced in accelerating reference frames. This is most commonly illustrated with a comparison of experiments done in a rocket at rest on Earth and similar experiments in the same rocket accelerating through empty space with a force equal to that of Earth's gravitational pull. This idea was further developed in [23] and promoted to the status of a principle of the theory of general relativity. From this idea, Einstein reached the conclusion that the experience of free fall should be indistinguishable from the experience in an inertial reference frame for an experimenter inside a closed laboratory. Essentially, if an observer was placed in a closed room, he or she could expect the same results from experiments whether that room was placed in free fall around a large mass or if the room was placed in a weightless environment. This concept is often referred to as the “weak” equivalence principle. Two further principles have since been developed from it. One, the “strong” equivalence principle, relates the above idea to a general range of scenarios and is more encompassing. The second is called Einstein's equivalence principle and it relates specifically to scenarios affected by gravity. In essence, it is the same concept as stated above—that the effects felt in an inertial frame are no different from those felt in free fall—but it clarifies that these effects are independent of the free-falling object's location or velocity.

The relevance for this in the black hole scenario is due to the presence of a large mass producing a substantial gravitational field. An observer falling into the black hole would experience the exact situation that Einstein's equivalence principle applies to. As outlined in [2], the curvature of spacetime at the horizon of a massive black hole would be

gentle. As such, the free-falling observer would experience no tidal forces until he or she was further inside. This means that the experience at the horizon, the one of free fall, should be similar to that experienced in an inertial reference frame which is characterized by empty space.

C. Quantum Monogamy

The following has been adapted from [24]. This refers to the condition that a quantum system may not be strongly entangled with multiple systems. This condition is a corollary of the strong-subadditivity statement that was proven in [25]. Strong subadditivity refers to an inequality that governs how the entropy of a system must be constrained with regard to the entropy of the subsystems which make up the whole. By tracing over individual subsystems, the entropies of specific sections of the system may be measured and then compared to one another.

The corollary in question—that relating the strong-subadditivity statement to quantum monogamy—was proven in [17] and can be expressed as

$$S(\rho^A) + S(\rho^B) \leq S(\rho^{AC}) + S(\rho^{CB}). \quad (\text{C.1})$$

Here, $S(\rho)$ denotes the entropy of the subsystem described by density matrix ρ , and the superscripts A, B , and C refer to three subsystems within a larger system. The entropy is compared between the subsystems. This is accomplished by tracing out either one or two subsystems from the total density matrix of the complete system. This allows (C.1) to limit possible entanglements within a group of three subsystems.

Now consider a situation in which Subsystem C is strongly entangled with both Subsystem A and Subsystem B . A strongly entangled system has low entropy, and each subsystem within the entangled system will have an individual entropy that is higher than the entropy of the entire entangled system. This results in $S(\rho^A)$ being greater than $S(\rho^{AC})$ and, similarly, $S(\rho^B)$ would be greater than $S(\rho^{CB})$. This combination violates (C.1) as the left-hand side, comprised of single-subsystem entropies, outweighs the right-hand side which consists of the entropies of entangled pairs. This outcome led to the conclusion that System C can only be strongly entangled with either System A or System B but not both. Hence a quantum system must respect monogamy and may be entangled strongly with only one other system at a time.

D. On Einstein-Rosen Bridges

The following has been adapted from [24]. The concept that entangled objects can be connected via a wormhole is relevant to the EPR argument in that it allows for actions at Alice's location to disturb Bob's system even though the experimenters are separated by space-like distances. If an entangled pair from the standard EPR setup is linked by a wormhole, then an action on one of the pairs can be felt by the other. Any disturbance caused by Alice's measurement on her system could then be transmitted through such a wormhole

to influence the system at Bob's location. By assuming that entangled pairs are linked in this way, the ER = EPR argument provides a mechanism through which the entangled pairs maintain their entangled correlations without requiring the spin directions to be determined when the particles are prepared. Alice's measurement will still result in a probabilistic outcome consistent with quantum mechanics. The wormhole allows the measurement at Alice's location to influence Bob's system over space-like distances, thus providing a mechanism for Alice's result to influence Bob's result instantaneously regardless of the distance between them. Bob's system would therefore be influenced by the actions at Alice's location. This essentially describes a mechanism which allows for "spooky action at a distance" between entangled particles.

Competing Interests

The authors declare that they have no competing interests.

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References

- [1] S. W. Hawking, "Breakdown of predictability in gravitational collapse," *Physical Review D*, vol. 14, no. 10, pp. 2460–2473, 1976.
- [2] L. Susskind, L. Thorlacius, and J. Uglum, "The stretched horizon and black hole complementarity," *Physical Review D. Third Series*, vol. 48, no. 8, pp. 3743–3761, 1993.
- [3] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, "Black holes: complementarity or firewalls?" *Journal of High Energy Physics*, vol. 2013, no. 2, article 062, 2013.
- [4] J. Maldacena and L. Susskind, "Cool horizons for entangled black holes," *Fortschritte der Physik. Progress of Physics*, vol. 61, no. 9, pp. 781–811, 2013.
- [5] S. W. Hawking, "Black hole explosions," *Nature*, vol. 248, pp. 30–31, 1974.
- [6] S. W. Hawking, "Particle creation by black holes," *Communications in Mathematical Physics*, vol. 43, no. 3, pp. 199–220, 1975.
- [7] G. T. Horowitz and J. Maldacena, "The black hole final state," *Journal of High Energy Physics*, vol. 0402, article no. 008, 2004.
- [8] S. Lloyd and J. Preskill, "Unitarity of black hole evaporation in final-state projection models," *Journal of High Energy Physics*, vol. 2014, no. 8, article 126, 2014.
- [9] S. Lloyd, L. MacCone, R. Garcia-Patron, V. Giovannetti, and Y. Shikano, "Quantum mechanics of time travel through post-selected teleportation," *Physical Review D - Particles, Fields, Gravitation and Cosmology*, vol. 84, no. 2, Article ID 025007, 2011.
- [10] W. K. Wootters and W. H. Zurek, "A single quantum cannot be cloned," *Nature*, vol. 299, no. 5886, pp. 802–803, 1982.
- [11] R. Bousso, "Positive vacuum energy and the N bound," *Journal of High Energy Physics*, vol. 2000, no. 11, article 038, 2000.
- [12] N. Itzhaki, "Is the black hole complementarity principle really necessary?" <https://arxiv.org/abs/hep-th/9607028>.

- [13] S. D. Mathur, “What exactly is the information paradox?” in *Physics of Black Holes*, vol. 769 of *Lecture Notes in Physics*, pp. 3–48, Springer, Berlin, Germany, 2009.
- [14] S. D. Mathur, “The information paradox: a pedagogical introduction,” *Classical and Quantum Gravity*, vol. 26, no. 22, 2009.
- [15] S. L. Braunstein, S. Pirandola, and K. Życzkowski, “Entangled black holes as ciphers of hidden information,” *Physical Review Letters*, vol. 110, Article ID 101301, 2013.
- [16] W. G. Unruh, “Notes on black-hole evaporation,” *Physical Review D*, vol. 14, no. 4, article 870, 1976.
- [17] H. Araki and E. H. Lieb, “Entropy inequalities,” *Communications in Mathematical Physics*, vol. 18, pp. 160–170, 1970.
- [18] A. Einstein, B. Podolsky, and N. Rosen, “Can quantum-mechanical description of physical reality be considered complete?” *Physical Review*, vol. 47, no. 10, pp. 777–780, 1935.
- [19] M. B. Plenio and V. Vedral, “Teleportation, entanglement and thermodynamics in the quantum world,” *Contemporary Physics*, vol. 39, no. 6, pp. 431–446, 1998.
- [20] Y. Aharonov, D. Z. Albert, and L. Vaidman, “How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100,” *Physical Review Letters*, vol. 60, no. 14, article 1351, 1988.
- [21] D. Dieks, “Communication by EPR devices,” *Physics Letters A*, vol. 92, no. 6, pp. 271–272, 1982.
- [22] A. Einstein, “On the relativity principle and the conclusions drawn from it,” *Jahrbuch der Radioaktivität und Elektronik*, vol. 4, pp. 411–462, 1907.
- [23] A. Einstein, “On the influence of gravitation on the propagation of light,” *Annalen der Physik*, vol. 35, pp. 898–906, 1911.
- [24] K. L. H. Bryan, *The EPR paradox: back from the future [M.S. thesis]*, Rhodes University, 2015.
- [25] E. H. Lieb and M. B. Ruskai, “Proof of the strong subadditivity of quantum-mechanical entropy,” in *Inequalities*, pp. 63–66, Springer, Berlin, Germany, 2002.

