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Research Article

A Study of Charged Cylindrical Gravitational Collapse with Dissipative Fluid

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The present works deals with gravitational collapse of cylindrical viscous heat conducting anisotropic fluid following the work of Misner and Sharp. Using Darmois matching conditions, the dynamical equations are derived and the effects of charge and dissipative quantities over the cylindrical collapse are analyzed. Finally, using the Miller-Israel-Steward causal thermodynamic theory, the transport equation for heat flux is derived and its influence on collapsing system has been studied.

1. Introduction

A challenging but curious issue in gravitational physics as well as in relativistic astrophysics is to know the final fate of a continual gravitational collapse. The stable configuration of a massive star persists as long as the inward pull of gravity is neutralized by the outward pressure of the nuclear fuel at the core of the star. Subsequently, when the star has exhausted its nuclear fuel there is no longer any thermonuclear burning and there will be endless gravitational collapse. However, depending on the mass of the collapsing star, the compact objects such as white dwarfs, neutron stars, and black holes are formed. White dwarf and neutron star gravity are counterbalanced by electron and neutron degeneracy pressure, respectively, while black hole is an example of the end state of collapse.

The study of gravitational collapse was initiated long back in 1939 by Oppenheimer and Snyder [1]. They have studied the collapse of a homogeneous spherical dust cloud in the frame work of general relativity. Then after a quarter century, a more realistic investigation was done by Misner and Sharp [2] with perfect fluid in the interior of a collapsing star. In both the studies, the exterior of the collapsing star was chosen as vacuum. Vaidya [3] formulated the nonvacuum exterior of a star having radiating fluid in the interior. An inhomogeneous spherically symmetric dust cloud was analytically studied by Joshi and Singh [4] and they have shown that the final fate of the collapsing star depends crucially on the initial density profile and the radius of the star. Debnath et al. [5] investigated collapse dynamics of the nonadiabatic fluid, considering quasi-spherical Szekeres space-time in the interior and plane symmetric Vaidya solution in the exterior region.

Although most of the works on collapse dynamics are related to spherical objects, still there is interesting information about self-gravitating fluids for collapsing object with different symmetries. The natural choice for nonspherical symmetry is axis-symmetric objects. The vacuum solution for Einstein field equations in cylindrically symmetric space-time was obtained first by Levi-Civita [6], but still it is a challenging issue of interpreting two independent parameters in the solution. Herrera and Santos [7] studied cylindrical collapse of nondissipative fluid with exterior Einstein-Rosen space-time and showed wrongly a nonvanishing radial pressure on the boundary surface and subsequently in collaboration with MacCallum [8] they corrected the result. Then Herrera and collaborators investigated cylindrical collapse of matter with [9] or without shear [10].

Further, the junction conditions due to Darmois [11] has a very active role in dealing collapsing problems. Sharif et al. [12–14] showed the effect of positive cosmological constant on the collapsing process by using junction conditions between static exterior and nonstatic interior with a cosmological constant. Also Herrera et al. [15], using junction conditions, were able to prove that any conformally flat cylindrically symmetric static source cannot be matched to the Levi-Civita space-time. Then Kurita and Nakao [16] formulated naked singularity along the axis of symmetry, considering cylindrical collapse with null dust.

Moreover, from realistic point of view, it is desirable to consider dissipative matter in the context of collapse dynamics [17–19]. Considering collapse of a radiating star with dissipation in the form of radial heat flow and shear viscosity, Chan [20] has showed that shear viscosity plays a significant role in the collapsing process. Collapse dynamics with dissipation of energy as heat flow and radiation have been studied by Herrera and Santos [18]. Subsequently, by considering of causal transport equations related to different dissipative components (heat flow, radiation, shear, and bulk viscosity), Herrera et al. [15, 21, 22] investigated the collapse dynamics. The same collapsing process with plane symmetric geometry or others has been examined by Sharif et al. [23, 24].

On the other hand, in the context of gravitational waves, the sources must have nonspherical symmetry. Further, cylindrical collapse of nondissipative fluid with exterior containing gravitational waves shows nonvanishing pressure on the boundary surface by using Darmois matching conditions. Recently, it has been verified [25, 26] in studying cylindrical collapse of anisotropic dissipative fluid with formation of gravitational waves outside the collapsing matter.

In the present work, following Misner and Sharp, collapse dynamics of viscous, heat conducting charged anisotropic fluid in cylindrically symmetric background will be studied. The paper is organized as follows. Section 2 deals with basic equations related to interior and exterior space-time. The junction conditions are evaluated and discussed in Section 3. The dynamical equations are derived and studied in Section 4. Finally, the process of mass, heat, and momentum transfer through transport equation is discussed in Section 5.

2. Interior and Exterior Space-Time: Basic Equations

Mathematically, the whole four-dimensional space-time manifold having a cylindrical collapsing process can be written as $M = M^{+}U\Sigma UM^{-}$ with $M^{-} \cap M^{+} = \phi$. Here, Σ, the collapsing cylindrical surface, is a time-like three-surface and is the boundary of the two four-dimensional submanifolds M^{-} (interior) and M^{+} (exterior).

In M^{-} choosing comoving coordinates, the line element can be written as follows [25, 26]:

$$ds_{-}^{2} = -A^{2}dt^{2} + B^{2}dr^{2} + C^{2}d\phi^{2} + D^{2}dz^{2}, \tag{1}$$

where the metric coefficients are functions of t and r, that is, $A = A(t,r)$ and so on. Also due to cylindrical symmetry, the coordinates are restricted as follows:

$$\begin{gathered} -\infty \leq t \leq +\infty, \\ r \geq 0, \\ -\infty < z < +\infty, \\ 0 \leq \phi \leq 2\pi. \end{gathered} \tag{2}$$

For compact notation, we write $\{x^{-\mu}\} \equiv [t, r, \phi, z]$, ($\mu = 0, 1, 2, 3$).

The anisotropic fluid having dissipation in the form of shear viscosity and heat flow has the energy-momentum tensor of the following form [7, 9]:

$$\begin{aligned} T_{\mu\nu} = {} & (\rho + p_t)\, v_\mu v_\nu + p_t g_{\mu\nu} + (p_r - p_t)\, \chi_\mu \chi_\nu - 2\eta\sigma_{\mu\nu} \\ & + 2q_{(\,\mu} v_{\nu)}. \end{aligned} \tag{3}$$

Here ρ, p_r, $p_t \eta$, and q_μ stand for energy density, the radial pressure, the tangential pressure, coefficient of shear viscosity, and radial heat flux vector, respectively. Also v_μ and χ_μ are unit time-like and space-like vectors satisfying the following relations:

$$\begin{aligned} v_\mu v^\mu &= -\chi_\mu \chi^\mu = -1, \\ \chi^\mu v_\mu &= 0, \\ q_\mu v^\mu &= 0. \end{aligned} \tag{4}$$

Moreover, the shear tensor $\sigma_{\mu\nu}$ has the following expression:

$$\sigma_{\mu\nu} = v_{(\mu;\nu)} + a_{(\,\mu} v_{\nu)} - \frac{1}{3}\Theta\left(g_{\mu\nu} + v_\mu v_\nu\right), \tag{5}$$

where $a_\mu = v_{\mu;\nu}v^\nu$ is the acceleration vector and $\Theta = v^\mu{}_{;\mu}$ is the expansion scalar.

For the above metric, one may choose the unit time-like vector, space-like vector, and heat flux vector in a simple form as

$$\begin{aligned} v^\mu &= A^{-1}\delta^\mu_0, \\ \chi^\mu &= B^{-1}\delta^\mu_1, \\ q^\mu &= q\delta^\mu_1. \end{aligned} \tag{6}$$

The shear tensor has only nonzero diagonal components as

$$\begin{aligned} \sigma_{11} &= \frac{B^2}{3A}\left[\Sigma_1 - \Sigma_3\right], \\ \sigma_{22} &= \frac{C^2}{3A}\left[\Sigma_2 - \Sigma_1\right], \\ \sigma_{33} &= \frac{D^2}{3A}\left[\Sigma_3 - \Sigma_2\right] \\ \text{with } \sigma^2 &= \frac{1}{6A^2}\left[\Sigma_1^2 + \Sigma_2^2 + \Sigma_3^2\right], \end{aligned} \tag{7}$$

where $\Sigma_1 = \dot{B}/B - \dot{C}/C$, $\Sigma_2 = \dot{C}/C - \dot{D}/D$, $\Sigma_3 = \dot{D}/D - \dot{B}/B$.

Also the acceleration vector and the expansion scalar have the following explicit expressions:

$$a_1 = \frac{A'}{A},$$
$$\Theta = \frac{1}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{D}}{D}\right). \tag{8}$$

In the above, by notation, we have used $\cdot \equiv \partial/\partial t$ and $' \equiv \partial/\partial r$.

If, in addition, we assume the above fluid distribution to be charged then the energy-momentum tensor for the electromagnetic field has the following form:

$$E_{\alpha\beta} = \frac{1}{4\pi}\left(F_{\mu}^{\alpha}F_{\nu\alpha} - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}g_{\mu\nu}\right), \tag{9}$$

where the Maxwell field tensor $F_{\alpha\beta}$ is related to the four potential ϕ_α as

$$F_{\alpha\beta} = \phi_{\beta,\alpha} - \phi_{\alpha,\beta} \tag{10}$$

and the evolution of the field tensor corresponds to Maxwell equations

$$F^{\alpha\beta}_{;\beta} = 4\pi J^{\alpha}, \tag{11}$$

where J^α is the four-current vector.

As the charge per unit length of the cylinder is at rest with respect to comoving coordinates so the magnetic field will be zero in this local coordinate system [27, 28]. Hence the four potentials and the four currents take the following simple form:

$$\phi_\alpha = \phi\delta^0_\alpha,$$
$$J^\alpha = \epsilon v^\alpha, \tag{12}$$

where $\phi = \phi(t,r)$ is the scalar potential and $\epsilon = \epsilon(t,r)$ is the charge density.

From the law of conservation of charge, $J^\alpha_{;\alpha} = 0$, one obtains the total charge distribution interior to radius r and per unit length of the cylinder as

$$s(r) = 2\pi\int_0^r \epsilon BCD\,dr. \tag{13}$$

Now the explicit form of Maxwell's equations (11) for the interior space-time M^- is given by

$$\phi'' - \left(\frac{A'}{A} + \frac{B'}{B} - \frac{C'}{C} - \frac{D'}{D}\right)\phi' = 4\pi\epsilon AB^2, \tag{14}$$

$$\dot{\phi}' - \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} - \frac{\dot{C}}{C} - \frac{\dot{D}}{D}\right)\phi' = 0 \tag{15}$$

A first integral of (14) gives

$$\phi' = \frac{2sAB}{CD}, \tag{16}$$

which satisfies identically the other Maxwell equation (15). Hence one obtains the electric field intensity as $E(t,r) = s(r)/2\pi C$.

Further, in the interior space-time M^-, the Einstein field equations $G_{\alpha\beta} = 8\pi(T_{\alpha\beta} + E_{\alpha\beta})$ have the following explicit form:

$$\frac{A^2}{B^2}\left(-\frac{C''}{C} - \frac{D''}{D} + \frac{B'}{B}\left(\frac{C'}{C} + \frac{D'}{D}\right) - \frac{C'D'}{CD}\right) + \left(\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}\dot{D}}{BD} + \frac{\dot{C}\dot{D}}{CD}\right) = 8\pi\left(\rho A^2 - 2\eta\sigma_{00}\right) + 4\frac{s^2A^2}{C^2D^2}. \tag{17}$$

$$-\frac{B^2}{A^2}\left(\frac{\ddot{C}}{C} + \frac{\ddot{D}}{D} + \frac{\dot{C}\dot{D}}{CD} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{A}\dot{D}}{AD}\right) + \left(\frac{C'D'}{CD} + \frac{A'C'}{AC} + \frac{A'D'}{AD}\right) = 8\pi\left(p_rB^2 - 2\eta\sigma_{11}\right) - 4\frac{s^2B^2}{C^2D^2} \tag{18}$$

$$-\frac{C^2}{A^2}\left[\frac{\ddot{B}}{B} + \frac{\ddot{D}}{D} - \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{D}}{D}\right) + \frac{\dot{B}\dot{D}}{BD}\right] + \frac{C^2}{B^2}\left[\frac{A''}{A} + \frac{D''}{D} - \frac{A'}{A}\left(\frac{B'}{B} - \frac{D'}{D}\right) - \frac{D'}{D}\frac{B'}{B}\right] = 8\pi\left(p_tC^2 - 2\eta\sigma_{22}\right) + 4\frac{s^2}{D^2} \tag{19}$$

$$-\frac{D^2}{A^2}\left[\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\dot{B}\dot{C}}{BC}\right] + \frac{D^2}{B^2}\left[\frac{A''}{A} + \frac{C''}{C} - \frac{A'}{A}\left(\frac{B'}{B} - \frac{C'}{C}\right) - \frac{C'}{C}\frac{B'}{B}\right] = 8\pi\left(p_tD^2 - 2\eta\sigma_{33}\right) + 4\frac{s^2}{C^2}, \tag{20}$$

$$\frac{1}{AB}\left(\frac{\dot{C}'}{C} + \frac{\dot{D}'}{D} - \frac{C'}{C}\frac{\dot{B}}{B} - \frac{\dot{B}}{B}\frac{D'}{D} - \frac{A'}{A}\frac{\dot{C}}{C} - \frac{A'}{A}\frac{\dot{D}}{D}\right) = 8\pi q \tag{21}$$

The gravitational energy per specific length in cylindrically symmetric space-time is defined as follows [29–31]:

$$E = \frac{\left(1 - l^{-2}\nabla^a r\nabla_a r\right)}{8}. \tag{22}$$

In principle, E is the charge associated with a general current which combines the energy-momentum of the matter and gravitational waves. It is usually referred to in the literature as C-energy for the cylindrical symmetric space-time. For cylindrically symmetric model with killing vectors, the circumference radius ρ and specific length l are defined as follows [29–31]:

$\rho^2 = \xi_{(1)a}\xi^a_{(1)}$, $l^2 = \xi_{(2)a}\xi^a_{(2)}$, so that $r = \rho l$ is termed as areal radius.

For the present model with the contribution of electromagnetic field in the interior region, the C-energy takes the following form:

$$E' = \frac{l}{8} + \frac{1}{8D}\left[\frac{1}{A^2}\left(C\dot{D} + \dot{C}D\right)^2 - \frac{1}{B^2}\left(CD' + C'D\right)^2\right] + \frac{s^2}{2C}. \quad (23)$$

It should be noted that the above energy is also very similar to Tabu's mass function in the plane symmetric space-time [32].

The exterior space-time manifold (M^+) of the cylindrical surface Σ is described by the metric in the retarded time coordinate as [33, 34]

$$ds_+^2 = -\left(\frac{-2M(\nu)}{R} + \frac{Q^2(\nu)}{R^2}\right)d\nu^2 - 2dRd\nu + R^2\left(d\phi^2 + \lambda^2 dz^2\right), \quad (24)$$

where ν is the usual retarded time, $M(\nu)$ is the total mass inside Σ, $Q(\nu)$ is the total charge bounded by Σ, and λ is an arbitrary constant. Further, from the point of view of the interior manifold (M^-), the bounding three-surface Σ (comoving surface) is described as

$$f_-(t, r) = r - r_\Sigma = 0, \quad (25)$$

and hence the interior metric on Σ takes the following form:

$$ds_-^2 \overset{\Sigma}{=} -d\tau^2 + C^2 dz^2 + D^2 d\phi^2, \quad (26)$$

where

$$d\tau \overset{\Sigma}{=} A dt \quad (27)$$

defines the time coordinate on Σ and $\overset{\Sigma}{=}$ by notation implies the equality of both sides on the surface Σ.

Similarly, from the perspective of the exterior manifold, the boundary three-surface Σ is characterized by

$$f_+(\nu, R) \equiv R - R_\Sigma(\nu) = 0 \quad (28)$$

so that the exterior metric on Σ takes the form

$$ds_+^2 \overset{\Sigma}{=} -\left(\frac{-2M(\nu)}{R} + \frac{Q^2(\nu)}{R^2} + \frac{2dR_\Sigma(\nu)}{d\nu}\right)d\nu^2 + R^2\left(d\phi^2 + \lambda^2 dz^2\right). \quad (29)$$

Here, by notation, we write $[x^{+\mu}] = [\nu, R, \phi, z]$.

3. Junction Conditions

In order to have a smooth matching of the interior and exterior manifolds over the bounding three surfaces (not a surface layer), the following conditions due to Darmois [11] are to be satisfied.

(i) The continuity of the first fundamental form is

$$\left(ds^2\right)_\Sigma = \left(ds_-^2\right)_\Sigma = \left(ds_+^2\right)_\Sigma. \quad (30)$$

(ii) The continuity of the second fundamental form is $K_{ij}d\xi^i d\xi^j$. This implies that the continuity of the extrinsic curvature K_{ij} over the hypersurface [11] is

$$\left[K_{ij}\right] \equiv K_{ij}^+ - K_{ij}^- = 0, \quad (31)$$

where $K_{ij}^\pm$ is given by

$$K_{ij}^\pm = -n_\sigma^\pm\left[\frac{\partial^2 x_\pm^\sigma}{\partial\xi^i\partial\xi^j} + \Gamma^\sigma_{\mu\nu}\frac{\partial x_\pm^\mu}{\partial\xi^i}\frac{\partial x_\pm^\nu}{\partial\xi^j}\right], \quad (\sigma, \mu, \nu = 0, 1, 2, 3). \quad (32)$$

In the above expression for extrinsic curvature, $n_\sigma^\pm$ are the components of the outward unit normal to the hypersurface with respect to the manifolds $M^\pm$ (i.e., in the coordinates $x^{\pm\mu}$) and have explicit expressions.

$n_\sigma^- \overset{\Sigma}{=} (0, B, 0, 0)$ and $n_\sigma^+ \overset{\Sigma}{=} \mu(-dR/d\nu, 1, 0, 0)$ with $\mu = [-2M(\nu)/R + Q^2(\nu)/R^2 + 2(dR/d\nu)]^{-1/2}$.

Also, in the above, the Christoffel symbols are evaluated for the metric in M^- or M^+ accordingly and we choose $\xi^0 = \tau$, $\xi^2 = z$, $\xi^3 = \phi$ as the intrinsic coordinates on Σ for convenience.

The continuity of the 1st fundamental form gives

$$\begin{aligned} C(t, r_\Sigma) &\overset{\Sigma}{=} R_\Sigma(\nu), \\ D(t, r_\Sigma) &\overset{\Sigma}{=} \lambda R_\Sigma(\nu) \\ \frac{dt}{d\tau} &= \frac{1}{A} \\ \frac{d\nu}{d\tau} &= \mu. \end{aligned} \quad (33)$$

Now the nonvanishing components of extrinsic curvature $K_{ij}^\pm$ are

$$K_{00}^- = -\left(\frac{A'}{AB}\right)_\Sigma$$

$$K_{00}^+ = \left[\left(\frac{d^2\nu}{d\tau^2}\right)\left(\frac{d\nu}{d\tau}\right)^{-1} - \left(\frac{d\nu}{d\tau}\right)\left(\frac{M}{R^2} - \frac{Q^2}{R^3}\right)\right]_\Sigma$$

$$K_{22}^{-} = \left(\frac{CC'}{B}\right)_{\Sigma}$$
$$K_{33}^{-} = \left(\frac{DD'}{B}\right)_{\Sigma}$$
$$K_{22}^{+} = \left[R\left(\frac{dR}{d\tau}\right) - \frac{(dv)}{d\tau}\left(2M - \frac{Q^2}{R}\right)\right]_{\Sigma} = \lambda^{-2}K_{33}^{+}. \quad (34)$$

Hence, continuity of the extrinsic curvature together with (33) gives the following relations over Σ [34, 35]:

$$M(v) \overset{\Sigma}{=} \frac{R}{2}\left[\left(\frac{\dot{R}}{A}\right)^2 - \left(\frac{R'}{B}\right)^2\right] + \frac{Q^2}{2R} \quad (35)$$

$$E \overset{\Sigma}{=} \frac{l}{8} + \lambda M, \quad (36)$$

$$q \overset{\Sigma}{=} p_r - \frac{2\eta\sigma_{11}}{B^2} - \frac{s^2}{2\pi c^4}\left(\frac{1}{\lambda^2} - \frac{1}{4}\right). \quad (37)$$

Thus (35) gives the total mass inside the boundary surface Σ, while (36) shows the linear relationship between the C energy for the cylindrically symmetric space-time with the bounding mass over Σ. Further, (37) shows a linear relationship among the fluid parameters (p_r, η, q) on the bounding surface Σ. Hence radial pressure is in general nonzero on the bounding surface due to dissipative nature of the fluid and the charge on the bounding surface. But when dissipative components of the fluid are switched off, then the above result (uncharged) agrees with the results of Herrera et al. [8]. Also it should be noted that the radial pressure on the boundary does not depend on the charge bounded by Σ; it depends only on the charge on the surface Σ.

4. Analysis of Dynamical Equations

From the conservation of energy-momentum, that is, $(T^{\alpha\beta} + E^{\alpha\beta})_{;\beta} = 0$, we can have two zero scalars, namely, $(T^{\alpha\beta} + E^{\alpha\beta})_{;\beta}v_{\alpha}$ and $(T^{\alpha\beta} + E^{\alpha\beta})_{;\beta}\chi_{\alpha}$

Using (3) and (9), the explicit expressions for these two scalars are

$$\frac{\dot{\rho}}{A} + \frac{\dot{B}}{A}\left(\frac{\rho}{B} + \frac{p_r}{B} - 2\eta\sigma^{11}\right) + \frac{\dot{C}}{A}\left(\frac{p_\perp}{C} + \frac{\rho}{C} - 2\eta\sigma^{22}\right)$$
$$+ \frac{\dot{D}}{A}\left(\frac{\rho}{D} + \frac{p_\perp}{D} - 2\eta\sigma^{33}\right) + \frac{q'}{B} \quad (38)$$
$$+ \frac{q}{B}\left(2\frac{A'}{A} + \frac{C'}{C} + \frac{D'}{D}\right) = 0,$$

$$\left(\frac{p_r}{B^2} - 2\eta\sigma^{11}\right)' + \frac{\dot{q}}{AB} + \frac{q}{AB}\left(\frac{\dot{C}}{C} + \frac{\dot{D}}{D}\right)$$
$$+ \frac{A'}{A}\left(\frac{\rho}{B^2} + \frac{p_r}{B^2} - 2\eta\sigma^{11}\right) + \frac{B'}{B}\left(\frac{p_r}{B^2} - 2\eta\sigma^{11}\right)$$
$$+ \frac{C'}{C}\left(\frac{p_r}{B^2} - \frac{p_\perp}{B^2} - 2\eta\sigma^{11} - 2\eta\sigma^{22}\frac{C^2}{B^2}\right)$$
$$+ \frac{D'}{D}\left(\frac{p_r}{B^2} - \frac{p_\perp}{B^2} - 2\eta\sigma^{11} + 2\eta\sigma^{33}\frac{D^2}{B^2}\right)$$
$$- \frac{ss'}{\pi C^2 D^2 B} = 0. \quad (39)$$

Now following the formulation of Misner and Sharp [2], we introduce the proper time derivative and proper radial derivative as

$$\mathrm{D}_T = \frac{1}{A}\frac{\delta}{\delta t},$$
$$D_R = \frac{1}{R'}\frac{\delta}{\delta r}, \quad (40)$$

so that the fluid velocity in the collapsing situation can be defined as follows [36]:

$$U = D_T(R) = D_T(C) < 0,$$
$$V = D_T(Rr) = D_T(D) < 0. \quad (41)$$

Using (18)–(23) and (41), we can obtain the acceleration of a collapsing matter inside Σ as

$$D_T(U) = -4\pi R\left(p_r - \frac{4\eta\sigma}{\sqrt{3}}\right) + \widetilde{E}\frac{A'}{AB}$$
$$+ \frac{s^2}{R^3}\left(2 + \frac{1}{2\lambda}\right) - \frac{1}{R^2\lambda}\left(E' - \frac{l}{8}\right). \quad (42)$$

Now combining (39) and (42), we obtain

$$\left(\rho + p_r - \frac{4\eta\sigma}{\sqrt{3}}\right)D_T(U) = \left(\rho + p_r - \frac{4\eta\sigma}{\sqrt{3}}\right)$$
$$\cdot\left[\frac{1}{R^2\lambda}\left(E' - \frac{l}{8}\right) + 4\pi R\left(p_r - \frac{4\eta\sigma}{\sqrt{3}}\right)\right.$$
$$\left.- \frac{s^2}{R^3}\left(2 + \frac{1}{2\lambda}\right)\right] - \widetilde{E}^2\left[D_R\left(p_r - \frac{4\eta\sigma}{\sqrt{3}}\right)\right. \quad (43)$$
$$\left.+ \frac{2}{R}\left(p_r - p_\perp - 2\sqrt{3}\eta\sigma\right) - \frac{s}{\pi R^4}D_R(s)\right]$$
$$- \frac{2q\widetilde{E}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) - \frac{\dot{q}\widetilde{E}}{A}.$$

Using (23) and the junction condition $D \overset{\Sigma}{=} \lambda C$, we write [24]

$$\widetilde{E} = \frac{C'}{B} = \left[U^2 + \frac{s^2}{\lambda c^2} - \frac{2}{\lambda c}\left(E' - \frac{l}{8}\right)\right]^{1/2}. \quad (44)$$

Hence, using the field equations for the interior manifold, we obtain the time rate of change of C-energy as

$$D_T E' = -4\pi R^2 \lambda \left[\left(p_r - \frac{4\eta\sigma}{\sqrt{3}} \right) U + q\widetilde{E} \right] + \frac{s^2 \dot{C}}{R^2 A} \left(2\lambda - \frac{1}{2} \right). \tag{45}$$

Also the above equation can be interpreted as the variation of the total energy inside the collapsing cylinder. Note that, due to negativity of the fluid velocity v, the first term on the r.h.s will contribute to the energy of the system, provided that the radial pressure is restricted as $p_r > 4\eta\sigma/\sqrt{3}$. Due to negativity, the second term indicates an outflow of energy in the form of radiation during the collapsing process. The third term is Coulomb-like force term and it will increase the energy of the system provided $\lambda > 1/4$.

Further, using the Einstein field equations (17), (21), and the expression for C- energy in (23), the radial derivative of the C energy takes the following form:

$$D_R E' = 4\pi\rho R^2 \lambda + \frac{s^2}{R^2} \left(2\lambda - \frac{1}{2} \right) + \frac{s}{R} D_R (s) + \frac{4\pi q B R^2 \lambda}{R'} D_T (C) + \frac{1}{8\rho R'}. \tag{46}$$

This radial derivative can be interpreted as the energy variation between the adjacent cylindrical surfaces within the matter distribution. The first term on the r.h.s. is the usual energy density of the fluid element, while the second term and third term are the conditions due to the electromagnetic field. The fourth term represents contribution due to the dissipative heat flux and the last term will increase or decrease the energy of the system during the collapse of the cylinder provided $R' >$ or < 0.

Finally, the collapse dynamics is completely characterized by the equation of motion in (43). Normally, for collapsing situation, $D_T U$ should be negative, that is, indicating an inward radial flow of the system. Consequently, terms on the r.h.s (of (43)) contributing negatively favour the collapse and positive terms oppose the collapsing process. In an extreme situation, the system will be in hydrostatic equilibrium if terms of both signs balance each other. Further, from dimensional analysis, the factor $(\rho + p_r - 4\eta\sigma/\sqrt{3})$ can be considered as an inertial mass density, independent of heat flux contribution. The first term on the r.h.s. of (43) can be identified as the gravitational force, indicating the effects of specific length and electric charge in the gravitational contribution. The second term has three contributing components: the pressure gradient (which is negative), local anisotropy of the fluid, and electromagnetic field term. The remaining terms represent the heat flux contribution and due to negativity they seem to leave the system along the radial outward streamlines.

5. Causal Thermodynamics: The Transport Equation

In causal thermodynamics, due to Miller-Israel-Stewart, the transport equation for heat flow is given by [21]

$$\tau h^{ab} V^c q_{b;c} + q^a = -\kappa h^{ab} \left(T_{,b} + a_b T \right) - \frac{1}{2} \kappa T^2 \left(\frac{\tau V^b}{\kappa T^2} \right)_{;b} q^a, \tag{47}$$

where $h^{ab} = g^{ab} + V^a V^b$ is the projection tensor of the 3-surface orthogonal to the unit time-like vector V^a, κ represents the thermal conductivity, T is the temperature, τ denotes the relaxation time, and $a_b T$ is the inertial term due to Tolman. Now, due to cylindrical symmetry, the above transport equation (47) simplifies to

$$\tau \dot{q} = -\frac{1}{2A} \kappa \frac{q T^2}{\tau} \left(\frac{\tau}{\kappa T^2} \right) - q \left[\frac{3U}{2R} + G + \frac{1}{\tau} \right] - \frac{\kappa \widetilde{E} D_R T}{\tau} - \frac{\kappa T D_T U}{\tau \widetilde{E}} - \frac{\kappa T}{\tau \widetilde{E} R^2} \left[\frac{1}{\lambda} \left(E' - \frac{l}{8} \right) + 4\pi R^3 \left(p_r - \frac{4\eta\sigma}{\sqrt{3}} \right) - \frac{S^2}{R} \left(2 + \frac{1}{2\lambda} \right) \right] \tag{48}$$

with $G = (1/A)(\dot{B}/B - \dot{C}/C)$.

Note that although heat dissipation is described above through a physically reasonable transport equation (47) or (48), the shear viscosity is described according to the standard (noncausal) irreversible thermodynamics. Thus, there is no such relaxation time for shear viscosity and as such no causal evolution equation for it. Further, the thermodynamic viscous/heat coupling coefficients are not taken into consideration in the present work; that is, the present approach is only partially causal. However, a full causal approach of the dissipative collapse has been considered by Herrera et al. [22].

Now considering proper derivatives in (40) of the above equation and using the field velocity (in (41)) and equation of motion (i.e., (43)), one obtains the effects of heat flux or dissipation in the collapsing process as

$$(1-\alpha) \left(\rho + p_r - \frac{4\eta\sigma}{\sqrt{3}} \right) D_T U = (1-\alpha) F_{\text{grav}} + F_{\text{hyd}} + \alpha \widetilde{E}^2 \left[D_R p_r + 2 \left(p_r - p_\perp - 2\sqrt{3}\eta\sigma \right) \frac{1}{R} - \frac{S D_R (S)}{\pi R^4 \lambda^2} \right] - \widetilde{E} \left[D_T q + 2qG + \frac{4qU}{R} \right] + \alpha \widetilde{E} \left[D_T q + \frac{4qU}{R} + 2qG \right] \tag{49}$$

with

$$\alpha = \frac{\kappa T}{\tau} \left(\rho + p_r - \frac{4\eta\sigma}{\sqrt{3}} \right)^{-1}$$

$$F_{\text{grav}} = -\left(\rho + p_r - \frac{4\eta\sigma}{\sqrt{3}} \right) \left[\left(E' - \frac{l}{8} \right) \frac{1}{\lambda} + 4\pi p_r R^3 \right.$$

$$
\left. - \left(2 + \frac{1}{2\lambda}\right)\frac{S^2}{R} \right] \left(\frac{1}{R^2}\right)
$$

$$
\begin{aligned}
F_{\text{hyd}} = {} & \widetilde{E}^2 \left[D_R \left(p_r - \frac{4\eta\sigma}{\sqrt{3}} \right) + \frac{2}{R} \left(p_r - p_\perp - 2\sqrt{3}\eta\sigma \right) \right. \\
& \left. - \frac{S}{\pi R^4} D_R (S) \right].
\end{aligned} \tag{50}
$$

The l.h.s. of (49) can be interpreted as Newtonian force F with $(\rho + p_r)(1 - \alpha)$ as the inertial mass density. So as $\alpha \rightarrow 1$, $F \rightarrow 0$; that is, there is no inertial force and collapse will be inevitable due to gravitational attraction. Further, the inertial mass density decreases as long as $0 < \alpha < 1$ and it increases for $\alpha > 1$. Moreover, due to equivalence principle, the gravitational mass also decreases or increases according to $\alpha <$ or > 1 and gives a clear distinction between the expanding and collapsing process due to dynamics of dissipative system. Note that although the gravitational force is affected by the same factor $(1 - \alpha)$, the hydrodynamical force is free from it. Further, combination of all these terms on the r.h.s of (49) results in the l.h.s, that is, $(1 - \alpha)(\rho + p_r - 4\pi\sigma/\sqrt{3})D_T U < 0$; there will be gravitational collapse, while there will be expansion if the l.h.s is to be positive. It should be mentioned that such effects were first discussed by Herrera et al. [37] in context of thermal conduction in hydrostatic equilibrium. Interestingly, if α continuously decreases from a value larger than unity to one less than unity, then there will be a phase transition (collapse to expansion) and bounce will occur. As a result, there will be loss of energy of the system and collapsing cylinder with nonadiabatic source causes emission of gravitational radiation. Therefore, there will be radiation outside the collapsing cylinder and hence the choice of the exterior metric (in (24)) is justified.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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