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*Research Article*

# $q$-Deformed Relativistic Fermion Scattering

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In this article, after introducing a kind of $q$-deformation in quantum mechanics, first, $q$-deformed form of Dirac equation in relativistic quantum mechanics is derived. Then, three important scattering problems in physics are studied. All results have satisfied what we had expected before. Furthermore, effects of all parameters in the problems on the reflection and transmission coefficients are calculated and shown graphically.

## 1. Introduction

$q$-Deformation for quantum group and physical system has been one of the remarkable and interesting issues of studies such as conformal quantum mechanics [1], nuclear and high energy physics [2–4], cosmic string and black holes [5], and fractional quantum Hall effect [6]. Applications of $q$-deformation emerged in physics and chemistry after introducing $q$-deformed harmonic oscillator [7, 8] such as investigation of electronic conductance in disordered metals and doped semiconductors [9], analyzing of the phonon spectrum in $^4$He [10], and expressing of the oscillatory-rotational spectra of diatomic and multiatomic molecules [11, 12]. Basically, $q$-calculus was established for the first time by Jackson [13] and then Arik and Coon used it. Arik and Coon studied generalized coherent states that are associated with generalization of the harmonic oscillator commutation relation [14]. They utilized

$$\begin{aligned} aa^\dagger - qa^\dagger a &= 1, \\ \left[N, a^\dagger\right] &= a^\dagger, \\ [N, a] &= -a, \end{aligned} \tag{1}$$

where the relation between number operator and step operators is given by

$$a^\dagger a = [N]_q, \tag{2}$$

where a $q$-number is defined as

$$[X]_q = \frac{1 - q^X}{1 - q}. \tag{3}$$

Another $q$-deformation exists that has been introduced by Tsallis [15] and has a different algebraic structure from Jackson's. For Tsallis's case, the $q$-derivative and $q$-integral were given by Borges [16].

In what follows, Section 2 is devoted to the introduction to the kind of $q$-deformation of quantum mechanics which will be used in the next sections. In Section 3, $q$-deformed version of Dirac equation is derived. As first relativistic scattering problem in $q$-deformed version of relativistic quantum mechanics, scattering from a Dirac delta potential is done in Section 4. Section 5 is devoted to the extended form of problem in Section 4, scattering problem from a double Dirac delta potential. At last, Ramsauer-Townsend effect is studied in considered formalism of quantum mechanics.

## 2. $q$-Deformed Quantum Mechanics

In this section, we want to introduce postulates of $q$-deformed quantum mechanics to use for the next sections. In this formalism of quantum mechanics, we deal with the following:

(1) In this formalism of quantum mechanics like the ordinary one, time-dependent form of Schrödinger equation in $q$-deformed quantum mechanics is written in form of

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = H(\hat{x},\hat{p})\psi(x,t) = \left(\frac{p^2}{2m}+V(\hat{x})\right)\psi(x,t), \tag{4}$$

in which we deal with the operators as

$$\hat{p} = -i\hbar D_x = -i\hbar\left(1+qx^2\right)\frac{d}{dx} \quad \hat{x} = x, \tag{5}$$

where $q$ is a positive constant and the wave function is $\psi(x,t)$.

(2) Inner product of Hilbert space in one-dimensional $q$-deformed quantum mechanics can be written as

$$\langle f \mid g\rangle = \int_{-\infty}^{\infty} g^*(x) f(x)\, d_q x, \quad d_q x = \frac{dx}{(1+qx^2)}. \tag{6}$$

(3) Expectation value of an operator $\hat{O}$ regarding the wave function $\psi(x,t)$ is given by

$$\langle\hat{O}\rangle = \langle\psi \mid \hat{O}\psi\rangle = \int_{-\infty}^{\infty}\psi^*(x,t)\hat{O}\psi(x,t)\, d_q x, \tag{7}$$

and also we have Hermitian definition for the operator if we get

$$\langle\psi \mid \hat{O}\psi\rangle = \langle\hat{O}\psi \mid \psi\rangle. \tag{8}$$

It should be noted that the deformation is considered only for the coordinate part; then, the time part has no deformation. This point can be checked in the first postulate.

In this formalism of quantum mechanics, commutation relation between coordinate and its momentum should be deformed in form of

$$[\hat{x},\hat{p}] = i\hbar\left(1+q\hat{x}^2\right). \tag{9}$$

Considering operator form of coordinate and momentum

$$\hat{x} \longleftrightarrow x, \qquad \hat{p} \longleftrightarrow -i\hbar D_x, \tag{10}$$

we can rewrite (4) in terms of the operators

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = \left(\frac{-\hbar^2}{2m}D_x^2+V(x)\right)\psi(x,t), \tag{11}$$

and to obtain time-independent form of Schrödinger equation in this formalism, we set $\psi(x,t)=e^{(-i/\hbar)Et}\phi(x)$; then, we have

$$\left(\frac{-\hbar^2}{2m}D_x^2+V(x)\right)\phi(x) = E\phi(x). \tag{12}$$

Using (11), we can easily find continuity relation in this formalism of quantum mechanics as

$$\frac{\partial\rho(x,t)}{\partial t} + D_x j(x,t) = 0, \tag{13}$$

where

$$\rho(x,t) = \psi^*(x,t)\psi(x,t), \qquad j(x,t) = \frac{\hbar}{2mi}\left(\psi^*(x,t)D_x\psi(x,t) - \psi(x,t)D_x\psi^*(x,t)\right). \tag{14}$$

By these considerations, we are in a position to study relativistic scattering of fermions in $q$-deformed relativistic quantum mechanics.

## 3. Scattering of Relativistic Fermions in $q$-Deformed Quantum Mechanics

In this section, we want to study scattering of fermions in $q$-deformed formalism of relativistic quantum mechanics. Study of fermions can be done by Dirac equation. This can be written as $\hbar=c=1$ [17]:

$$i\frac{\partial\Psi(x,t)}{\partial t} = \left(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta(m+S(x))+V(x)\right)\Psi(x,t), \tag{15}$$

in which the matrices are

$$\boldsymbol{\alpha} = \begin{pmatrix}0 & \boldsymbol{\sigma}\\ \boldsymbol{\sigma} & 0\end{pmatrix}, \qquad \beta = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{16}$$

where $\sigma$ stands for Pauli matrices. We have considered $x$ direction as interaction direction. To obtain stationary states, we choose the wave function as

$$\Psi(x,t) = e^{-iEt}\Phi(x) = e^{-iEt}\begin{pmatrix}\Phi_u(x)\\ \Phi_d(x)\end{pmatrix}; \tag{17}$$

also, we would like to consider $S(x)=V(x)$ for simplicity. These assumptions give us a system of equation like

$$(m+2V(x)-E)\Phi_u(x) + \sigma_x\left(-i\left(1+qx^2\right)\frac{d\Phi_d(x)}{dx}\right) = 0, \tag{18}$$

$$\sigma_x\left(-i\left(1+qx^2\right)\frac{d\Phi_u(x)}{dx}\right) - (E+m)\Phi_d(x) = 0. \tag{19}$$

From (19), we find that

$$\Phi_d(x) = \sigma_x \frac{-i\left(1+qx^2\right)}{E+m}\frac{d\Phi_u(x)}{dx}, \tag{20}$$

If one substitutes (20) into (18), one easily can derive

$$\begin{aligned}&\left(1+qx^2\right)^2 \frac{d^2\Phi_u(x)}{dx^2} + 2qx\left(1+qx^2\right)\frac{d\Phi_u(x)}{dx}\\&\quad + \left(p^2 - 2(E+m)V(x)\right)\Phi_u(x) = 0,\\&p^2 = E^2 - m^2.\end{aligned} \tag{21}$$

In the next sections, we will study three important and famous types of scattering.

## 4. Scattering due to Single Dirac Delta Potential

As first scattering study, we want to consider single Dirac delta potential as

$$V(x) = V_1\delta(x-a_1), \tag{22}$$

where $V_1$ and $a_1$ are real constants. This point can be derived that this potential produces a discontinuity for the first derivative of wave function as

$$\begin{aligned}&\frac{d\Phi_u\ (x=a_1^+)}{dx} - \frac{d\Phi_u\ (x=a_1^-)}{dx}\\&\quad = \frac{2(m+E)}{\left(1+qa_1^2\right)^2}\Phi_u\ (x=a_1).\end{aligned} \tag{23}$$

We assume that particles come from $x < a_1$; then, because of Dirac delta existence, they scatter. Consequently, some of them are reflected to region I ($x < a_1$) and the others are transmitted to region II ($x > a_1$). According to this assumption, we can find wave functions of the regions as

$$\begin{aligned}\Phi_{u,\mathrm{I}}(x) &= e^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}x))} + re^{-ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}x))},\\ \Phi_{u,\mathrm{II}}(x) &= te^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}x))}.\end{aligned} \tag{24}$$

The coefficients $r_1$ and $t_1$ can be determined by using boundary condition of continuity and discontinuity of wave functions at $x = a_1$. These are

$$\begin{aligned}&e^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}a_1))} + re^{-ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}a_1))}\\&\quad = te^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}a_1))},\\&e^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}a_1))}(t-1) + re^{-ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}a_1))}\\&\quad = \frac{2(m+E)V_1}{\left(1+qa_1^2\right)ip}te^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}a_1))},\end{aligned} \tag{25}$$

whereby, solving them, we can find that

$$\begin{aligned}r_1 &= -\frac{V(e+m)e^{2ip\tan^{-1}(a_1\sqrt{q})/\sqrt{q}}}{V(e+m) - ip\left(a_1^2q+1\right)},\\ t_1 &= \frac{a_1^2pq + p}{a_1^2pq + iV(e+m) + p}.\end{aligned} \tag{26}$$

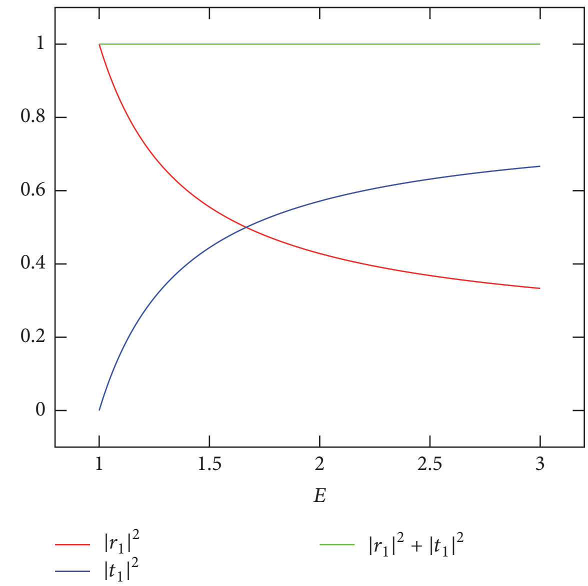



Figure 1: Plots of $|r_1|^2$, $|t_1|^2$, and $|r_1|^2 + |t_1|^2$ as energy varies.

On the other hand, current density of fermions can be derived by $j = \Psi^\dagger\alpha\Psi$. Because there is no sink or source, we have from current density that

$$\left|r_1\right|^2 + \left|t_1\right|^2 = 1. \tag{27}$$

We plot this equation considering $V_1 = 2$, $q = 3$, $a_1 = 1$, and $m = 1$ in Figure 1.

For further study about effects of different parameters on the reflection and transmission coefficients, we have plotted Figure 2 in which readers can see how different parameters in our system can affect these confinements.

As it can be seen, there are no fluctuations in the figures and each curve has a smooth treatment. But in the next section we will deal with interesting results. Because kind of scatter potential makes interesting results resemble one of the most important and famous effects in physics.

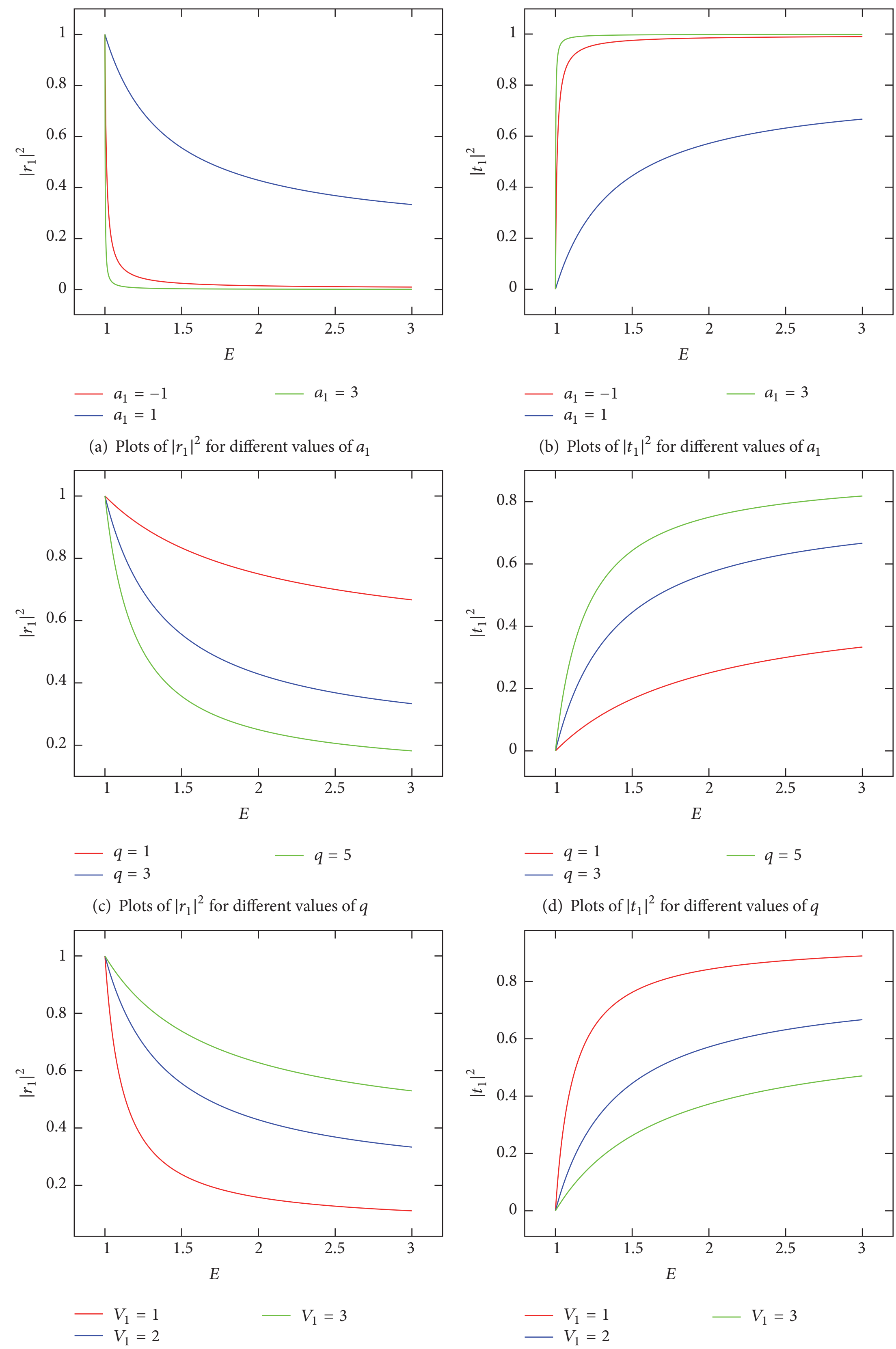
## 5. Scattering from Double Dirac Delta Potential

In this section, we suppose that particles are scattered from a double Dirac delta potential in form of

$$V(x) = V_2\left(\delta(x+a_2) + \delta(x-a_2)\right), \tag{28}$$

where $V_2$ and $a_2$ are real constants. From the previous section, we know that this kind of potential makes discontinuity for derivative of wave functions at $x = a_2$ and $x = -a_2$. They are

$$\begin{aligned}&\frac{d\Phi_u\ (x=a_2^+)}{dx} - \frac{d\Phi_u\ (x=a_2^-)}{dx}\\&\quad = \frac{2(m+E)}{\left(1+qa_2^2\right)^2}\Phi_u\ (x=a_2), \quad x = a_2,\end{aligned}$$




(a) Plots of $|r_1|^2$ for different values of $a_1$

(b) Plots of $|t_1|^2$ for different values of $a_1$

(c) Plots of $|r_1|^2$ for different values of $q$

(d) Plots of $|t_1|^2$ for different values of $q$

(e) Plots of $|r_1|^2$ for different values of $V_1$

(f) Plots of $|t_1|^2$ for different values of $V_1$

FIGURE 2: In this figure, different treatments of $|t_1|^2$ and $|r_1|^2$ as parameters $a_1$, $q$, and $V_1$ vary have been plotted. We set the parameters in (a) and (b) $V_1 = 2$, $q = 3$, (c) and (d) $V_1 = 2$, $a_1 = 1$, and (e) and (f) $q = 3$, $a_1 = 1$.

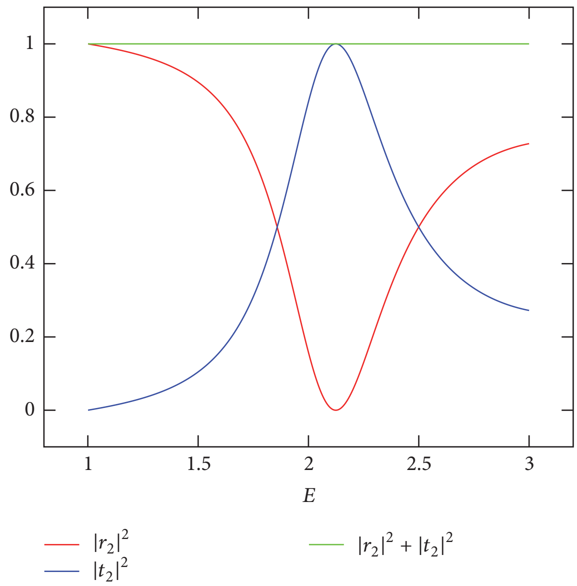



FIGURE 3: Plots of $|r_2|^2$, $|t_2|^2$, and $|r_2|^2 + |t_2|^2$ as energy varies.

$$\frac{d\Phi_u\left(x=-a_2^+\right)}{dx}-\frac{d\Phi_u\left(x=-a_2^-\right)}{dx}=\frac{2\left(m+E\right)}{\left(1+qa_2^2\right)^2}\Phi_u\left(x=-a_2\right),\quad x=-a_2. \tag{29}$$

Similar to the previous section, we suppose that particles come from region I ($x < -a_2$). Then, they are scattered in region II ($-a_2 < x < a_2$). So some of them will be reflected into region I and the others will be transmitted into region III ($x > a_2$). According to this assumption, we have wave functions

$$\begin{aligned}\Phi_{u,\mathrm{I}}(x)&=e^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}x))}+re^{-ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}x))},\\ \Phi_{u,\mathrm{II}}(x)&=Ae^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}x))}+Be^{-ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}x))},\\ \Phi_{u,\mathrm{III}}(x)&=te^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}x))},\end{aligned} \tag{30}$$

where the coefficients are constant which can be determined from continuity and discontinuity condition at $x = a_2$ and $x = -a_2$. Using these conditions leads to the system of four equations:

$$\begin{aligned}&e^{ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_2))}+re^{-ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_2))}\\&\quad=Ae^{ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_2))}+Be^{-ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_2))},\\&e^{ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_2))}\left(A-1\right)+e^{-ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_2))}\left(r\right.\\&\quad\left.-B\right)=\frac{2V_2\left(E+m\right)}{ip\left(1+qa_2^2\right)}\left(e^{ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_2))}\right.\\&\quad\left.+re^{-ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_2))}\right),\\&\qquad x=-a_2,\\&Ae^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}a_2))}+Be^{-ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}a_2))}\\&\quad=te^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}a_2))},\\&e^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}a_2))}\left(t-A\right)+Be^{-ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}a_2))}\\&\quad=\frac{2V_2\left(E+m\right)}{ip\left(1+qa_2^2\right)}\left(te^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}a_2))}\right),\\&\qquad x=a_2.\end{aligned} \tag{31}$$

Solving this system of equations for the constants of wave functions, we obtain

$$\begin{aligned}r_2&=-\frac{V\left(e+m\right)e^{-2ip\tan^{-1}(a_2\sqrt{q})/\sqrt{q}}\left(e^{4ip\tan^{-1}(a_2\sqrt{q})/\sqrt{q}}\left(V\left(e+m\right)+ip\left(a_2^2q+1\right)\right)+i\left(a_2^2pq+imV+p\right)-eV\right)}{\left(a_2^2pq+iV\left(e+m\right)+p\right)^2+V^2\left(e+m\right)^2e^{4ip\tan^{-1}(a_2\sqrt{q})/\sqrt{q}}},\\ t_2&=\frac{\left(a_2^2pq+p\right)^2}{\left(a_2^2pq+iV\left(e+m\right)+p\right)^2+V^2\left(e+m\right)^2e^{4ip\tan^{-1}(a_2\sqrt{q})/\sqrt{q}}}.\end{aligned} \tag{32}$$

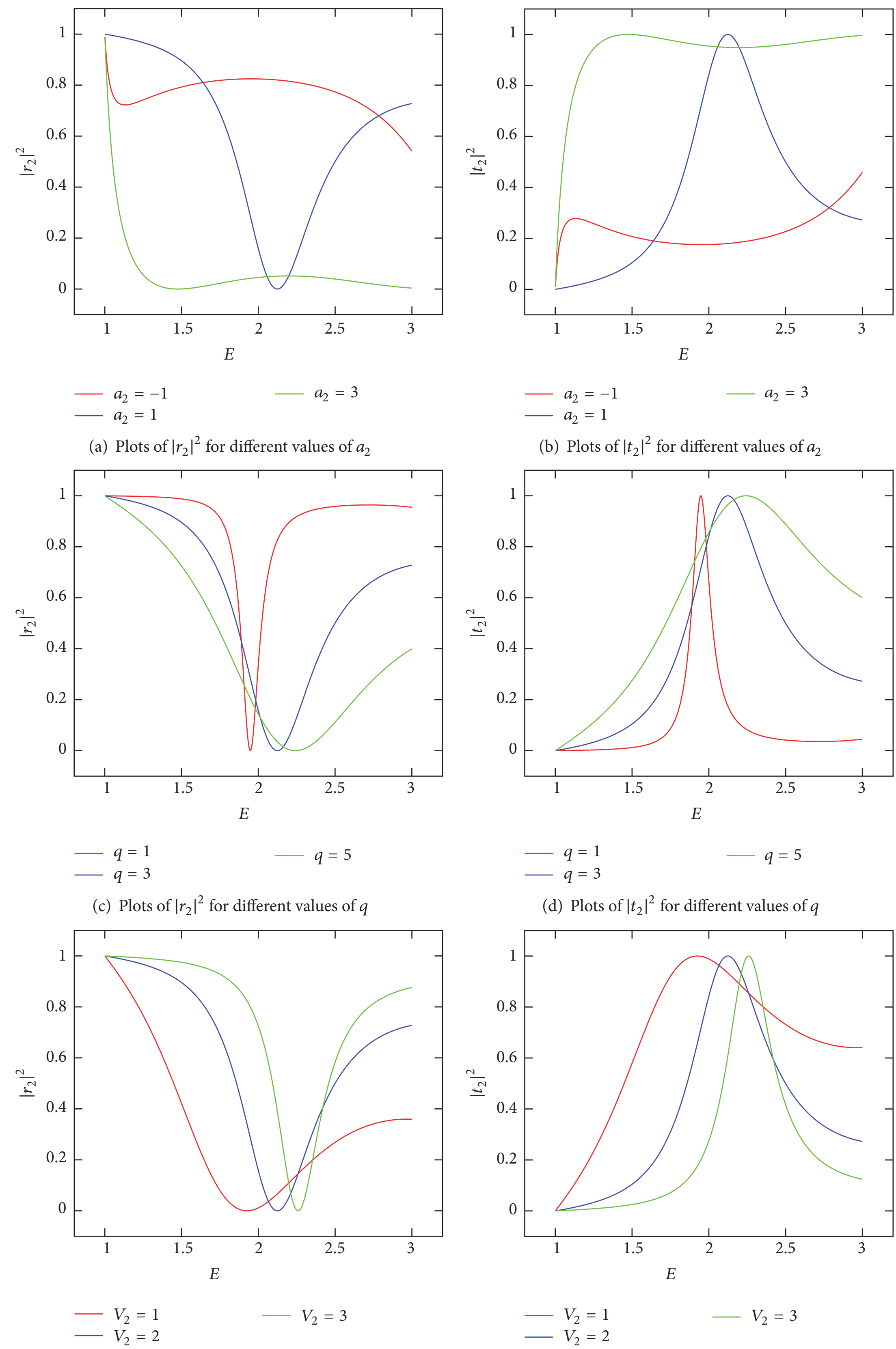
Using the similar manner of previous section, we have found the constraint

$$\left|r_2\right|^2+\left|t_2\right|^2=1. \tag{33}$$

By plotting this equation using (31), we can check validity of it. This point can be seen in Figure 3.

It is instructive if we check treatments of reflection and transmission coefficients as different parameters in problem vary. This one is done and plotted in Figure 4.

The main difference between this section and the previous section is shown in the results. In this section, by considering a double Dirac potential as scatter potential, we find out some fluctuation in reflection and transmission coefficients; however, in the previous section, we dealt with a smooth treatment. On the other hand, such fluctuations remind us of one of the most famous and important effect in physics, Ramsauer-Townsend effect. In the next section, we will investigate this effect in $q$-deformed relativistic version.




(a) Plots of $|r_2|^2$ for different values of $a_2$

(b) Plots of $|t_2|^2$ for different values of $a_2$

(c) Plots of $|r_2|^2$ for different values of $q$

(d) Plots of $|t_2|^2$ for different values of $q$

(e) Plots of $|r_2|^2$ for different values of $V_2$

(f) Plots of $|t_2|^2$ for different values of $V_2$

FIGURE 4: In this figure, different treatments of $|t_2|^2$ and $|r_2|^2$ as parameters $a_2$, $q$, and $V_2$ vary have been plotted. We set the parameters in (a) and (b) $V_2 = 2$, $q = 3$, (c) and (d) $V_2 = 2$, $a_2 = 1$, and (e) and (f) $q = 3$, $a_2 = 1$.

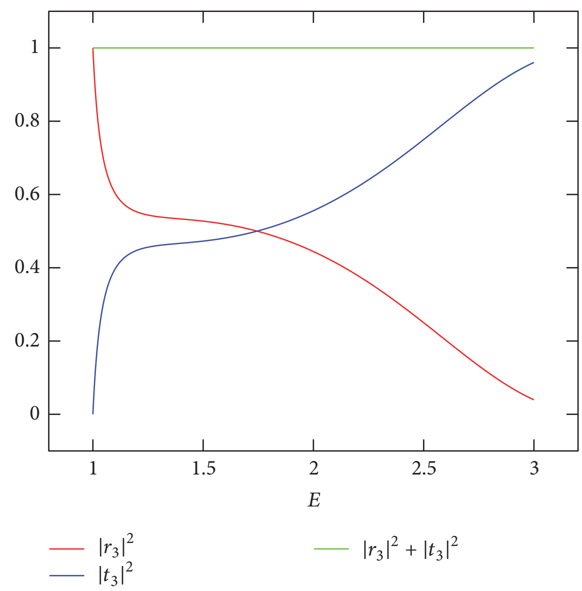



FIGURE 5: Plots of $|r_3|^2$, $|t_3|^2$, and $|r_3|^2 + |t_3|^2$ as energy varies.

## 6. Ramsauer-Townsend Effect in $q$-Deformed Relativistic Quantum Mechanics

Ramsauer-Townsend effect is a face of electron scattering. This scattering due to a simple potential well. Actually when an electron is moving through noble gas such as Xenon with low energy (like 0.1 eV) something strange happens. There is an anomalously large transmission in this scattering [1]. Importance of this simple scattering is that this effect can only be decried by quantum mechanics. Now, in considered formalism of relativistic quantum mechanics, we want to study Ramsauer-Townsend effect. Considering a potential well such that

$$\begin{aligned} -V &\quad \text{for } -a_3 < x < a_3, \\ 0 &\quad \text{for elsewhere,} \end{aligned} \tag{34}$$

and following previous assumptions, we can derive wave functions of our three regions in the problem as follows:

$$\begin{aligned} \Phi_{u,\mathrm{I}}(x) &= e^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}x))} + r_3 e^{-ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}x))}, \\ \Phi_{u,\mathrm{II}}(x) &= A_3 e^{i\eta((1/\sqrt{q})\tan^{-1}(\sqrt{q}x))} \\ &\quad + B_3 e^{-i\eta((1/\sqrt{q})\tan^{-1}(\sqrt{q}x))}, \\ \Phi_{u,\mathrm{III}}(x) &= t_3 e^{ip((1/\sqrt{q})\tan^{-1}(\sqrt{q}x))}, \\ \eta^2 &= p^2 + 2(E+m)V, \end{aligned} \tag{35}$$

in which the coefficients can be given by using continuity conditions of wave functions and their derivatives at $x = -a_3$ and $x = a_3$. Using these conditions results in

$$\begin{aligned} & e^{ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))} + r_3 e^{-ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))} \\ &\quad = A_3 e^{i\eta((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))} \\ &\quad + B_3 e^{-i\eta((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))}, \\ & p\left(e^{ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))} - r_3 e^{-ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))}\right) \\ &\quad = \eta\left(A_3 e^{i\eta((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))}\right. \\ &\quad \left. - B_3 e^{-i\eta((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))}\right), \\ &\qquad\qquad x = a_3, \\ & A_3 e^{i\eta((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))} + B_3 e^{-i\eta((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))} \\ &\quad = t_3 e^{ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))}, \\ & \eta\left(A_3 e^{i\eta((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))} - B_3 e^{-i\eta((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))}\right) \\ &\quad = p t_3 e^{ip((1/\sqrt{q})\tan^{-1}(-\sqrt{q}a_3))}, \\ &\qquad\qquad x = a_3. \end{aligned} \tag{36}$$

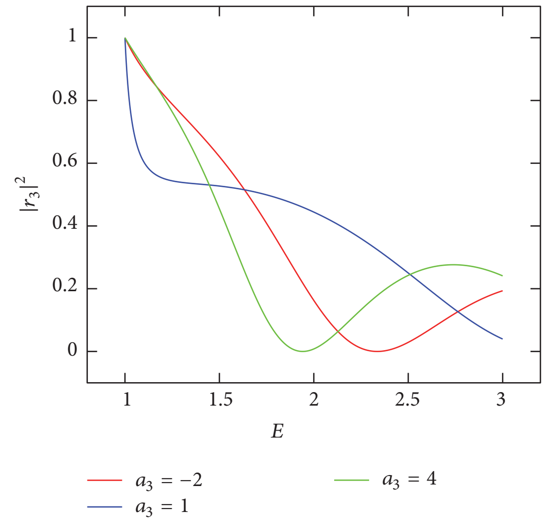
From this system of equation, we can determine the coefficients as

$$\begin{aligned} r_3 &= \frac{(p-\eta)(\eta+p)\, e^{-2ip\tan^{-1}(a_3\sqrt{q})/\sqrt{q}}\left(-1 + e^{4i\eta\tan^{-1}(a_3\sqrt{q})/\sqrt{q}}\right)}{p^2 e^{4i\eta\tan^{-1}(a_3\sqrt{q})/\sqrt{q}} - 2\eta p e^{4i\eta\tan^{-1}(a_3\sqrt{q})/\sqrt{q}} + \eta^2 e^{4i\eta\tan^{-1}(a_3\sqrt{q})/\sqrt{q}} - \eta^2 - p^2 - 2\eta p}, \\ t_3 &= -\frac{4\eta p e^{(2i\eta\tan^{-1}(a_3\sqrt{q})/\sqrt{q} - 2ip\tan^{-1}(a_3\sqrt{q})/\sqrt{q})}}{p^2 e^{4i\eta\tan^{-1}(a_3\sqrt{q})/\sqrt{q}} - 2\eta p e^{4i\eta\tan^{-1}(a_3\sqrt{q})/\sqrt{q}} + \eta^2 e^{4i\eta\tan^{-1}(a_3\sqrt{q})/\sqrt{q}} - \eta^2 - p^2 - 2\eta p}. \end{aligned} \tag{37}$$

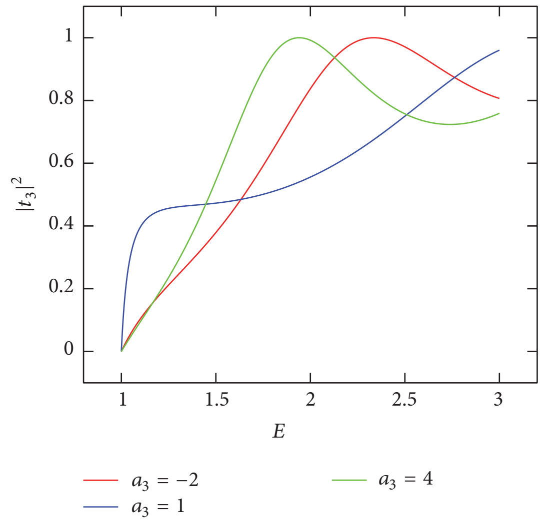
Like previous sections, by using definition of current density, we can find out that the constraint

$$|r_2|^2 + |t_2|^2 = 1 \tag{38}$$

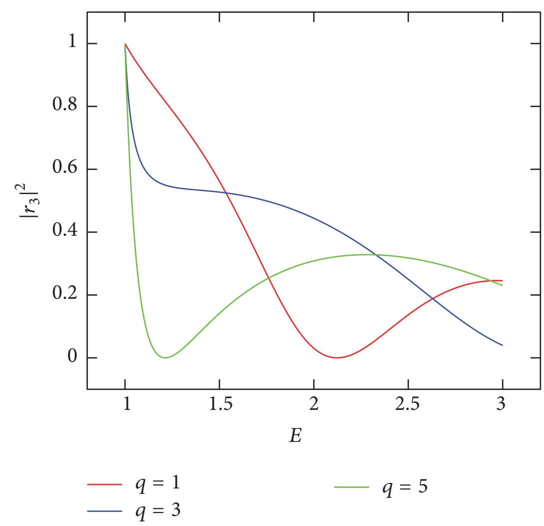
is governed here. By solving (36) and plotting (38) in Figure 6, we have Figure 5 which is similar to what we faced in the previous sections. Furthermore, treatments of reflection and transmission coefficients in terms of different parameters are plotted in Figure 6.



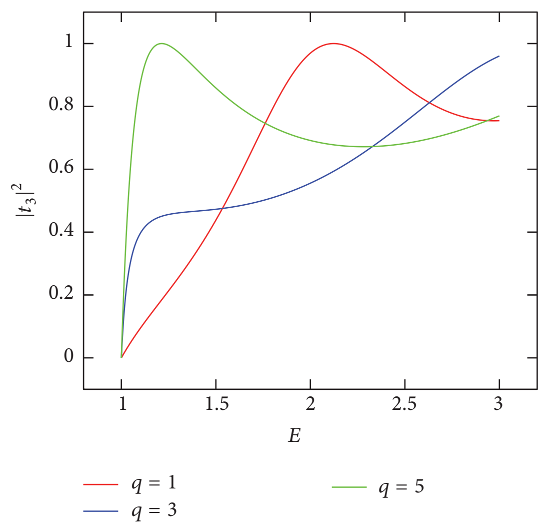

(a) Plots of $|r_3|^2$ for different values of $a_3$



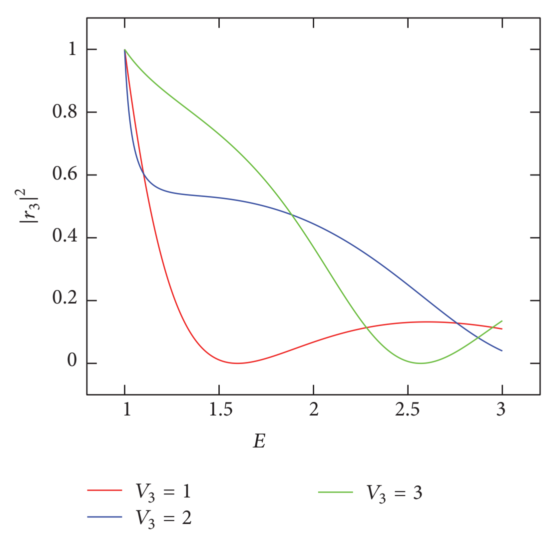

(b) Plots of $|t_3|^2$ for different values of $a_3$



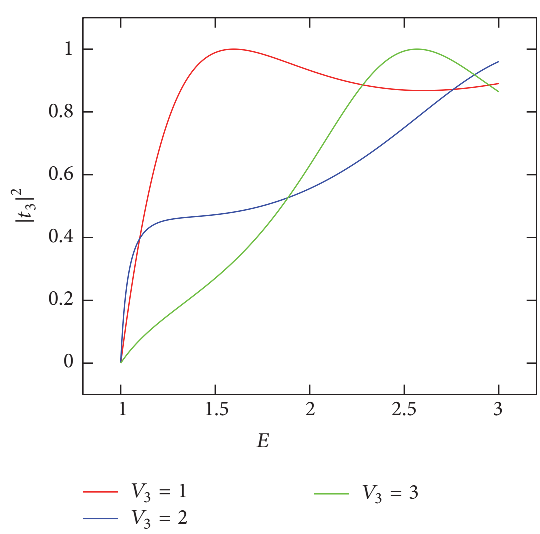

(c) Plots of $|r_3|^2$ for different values of $q$




(d) Plots of $|t_3|^2$ for different values of $q$




(e) Plots of $|r_3|^2$ for different values of $V_3$




(f) Plots of $|t_3|^2$ for different values of $V_3$

FIGURE 6: In this figure, different treatments of $|t_3|^2$ and $|r_3|^2$ as parameters $a_3$, $q$, and $V_3$ vary have been plotted. We set the parameters in (a) and (b) $V_3 = 2$, $q = 3$, (c) and (d) $V_3 = 2$, $a_3 = 1$, and (e) and (f) $q = 3$, $a_3 = 1$.

## 7. Conclusions

In this article, we introduced a $q$-deformation of quantum mechanics. Then, in such formalism of quantum mechanics, we studied three important and famous scattering problems in relativistic region. We first rewrote Dirac equation in $q$-deformation; then, as first case, scattering due to a single Dirac delta potential was studied. In this case, we dealt with smooth treatments in the reflection and transmission coefficients. In the next case, a double Dirac delta potential was considered. In this case, we saw that there was some fluctuation in reflection and transmission coefficients which were similar to Ramsauer-Townsend effect. To check this point, we also investigated this effect in relativistic region. By plotting the coefficients, we found out that effect of scattering from a potential well in $q$-deformed version of relativistic quantum mechanics could be simulated by considering double Dirac delta potential.

## Competing Interests

The authors declare that they have no competing interests.

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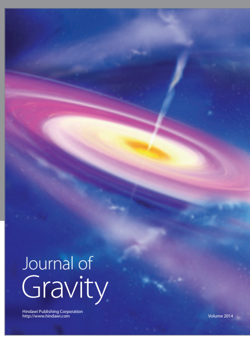

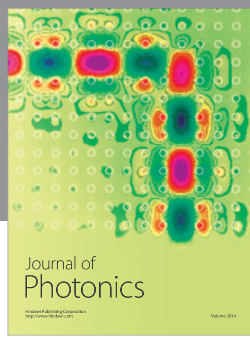

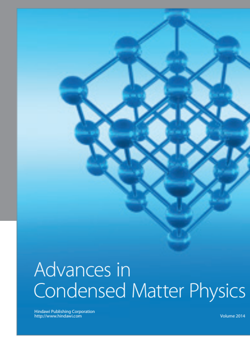

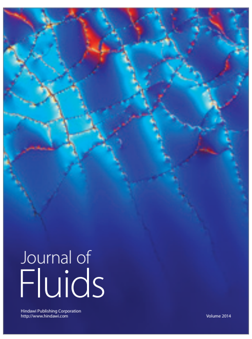


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