

## Research Article

# Thermodynamics of the FRW Universe at the Event Horizon in Palatini $f(R)$ Gravity

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Received 21 July 2018; Revised 18 September 2018; Accepted 2 October 2018; Published 8 November 2018

Academic Editor: Elias C. Vagenas

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In an accelerated expanding universe, one can expect the existence of an event horizon. It may be interesting to study the thermodynamics of the Friedmann-Robertson-Walker (FRW) universe at the event horizon. Considering the usual Hawking temperature, the first law of thermodynamics does not hold on the event horizon. To satisfy the first law of thermodynamics, it is necessary to redefine Hawking temperature. In this paper, using the redefinition of Hawking temperature and applying the first law of thermodynamics on the event horizon, the Friedmann equations are obtained in  $f(R)$  gravity from the viewpoint of Palatini formalism. In addition, the generalized second law (GSL) of thermodynamics, as a measure of the validity of the theory, is investigated.

## 1. Introduction

The existence of a deep connection between gravity and thermodynamics is one of the greatest discoveries in theoretical physics [1–5]. Based on the Hawking proposal, a Schwarzschild black hole emits a thermal radiation. For a black hole with mass  $M$ , the Hawking temperature is given by  $T = 1/8\pi M$  [4, 5]. Moreover, the black hole entropy is given by  $S = A/4l_p^2$ , which is introduced by Bekenstein. The parameter  $A$  is the area of the black hole horizon and  $l_p$  is the Planck length. The black hole mass, temperature, and the horizon entropy obey the first law of thermodynamics. Hence, one can conclude that a black hole can be assumed as a thermodynamical object [6–8]. In 1995, Jacobson has considered the relation between the horizon area and the entropy and derived the Einstein field equations from the first law of thermodynamics [9]. Following Jacobson, the relation between the thermodynamics and the field equations has been investigated in modified theories of gravity [10–15]. The thermodynamical interpretation of gravity is also important in modern cosmology. It is possible to derive the Friedmann equations from the first law of thermodynamics and vice versa [16–18]. The deep connection between the

first law of thermodynamics and Friedmann equations has been investigated in gravity with Gauss-Bonnet term [19], Lovelock gravity theory [19], the braneworld scenarios [20], rainbow gravity [21], and scalar-tensor gravity and  $f(R)$  gravity [22].

The thermodynamics of the FRW universe bounded by the apparent horizon has been studied by many authors [16–18]. However, astronomical observations show that the universe is currently in an accelerated expanding phase [23]. In an accelerated expanding universe, there may exist an event horizon [24, 25]. Certainly, it is necessary to investigate the thermodynamics on the event horizon. Through the researches about the thermodynamics of the universe, Wang et al. have found that the laws of thermodynamics do not hold on the event horizon. Therefore, one may conclude that the event horizon is unphysical [24]. Chacrabarty has proposed a redefinition of Hawking temperature to obtain the universe bounded by the event horizon as a Bekenstein system [26]. Using the Chacrabarty redefinition of Hawking temperature, Tu and Chen have considered a universe dominated by tachyon fluid and obtained a good thermodynamical description on the event horizon [27]. However, the temperature redefinition proposed by Chacrabarty can be

applied only in a flat universe. Tu and Chen have introduced a new redefinition of the temperature in a universe with an arbitrary spatial curvature [25]. They have investigated the thermodynamics on the event horizon in metric  $f(R)$  gravity. Utilizing the redefinition of the temperature, it is possible to study the thermodynamics of the event horizon in Palatini  $f(R)$  gravity.

In this paper, we are going to apply Hawking temperature redefinition introduced by Tu and Chen in [25] to investigate the event horizon thermodynamics from the viewpoint of Palatini  $f(R)$  gravity. Redefining the gravitational energy density and the gravitational pressure density, the continuity equation has been satisfied. Then, we have considered the new gravitational source accompanying with the ordinary matter as a modified energy-momentum source in the context of general relativity. Applying the first law of thermodynamics on the event horizon and using the usual entropy-area relation, we have derived the Friedmann equations the same as the ones obtained via other approaches. Since the first law of thermodynamics is reasonably applicable on the event horizon, one can consider the event horizon as a physical system with an equilibrium thermodynamics in Palatini  $f(R)$  gravity. Moreover, the equivalency between the first law of thermodynamics and Friedmann equations has been appeared. Finally, the generalized second law of thermodynamics (GSL) has been studied. The generalized second law of thermodynamics can be satisfied by choosing suitable  $f(R)$  functions.

## 2. Hawking Temperature Redefinition on the Event Horizon

Considering a homogeneous and isotropic FRW universe, one can write the metric as follows:

$$ds^2 = h_{ij}dx^i dx^j + R^2 d\Omega_2^2, \quad (1)$$

where  $h_{ij}$  is the two-dimensional metric

$$h_{ij} = \text{diag} \left( -1, \frac{a^2(t)}{1 - Kr^2} \right). \quad (2)$$

The parameters  $i$  and  $j$  take the values 0 and 1. The parameter  $K$  is the spatial curvature constant, the parameter  $R = a(t)r$  is the area radius, and  $a(t)$  is the scale factor. Using  $R$  as a scalar field in the normal 2-dimensional space, one can define a scalar quantity

$$\chi = h^{ij} \partial_i R \partial_j R. \quad (3)$$

The apparent horizon can be found via  $\chi = 0$  as

$$R_A = \frac{1}{\sqrt{H^2 + K/a^2}}, \quad (4)$$

where  $H = (1/a)(da/dt) = \dot{a}/a$  is the Hubble parameter.

The surface gravity on the apparent horizon is given by

$$\kappa_A = - \left. \frac{1}{2} \frac{\partial \chi}{\partial R} \right|_{R=R_A} = \frac{1}{R_A}. \quad (5)$$

Then, one can derive Hawking temperature on the apparent horizon as

$$T_A = \frac{|\kappa_A|}{2\pi} = \frac{1}{2\pi R_A}. \quad (6)$$

On the other hand, one can consider the definition of the event horizon

$$R_E = a(t) \int_t^\infty \frac{dt}{a(t)}. \quad (7)$$

In an accelerated expanding universe, one can find that the infinite integral converges. In other words, it is possible to have a cosmological event horizon in an accelerated expanding universe. Then, the thermodynamics on the event horizon is important to be studied.

Through the researches about the thermodynamics on the event horizon, Wang et al. found that the thermodynamical description based on the standard definitions of boundary entropy and temperature breaks down in a universe bounded by the cosmological event horizon [24]. They concluded that the cosmological event horizon is unphysical from the viewpoint of thermodynamic laws. However, a redefinition of Hawking temperature has been introduced by Chakraborty in a flat universe to preserve the laws of thermodynamics on the event horizon [26]. The temperature redefinition has been upgraded by Tu et al. to be applicable in a universe with an arbitrary curvature constant [25]. According to [25], one can write the surface gravity on the event horizon as

$$\kappa_E = - \left. \frac{1}{2} \frac{\partial \chi}{\partial R} \right|_{R=R_E} = \frac{\dot{R}_A R_E}{R_A \dot{R}_E}. \quad (8)$$

Using the relation between the surface gravity and Hawking temperature on spacetime horizons, one can find

$$T_E = \frac{|\kappa_E|}{2\pi} = \frac{H}{2\pi} \left( \frac{K}{a^2} - \dot{H} \right) \frac{R_E^2}{\dot{R}_E}. \quad (9)$$

To investigate the universality of the redefinition of Hawking temperature on the event horizon, one can investigate the validity of the first law of thermodynamics. During an infinitesimal time interval, one can write the energy flux across the event horizon as [24–26, 28, 29]

$$\delta Q = AT_{\mu\nu} k^\mu k^\nu dt \Big|_{r=R_E}, \quad (10)$$

where  $k^\mu$  is a null vector,  $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$  is the perfect fluid energy-momentum tensor, and  $u^\mu$  is the timelike 4-velocity vector. Thus, the energy flux can be obtained as

$$\delta Q = 4\pi R_E^3 H (\rho + p) dt. \quad (11)$$

It can also be written as

$$\delta Q = \frac{HR_E^3}{G} \left( \frac{K}{a^2} - \dot{H} \right) dt, \quad (12)$$

in which the Friedmann equation in Einstein gravity;  $\dot{H} - K/a^2 = -4\pi G(\rho + p)$  has been used. On the other hand, the

Bekenstein entropy-area relation in Einstein gravity is given by

$$S_E = \frac{A}{4G}. \quad (13)$$

Using (9) and (13), one can find

$$T_E dS_E = \frac{HR_E^3}{G} \left( \frac{K}{a^2} - \dot{H} \right) dt. \quad (14)$$

Comparing (12) and (14), the validity of the first law of thermodynamics,  $\delta Q = T_E dS_E$ , can be concluded on the event horizon [25]. It seems that the temperature redefinition can lead to a good thermodynamical description on the event horizon.

### 3. From the First Law of Thermodynamics into Friedmann Equations in Palatini $f(R)$ Gravity

The action of  $f(R)$  gravity in Palatini formalism is written as

$$I = \int d^4x \sqrt{-g} \left[ \frac{f(\mathcal{R})}{2k^2} + \mathcal{L}_{matter} \right], \quad (15)$$

where  $g$  is the determinant of the metric tensor,  $\mathcal{L}_{matter}$  is the matter Lagrangian, and  $k^2 = 8\pi G$ . The connection and the metric tensor are considered as independent variables in Palatini formalism. The Ricci tensor and the Ricci scalar constructed with the independent connection are denoted, respectively, by  $\mathcal{R}_{\mu\nu}$  and  $\mathcal{R} = g^{\mu\nu} \mathcal{R}_{\mu\nu}$ . Certainly,  $\mathcal{R}_{\mu\nu}$  is different from the Ricci tensor  $R_{\mu\nu}$  which is constructed with the Levi-Civita connection of the metric. Variation of the action with respect to the metric leads to

$$F(\mathcal{R}) \mathcal{R}_{\mu\nu} - \frac{1}{2} f(\mathcal{R}) g_{\mu\nu} = k^2 T_{\mu\nu}^m, \quad (16)$$

in which  $F(\mathcal{R}) \equiv df(\mathcal{R})/d\mathcal{R}$  and  $T_{\mu\nu}^m$  is the energy-momentum tensor related to the ordinary matter. Varying the action with respect to the connection and using (16) yield

$$\begin{aligned} FG_{\mu\nu} &= k^2 T_{\mu\nu}^m - \frac{1}{2} g_{\mu\nu} (F\mathcal{R} - f) + \nabla_\mu \nabla_\nu F \\ &\quad - g_{\mu\nu} g^{\alpha\beta} \nabla_\alpha \nabla_\beta F \\ &\quad - \frac{3}{2F} \left[ \nabla_\mu F \nabla_\nu F - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \nabla_\alpha F \nabla_\beta F \right]. \end{aligned} \quad (17)$$

Here  $G_{\mu\nu} = R_{\mu\nu} - (1/2)Rg_{\mu\nu}$  is the Einstein tensor and  $T_{\mu\nu}^m = (\rho_m + p_m)u_\mu u_\nu + p_m g_{\mu\nu}$  is the energy-momentum tensor of the ordinary matter. One can rewrite the field equations as

$$G_{\mu\nu} = \frac{k^2}{F} (T_{\mu\nu}^m + T_{\mu\nu}^g), \quad (18)$$

where

$$\begin{aligned} T_{\mu\nu}^g &= \frac{1}{k^2} \left( \frac{f - F\mathcal{R}}{2} g_{\mu\nu} + \nabla_\mu \nabla_\nu F - g_{\mu\nu} g^{\alpha\beta} \nabla_\alpha \nabla_\beta F \right) \\ &\quad - \frac{3}{2Fk^2} \left[ \nabla_\mu F \nabla_\nu F - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \nabla_\alpha F \nabla_\beta F \right] \end{aligned} \quad (19)$$

is the gravitational energy-momentum tensor arising from  $f(R)$  gravity. Now, the gravitational energy density and the gravitational pressure can be written respectively as

$$\rho_g = \frac{1}{k^2} \left[ \frac{1}{2} (F\mathcal{R} - f) - 3H\dot{F} - \frac{3}{4} \frac{\dot{F}^2}{F} \right], \quad (20)$$

and

$$p_g = \frac{1}{k^2} \left[ -\frac{1}{2} (F\mathcal{R} - f) + \ddot{F} + 2H\dot{F} - \frac{3}{4} \frac{\dot{F}^2}{F} \right]. \quad (21)$$

The perfect fluid energy-momentum tensor satisfies the continuity equation

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0. \quad (22)$$

It is easy to investigate the continuity equation for the gravitational sector of the energy-momentum tensor arising from  $f(R)$  gravity. Clearly, one can find

$$\dot{\rho}_g + 3H(\rho_g + p_g) = \frac{3}{k^2} \left( H^2 + \frac{K}{a^2} \right) \dot{F}. \quad (23)$$

To satisfy the continuity equation, it is possible to redefine the gravitational energy density and the gravitational pressure density as

$$\begin{aligned} \hat{\rho}_g &= \frac{1}{k^2} \left[ \frac{1}{2} (F\mathcal{R} - f) - 3H\dot{F} - \frac{3}{4} \frac{\dot{F}^2}{F} \right. \\ &\quad \left. + 3(1-F) \left( H^2 + \frac{K}{a^2} \right) \right], \end{aligned} \quad (24)$$

and

$$\begin{aligned} \hat{p}_g &= \frac{1}{k^2} \left[ -\frac{1}{2} (F\mathcal{R} - f) + \ddot{F} + 2H\dot{F} - \frac{3}{4} \frac{\dot{F}^2}{F} \right. \\ &\quad \left. - (1-F) \left( 2\dot{H} + 3H^2 + \frac{K}{a^2} \right) \right]. \end{aligned} \quad (25)$$

Now, one can rewrite the continuity equation for the gravitational energy density and gravitational pressure density as

$$\dot{\hat{\rho}}_g + 3H(\hat{\rho}_g + \hat{p}_g) = 0. \quad (26)$$

From (22) and (26), one can write the total continuity equation as

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0, \quad (27)$$

in which  $\rho_T = \rho_m + \hat{\rho}_g$  and  $p_T = p_m + \hat{p}_g$ . It is clear that the redefinition of the energy density in (24) and the pressure density in (25) lead to the standard continuity equation. It is now possible to consider  $\rho_T$  and  $p_T$  as the energy and the pressure density of a modified source in the context of general relativity. Hence, one can apply the usual entropy-area relation in (13).

To derive the Friedmann equation, one can start with the first law of thermodynamics. Using (10), the energy flux can be written as

$$\delta Q = 4\pi R_E^3 H (\rho_T + p_T) dt. \quad (28)$$

Utilizing (14) and (28), the first law of thermodynamics yields

$$\left( \frac{K}{a^2} - \dot{H} \right) = 4\pi G (\rho_T + p_T). \quad (29)$$

Now, one can substitute (24) and (25) into (29) to find the Friedmann equation

$$-2 \left( \dot{H} - \frac{K}{a^2} \right) F = k^2 (\rho_m + p_m) + \ddot{F} - H\dot{F} - \frac{3}{2} \frac{\dot{F}^2}{F}. \quad (30)$$

The other Friedmann equation can be obtained via (27) and (30)

$$3 \left( H^2 + \frac{K}{a^2} \right) F = k^2 \rho_m + \frac{1}{2} (F\mathcal{R} - f) - 3H\dot{F} - \frac{3}{4} \frac{\dot{F}^2}{F}. \quad (31)$$

The modified Friedmann equations in (30) and (31) are the same as the ones obtained in [30]. In other words, the first law of thermodynamics on the event horizon leads to the Friedmann equations which are consistent with the ones obtained via the other approach in [30]. Hence, one can rely on the validity of the first law of thermodynamics on the event horizon and consider the event horizon as a thermodynamical system in Palatini  $f(R)$  gravity. It is also clear that the first law of thermodynamics is equivalent with Friedmann equations.

Equation (31) can also be rewritten as

$$\left( H^2 + \frac{K}{a^2} \right) = \frac{8\pi G}{3} \rho_{eff}, \quad (32)$$

where

$$\rho_{eff} = \frac{1}{F} \left[ \rho_m + \frac{1}{8\pi G} \left( \frac{1}{2} (F\mathcal{R} - f) - 3H\dot{F} - \frac{3}{4} \frac{\dot{F}^2}{F} \right) \right] \quad (33)$$

can be considered as an effective energy density. Such an effective energy density may provide an accelerated expanding universe to solve the problem of dark energy. The modified Friedmann equations can be reduced to the Friedmann equations in Einstein gravity by putting  $f(\mathcal{R}) = \mathcal{R}$ .

#### 4. The Generalized Second Law of Thermodynamics

Applying the usual Hawking temperature, the first law of thermodynamics does not hold on the event horizon. However, using the redefinition of Hawking temperature, it has been shown that the first law of thermodynamics may hold on the event horizon in Einstein gravity and metric  $f(R)$  gravity

[25]. Hence, the FRW universe bounded by the event horizon may be described by equilibrium thermodynamics. In previous section, we have considered Hawking temperature redefinition on the event horizon in Palatini  $f(R)$  gravity. We have shown that the first law of thermodynamics can yield the Friedmann equations just the same as the ones obtained via the other approach in [30]. It means that the FRW universe bounded by the event horizon can be described by an equilibrium thermodynamics in Palatini approach too.

It is now possible to investigate the generalized second law of thermodynamics in the FRW universe bounded by the event horizon in Palatini approach. Considering a universe with the dust energy-momentum tensor in  $f(R)$  gravity, one can write the continuity equations as

$$\dot{\rho}_d + 3H\rho_d = 0, \quad (34)$$

and

$$\dot{\hat{\rho}}_g + 3H(\hat{\rho}_g + \hat{p}_g) = 0. \quad (35)$$

The parameter  $\rho_d$  is the energy density of dust and the pressure of dust is  $p_d = 0$ . The energy flux can be obtained from (10) as

$$\delta Q = 4\pi R_E^3 H (\rho_d + \hat{\rho}_g + \hat{p}_g) dt. \quad (36)$$

Based on the generalized second law of thermodynamics, the sum of the entropy on the event horizon and the one inside the bulk must never decrease. To find the total entropy changes, one can start with Gibb's relation [31–33]

$$T_E dS_{in} = dE_{in} + \hat{p}_g dV, \quad (37)$$

where  $V = 4\pi R_E^3/3$ . The parameters  $E_{in} = (4\pi R_E^3/3)(\rho_d + \hat{\rho}_g)$  and  $S_{in}$  are the energy and the entropy inside the boundary respectively. From (7), one can find

$$dR_E = (HR_E - 1) dt. \quad (38)$$

Now, (37) leads to

$$\frac{dS_{in}}{dt} = -\frac{4\pi R_E^2}{T_E} (\rho_d + \hat{\rho}_g + \hat{p}_g), \quad (39)$$

in which we have used relation (38). Since the universe is assumed to be in thermal equilibrium, the temperature inside the universe and the temperature on the event horizon are considered the same. Using the definition  $S_{tot} = S_{in} + S_E$  and applying (14), one can find

$$\frac{dS_{tot}}{dt} = \frac{4\pi R_E^2}{T_E} (\rho_d + \hat{\rho}_g + \hat{p}_g) (HR_E - 1). \quad (40)$$

It is possible to substitute (24) and (25) into (40) to find

$$\begin{aligned} \frac{dS_{tot}}{dt} &= \frac{R_E^2}{2GT_E} (HR_E - 1) \left( \rho_d + \ddot{F} - H\dot{F} - \frac{3}{2} \frac{\dot{F}^2}{F} \right) \\ &\quad - \frac{R_E^2}{GT_E} (1 - F) (HR_E - 1) \left( \dot{H} - \frac{K}{a^2} \right). \end{aligned} \quad (41)$$

Based on the redefinition of Hawking temperature, we have found the event horizon as an equilibrium thermodynamical system. Hence, the first law of thermodynamics and Gibb's relation have been applied to obtain the change of the total entropy. To satisfy GSL, it is necessary to have  $\dot{S}_{tot} \geq 0$ . Of course, the validity of the generalized second law of thermodynamics depends on the  $f(R)$  functions. In other words, GSL can provide some constraints on the choice of  $f(R)$  gravity models.

To investigate GSL, one can substitute  $T_E$  from (9) into (40) to find

$$\frac{dS_{tot}}{dt} = \frac{8\pi^2}{H(K/a^2 - \dot{H})} (\rho_d + \hat{\rho}_g + \hat{P}_g) \dot{R}_E^2. \quad (42)$$

Based on (29), one can conclude that  $(K/a^2 - \dot{H}) \geq 0$  in the weak energy condition. Hence, GSL can be satisfied, when

$$(\rho_d + \hat{\rho}_g + \hat{P}_g) \geq 0. \quad (43)$$

It means that the function  $f$  should satisfy the condition

$$\rho_d + \ddot{F} - H\dot{F} - \frac{3}{2} \frac{\dot{F}^2}{F^2} + 2(1-F) \left( \frac{K}{a^2} - \dot{H} \right) \geq 0. \quad (44)$$

In the case  $f(R) = R$ , one can substitute  $F = 1$  in (44) to investigate the validity of GSL in general relativity. Since  $\rho_d \geq 0$ , one can find  $\dot{S}_{tot} \geq 0$  in the dust dominated universe bounded by the event horizon in general relativity. The Hawking temperature redefinition has provided the validity of the GSL in the universe bounded by the event horizon in general relativity.

## 5. Conclusions

Although the apparent horizon thermodynamics has been studied by many authors in FRW universe, the event horizon thermodynamics is not investigated enough. Since the universe is undergoing an accelerated expansion phase, one may expect the existence of an event horizon. Certainly, the thermodynamics of the event horizon is important to be studied. It has been shown that applying the usual Hawking temperature leads to a nonequilibrium thermodynamics on the event horizon [24]. Hence, the event horizon may be considered unphysical from the view point of the thermodynamic laws. However, redefining Hawking temperature can solve this problem [25, 26]. The Hawking temperature redefinition introduced by Tu and Chen [25] leads to an equilibrium thermodynamics on the event horizon in general relativity and metric  $f(R)$  gravity. In this paper, the thermodynamics of the event horizon in FRW universe is investigated in Palatini  $f(R)$  gravity. Apparently, the continuity equation does not hold in Palatini formalism. To satisfy the continuity equation, redefinitions of the energy density and the pressure density have been introduced in the gravitational sector of the energy-momentum tensor. The redefined gravitational source accompanying with the ordinary matter may play the role of a new modified source in the context of Einstein gravity. Considering this modified energy-momentum source,

using the first law of thermodynamics on the event horizon and applying Bekenstein-Hawking entropy-area relation, the Friedmann equations have been derived. The corrections to the Friedmann equations are the same as the ones obtained via other approaches in [30]. It means that considering the first law of thermodynamics on the event horizon is reliable. In addition, the equivalency of the first law of thermodynamics and the Friedmann equations has been shown. The generalized second law of thermodynamics as a measure of the validity of the gravitational models is investigated in Palatini  $f(R)$  gravity. Of course, GSL can provide some constraints on choosing  $f(R)$  functions.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## References

- [1] J. D. Bekenstein, "Black holes and the second law," *Lettere al Nuovo Cimento (1971-1985)*, vol. 4, no. 15, pp. 737–740, 1972.
- [2] J. D. Bekenstein, "Generalized second law of thermodynamics in black-hole physics," *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 9, article 3292, 1974.
- [3] J. D. Bekenstein, "Black holes and entropy," *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 7, pp. 2333–2346, 1973.
- [4] S. W. Hawking, "Black hole explosions?" *Nature*, vol. 248, no. 5443, pp. 30–31, 1974.
- [5] S. W. Hawking, "Particle creation by black holes," *Communications in Mathematical Physics*, vol. 43, no. 3, pp. 199–220, 1975.
- [6] P. C. W. Davies, "Scalar production in Schwarzschild and Rindler metrics," *Journal of Physics A: Mathematical and General*, vol. 8, no. 4, p. 609, 1975.
- [7] W. G. Unruh, "Notes on black-hole evaporation," *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 14, no. 4, p. 870, 1976.
- [8] L. Susskind, "A predictive Yukawa unified SO(10) model: higgs and sparticle masses," *Journal of Mathematical Physics*, vol. 36, no. 7, article 139, pp. 6377–6396, 1995.
- [9] T. Jacobson, "Thermodynamics of spacetime: the Einstein equation of state," *Physical Review Letters*, vol. 75, p. 1260, 1995.
- [10] C. Eling, R. Guedens, and T. Jacobson, "Nonequilibrium thermodynamics of spacetime," *Physical Review Letters*, vol. 96, no. 12, Article ID 121301, 2006.
- [11] A. Sheykhi, B. Wang, and R.-G. Cai, "Thermodynamical properties of apparent horizon in warped DGP braneworld," *Nuclear Physics B*, vol. 779, no. 1-2, pp. 1–12, 2007.
- [12] R. G. Cai and L. M. Cao, "Thermodynamics of apparent horizon in brane world scenario," *Nuclear Physics B*, vol. 785, no. 1-2, pp. 135–148, 2007.
- [13] A. Sheykhi, B. Wang, and R.-G. Cai, "Deep connection between thermodynamics and gravity in Gauss-Bonnet braneworlds," *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 76, no. 2, Article ID 023515, 5 pages, 2007.

- [14] A. Sheykhi and B. Wang, "Generalized second law of thermodynamics in Gauss-Bonnet braneworld," *Physics Letters B*, vol. 678, no. 5, pp. 434–437, 2009.
- [15] T. Padmanabhan, "Classical and quantum thermodynamics of horizons in spherically symmetric spacetimes," *Classical and Quantum Gravity*, vol. 19, no. 21, pp. 5387–5408, 2002.
- [16] R.-G. Cai and L. M. Cao, "Unified first law and the thermodynamics of the apparent horizon in the FRW universe," *Physical Review D*, vol. 75, Article ID 064008, 2007.
- [17] S. Mitra, S. Saha, and S. Chakraborty, "Universal thermodynamics in different gravity theories: modified entropy on the horizons," *Physics Letters B*, vol. 734, pp. 173–177, 2014.
- [18] M. Akbar and R.-G. Cai, "Thermodynamic behavior of the Friedmann equation at the apparent horizon of the FRW universe," *Physical Review D*, vol. 75, Article ID 084003, 2007.
- [19] R.-G. Cai and S. P. Kim, "First Law of Thermodynamics and Friedmann Equations of Friedmann-Robertson-Walker Universe," *Journal of High Energy Physics*, vol. 2005, article 02, 2005, arXiv.
- [20] X.-H. Ge, "First law of thermodynamics and Friedmann-like equations in braneworld cosmology," *Physics Letters B*, vol. 651, no. 1, pp. 49–53, 2007.
- [21] A. S. Sefiedgar and M. Daghigh, "Thermodynamics of the FRW universe in rainbow gravity," *International Journal of Modern Physics D*, vol. 26, no. 13, Article ID 1750139, 2017.
- [22] M. Akbar and R. G. Cai, "Friedmann equations of FRW universe in scalar-tensor gravity,  $f(R)$  gravity and first law of thermodynamics," *Physics Letters B*, vol. 635, no. 1, pp. 7–10, 2006.
- [23] R. A. Knop, G. Aldering, R. Amanullah et al., "New Constraints on  $\Omega_M$ ,  $\Omega_\Lambda$ , and  $w$  from an Independent Set of 11 High-Redshift Supernovae Observed with the Hubble Space Telescope," *The Astrophysical Journal*, vol. 598, no. 1, pp. 102–137, 2003, The Astrophysical Journal.
- [24] B. Wang, Y. G. Gong, and E. Abdalla, "Thermodynamics of an accelerated expanding universe," *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 74, Article ID 083520, 2006.
- [25] F.-Q. Tu and Y.-X. Chen, "A general thermodynamical description of the event horizon in the FRW universe," *The European Physical Journal C*, vol. 76, article 28, 2016.
- [26] S. Chakraborty, "Is thermodynamics of the universe bounded by event horizon a Bekenstein system?" *Physics Letters B*, vol. 718, no. 2, pp. 276–278, 2012.
- [27] F.-Q. Tu and Y.-X. Chen, "Thermodynamics of the universe bounded by the cosmological event horizon and dominated by the tachyon fluid," <https://arxiv.org/abs/1310.7295>.
- [28] Q.-J. Cao, Y.-X. Chen, and K.-N. Shao, "Clausius relation and Friedmann equation in FRW universe model," *Journal of Cosmology and Astroparticle Physics*, vol. 2010, article 030, 2010.
- [29] R. S. Bousso, "Cosmology and the S matrix," *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 71, no. 6, Article ID 064024, 2005.
- [30] K. Bamba and C.-Q. Geng, "Thermodynamics in  $f(R)$  gravity in the Palatini formalism," *Journal of Cosmology and Astroparticle Physics*, vol. 2010, article 014, 2010.
- [31] S. H. Pereira and J. A. S. Lima, "On phantom thermodynamics," *Physics Letters B*, vol. 669, no. 5, pp. 266–270, 2008.
- [32] E. N. Saridakis, P. F. González-Díaz, and C. L. Sigüenza, "Unified dark energy thermodynamics: varying  $w$  and the  $-1$ -crossing," *Classical and Quantum Gravity*, vol. 26, Article ID 165003, 8 pages, 2009.
- [33] G. Izquierdo and D. Pavon, "Dark energy and the generalized second law," *Physics Letters B*, vol. 633, no. 4-5, pp. 420–426, 2006.



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