

Research Article

Hawking Radiation of a Single-Partition Black Hole

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Brian Kong and the present author recently presented a new area spectrum and showed that the new area spectrum implies that the decay time of a single-partition black hole (i.e., a black hole with the area not big enough to have two or more partitions) is roughly constant. In this article, we show why the decay time of a single-partition black hole is roughly constant.

1. Introduction

It is now very well-known that black hole emits particles thanks to Hawking's semiclassical treatment of black hole [1]. However, as Rovelli and Smolin discovered loop quantum gravity as a possible method to quantize gravity in late 80s [2], it was destined that Hawking's semiclassical treatment of Hawking radiation needed some modification. Soon, it was discovered that area is discrete according to loop quantum gravity [3–5], and Rovelli found the first connection between the Bekenstein-Hawking entropy and loop quantum gravity in his seminal paper in 1996 [6]. Soon, Hawking radiation spectrum was also approached from the loop quantum gravity framework using the discreteness of area [7–9]. In mid-2010s, the present author approached the problem of Hawking radiation spectrum from the discreteness of area as well and found surprising results such as full discreteness of Hawking radiation spectrum [10] and Maxwell-Boltzmann nature of Hawking radiation spectrum [11]. Also, Brian Kong and the present author suggested a new area spectrum in [12], boldly claiming that the conventional area spectrum known in loop quantum gravity community could be wrong, if one considers that, for area two-form, one needs to use Levi-Civita tensor instead of Levi-Civita symbol as conventionally used in loop quantum gravity community. This consideration predicted that the area spectrum was the square root of the conventional one. As for the evidence of this area spectrum, we showed that the Bekenstein-Hawking entropy was approximately satisfied (we conjectured that it would be

exactly satisfied if the effect of extra dimension is considered, which can lead to the prediction of fine structure constant) and found a strange, coincidental numerical relation that implied that the decay time of a single-partition black hole is roughly constant. In this article, we show why the decay time of a single-partition black hole is roughly constant. This new further evidence will give another support the area spectrum proposed by Brian Kong and us. The organization of this paper is as follows. In Section 2, we briefly review loop quantum gravity and its approach to Hawking radiation, particularly focusing on our previous work. In Section 3, we briefly review Brian Kong and the present author's work on new area spectrum focusing on the strange, coincidental numerical relation. In Section 4, we explain our own work on Hawking radiation briefly mentioned in Section 2 in more detail. In Section 5, we explain what is special about single-partition black hole. In Section 6, we derive the constancy of decay time for single-partition black hole. In Section 7, we conclude our paper.

2. Loop Quantum Gravity and Its Approach to Hawking Radiation

According to loop quantum gravity, the eigenvalues of the area operator are quantized [3–5] and the black hole area, as much as any area, is the sum of these eigenvalues. For example, let us say that we have the following area eigenvalues (i.e., the unit areas):

$$A_i = A_1, A_2, A_3, A_4, A_5, A_6 \dots \quad (1)$$

TABLE 1: C(A).

y	A	K(A)	I(A)	C(A)
1	17.8	4	767.4	191.8
2	21.1	14	2740	195.7
3	23.4	32	5552	173.5
4	25.1	50	9276	185.5
5	26.6	72	14000	194.4
6	27.8	110	19814	180.1
7	28.9	154	26817	174.1
8	29.9	204	35109	172.1
9	30.8	262	44797	171.0
10	31.6	326	55990	171.7
11	32.4	388	68803	177.3
12	33.1	474	83353	175.8
13	33.7	584	99761	170.8
14	34.4	684	118155	172.7
15	35.0	804	138664	172.5

Then, the black hole area A must be given by the following formula:

$$A = \sum_i N_i A_i, \quad (2)$$

where N_i s are nonnegative integers. Here, we can regard the black hole as having $\sum N_i$ partitions, each of which has one of the A_i as its area.

In [10], the present author showed that the black hole area decrease upon a single emission of photon during Hawking radiation must be given by the unit area. In other words, we have

$$\Delta A = -A_i \quad (3)$$

As the Bekenstein-Hawking entropy is given by $S = kA/4$, [1, 13] and we know $\Delta Q = T\Delta S$, the energy decrease is given by

$$\Delta Q = -\frac{A_i}{4}kT \quad (4)$$

Since this energy must be equal to the energy of photon emitted (i.e., $\Delta Q = -E_{\text{photon}}$) the energy of the photon emitted during the Hawking radiation is given by

$$E_{\text{photon}} = \frac{A_i}{4}kT \quad (5)$$

In particular, the Hawking radiation spectrum is discrete.

3. Area Spectrum and Single-Partition Black Holes

In [12] (see also [14]), Brian Kong and the present author showed that the area spectrum (i.e., the unit areas) in black hole horizon is given by

$$A = 8\pi \sum_i \sqrt{\frac{1}{2} \sqrt{2j_i^u (j_i^u + 1) + 2j_i^d (j_i^d + 1) - j_i^t (j_i^t + 1)}} \quad (6)$$

with the degeneracy $(2j_i^u + 1) + (2j_i^d + 1)$ where j_i^u , j_i^d , and j_i^t satisfy the triangle inequality (i.e., $j_i^1 + j_i^2 \geq j_i^3$) and j_i^t is an integer, and both j_i^u and j_i^d are integers or half-integers at the same time. We want to remark that (6) is not a conventional one and it has an extra square root compared to [14]. However, Brian Kong and the present author argued in [12] that we arrive at this formula, if we use, in the equation for the area two-form, a Levi-Civita tensor instead of a Levi-Civita symbol as conventionally done in loop quantum gravity community.

Using this, we calculated the degeneracy of area spectrum using Java. This yielded $K(A)$ in Table 1 where $K(A)$ is the number of states for area equal to or below A , and $I(A)$ is given by

$$I(A) = \int_{A_{\text{cut}}}^A \sqrt{A'} (e^{A'/4} - 1) dA' \quad (7)$$

and y is conveniently defined as follows:

$$y = 2j_i^u (j_i^u + 1) + 2j_i^d (j_i^d + 1) - j_i^t (j_i^t + 1) \quad (8)$$

Of course, using y , we have the following relation:

$$A(y) = 8\pi \sqrt{\frac{1}{2} \sqrt{y}} \quad (9)$$

$C(A)$ is given by

$$C(A) = \frac{I(A)}{K(A)} \quad (10)$$

The point of this table is that $C(A)$ is roughly constant. We found this accidentally. Of course $C(A)$ is not exactly but only approximately constant, but the biggest value for $C(A)$ in our result is 195.7, deviating from the ‘‘right value’’ of C , i.e., the value of C for large A by only about 13 percent. As $K(A)$ at this biggest value of $C(A)$ is only 14, it necessarily has a big ‘‘statistical’’ variation. Therefore, 195.7 is not a big deviation.

Then, in Sections X and XI of the same paper, we showed that this constancy can be explained if the decay time of a single-partition black hole is constant. Let us briefly summarize some of the main reasonings.

Let us denote the average number of photons emitted from a black hole during the infinitesimal time Δt by $j_{\Delta t}$. Then, we have

$$j_{\Delta t} = \frac{\Delta t}{\tau} \quad (11)$$

For macroscopic black hole, if dn_{photon} is the number of photons with a frequency between ν and $\nu + d\nu$ emitted during unit time, from Planck's blackbody radiation formula, we have

$$dn_{\text{photon}} = \frac{2\pi\nu^2}{c^2} \frac{A_{BH}d\nu}{e^{h\nu/kT} - 1} \quad (12)$$

where A_{BH} is the area of black hole. Of course, we naturally have

$$\frac{1}{\tau} = \int dn_{\text{photon}} \quad (13)$$

Now let us say that, during the time Δt , the number of emitted photons which correspond to a decrease in the black hole area by amount a is given by $x_{a(y),\Delta t}$. Here, y is given by (8), and $a(y)$ is given by (9).

Then, in section XI, by some calculations we showed

$$x_{a,\Delta t} = \frac{3\sqrt{\pi c}}{2\sqrt{A_{BH}}} \frac{1}{e^{a/4} - 1} \Delta t \quad (14)$$

We proceeded by borrowing the idea from a paper by Bekenstein and Mukhanov [7]. The idea is that $x_{a,\Delta t}/j_{\Delta t}$ is proportional to the degeneracy of the black hole after emission divided by the degeneracy of the black hole before emission. Let us denote the degeneracy of the black hole with area A_{BH} by $W(A_{BH})$. Then, from (11) and (14), we have

$$\frac{W(A_{BH} - a)}{W(A_{BH})} \propto \frac{\tau}{\sqrt{A_{BH}}} \frac{1}{e^{a/4} - 1} \quad (15)$$

Now, if a black hole is a single-partition black hole (a black hole not big enough to have two or more partitions), upon emission of a photon, there is no other value for area decrease than $a = A_{BH}$. Plugging this value to the above formula, we get

$$\frac{W(0)}{W(A_{BH})} \propto \frac{\tau}{\sqrt{A_{BH}}} \frac{1}{e^{A_{BH}/4} - 1} \quad (16)$$

which implies

$$W(A_{BH}) \propto \frac{\sqrt{A_{BH}}}{\tau} (e^{A_{BH}/4} - 1) \quad (17)$$

If τ is constant for single-partition black hole, then it is precisely our earlier relations (7) and (10). Also, one thing very noticeable is that these relations do not hold for

multipartitioned black holes. The minimum area a black hole can have is given by $A(y = 1) = 4\pi\sqrt{2}$. For it to be a single-partition black hole, its area should be smaller than $2A(y = 1) = A(y = 16)$. Our relations suddenly do not hold beginning from $y = 16$ even though it holds very well for $y < 16$. This is the reason why the table is presented up to $y = 15$.

However, we later realized formula (14), which we derived in section XI of [12], is due to an error. Nevertheless, we obtained a strong hint that Table 1 is due to Hawking radiation of single-partition black hole.

4. Quantum Corrections to the Hawking Radiation Spectrum

As mentioned in Section 2, the present author showed in [10] that, upon emission of a single photon during Hawking radiation, only single area quanta can decrease. Let us briefly repeat the main argument in that paper. (In an earlier work [11], the author showed that Hawking radiation should follow Maxwell-Boltzmann statistics rather than Bose-Einstein statistics as considered in [10]. We adjust our presentation here considering this earlier work.)

Consider the following problem [15].

Let us say that the unit areas A_1, A_2, A_3, \dots have degeneracies d_1, d_2, d_3, \dots . Suppose we have a black hole with area A which satisfies $A = \sum_i N_i A_i$ as explained before in (2). For a given configuration ($N_i = N_1, N_2, N_3, \dots$), how many different ways can this be achieved?"

According to Rovelli [6], the area quanta are distinguishable, as they have fixed location on the black hole horizon, and, only if they are so, the Bekenstein-Hawking entropy can be satisfied. Using this distinguishability, the answer to the above question is given by

$$Q = N! \prod_{i=1}^{\infty} \frac{d_i^{N_i}}{N_i!} \quad (18)$$

where $N = \sum_i N_i$.

By maximizing Q and using (5), one obtains that the Hawking radiation for macroscopic black hole is given by the following Maxwell-Boltzmann distribution:

$$N_i = N d_i e^{-hf_i/(kT)} = N d_i e^{-A_i/4} \quad (19)$$

(Note that if we do not follow the present author's earlier work [11], this would be the familiar Einstein-Bose distribution of Hawking radiation, namely, $N_i/N = d_i/(\exp(hf_i/kT) - 1)$.)

We can do better than this. We can express the above expression in terms of the black hole horizon area A . By plugging the above formula to (2), we get

$$N = \frac{A}{\sum_i d_i A_i e^{-A_i/4}} \quad (20)$$

By plugging the above formula back to (19), we get

$$N_i = \frac{A d_i e^{-A_i/4}}{\sum_i d_i A_i e^{-A_i/4}} \quad (21)$$

Notice that the above expression is proportional to the black hole horizon area A . Therefore, we conclude that the Hawking radiation for macroscopic black hole is proportional to the area of black hole, as much as Planck's blackbody radiation spectrum is proportional to the area of blackbody.

5. Single-Partition Black Hole

Let us see what happens for (2) for a single-partition black hole. In such a case we have $N = 1$. This implies $N_i = 1$ for the concerned i and 0 for other unconcerned values of i . Now, remember what N_i meant in (19). It is exactly Hawking radiation spectrum of given frequency. As the sum of N_i is N , N is the sum of Hawking radiation of every frequency, namely, total Hawking radiation. Thus, we conclude that the decay time of the single-partition black hole is constant as N , which is 1, is constant. In other words, τ is constant as desired.

6. Derivation

In this section, we derive (7) and (10) using the correct method.

At the end of Section 4, we showed that Hawking radiation is proportional to the black hole area A . However, this is only half correct as this holds true only if the temperature of black hole were fixed constant. In reality, the temperature of black hole depends on the black hole area A .

So, let us consider this dependence. The number of black hole area quanta is given by (20), and the number of photons emitted per a second is given by (12) and (13). That is,

$$\frac{1}{\tau} = A_{BH} \frac{2\pi}{c^2} \int_{\pi\sqrt{2}}^{\infty} \frac{u^2 du}{e^u - 1} \left(\frac{kT}{h} \right)^3 \quad (22)$$

where the integration range is not from 0 but from $\pi\sqrt{2}$ because the minimum unit area satisfies $A(y=1)/4 = \pi\sqrt{2}$ which is nonzero. As $kT = 1/(8\pi M)$ and $A = 16\pi M^2$ (in this article and earlier ones we only consider Schwarzschild black hole for simplicity), we can write

$$\frac{1}{\tau} = N \frac{\alpha_0}{A_{BH}^{3/2}} \quad (23)$$

for some constant α_0 calculable from (20) and (22). Therefore, for macroscopic black hole, we can write

$$\frac{x_{A_i, \Delta t}}{\Delta t} = N_i \frac{\alpha_0}{A_{BH}^{3/2}} \quad (24)$$

which, when plugged in (21), is given by

$$\frac{x_{A_i, \Delta t}}{\Delta t} = \frac{d_i e^{-A_i/4}}{\sqrt{A_{BH}}} \left(\frac{\alpha_0}{\sum_i d_i A_i e^{-A_i/4}} \right) \quad (25)$$

Now, let us assume that this formula holds for single-partition black hole and further assume that the left-hand side of the above equation is constant, which is reasonable as N_i , being 1, is constant. Then, as the factors in the parenthesis in the above equation are merely constant, we immediately see that (7) and (10) are reproduced.

7. Discussions and Conclusions

One may argue that the table is not fitted correctly as it is fitted with Bose-Einstein distribution factor $e^{A/4} - 1$ while the table should be fitted with the Maxwell-Boltzmann distribution factor $e^{A/4}$ if author's earlier work [11] is correct. This is true, but the difference is insignificant as the lowest value for $e^{A/4}$ is about 85, which means that extra “-1” would not significantly affect the fitting.

In this paper, we showed what the thermodynamics of single-partition black hole implies about the degeneracy of area spectrum up from $y = 1$ to $y = 15$. For your information, the number of degeneracies for $A(y)$ up to $y = 65$ is available at (<http://youngsubyoon.com/65.csv>). If you add them up, you get $K(A)$.

In conclusion, we strongly believe that the extremely strange behaviors of Table 1 (the approximate constancy of C and the relation not holding beginning from exactly $y = 16$) are not a mere coincidence, instead they indicate the correctness of the area spectrum presented in [12].

Data Availability

The data are available at youngsubyoon.com/65.csv.

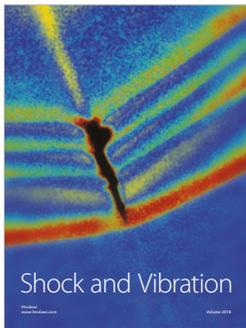
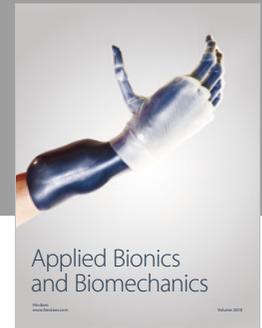
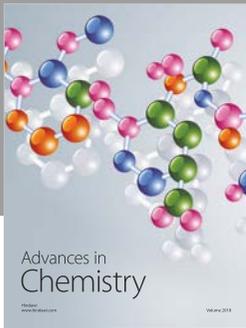
Conflicts of Interest

The authors declare that they have no conflicts of interest.

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