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Research Article

Thermodynamics of Ricci-Gauss-Bonnet Dark Energy

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We investigate the validity of generalized second law of thermodynamics of a physical system comprising newly proposed dark energy model called Ricci-Gauss-Bonnet and cold dark matter enveloped by apparent horizon and event horizon in flat Friedmann-Robertson-Walker (FRW) universe. For this purpose, Bekenstein entropy, Renyi entropy, logarithmic entropy, and power law entropic corrections are used. It is found that this law exhibits the validity on both apparent and event horizons except for the case of logarithmic entropic correction at apparent horizon. Also, we check the thermodynamical equilibrium condition for all cases of entropy and found its vitality in all cases of entropy.

1. Introduction

The revelation of black holes thermodynamics motivated the physicist to examine the thermodynamics of cosmological models in accelerated expanding universe [1–3]. Bekenstein and Hawking determined that the entropy of black hole is proportional to its event horizon [4, 5] which leads to important law named generalized second law of thermodynamics (GSLT) for black hole physics. This law can be defined as the entropy of black hole and its exterior is always increasing. The primitive level of thermodynamics properties of horizons is exhibited by considering Einstein field equations as an alternate of first law of thermodynamics [6, 7]. Gibbons and Hawking developed the Beckenstein's idea for cosmological system by exhibiting that the entropy of cosmological event horizon is proportional to horizon area [8]. They represented the equality of apparent horizon and event horizon for de Sitter universe. The validity of GSLT was deeply studied later [9–11]. GSLT in cosmological scenario implies that the rate of change of entropy of horizon along with that of fluid inside it will always be greater than or equal to zero. Its mathematical expression is

$$\frac{dS_{\text{horizon}}}{dt} + \frac{dS_{\text{inside}}}{dt} \geq 0. \tag{1}$$

In addition, the holographic dark energy (HDE) is an interesting effort in exploring the nature of dark energy in the framework of quantum gravity. This model is motivated from the fundamental holographic principle that arises from black hole thermodynamics and string theory [12–15]. HDE fascinated a large amount of research despite some objections [16, 17]. The choice of the length scale L appearing in the holographic dark energy density $\rho_{de} = 3M_{pl}L^{-2}$ gives rise to different dark energy models. One of the crucial models is holographic Ricci dark energy model which is developed by assuming IR length scale as the average radius of Ricci scalar curvature, $R^{-1/2}$ [18–20]. Moreover, its modified form is also presented and discussed widely [21–23].

Further, Wang et al. [24] observed that GSLT is verified at apparent horizon but not at event horizon for a specific model of dark energy. In case of new holographic dark energy, GSLT is valid fully on apparent horizon but partially on event horizon of universe [25]. The breakdown of GSLT was argued in case of event horizon enveloping the universe as compared to apparent horizon [26]. Setare [27] has derived the constraints on deceleration parameter in order to fulfill GSLT in case of nonflat universe enveloped by event horizon. The GSLT of thermodynamics has also been analyzed in case of Braneworld [28, 29] and generally Levelock gravity [30].

Moreover, modified matter part of Einstein Hilbert action results in dynamical models such as cosmological constants, quintessence, k-essence, Chaplygin gas, and holographic dark energy (HDE) models [31–39]. Moreover, several modified theories of gravity are $f(R)$, $f(T)$ [40–42], $f(R,\mathcal{T})$ [43, 44], $f(G)$ [45–47], $f(T,T_G)$ [48–53], and $f(T,T)$ [54, 55] (where R is the curvature scalar, T denotes the torsion scalar, $\mathcal{T}$ is the trace of the energy momentum tensor, and G is the invariant of Gauss-Bonnet defined as $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$). For clear review of DE models and modified theories of gravity, see [39]. Some authors [56–66] have also discussed various DE models in different frameworks and found interesting results.

Recently, Saridakis [67] proposed Ricci-Gauss-Bonnet holographic dark energy in which Infrared cutoff consists of both Ricci scalar and the Gauss-Bonnet invariant. Such a construction has the significant advantage that the Infrared cutoff and consequently the HDE density do not depend on the future or the past evolution of the universe, but only on its current features, and moreover it is determined by invariants, whose role is fundamental in gravitational theories. This model has IR cutoff form as $1/L^2 = -\alpha R + \beta\sqrt{|G|}$ where α and β are model parameters. In flat FRW geometry, the Ricci scalar (R) and the Gauss-Bonnet invariant (G) are given as $R = -6(2H^2 + \dot{H})$ and $G = 24H^2(H^2 + \dot{H})$, respectively [67].

In the present work, we examine the validity of GSLT by assuming various forms of entropy on apparent and event horizons. We have also examined whether each entropy attain maximum (thermodynamic equilibrium) by satisfying the condition $\ddot{S}_{\text{tot}} < 0$. The plan of the paper is as follows. In Sections 2 and 3, we have examined the validity of GSLT as well as thermal equilibrium condition at apparent and event horizons, respectively. The results are summarized in the last section.

2. Generalized Second Law of Thermodynamics at Apparent Horizon

According to GSLT, the entropy of horizon and entropy of matter resources inside horizon does not decrease with respect to time. Following (1), we can write

$$\dot{S}_{\text{tot}} = \dot{S}_h + \dot{S}_{\text{in}} \geq 0. \tag{2}$$

Here $\dot{S}_h$ gives entropy of horizon and entropy of matter inside horizon is represented by $\dot{S}_{\text{in}}$. Now considering spatially flat FRW universe, the first Friedmann equation is

$$H^2 = \frac{\kappa^2}{3}\left(\rho_{\text{eff}} + P_{\text{eff}}\right). \tag{3}$$

Here ρ_{eff} and P_{eff} are effective density and pressure, respectively. We have made the following two assumptions: (i) an entropy is associated with the horizon in addition to the entropy of the universe inside the horizon and (ii) according to the local equilibrium hypothesis, there is no spontaneous exchange of energy between the horizon and fluid inside. Moreover, Gibb's equation can be written as

$$TdS_{\text{in}} = P_{\text{eff}}dV + dE_{\text{in}}. \tag{4}$$

Here $E_{\text{in}} = \rho_{\text{eff}}V$, $V = (4\pi/3)R_h^2$, and $T = 1/2\pi R_h$ which modified the above equation as follows:

$$\dot{S}_{\text{in}} = 8\pi^2 R_h^3\left(\rho_{\text{eff}} + P_{\text{eff}}\right)\left(\dot{R}_h - HR_h\right). \tag{5}$$

For flat FRW universe, the Hubble horizon can be defined as

$$R_h = \frac{1}{H}. \tag{6}$$

By utilizing the above horizon, $P_{\text{eff}} = p_{de}$ (for cold dark matter $p_m = 0$) and $\rho_{\text{eff}} = \rho_d + \rho_m$ in (5), we can get

$$\dot{S}_{\text{in}} = 8\pi^2 R_h^3\left(\rho_{\text{eff}} + \rho_d\omega_{de}\right)\left(\dot{R}_h - 1\right). \tag{7}$$

From conservation equation, one can obtain

$$\begin{aligned} \dot{\rho}_d &= 3H\left(1 + \omega_d\right)\rho_d \Longrightarrow \\ \omega_d &= -1 - \frac{\dot{\rho}_d}{3H\rho_d}. \end{aligned} \tag{8}$$

Substituting the value of ω_{de} in (7), we get

$$\dot{S}_{\text{in}} = 8\pi^2 R_h^3\left(\rho_m - \frac{\dot{\rho}_d}{3H}\right)\left(\dot{R}_h - 1\right). \tag{9}$$

Moreover, Ricci-Gauss Bonnet dark energy can be defined as follows [67]:

$$\rho_d = 3\left(6\alpha\left(2H^2 + \dot{H}\right) + 2\sqrt{3}\beta H\sqrt{\left|H^2 + \dot{H}\right|}\right). \tag{10}$$

Here α and β are the model parameters. Standard Ricci dark energy can be obtained by substituting $\beta = 0$ and $\alpha = 0$ yields a pure Gauss-Bonnet HDE. The density parameters can be introduced as

$$\begin{aligned} \Omega_m &= \frac{\rho_m}{3H^2}, \\ \Omega_d &= \frac{\rho_d}{3H^2}. \end{aligned} \tag{11}$$

According to first Friedman equation, we can obtain

$$\Omega_d + \Omega_m = 1. \tag{12}$$

Also, ρ_m can be evaluated by using conservation equation as follows:

$$\rho_m = \frac{\rho_{m_0}}{a^3}, \tag{13}$$

with $\rho_{m_0} = 3H_0^2\Omega_{m_0}$. By using this value of ρ_m, Ω_m takes the following form:

$$\Omega_m = \frac{\Omega_{m_0}H_0^2}{a^3H^2}. \tag{14}$$

Using (12) and (14), we can find H as

$$H = \frac{H_0\sqrt{\Omega_{m_0}}}{a^3\left(1 - \Omega_d\right)}. \tag{15}$$

Differentiating H, we obtain

$$\dot{H} = -\frac{H^2}{2}\left(3 - \frac{\acute{\Omega}_d}{1-\Omega_d}\right), \tag{16}$$

where prime denotes the differentiation with respect to $x = \ln a$. Also, differentiation of R_A with respect to t leads to

$$\dot{R}_h = -\frac{\dot{H}}{H^2} = \frac{1}{2}\left(3 - \frac{\acute{\Omega}_d}{1-\Omega_d}\right). \tag{17}$$

We get the following value of $\acute{\Omega}_{de}$ by differentiating (11):

$$\acute{\Omega}_d = \left(\frac{\dot{\rho}_d}{3H^3} + \frac{\rho_d}{H^2}\right)\left(1 + \frac{\rho_d}{3H^2(1-\Omega_d)}\right)^{-1}. \tag{18}$$

Now, $\dot{R}_h$ takes the form

$$\dot{R}_h = \frac{1}{2}\left(3 - \left(\frac{1}{1-\Omega_d}\right)\left(\frac{\dot{\rho}_d}{3H^3} + \frac{\rho_d}{H^2}\right) \cdot \left(1 + \frac{\rho_d}{3H^2(1-\Omega_d)}\right)^{-1}\right). \tag{19}$$

Also, Friedman first equation gives $\rho_m = 3H^2 - \rho_d$ and hence we can write

$$\dot{S}_{\text{in}} = 8(\pi)^2 R_h^3\left(3H^2 - \rho_d - \frac{\dot{\rho}_d}{3H}\right)\left(\dot{R}_h - 1\right). \tag{20}$$

By inserting (6) in above equation, we have

$$\dot{S}_{\text{in}} = \frac{8(\pi)^2}{H^3}\left(3H^2 - \rho_d - \frac{\dot{\rho}_d}{3H}\right)\left(\dot{R}_h - 1\right). \tag{21}$$

By using value of $\dot{R}_h$ from (19), we get

$$\dot{S}_{\text{in}} = \frac{8\pi^2}{H^3}\left(3H^2 - \rho_d - \frac{\dot{\rho}_d}{3H}\right)\left(\frac{1}{2} - \left(\frac{1}{2(1-\Omega_d)}\right)\left(\frac{\dot{\rho}_d}{3H^3} + \frac{\rho_d}{H^2}\right) \times \left(1 + \frac{\rho_d}{3H^2(1-\Omega_d)}\right)^{-1}\right). \tag{22}$$

Next, we will discuss the various expressions of entropy-area relations in order analyze the validity of GSLT on Hubble horizon.

2.1. Bekenstein Entropy. The Bekenstein entropy is given by

$$S_h = \frac{A}{4G}. \tag{23}$$

By using $G = 1$, $c = 1$, and $A = 4\pi R_h^2$ being the area of horizon, we get

$$S_h = \pi R_h^2 \Longrightarrow \dot{S}_h = 2\pi R_h \dot{R}_h. \tag{24}$$

By using the expressions of R_h and $\dot{R}_h$, we have

$$\dot{S}_h = \frac{\pi}{H}\left(3 - \left(\frac{1}{1-\Omega_d}\right)\left(\frac{\dot{\rho}_d}{3H^3} + \frac{\rho_d}{H^2}\right) \cdot \left(1 + \frac{\rho_d}{3H^2(1-\Omega_d)}\right)^{-1}\right). \tag{25}$$

Equations (22) and (25) join to form

$$\dot{S}_{\text{tot}} = \frac{8\pi^2}{H^3}\left(\frac{1}{2} - \left(\frac{1}{2(1-\Omega_{de})}\right)\left(3 - \frac{\rho_{de}}{H^2}\right) \cdot \left(1 + \frac{\rho_{de}}{3H^2(1-\Omega_{de})}\right)^{-1}\right) \times \left(6H^2 - \rho_{de}\right) + \frac{\pi}{H}\left(3 - \left(\frac{1}{1-\Omega_{de}}\right)\left(1 + \frac{\rho_{de}}{3H^2(1-\Omega_{de})}\right)^{-1} \times \left(3 - \frac{\rho_{de}}{H^2}\right)\right), \tag{26}$$

where $\dot{S}_{\text{tot}}$ represents the total entropy; that is, $\dot{S}_{\text{tot}} = \dot{S}_{\text{in}} + \dot{S}_h$.

Now, we assume the power law form of scale factor; that is, $a = a_0 t^n$, where n and a_0 appear as constant parameters. Under this assumption, the values of H and R_h turn out to be n/t, t/n respectively. In this way, $\dot{S}_{\text{tot}}$ reduces to

$$\dot{S}_{\text{tot}} = \frac{8\pi^2 t}{n^3}\left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right)\left(\frac{1}{2} + \frac{U}{2n^2}\left(\frac{2}{3n} - 1\right)\right) + \frac{\pi t}{n}\left(3 + \frac{U}{n^2}\left(\frac{2}{3n} - 1\right)\right), \tag{27}$$

where $U = 3(6\alpha(2n^2 - n) + 2\sqrt{3}\beta n\sqrt{n^2 - n})$. In order to analyze the clear picture of validity of GSLT for this entropy on the Hubble horizon, we plot $\dot{S}_{\text{tot}}$ against cosmic time (t) by fixing constant parameters as $\alpha = 0.2$, $\beta = 0.001$, and $n = 4$ as shown in Figure 1. This shows that $\dot{S}_{\text{tot}}$ remains positive with increasing value of t which confirms the validity of GSLT at apparent horizon with Bekenstein entropy.

To examine the thermodynamic equilibrium, we differentiate (27) to get $\ddot{S}_{\text{tot}}$ given below

$$\ddot{S}_{\text{tot}} = \frac{8\pi^2}{n^3}\left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right)\left(\frac{1}{2} + \frac{U}{2n^2}\left(\frac{2}{3n} - 1\right)\right) + \frac{\pi}{n}\left(3 + \frac{U}{n^2}\left(\frac{2}{3n} - 1\right)\right). \tag{28}$$

We plot $X = \ddot{S}_{\text{tot}}$ versus n in Figure 2 which shows that $\ddot{S}_{\text{tot}} < 0$ for the selected range of n. Hence, thermal equilibrium condition is satisfied for Bekenstein entropy at apparent horizon.

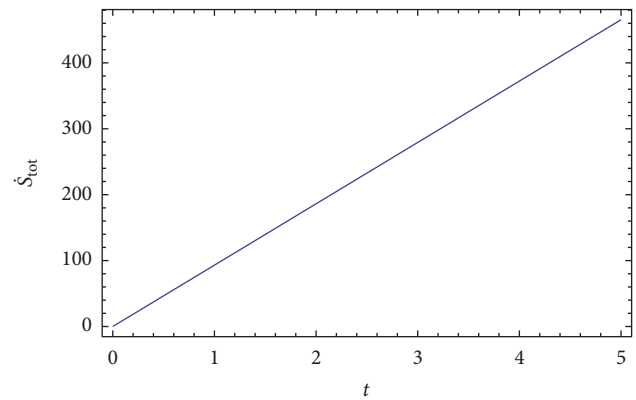

FIGURE 1: Plot of $\dot{S}_{tot}$ by taking Bekenstein entropy as entropy at apparent horizon, where time is measured in seconds.

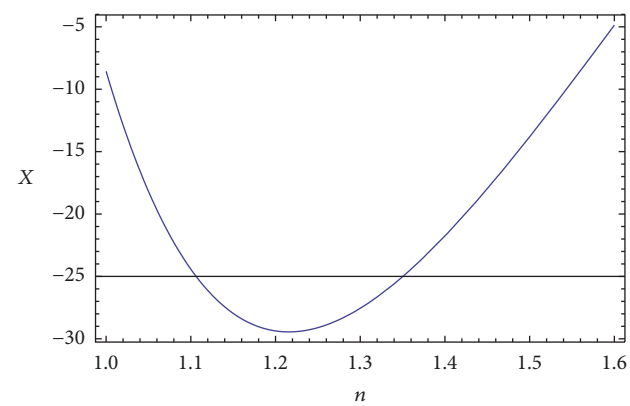

FIGURE 2: Plot of $X = \ddot{S}_{tot}$ by taking Bekenstein entropy as entropy at apparent horizon.

2.2. Logarithmic Corrections to Entropy. Logarithmic corrections arises from loop quantum gravity due to thermal equilibrium and quantum fluctuations [68–74]. The entropy on apparent horizon can be defined as follows:

$$S_h = \frac{A}{4G} + \eta \ln\left[\frac{A}{4G}\right] - \xi\frac{4G}{A} + \gamma, \tag{29}$$

where η, ξ, and γ are dimensionless constants. Differentiating with respect to t, we get

$$\dot{S}_h = \left(\frac{2\pi}{H} + 2\eta H + \frac{2\xi H^3}{\pi}\right)\dot{R}_h, \tag{30}$$

which takes the following form by inserting value of $\dot{R}_h$ from (19):

$$\dot{S}_h = \left(\frac{\pi}{H} + \eta H + \frac{\xi H^3}{\pi}\right)\left(3 - \left(\frac{1}{1-\Omega_d}\right) \cdot \left(\frac{\dot{\rho}_d}{3H^3} + \frac{\rho_d}{H^2}\right)\left(1 + \frac{\rho_d}{3H^2(1-\Omega_d)}\right)^{-1}\right). \tag{31}$$

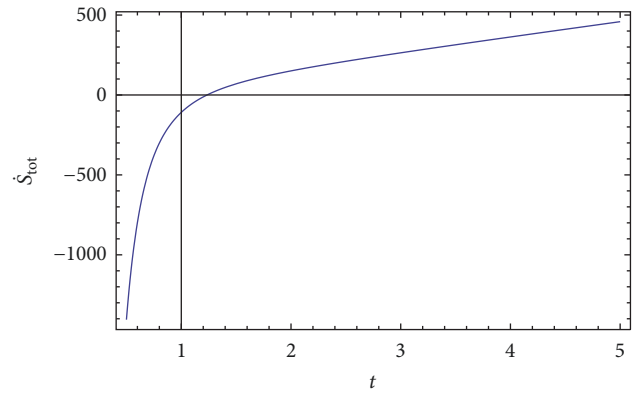

FIGURE 3: Plot of $\dot{S}_{tot}$ by taking Logarithmic entropy as entropy at apparent horizon, where time is measured in second.

In the presence of logarithmic entropy, $\dot{S}_{tot}$ can be obtained by using (22) and (31):

$$\begin{aligned}\dot{S}_{tot} = {} & \frac{8\pi^2}{H^3}\left(3H^2 - \rho_d - \frac{\dot{\rho}_d}{3H}\right)\left(\frac{1}{2} - \left(\frac{1}{2(1-\Omega_d)}\right)\right. \\ & \times\left(\frac{\dot{\rho}_d}{3H^3} + \frac{\rho_d}{H^2}\right)\left(1 + \frac{\rho_d}{3H^2(1-\Omega_d)}\right)^{-1}\Bigg) + \left(\frac{\pi}{H}\right. \\ & \left. + \eta H + \frac{\xi H^3}{\pi}\right) \times \left(3 - \left(\frac{1}{1-\Omega_d}\right)\left(\frac{\dot{\rho}_d}{3H^3} + \frac{\rho_d}{H^2}\right)\right. \\ & \left. \cdot \left(1 + \frac{\rho_d}{3H^2(1-\Omega_d)}\right)^{-1}\right).\end{aligned} \tag{32}$$

By substituting value of scale factor, the above equation reduces to

$$\begin{aligned}\dot{S}_{tot} & = \frac{8\pi^2 t}{n^3}\left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right)\left(\frac{1}{2} + \frac{U}{2n^2}\left(\frac{2}{3n} - 1\right)\right) \\ & + \left(\frac{\pi t}{n} + \frac{\eta n}{t} + \frac{\xi n^3}{\pi t^3}\right)\left(3 + \frac{U}{n^2}\left(\frac{2}{3n} - 1\right)\right).\end{aligned} \tag{33}$$

Differentiating the above equation, we get

$$\begin{aligned}\ddot{S}_{tot} & = \frac{8\pi^2}{n^3}\left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right)\left(\frac{1}{2} + \frac{U}{2n^2}\left(\frac{2}{3n} - 1\right)\right) \\ & + \left(\frac{\pi}{n} - \frac{\eta n}{t^2} - \frac{3\xi n^3}{\pi t^4}\right)\left(3 + \frac{U}{n^2}\left(\frac{2}{3n} - 1\right)\right).\end{aligned} \tag{34}$$

Figure 3 presents the plot of $\dot{S}_{tot}$ at apparent horizon by taking logarithmic entropy at apparent horizon. Here we have taken $\eta = 3.8$ and $\xi = 3$ along with the same values of α, β, and n as in the above-mentioned case. Here $\dot{S}_{tot}$ remains negative for $t < 1.5$ while it moves in positive direction $t \geq 1.5$.

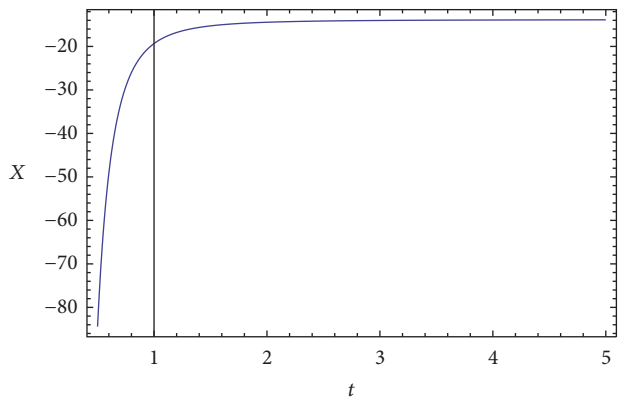

FIGURE 4: Plot of $X = \ddot{S}_{tot}$ by taking Logarithmic entropy as entropy at apparent horizon, where time is measured in second.

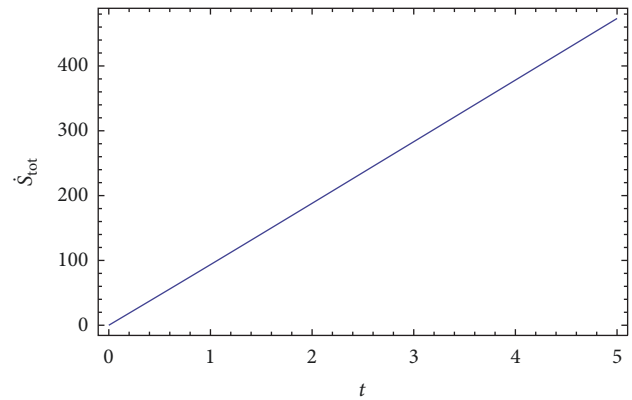

FIGURE 5: Plot of $\dot{S}_{tot}$ by taking Renyi entropy as entropy at apparent horizon, where time is measured in seconds.

Hence, validity of GSLT is verified for $t \geq 1.5$ at apparent horizon with logarithmic entropy. Figure 4 shows that $X = \ddot{S}_{tot} < 0$ with increasing value of t and $n = 1.5$. Hence, for logarithmic entropy at apparent horizon, the condition of thermal equilibrium is satisfied.

2.3. Renyi Entropy. A novel type of Renyi entropy was recommended by Biro and Czinner [75] on black hole horizons by considering Bekenstein-Hawking entropy as nonextensive Tsalis entropy. The modified Renyi entropy can be defined as [76]

$$S_h = \frac{1}{\lambda} \ln \left[1 + \lambda \frac{A}{4G} \right]. \tag{35}$$

It behaves as Bekenstein entropy for $\lambda = 0$. Differentiating with respect to t, we obtain

$$\dot{S}_h = \frac{2\pi H}{H^2 + \lambda\pi} \dot{R}_h. \tag{36}$$

Using (19) in the above equation, we get

$$\dot{S}_h = \frac{\pi H}{H^2 + \lambda\pi} \left(3 - \left(\frac{1}{1 - \Omega_d} \right) \left(\frac{\dot{\rho}_d}{3H^3} + \frac{\rho_{de}}{H^2} \right) \cdot \left(1 + \frac{\rho_d}{3H^2 (1 - \Omega_d)} \right)^{-1} \right). \tag{37}$$

Combine (22) and (37) to get

$$\dot{S}_{tot} = \frac{8(\pi)^2}{H^3} \left(3H^2 - \rho_d - \frac{\dot{\rho}_d}{3H} \right) \left(\frac{1}{2} - \left(\frac{1}{2(1 - \Omega_d)} \right) \left(\frac{\dot{\rho}_d}{3H^3} + \frac{\rho_d}{H^2} \right) \times \left(1 + \frac{\rho_d}{3H^2 (1 - \Omega_d)} \right)^{-1} \right) + \frac{\pi H}{H^2 + \lambda\pi} \left(3 - \left(\frac{1}{1 - \Omega_d} \right) \times \left(\frac{\dot{\rho}_d}{3H^3} + \frac{\rho_d}{H^2} \right) \left(1 + \frac{\rho_d}{3H^2 (1 - \Omega_d)} \right)^{-1} \right). \tag{38}$$

For power law scale factor, we obtain

$$\dot{S}_{tot} = \frac{8\pi^2 t}{n^3} \left(3n^2 + U \left(\frac{2}{3n} - 1 \right) \right) \left(\frac{1}{2} + \frac{U}{2n^2} \left(\frac{2}{3n} - 1 \right) \right) + \left(\frac{n\pi t}{n^2 + \pi\lambda t^2} \right) \left(3 + \frac{U}{n^2} \left(\frac{2}{3n} - 1 \right) \right). \tag{39}$$

The plot of $\dot{S}_{tot}$ by taking Renyi entropy at apparent horizon is presented by Figure 5. Here α, β, and n have the same values like previous case and $\lambda = 1.5$. In this case, $\dot{S}_{tot}$ behaves positively with the passage of time which verifies the validity of GSLT for the present case. Further, differentiating the above equation, we get

$$\ddot{S}_{tot} = \frac{8\pi^2}{n^3} \left(3n^2 + U \left(\frac{2}{3n} - 1 \right) \right) \left(\frac{1}{2} + \frac{U}{2n^2} \left(\frac{2}{3n} - 1 \right) \right) + \left(\frac{n^3\pi - \lambda\pi^2 n t^2}{(n^2 + \pi\lambda t^2)^2} \right) \left(3 + \frac{U}{n^2} \left(\frac{2}{3n} - 1 \right) \right). \tag{40}$$

The plot of this expression is shown in Figure 6 which shows that $\ddot{S}_{tot} < 0$ for $n = 1.5$ with the passage of time. Hence, the condition for thermal equilibrium is satisfied in case of Renyi entropy at apparent horizon.

2.4. Power Law Entropic Correction. The power law corrections to entropy appear in dealing with entanglement of

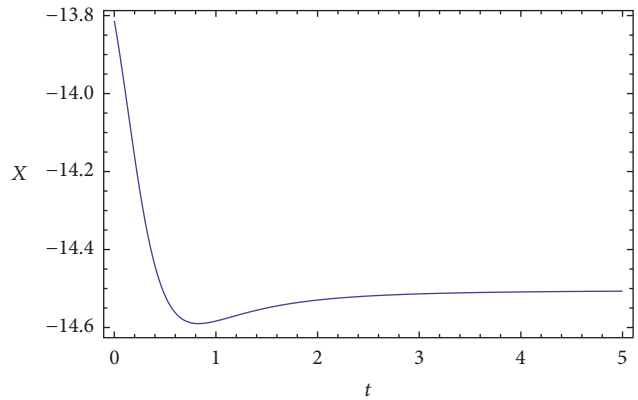

FIGURE 6: Plot of $X = \ddot{S}_{\text{tot}}$ by taking Renyi entropy as entropy at apparent horizon, where time is measured in seconds.

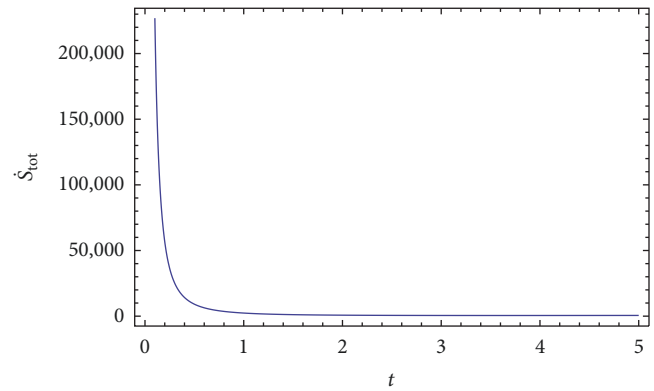

FIGURE 7: Plot of $\dot{S}_{\text{tot}}$ by taking power law entropy as entropy at apparent horizon, where time is measured in seconds.

quantum fields in and out of the horizon [77]. The corrected entropy takes the form [78]

$$S_h = \frac{A}{4G}\left(1 - k_\mu A^{1-\mu/2}\right), \tag{41}$$

with $k_\mu = \mu(4\pi)^{\mu/2-1}/(4-\mu)r_c^{(2-\mu)}$; r_c is crossover length and μ appears as a constant.

$$\dot{S}_h = \frac{\pi \dot{R}_h}{H}\left(2 - k_\mu (4-\mu)\left(\frac{4\pi}{H^2}\right)^{1-\mu/2}\right). \tag{42}$$

Utilization of (19) in the above equation leads to

$$\begin{aligned}\dot{S}_h = \frac{\pi}{2H}&\left(2 - k_\mu (4-\mu)\left(\frac{4\pi}{H^2}\right)^{1-\mu/2}\right)\left(3\right.\\ &-\left(\frac{1}{1-\Omega_d}\right)\left(\frac{\dot{\rho}_d}{3H^3} + \frac{\rho_d}{H^2}\right)\\ &\left.\times\left(1 + \frac{\rho_d}{3H^2(1-\Omega_d)}\right)^{-1}\right).\end{aligned} \tag{43}$$

Join (22) and (43) to obtain

$$\begin{aligned}\dot{S}_{\text{tot}} = &\frac{8(\pi)^2}{H^3}\left(3H^2 - \rho_d - \frac{\dot{\rho}_d}{3H}\right)\left(\frac{1}{2} - \left(\frac{1}{2(1-\Omega_d)}\right)\right.\\ &\left.\cdot\left(\frac{\dot{\rho}_d}{3H^3} + \frac{\rho_d}{H^2}\right)\times\left(1 + \frac{\rho_d}{3H^2(1-\Omega_d)}\right)^{-1}\right)\\ &+\frac{\pi}{2H}\left(2 - k_\mu(4-\mu)\left(\frac{4\pi}{H^2}\right)^{1-\mu/2}\right)\times\left(3\right.\\ &-\left(\frac{1}{1-\Omega_d}\right)\left(\frac{\dot{\rho}_d}{3H^3} + \frac{\rho_d}{H^2}\right)\\ &\left.\cdot\left(1 + \frac{\rho_d}{3H^2(1-\Omega_d)}\right)^{-1}\right).\end{aligned} \tag{44}$$

In the presence of scale factor, the above expression turns out to be

$$\begin{aligned}\dot{S}_{\text{tot}} &= \frac{8\pi^2 t}{n^3}\left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right)\left(\frac{1}{2} + \frac{U}{2n^2}\left(\frac{2}{3n} - 1\right)\right)\\ &+\left(\frac{\pi t}{2n}\right)\left(2 - \frac{\mu}{(nr_c/t)^{2-\mu}}\right)\left(3 + \frac{U}{n^2}\left(\frac{2}{3n} - 1\right)\right).\end{aligned} \tag{45}$$

By taking power Law entropy at apparent horizon, $\dot{S}_{\text{tot}}$ is plotted at apparent horizon as shown in Figure 7. With the same values for α, β, and n, we have taken $\mu = 5$ and $r_c = 2$. Here the effectiveness of GSLT at apparent horizon is certified by positive moves of $\dot{S}_{\text{tot}}$ with increasing t. Differentiating the above equation, we get

$$\begin{aligned}\ddot{S}_{\text{tot}} = &\frac{8\pi^2}{n^3}\left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right)\\ &\cdot\left(\frac{1}{2} + \frac{U}{2n^2}\left(\frac{2}{3n} - 1\right)\right)\\ &+\left(\frac{\pi t}{n} - \frac{\pi\mu}{2n}(nr_c)^{(2-\mu)}t^{(\mu-1)}\right)\\ &\cdot\left(3 + \frac{U}{n^2}\left(\frac{2}{3n} - 1\right)\right).\end{aligned} \tag{46}$$

Just like the above-mentioned three cases, in case of power law entropy at apparent horizon, the condition for thermal equilibrium is satisfied with the passage of cosmic time as shown in Figure 8.

3. Generalized Second Law of Thermodynamics at Event Horizon

In this section, we study GSL of thermodynamics at event horizon which is defined as $R_h = a(t)\int_t^\infty d\hat{t}/a(\hat{t})$. Its derivative with respect to time is given by $\dot{R}_h = HR_h - 1$. The temperature we used in this section is $T = bH/2\pi$, where b is a constant. For the present case, rewriting (4) by using

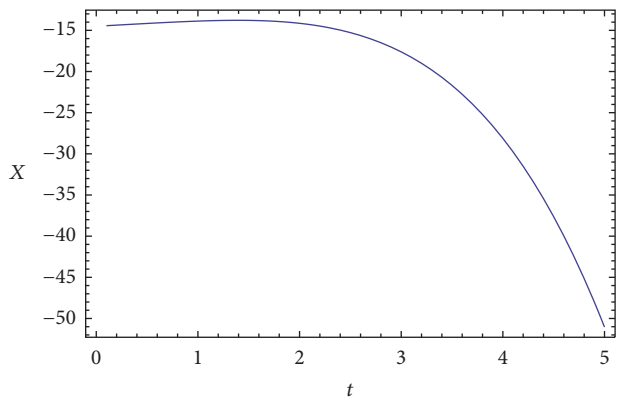

FIGURE 8: Plot of $X = \ddot{S}_{\text{tot}}$ by taking power law entropy as entropy at apparent horizon, where time is measured in seconds.

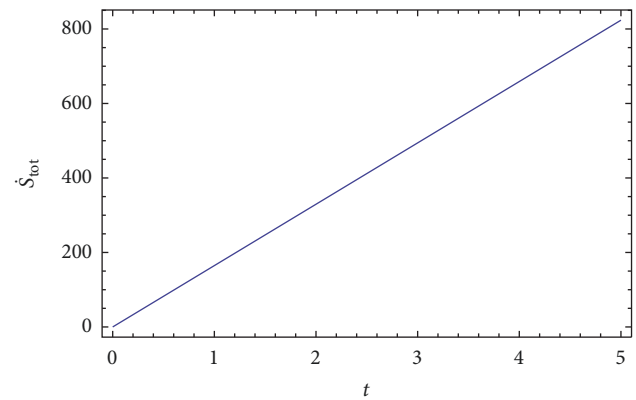

FIGURE 9: Plot of $\dot{S}_{\text{tot}}$ by taking Bekenstein entropy as entropy at event horizon, where time is measured in seconds.

value of T and $\dot{R}_h$, we have the following equation for entropy inside horizon:

$$\dot{S}_{\text{in}} = -\frac{8\pi^2}{bH} R_h^2 \left(3H^2 - \rho_d - \frac{\dot{\rho}_d}{3H}\right). \tag{47}$$

3.1. Bekenstein Entropy. Under this scenario, (24) can be written as

$$\dot{S}_h = 2\pi R_h \left(HR_h - 1\right). \tag{48}$$

The equation for $\dot{S}_{\text{tot}}$ can be obtained by using (43) and (48) as follows:

$$\begin{aligned} \dot{S}_{\text{tot}} = &-\frac{8\pi^2}{bH} R_h^2 \left(3H^2 - \rho_{de} - \frac{\dot{\rho}_{de}}{3H}\right) \\ &+ 2\pi R_h \left(HR_h - 1\right). \end{aligned} \tag{49}$$

By putting values of scale factor and R_h in the above equation, we have

$$\dot{S}_{\text{tot}} = -\frac{8\pi^2 t}{n(n-1)^2 b} \left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right) + 2\pi t. \tag{50}$$

Differentiating the above equation with respect to t, we get

$$\dot{S}_{\text{tot}} = -\frac{8\pi^2}{n(n-1)^2 b} \left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right) + 2\pi. \tag{51}$$

Figure 9 contains the plot of $\dot{S}_{\text{tot}}$ by taking Bekenstein entropy at event horizon. Here we have taken $\alpha = 0.2$, $\beta = 0.001$, and $n = 4$. It is clear from figure that $\dot{S}_{\text{tot}}$ remains positive with increasing value of t. This confirms the validity of GSLT at event horizon with Bekenstein entropy. Figure 10 shows that $\ddot{S}_{\text{tot}} < 0$ for increasing values of n. Hence, at event horizon, the Bekenstein entropy fulfilled the condition of thermodynamic equilibrium.

3.2. Logarithmic Entropy. For this entropy at event horizon, (29) leads to

$$\dot{S}_h = \left(\frac{2\pi}{H} + 2\eta H + \frac{2\xi H^3}{\pi}\right) \dot{R}_h. \tag{52}$$

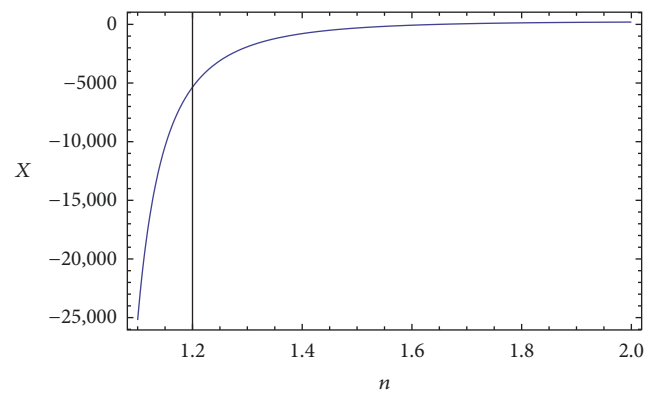

FIGURE 10: Plot of $X = \ddot{S}_{\text{tot}}$ by taking Bekenstein entropy as entropy at event horizon.

By using (47) and (52), the expression of $\dot{S}_{\text{tot}}$ can be written as

$$\begin{aligned} \dot{S}_{\text{tot}} = &-\frac{8\pi^2}{bH} R_h^2 \left(3H^2 - \rho_{de} - \frac{\dot{\rho}_{de}}{3H}\right) \\ &+ \left(\frac{2\pi}{H} + 2\eta H + \frac{2\xi H^3}{\pi}\right) \dot{R}_h. \end{aligned} \tag{53}$$

The following equation is obtained by using values of scale factor and R_h

$$\begin{aligned} \dot{S}_{\text{tot}} = &-\frac{8\pi^2 t}{n(n-1)^2 b} \left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right) + \frac{2\pi t}{(n-1)^2} \\ &+ \frac{2\eta}{t} + \frac{2\xi (n-1)^2}{\pi t^3}. \end{aligned} \tag{54}$$

Differentiating the above equation with respect to t, we obtain

$$\begin{aligned} \ddot{S}_{\text{tot}} = &-\frac{8\pi^2}{n(n-1)^2 b} \left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right) + \frac{2\pi}{(n-1)^2} \\ &- \frac{2\eta}{t^2} - \frac{6\xi (n-1)^2}{\pi t^4}. \end{aligned} \tag{55}$$

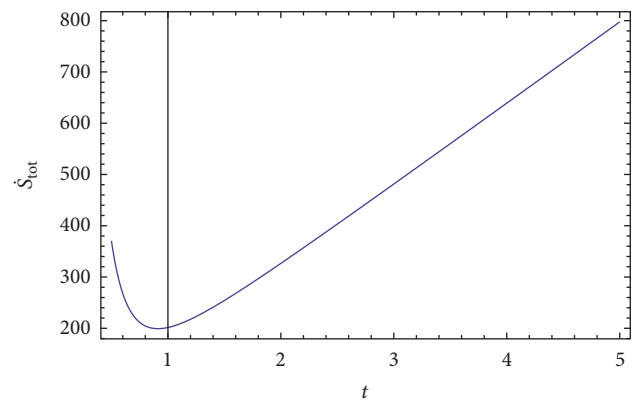

FIGURE 11: Plot of $\dot{S}_{tot}$ by taking logarithmic entropy as entropy at event horizon, where time is measured in seconds.

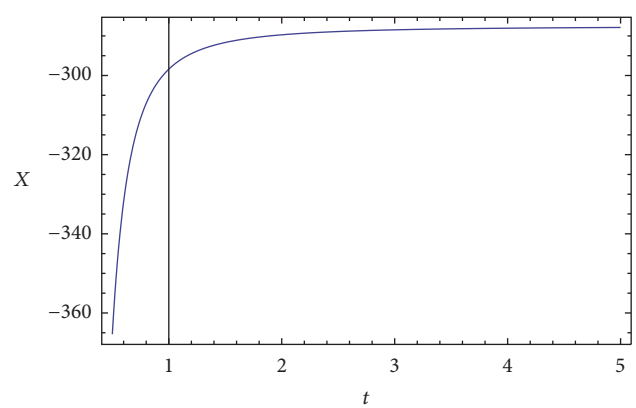

FIGURE 12: Plot of $X = \ddot{S}_{tot}$ by taking logarithmic entropy as entropy at event horizon, where time is measured in seconds.

Figure 11 presents the plot of $\dot{S}_{tot}$ by taking logarithmic entropy at event horizon. Here we have taken $\eta = 4$ and $\xi = 6$ along with the same values of α, β, and n as in the above-mentioned case. Clearly, $\dot{S}_{tot}$ moves in positive direction as value of t increases. The validity of GSLT is verified at event horizon in the presence of logarithmic entropy. From Figure 12, we can see that $\ddot{S}_{tot} < 0$ for $n = 1.5$. Hence, for this case, thermodynamic equilibrium condition holds.

3.3. Renyi Entropy. The following form is obtained from (34), by substituting value for $\dot{R}_h$:

$$\dot{S}_h = \frac{2\pi H}{H^2 + \lambda\pi}\left(HR_h - 1\right). \tag{56}$$

Joining (47) and (56), we get

$$\dot{S}_{tot} = -\frac{8\pi^2}{bH}R_h^2\left(3H^2 - \rho_{de} - \frac{\dot{\rho}_{de}}{3H}\right) + \frac{2\pi H}{H^2 + \lambda\pi}\left(HR_h - 1\right). \tag{57}$$

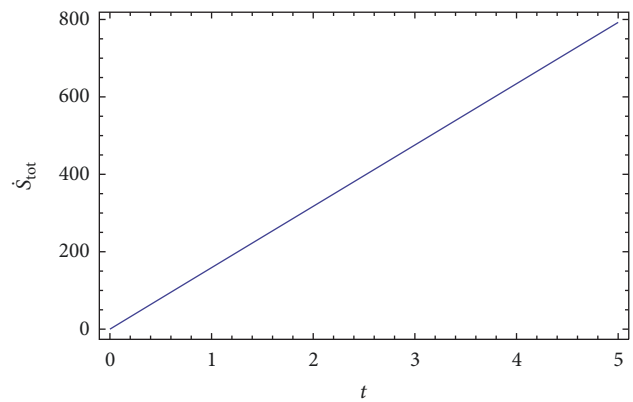

FIGURE 13: Plot of $\dot{S}_{tot}$ by taking Renyi entropy as entropy at event horizon.

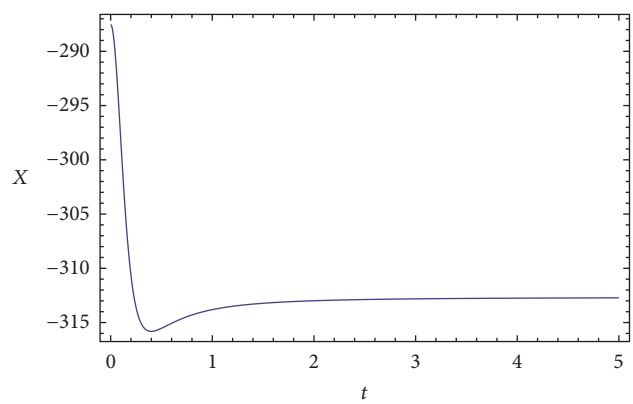

FIGURE 14: Plot of $X = \ddot{S}_{tot}$ by taking Renyi entropy as entropy at event horizon.

By using values of scale factor and R_h, the above equation reduces to

$$\dot{S}_{tot} = -\frac{8\pi^2 t}{n(n-1)^2 b}\left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right) + \frac{2\pi t}{(n-1)^2 + \lambda\pi t^2}. \tag{58}$$

Differentiating the above equation with respect to t, we get

$$\ddot{S}_{tot} = -\frac{8\pi^2}{n(n-1)^2 b}\left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right) + \frac{2\pi(n-1)^2 - 2\pi^2\lambda t^2}{\left((n-1)^2 + \lambda\pi t^2\right)^2}. \tag{59}$$

The plot of $\dot{S}_{tot}$ for Renyi entropy at event horizon is presented in Figure 13.

Here α, β, and n have the same values like the previous case while $\lambda = 1.5$. In this case, $\dot{S}_{tot}$ behaves positively with the passage of time which verifies the validity of GSLT. Figure 14 shows that the trajectories of $\ddot{S}_{tot}$ remain negative for increasing of t with $n = 1.5$. This means that the present scenario obeys the condition for thermodynamic equilibrium.

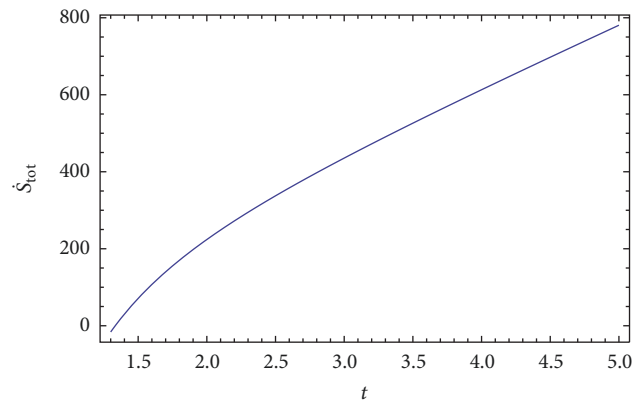

FIGURE 15: Plot of $\dot{S}_{tot}$ by taking power law entropy as entropy at event horizon.

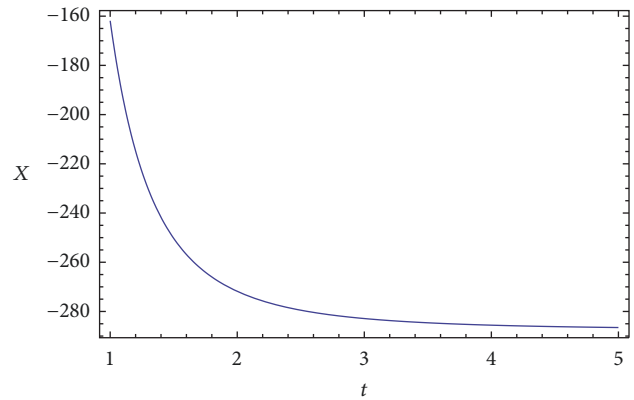

FIGURE 16: Plot of $X = \ddot{S}_{tot}$ by taking power law entropy as entropy at event horizon.

3.4. Power Law Entropy. Under conditions of present section, (41) reduces to

$$\dot{S}_h = \frac{\pi}{H}\left(2 - k_\mu (4-\mu)\left(\frac{4\pi}{H^2}\right)^{1-\mu/2}\right)(HR_h - 1). \quad (60)$$

Joining (47) and (60), we get the following equation:

$$\begin{aligned}\dot{S}_{tot} = &-\frac{8\pi^2}{bH}R_h^2\left(3H^2 - \rho_{de} - \frac{\dot{\rho}_{de}}{3H}\right)\\ &+\frac{\pi}{H}\left(2 - k_\mu (4-\mu)\left(\frac{4\pi}{H^2}\right)^{1-\mu/2}\right)(HR_h - 1).\end{aligned} \quad (61)$$

Inserting conditions for scale factor and R_h in the above equation, we get

$$\begin{aligned}\dot{S}_{tot} = &-\frac{8\pi^2 t}{n(n-1)^2 b}\left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right)\\ &+\left(2 - \mu\left(\frac{t}{r_c(n-1)}\right)^{2-\mu}\right)\frac{\pi t}{(n-1)^2}.\end{aligned} \quad (62)$$

The plot of this expression is displayed in Figure 15 with the same values for α, β, and n while $\mu = 5$ and $r_c = 2$. Here the effectiveness of GSLT at event horizon is certified by positive moves of $\dot{S}_{tot}$ with increasing t. Differentiating with respect to t, we obtain

$$\begin{aligned}\ddot{S}_{tot} = &-\frac{8\pi^2}{n(n-1)^2 b}\left(3n^2 + U\left(\frac{2}{3n} - 1\right)\right) + \frac{2\pi}{(n-1)^2}\\ &-\frac{\mu\pi(3-\mu)t^{2-\mu}}{(r_c(n-1))^{2-\mu}(n-1)^2}.\end{aligned} \quad (63)$$

Figure 16 shows that the present scenario fulfils the thermodynamic equilibrium condition for power law entropy at event horizon.

4. Conclusion

The concept of thermodynamics in cosmological system originates through black hole physics. It was suggested [79] that the temperature of Hawking radiations emitting from black holes is proportional to their corresponding surface gravity on the event horizon. Jacobson [80] found a relation between thermodynamics and the Einstein field equations. He derived this relation on the basis of entropy-horizon area proportionality relation along with first law of thermodynamics (also called Clausius relation) $dQ = TdS$, where dQ, T, and dS indicate the exchange in energy, temperature, and entropy change for a given system. It was shown that the field equations for any spherically symmetric spacetime can be expressed as $TdS = dE + PdV$ (E, P, and V represent the internal energy, pressure, and volume of the spherical system) for any horizon [81]. By utilizing this relation, GSLT has been studied extensively in the scenario of expanding behavior of the universe. In order to discuss GSLT, horizon entropy of the universe can be taken as one quarter of its horizon area [82] or power law corrected [83–85] or logarithmic corrected [86] forms. Many people have explored the validity of GSLT of different systems including interaction of two fluid components like DE and dark matter [87–90], as well as interaction of three components of fluid [91–93] in the FRW universe by using simple horizon entropy of the universe. The thermodynamical analysis was widely performed in modified theories of gravity [94–97].

Motivated by the above-mentioned works, we have considered a newly proposed DE model named Ricci-Gauss-Bonnet DE in flat FRW universe. We have developed thermodynamical quantities and analyzed the validity of GSLT and thermodynamic equilibrium. For dense elaboration of thermodynamics of present DE model, we have assumed various entropy corrections such as Bekenstein entropy, logarithmic corrected entropy, Renyi entropy, and power law entropy at apparent horizon as well as event horizon of the universe. We have found that GSLT holds for all cases of entropies as well as horizons. Also, thermal equilibrium condition was satisfied under certain conditions on constant parameters. The detailed results are as follows.

On Apparent Horizon. By utilizing usual entropy, GSLT on the apparent horizon was shown in Figure 1 which shows that $\dot{S}_{tot}$ remains positive with increasing value of t and

confirms its validity. Figure 2 has also indicated that thermal equilibrium condition is satisfied for Bekenstein entropy at apparent horizon. For logarithmic corrected entropy, GSLT on apparent horizon was displayed in Figure 3 which exhibits that GSLT remains valid for $t \geq 1.5$. However, Figure 4 shows that $X = \ddot{S}_{\text{tot}} < 0$ with increasing value of t and $n = 1.5$. Hence, for logarithmic entropy at apparent horizon, the condition of thermal equilibrium is satisfied.

The plot of $\dot{S}_{\text{tot}}$ by taking Renyi entropy at apparent horizon was displayed in Figure 5 which behaves positively with the passage of time and exhibits the validity of GSLT. Also, for this entropy, the condition for thermal equilibrium has been satisfied in case of Renyi entropy at apparent horizon (Figure 6). By taking power law entropy at apparent horizon, $\dot{S}_{\text{tot}}$ is plotted at apparent horizon as shown in Figure 7. Here the effectiveness of GSLT at apparent horizon is certified by positive moves of $\dot{S}_{\text{tot}}$ with increasing t. Just like the above-mentioned three cases, in case of power law entropy at apparent horizon, the condition for thermal equilibrium is satisfied with the passage of cosmic time as shown in Figure 8.

On Event Horizon. It has been observed from Figure 9 that GSLT remains valid at event horizon with Bekenstein entropy. Also, at event horizon, the Bekenstein entropy fulfilled the condition of thermodynamic equilibrium (Figure 10). The validity of GSLT is verified at event horizon in the presence of logarithmic entropy (Figure 11). From Figure 12, we can see that $\ddot{S}_{\text{tot}} < 0$ for $n = 1.5$ which leads to the validity of thermal equilibrium condition.

The plot of $\dot{S}_{\text{tot}}$ for Renyi entropy at event horizon is presented in Figure 13. It is observed that $\dot{S}_{\text{tot}}$ behaves positively with the passage of time which verifies the validity of GSLT. Figure 14 shows that the trajectories of $\ddot{S}_{\text{tot}}$ remain negative for increasing of t with $n = 1.5$. This means that the present scenario obeys the condition for thermodynamic equilibrium. The plot of $\dot{S}_{\text{tot}}$ for power law corrected entropy is displayed in Figure 15 and observe that GSLT holds in this case. Figure 16 shows that the present scenario fulfils the thermodynamic equilibrium condition for power law entropy at event horizon.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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