

Research Article

$f(T)$ Corrected Instability of Cylindrical Collapsing Object with Harrison-Wheeler Equation of State

Abdul Jawad ¹ and M. Azam²

¹Department of Mathematics, COMSATS University Islamabad, Lahore Campus, Pakistan

²University of Education Lahore, Multan Campus, Pakistan

Correspondence should be addressed to Abdul Jawad; jawadab181@yahoo.com

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We study the dynamical instability of a collapsing object in the framework of generalized teleparallel gravity. We assume a cylindrical object with a specific matter distribution. This distribution contains energy density and isotropic pressure component with heat conduction. We take oscillating states scheme up to first order to check the instable behavior of the object. We construct a general collapse equation for underlying case with nondiagonal tetrad depending on the matter, metric functions, heat conducting term, and torsional terms. The Harrison-Wheeler equation of state which contains adiabatic index is used to explore the dynamical instability ranges for Newtonian and post-Newtonian constraints. These ranges depend on perturbed part of metric coefficients, matter parts, and torsion.

1. Introduction

General relativity is one of the most acceptable theories of gravity which describes many natural phenomena of the universe. However, this theory faces some issues such as dark matter and dark energy. On the same side, dark matter and dark energy are the important ingredients for the dynamics of the entire universe. That is, 25% of the universe consists of dark matter while dark energy reaches up to 68%. On the other hand, dark energy is responsible for the expansion of the universe. These problems are the main reason behind the modification of general relativity (GR). The modification of GR is as old as this theory itself. In recent era, $f(R)$ theory of gravity is the simplest modification to GR based on curvature tensor. However, this theory contains fourth-order field equations which are very difficult to handle.

An alternative theory to GR is teleparallel gravity which is based on torsion using Weitzenböck connection where curvature is zero. The modification of teleparallel gravity results in a generalized teleparallel gravity, that is, $f(T)$ theory of gravity [1–11]. One of the advantages of this theory is that it gives field equations of second order which are easier to handle for the discussion of any underlying scenario as

compared to $f(R)$ gravity. In this context, Ferraro and Fiorini introduced the $f(T)$ theory of gravity to solve the partial horizon problem in Born-Infeld strategy which gave singularity free solution [9]. It is also known as nonlocal Lorentz invariant theory. To figure out this problem, Nashed [2] used two tetrad matrices for the regularization of $f(T)$ field equations. He proposed regularized process with general tetrad field to figure out the effect of local Lorentz invariance. Some authors also investigated this problem (for reference, see [10, 11]).

The dynamical collapse of self-gravitating spherically symmetric object has been widely discussed in $f(T)$ theory of gravity [12–17]. The collapse process occurs when the balanced matter of object becomes imbalanced and the object fails to maintain its equilibrium. In this way, various dynamical states occur, which may be analyzed through dynamical equations. Chandrasekhar [18] was the first who introduced the concept of dynamical instability analysis with the help of adiabatic index, Γ . Instability ranges through adiabatic index have been critically examined for cylindrically and spherically symmetric collapsing matter in $f(R)$ gravity [19–22]. Skripkin [23] analyzed the collapsing nondissipative spherically symmetric fluid with constant energy density and

isotropy. He concluded that, under expansion-free condition, Minkowskian cavity is located at the center of the fluid. Similarly, under the same conditions, dynamical collapse analysis of spherically and cylindrically symmetric anisotropic fluid is explored by Herrera et al. [24–27]. Some authors [21, 28] also studied the instability ranges of spherically symmetric collapsing star with the presence of charge and without charge in $f(R)$ gravity and concluded that these instability ranges are based on geometry, matter, and curvature.

In this scenario, Kausar worked on the effects of CDTT model having inverse curvature term on the unpredictable behavior for the cylindrical symmetric object. It was shown that dynamical instability of expansion-free gravitational collapse with spherical geometry can be investigated without adiabatic index Γ [29, 30]. It was concluded that instability ranges depend on energy density, electromagnetic field, and anisotropic pressure. In Brans-Dicke gravity, Sharif and Manzoor [31] explored the dynamical instability of cylindrically symmetric collapsing star and concluded the range of adiabatic index which is greater than one in special cases and remains less than one for unstable behavior. In $f(T)$ gravity, dynamics of collapsing spherical symmetric object with expansion and expansion-free, shear-free conditions have been explored [16, 17].

Jawad et al. [16] discussed the dynamical instability ranges of a cylindrical symmetric object in the framework of $f(T)$ gravity. They used anisotropic matter distribution and concluded that the instability ranges were affected by the presence of matter, metric coefficients, and torsional terms. We extend this paper taking matter contribution which contains isotropic pressure with heat conducting term. The scheme of the paper is as follows. In Section 2, we construct the field equations for cylindrical symmetric collapsing object for $f(T)$ gravity taking nondiagonal tetrad. Section 3 provides the perturbation scheme up to first order to develop a general collapse equation. In Section 4, we obtain instability ranges under Newtonian and post-Newtonian constraints. The last section contains the concluding remarks.

2. Field Equations of $f(T)$ Gravity for Collapsing Star

In this section, we provide the basic formulation of $f(T)$ gravity. We discuss the basics of collapsing star in cylindrical symmetry and obtain the $f(T)$ field equations in this scenario. We have taken cylindrically symmetric line element as interior space-time and exterior space-time in retarded time coordinate in the framework of $f(T)$ gravity. The collapse process happens when stability of matter is disturbed and at long last experiences collapse which leads to different structures. We have taken the self-gravitating object as cylindrically symmetric collapsing star. We consider cylindrically symmetric collapsing star as an interior region defined by

$$ds_{(-)}^2 = -\mathcal{A}^2 dt^2 + \mathcal{B}^2 dr^2 + \mathcal{C}^2 d\phi^2 + dz^2, \quad (1)$$

where \mathcal{A} , \mathcal{B} and \mathcal{C} are functions of t, r and cylindrical coordinates satisfy the following constraints: $-\infty \leq t \leq \infty$,

$0 \leq r < \infty$, $-\infty < z < \infty$, $0 \leq \phi \leq 2\pi$. In order to keep the cylindrical symmetry, it must satisfy some conditions in the beginning of collapse which is not a trivial process. That is, if the symmetry does not contain any curvature singularity, then we know the conditions to apply. Otherwise, on the singular symmetry, it is hard to impose any condition. In the underlying case, the inside region of cylinder is assumed to be flat; that is, axis is regular. Thus, we may apply the following conditions [32, 33].

- (i) Consider the radial coordinate r in such a manner that axis is located at $r = 0$, and then there exists an axially symmetric axis. This can be written as

$$\mathcal{X} \equiv \left| \xi_{(\phi)}^\alpha \xi_{(\phi)}^\beta g_{\alpha\beta} \right| = \left| g_{\phi\phi} \right| \longrightarrow 0, \quad (2)$$

when $r \rightarrow 0^+$, where $\xi_{(\phi)}$ represents the killing vector.

- (ii) The space-time must hold the following condition in order to preserve the flatness near the symmetry axis. When $r \rightarrow 0^+$, the condition is given by

$$\frac{1}{4\mathcal{X}} \frac{\partial \mathcal{X}}{\partial x^\alpha} \frac{\partial \mathcal{X}}{\partial x^\beta} g^{\alpha\beta} \longrightarrow 1. \quad (3)$$

- (iii) There must not be any closed time-like curves (rather, this is easy to introduce in cylindrical space-time). We impose the following condition in order to avoid these curves:

$$\xi_{(\phi)}^\alpha \xi_{(\phi)}^\beta g_{\alpha\beta} < 0, \quad (4)$$

throughout the space-time.

- (iv) The cylindrical space-time cannot be asymptotically flat in the axial direction, but it must be along radial direction.

The exterior region in terms of retarded time coordinates v and gravitational mass \mathcal{M} is given by [34]

$$ds_{(+)}^2 = - \left(-\frac{2\mathcal{M}(v)}{\mathcal{R}} \right) dv^2 - 2dv d\mathcal{R} + \mathcal{R}^2 (d\phi^2 + \alpha^2 dz^2), \quad (5)$$

where α is a constant having dimension L . The action of $f(T)$ gravity [9] is defined as

$$S = \frac{1}{2\kappa} \int h (f(T) + L_m) d^4x, \quad (6)$$

where $h = \sqrt{-g} = \det(h_\alpha^a)$, h_α^a represents tetrad components, L_m is Lagrangian density of matter, f is the function of torsion scalar, and $\kappa = 8\pi G$ is the coupling constant with G being the gravitational constant. The variation of action (6) with respect to tetrad leads to the following field equations:

$$\begin{aligned} \frac{1}{2} \kappa^2 h_a^\alpha \phi_\alpha^\gamma &= h_a^\alpha S_\alpha^{\mu\gamma} \partial_\mu T f_{TT} \\ &+ \left[\frac{1}{2} \partial_\mu (h h_a^\alpha S_\alpha^{\mu\gamma}) + h_a^\alpha T^\lambda{}_{\mu\alpha} S_\lambda{}^{\gamma\mu} \right] f_T \quad (7) \\ &+ \frac{1}{4} h_a^\gamma f, \end{aligned}$$

where f_T, f_{TT} stand for first- and second-order derivatives with respect to T and torsion scalar is $T \equiv S_{\sigma}^{\mu\nu} T^{\sigma}_{\mu\nu}$. The torsion tensor and superpotential tensor are defined as

$$\begin{aligned} T^{\lambda}_{\mu\nu} &= h^{\lambda}_{\alpha} (\partial_{\nu} h^{\alpha}_{\mu} - \partial_{\mu} h^{\alpha}_{\nu}), \\ S_{\sigma}^{\mu\nu} &= \frac{1}{2} (K^{\mu\nu}_{\sigma} + \delta^{\mu}_{\sigma} T^{\xi\nu}_{\xi} - \delta^{\nu}_{\sigma} T^{\xi\mu}_{\xi}), \end{aligned} \quad (8)$$

where

$$K^{\rho}_{\mu\nu} = \frac{1}{2} (T^{\rho}_{\mu\nu} + T^{\rho}_{\nu\mu} - T^{\rho}_{\nu\nu}). \quad (9)$$

In terms of Einstein tensor, the $f(T)$ field equations take the following form:

$$G_{\mu\nu} = \frac{k^2}{f_T} (\Phi_{\mu\nu}^{(M)} + \Phi_{\mu\nu}^{(T)}), \quad (10)$$

where $\Phi_{\mu\nu}^{(T)}$ represents the torsion part which is defined by

$$\begin{aligned} \Phi_{\mu\nu}^{(T)} &= \frac{1}{\kappa^2} \left[-D_{\mu\nu} f_{TT} - \frac{1}{4} g_{\mu\nu} (\phi - Df_{TT} + Rf_T) \right], \\ D_{\mu\nu} &= S_{\gamma\mu}^{\beta} \nabla_{\beta} T. \end{aligned} \quad (11)$$

The term $\Phi_{\mu\nu}^{(M)}$ represents the matter of the universe. Here, we take the matter under consideration as

$$\Phi_{\mu\nu}^{(M)} = (\rho + p) u_{\mu} u_{\nu} + p g_{\mu\nu} + q_{\mu} u_{\nu} + q_{\nu} u_{\mu}, \quad (12)$$

where u_{μ} and q_{μ} are the four velocities and heat conduction that satisfy the following relations: $u^{\mu} = \mathcal{A}^{-1} \delta^{\mu}_0$, $q^{\mu} = q \mathcal{B}^{-1} \delta^{\mu}_1$, $q_{\mu} u^{\mu} = 0$, $u^{\mu} u_{\mu} = -1$, ρ is the energy density of fluid, and p denotes the isotropic pressure.

The selection of vierbein field is crucial for the framework of $f(T)$ gravity. For cylindrical and spherical geometries, the good tetrads are nondiagonal tetrads that keep the modification of theory with no condition applied on torsion scalar. The diagonal tetrads take the torsion scalar to be constant or vanish in these geometries named as bad choice of tetrad in $f(T)$ gravity. Therefore, we take the nondiagonal tetrad for the interior space-time. In order to construct nondiagonal tetrads which are suitable for cylindrical symmetry, different methods are adopted in the literature like the local Lorentz rotations forming rotated tetrad and local Lorentz boosts introducing boosted tetrad. We choose here another approach in which general coordinate transformation is used [35]. Consider static cylindrically symmetric space-time (1). The general coordinate transformation law is given as

$$h^i_{\mu} = \mathcal{M}^T h^i_{\nu'}, \quad (13)$$

where $\mathcal{M}^T = \partial \mathcal{X}^{\nu'} / \partial \mathcal{X}^{\mu}$ denotes the transformation matrix from Cartesian to cylindrical polar coordinates while $\{\mathcal{X}^{\nu'}\}$ and $\{\mathcal{X}^{\mu}\}$ are Cartesian and cylindrical polar coordinates, respectively; i.e., $\mathcal{X}^{1'} = r \cos \phi$, $\mathcal{X}^{2'} = r \sin \phi$, $\mathcal{X}^{3'} = z$,

$\mathcal{X}^1 = r$, $\mathcal{X}^2 = \phi$, $\mathcal{X}^3 = z$, and $\mathcal{X}^{0'} = \mathcal{X}^0 = t$. The matrix of transformation takes the form

$$h^a_{\mu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \phi & -r \sin \phi & 0 \\ 0 & \sin \phi & r \cos \phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (14)$$

Now we compare metric (1) with Minkowski metric in cylindrical polar coordinates given as

$$ds^2 = dt^2 - dr^2 - r^2 d\phi^2 - dz^2. \quad (15)$$

Notice that g_{00} and g_{11} of metric (1) are obtained by multiplying the corresponding g_{00} and g_{11} of Minkowski metric by \mathcal{A}^2 and \mathcal{B}^2 , respectively, and replacing r by \mathcal{C} for g_{22} . Thus, multiplying zeroth and first columns of the above matrix by \mathcal{A} and \mathcal{B} , respectively, while replacing r by \mathcal{C} , we obtain the tetrad components of static cylindrically symmetric space-time:

$$h^a_{\mu} = \begin{pmatrix} \mathcal{A} & 0 & 0 & 0 \\ 0 & \mathcal{B} \cos \phi & -\mathcal{C} \sin \phi & 0 \\ 0 & \mathcal{B} \sin \phi & \mathcal{C} \cos \phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (16)$$

Using the nonzero components of torsion and superpotential tensors, we get torsion scalar as

$$T = -2 \left[\frac{\mathcal{A}'}{\mathcal{A} \mathcal{B}^2} \left(\frac{\mathcal{B}}{\mathcal{C}} - \frac{\mathcal{C}'}{\mathcal{C}} \right) + \frac{\dot{\mathcal{B}} \dot{\mathcal{C}}}{\mathcal{A}^2 \mathcal{B} \mathcal{C}} \right], \quad (17)$$

where dot and prime represent derivatives with respect to time and radial coordinates, respectively. The $f(T)$ field equations of cylindrically symmetric collapsing star using Eqs. (1) and (10) yield

$$\left(\frac{\mathcal{A}}{\mathcal{B}} \right)^2 \left(\frac{\mathcal{B}' \mathcal{C}'}{\mathcal{B} \mathcal{C}} - \frac{\mathcal{C}''}{\mathcal{C}} \right) + \frac{\dot{\mathcal{B}} \dot{\mathcal{C}}}{\mathcal{B} \mathcal{C}} = \frac{\kappa^2 \mathcal{A}^2}{f_T} \left[\rho \right. \quad (18)$$

$$\left. + \frac{1}{\kappa^2} \left\{ \frac{T f_T - f}{2} + \frac{1}{2 \mathcal{B}^2} \times \left(\frac{\mathcal{B}}{\mathcal{C}} - \frac{\mathcal{C}'}{\mathcal{C}} \right) f_T' \right\} \right].$$

$$\frac{\dot{\mathcal{C}}'}{\mathcal{C}} - \frac{\dot{\mathcal{C}} \mathcal{A}'}{\mathcal{C} \mathcal{A}} - \frac{\dot{\mathcal{B}} \mathcal{C}'}{\mathcal{B} \mathcal{C}} = \frac{\kappa^2}{f_T} \left\{ -q \mathcal{A} \mathcal{B} + \frac{\dot{\mathcal{C}}}{2 \kappa^2 \mathcal{C}} f_T' \right\}. \quad (19)$$

$$\frac{\dot{\mathcal{C}}'}{\mathcal{C}} - \frac{\dot{\mathcal{C}} \mathcal{A}'}{\mathcal{C} \mathcal{A}} - \frac{\dot{\mathcal{B}} \mathcal{C}'}{\mathcal{B} \mathcal{C}} = \frac{\kappa^2}{f_T} \left\{ -q \mathcal{A} \mathcal{B} \right. \quad (20)$$

$$\left. + \frac{1}{2 \kappa^2} \left(\frac{\mathcal{C}'}{\mathcal{C}} - \frac{\mathcal{B}}{\mathcal{C}} \right) \frac{\dot{T}}{T} f_T' \right\},$$

$$\left(\frac{\mathcal{B}}{\mathcal{A}} \right)^2 \left(\frac{\dot{\mathcal{A}} \dot{\mathcal{C}}}{\mathcal{A} \mathcal{C}} - \frac{\ddot{\mathcal{C}}}{\mathcal{C}} \right) + \frac{\mathcal{A}' \mathcal{C}'}{\mathcal{A} \mathcal{C}} = \frac{\kappa^2 \mathcal{B}^2}{f_T} \left[P \right. \quad (21)$$

$$\left. + \frac{1}{\kappa^2} \left\{ \frac{f - T f_T}{2} - \frac{\dot{\mathcal{C}} \dot{T}}{2 \mathcal{A}^2 \mathcal{C} T} f_T' \right\} \right].$$

$$\begin{aligned}
& \left(\frac{\mathcal{E}^2}{\mathcal{A}\mathcal{B}} \right) \left(\frac{\mathcal{A}''}{\mathcal{B}} - \frac{\ddot{\mathcal{B}}}{\mathcal{A}} + \frac{\dot{\mathcal{A}}\dot{\mathcal{B}}}{\mathcal{A}^2} - \frac{\mathcal{A}'\mathcal{B}'}{\mathcal{B}^2} \right) = \frac{\kappa^2 \mathcal{E}^2}{f_T} \left[P \right. \\
& \left. + \frac{1}{\kappa^2} \left\{ \frac{f - Tf_T}{2} - \left(\frac{\dot{\mathcal{B}}\dot{T}}{2\mathcal{A}^2\mathcal{B}T'} - \frac{\mathcal{A}'}{2\mathcal{A}\mathcal{B}^2} \right) f_T' \right\} \right]. \quad (22) \\
& \frac{\mathcal{A}''}{\mathcal{A}\mathcal{B}^2} - \frac{\ddot{\mathcal{B}}}{\mathcal{A}^2\mathcal{B}} + \frac{\dot{\mathcal{A}}\dot{\mathcal{B}}}{\mathcal{A}^3\mathcal{B}} - \frac{\mathcal{A}'\mathcal{B}'}{\mathcal{A}\mathcal{B}^3} + \frac{\dot{\mathcal{A}}\dot{\mathcal{C}}}{\mathcal{A}^3\mathcal{C}} - \frac{\ddot{\mathcal{C}}}{\mathcal{A}^2\mathcal{C}} \\
& - \frac{\mathcal{B}'\mathcal{C}'}{\mathcal{B}^3\mathcal{C}} + \frac{\mathcal{C}''}{\mathcal{B}^2\mathcal{C}} + \frac{\mathcal{A}'\mathcal{C}'}{\mathcal{A}\mathcal{B}^2\mathcal{C}} - \frac{\dot{\mathcal{B}}\dot{\mathcal{C}}}{\mathcal{A}^2\mathcal{B}\mathcal{C}} = \frac{\kappa^2}{f_T} \left[P \right. \\
& \left. + \frac{1}{\kappa^2} \left\{ \frac{f - Tf_T}{2} - \left(\frac{\dot{T}}{2\mathcal{A}^2T'} \left(\frac{\dot{\mathcal{B}}}{\mathcal{B}} \right. \right. \right. \\
& \left. \left. \left. + \frac{\dot{\mathcal{C}}}{\mathcal{C}} \right) - \frac{1}{2\mathcal{B}^2} \left(\frac{\mathcal{A}'}{\mathcal{A}} - \frac{\mathcal{B}'}{\mathcal{B}} + \frac{\mathcal{C}'}{\mathcal{C}} \right) \right) \right\} f_T' \right]. \quad (23)
\end{aligned}$$

Using (19)-(20), we get the following relation:

$$\frac{\dot{\mathcal{C}}}{\mathcal{C}} = \frac{\dot{T}}{T'} \left(\frac{\mathcal{C}'}{\mathcal{C}} - \frac{\mathcal{B}'}{\mathcal{B}} \right). \quad (24)$$

In order to match interior and exterior regions of cylindrical symmetric collapsing star, we use junction conditions defined by Darmois. For this purpose, we consider the C-energy; i.e., mass function representing matter inside the cylinder is given by [36]

$$m(t, r) = \frac{\mathcal{L}}{8} \left(1 - \frac{1}{\mathcal{L}^2} \nabla^\alpha \hat{r} \nabla_\alpha \hat{r} \right) = \mathcal{E}(t, r), \quad (25)$$

where \mathcal{E} is the gravitational energy per unit specific length \mathcal{L} of the cylinder. The areal radius \hat{r} is defined as $\hat{r} = \mu\mathcal{L}$ where circumference radius has the relation $\mu^2 = \chi_{(1)i}\chi_{(1)}^i$ while $\mathcal{L}^2 = \chi_{(2)i}\chi_{(2)}^i$. The terms $\chi_{(1)} = \partial/\partial\phi$ and $\chi_{(2)} = \partial/\partial z$ are the killing vectors corresponding to cylindrical systems. For interior region, the C-energy turns out as

$$m(t, r) = \frac{l}{8} \left[1 + \left(\frac{\dot{\mathcal{C}}}{\mathcal{A}} \right)^2 - \left(\frac{\mathcal{C}'}{\mathcal{B}} \right)^2 \right]. \quad (26)$$

The continuity of Darmois conditions (junction conditions) establishes the following constraints

$$\begin{aligned}
\frac{\mathcal{L}}{8} \Sigma^{(e)} &= m(t, r) - M, \\
\mathcal{L} \Sigma^{(e)} &= 4C, \\
-P &= \frac{\Sigma^{(e)} \Phi_{11}^T}{\mathcal{B}^2} - \frac{\Phi_{01}^T}{\mathcal{A}\mathcal{B}},
\end{aligned} \quad (27)$$

where $\Sigma^{(e)}$ indicates the outer region for measurements with $r = r_{\Sigma^{(e)}} = \text{constant}$. The conservation of total energy of a system is obtained from Bianchi identities through dynamical equations.

This relation might be useful in order to simplify the collapse equation. To analyze the conservation of total energy

of a collapsing star, contracted Bianchi identities in the framework of $f(T)$ gravity are used. These are defined as

$$(\Phi^{(T)\mu\gamma} + \Phi^{(M)\mu\gamma})_{;\gamma} u_\mu = (\Phi^{(T)0\gamma} + \Phi^{(M)0\gamma})_{;\gamma} u_0 = 0, \quad (28)$$

$$(\Phi^{(T)\mu\gamma} + \Phi^{(M)\mu\gamma})_{;\gamma} v_\mu = (\Phi^{(T)1\gamma} + \Phi^{(M)1\gamma})_{;\gamma} v_1 = 0. \quad (29)$$

Consequently, we obtain the following two dynamical equations:

$$\begin{aligned}
& \mathcal{A}^2 \left[\frac{1}{\mathcal{A}^2} \left\{ \frac{Tf_T - f}{2} + \frac{1}{2\mathcal{B}^2} \left(\frac{\mathcal{B}}{\mathcal{C}} - \frac{\mathcal{C}'}{\mathcal{C}} \right) f_T' \right\} \right]_{,0} \\
& + \mathcal{A}^2 \left\{ \frac{\dot{\mathcal{C}}}{2\mathcal{A}^2\mathcal{B}^2\mathcal{C}} f_T' \right\}_{,1} + \frac{\dot{\mathcal{A}}}{\mathcal{A}} (Tf_T - f) \\
& + \frac{1}{\mathcal{B}\mathcal{C}} \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} - \frac{\dot{\mathcal{A}}\mathcal{C}'}{\mathcal{A}\mathcal{B}} - \frac{3\mathcal{A}'\dot{\mathcal{C}}}{2\mathcal{A}\mathcal{B}} + \frac{\dot{\mathcal{B}}}{2\mathcal{B}} + \frac{\dot{\mathcal{C}}}{2\mathcal{C}} \right. \\
& \left. - \frac{\dot{\mathcal{B}}\mathcal{C}'}{2\mathcal{B}^2} + \frac{\mathcal{B}'\dot{\mathcal{C}}}{2\mathcal{B}^2} \right) + \left(\frac{\mathcal{A}'\mathcal{C}'}{2\mathcal{A}\mathcal{B}^2\mathcal{C}} - \frac{\mathcal{A}'\mathcal{B}'}{2\mathcal{A}\mathcal{B}^2\mathcal{C}} \right) \frac{\dot{T}}{T'} \\
& \cdot f_T' + \dot{\rho} + q' \mathcal{A} + \left(\frac{2\mathcal{A}'}{\mathcal{A}} + \frac{\mathcal{B}'}{\mathcal{B}} + \frac{\mathcal{C}'}{\mathcal{C}} \right) q \mathcal{A} + (\rho \\
& + p) \left(\frac{\dot{\mathcal{B}}}{\mathcal{B}} + \frac{\dot{\mathcal{C}}}{\mathcal{C}} \right) = 0. \\
& \mathcal{B}^2 \left\{ \frac{\mathcal{B}^2}{2\mathcal{A}^2} \left(\frac{\mathcal{C}'}{\mathcal{C}} - \frac{\mathcal{B}'}{\mathcal{B}} \right) \frac{\dot{T}}{T'} f_T' \right\}_{,0} \\
& + \mathcal{B}^2 \left\{ \frac{1}{\mathcal{B}^2} \left(\frac{f - Tf_T}{2} - \frac{\dot{\mathcal{C}}\dot{T}}{2\mathcal{A}^2\mathcal{C}T'} f_T' \right) \right\}_{,1} \\
& + \frac{\mathcal{B}'}{\mathcal{B}} (f - Tf_T) \\
& + \left\{ \left(\frac{3\dot{\mathcal{B}}\mathcal{C}'}{2\mathcal{A}^2\mathcal{B}\mathcal{C}} - \frac{\dot{\mathcal{B}}}{\mathcal{A}^2\mathcal{C}} - \frac{\mathcal{B}'\dot{\mathcal{C}}}{\mathcal{A}^2\mathcal{B}\mathcal{C}} + \frac{\mathcal{A}'}{2\mathcal{A}\mathcal{B}\mathcal{C}} \right. \right. \\
& \left. \left. - \frac{\mathcal{A}'\mathcal{C}'}{2\mathcal{A}\mathcal{B}^2\mathcal{C}} \right. \right. \\
& \left. \left. + \frac{\mathcal{A}'\mathcal{C}'}{2\mathcal{A}^3\mathcal{C}} - \frac{\dot{\mathcal{A}}\mathcal{B}}{2\mathcal{A}^3\mathcal{C}} - \frac{\mathcal{B}\dot{\mathcal{C}}}{2\mathcal{A}^2\mathcal{C}^2} - \frac{\mathcal{A}'\mathcal{C}'}{2\mathcal{A}^2\mathcal{C}} \right) \frac{\dot{T}}{T'} \right. \\
& \left. - \frac{\mathcal{A}'\mathcal{C}'}{\mathcal{A}\mathcal{B}^2} \mathcal{C} + \frac{\dot{\mathcal{B}}\dot{\mathcal{C}}}{2\mathcal{A}^2\mathcal{B}\mathcal{C}} \right\} f_T' + p' + \frac{\dot{q}\mathcal{A}}{\mathcal{B}^2} + \left(\frac{3\dot{\mathcal{B}}}{\mathcal{B}} \right. \\
& \left. + \frac{\dot{\mathcal{C}}}{\mathcal{C}} \right) \frac{q}{\mathcal{A}} \mathcal{B}^2 + (\rho + p) \frac{\mathcal{A}'}{\mathcal{A}} = 0.
\end{aligned} \quad (30)$$

3. Oscillating States and Collapse Equation

The selection of $f(T)$ model is very important to establish the collapse equation representing the instability dynamics of cylindrically symmetric object in the framework of $f(T)$ gravity. Here we assume the $f(T)$ model in power-law form

up to quadratic order which is defined by $f(T) = T + \chi T^2$, where χ is an arbitrary constant. This model is widely used in the literature as it indicates the accelerated expansion of the universe in the phantom phase. The possibility of realistic wormhole solutions under this model is found. Also, the instability conditions for a collapsing star under spherically symmetric collapsing object are discussed. In order to construct the dynamical equations and explore instability ranges, we assume the linear perturbation strategy to find the instable behavior. For this we choose the perturbation scheme in such a way that, in the static configuration (nonperturbed part), metric and matter parts are only radially dependent while perturbed part contains both radial and time dependency up to first order where $0 < \lambda \leq 1$ [28]. The metric functions under perturbation strategy become

$$\begin{aligned}\mathcal{A}(t, r) &= \mathcal{A}_0(r) + \lambda \hat{a}(r) \Delta(t), \\ \mathcal{B}(t, r) &= \mathcal{B}_0(r) + \lambda \hat{b}(r) \Delta(t), \\ \mathcal{C}(t, r) &= \mathcal{C}_0(r) + \lambda \hat{c}(r) \Delta(t).\end{aligned}\quad (32)$$

The static and perturbed parts of matter components are

$$\begin{aligned}\rho(t, r) &= \rho_0(r) + \lambda \bar{\rho}(t, r), \\ p(t, r) &= p_0(r) + \lambda \bar{p}(t, r), \\ q(t, r) &= \lambda \bar{q}(t, r),\end{aligned}\quad (33)$$

while mass function, torsion scalar, and $f(T)$ model are perturbed as follows:

$$\begin{aligned}m(t, r) &= m_0(r) + \lambda \bar{m}(t, r), \\ T(t, r) &= T_0(r) + \lambda \omega(r) \Delta(t), \\ f(T) &= T_0(1 + \chi T_0) + \omega \lambda \Delta(1 + 2\chi T_0), \\ f_T(T) &= 1 + 2\chi T_0 + 2\chi \lambda \Delta \omega.\end{aligned}\quad (34)$$

The quantities having zero subscript show the zero-order perturbation. Applying the above set of equations, the perturbed part of the first dynamical equation is

$$\begin{aligned}\left\{ \omega \chi T_0 + \frac{\chi \omega' (\mathcal{B}_0 - 1)}{\mathcal{C}_0 \mathcal{B}_0^2} + \frac{\chi T_0'}{\mathcal{C}_0 \mathcal{B}_0^2} \left(\hat{b} - \hat{c}' \right) \right. \\ + \frac{\hat{c}(1 - \mathcal{B}_0)}{\mathcal{C}_0} - \frac{2\hat{b}(\mathcal{B}_0 - 1)}{\mathcal{B}_0} \left. \right) - \frac{\hat{a}}{\mathcal{A}_0} \left(\chi T_0^2 \right. \\ + \frac{2\chi T_0' (\mathcal{B}_0 - 1)}{\mathcal{C}_0 \mathcal{B}_0^2} \left. \right) + \mathcal{A}_0^2 \left(\frac{\hat{c} \chi T_0'}{\mathcal{C}_0 \mathcal{A}_0^2 \mathcal{B}_0^2} \right) + \frac{\chi T_0^2 \hat{a}}{\mathcal{A}_0} \\ + \frac{2\chi T_0'}{\mathcal{C}_0 \mathcal{B}_0} \left(\frac{\hat{a}}{\mathcal{A}_0} - \frac{\hat{a}}{\mathcal{A}_0 \mathcal{B}_0} + \frac{\hat{c} \mathcal{A}_0'}{2\mathcal{A}_0 \mathcal{B}_0} + \frac{\hat{b}}{2\mathcal{B}_0} \right. \\ + \frac{\hat{c}}{2\mathcal{C}_0} - \frac{\hat{b}}{2\mathcal{B}_0^2} + \frac{\hat{c} \mathcal{A}_0'}{\mathcal{A}_0 \mathcal{B}_0} + \frac{\hat{c} \mathcal{B}_0'}{2\mathcal{B}_0^2} \\ \left. + \frac{\omega \mathcal{A}' (1 - \mathcal{B}_0) T_0'}{2\mathcal{A}_0 \mathcal{B}_0} \right) \left. \right\} \dot{\Delta} + \dot{\bar{\rho}} + \mathcal{A}_0 \bar{q}'\end{aligned}$$

$$\begin{aligned}+ \bar{q} \mathcal{A}_0 \left(\frac{2\mathcal{A}'_0}{\mathcal{A}_0} + \frac{\mathcal{B}'_0}{\mathcal{B}_0} + \frac{\mathcal{C}'_0}{\mathcal{C}_0} \right) + (\rho_0 + p_0) \left(\frac{2\hat{b}}{\mathcal{B}_0} \right. \\ \left. + \hat{c} \right) \dot{\Delta} = 0.\end{aligned}\quad (35)$$

Similarly, the second dynamical equation yields

$$\begin{aligned}\frac{\omega \chi T_0^2 (1 - \mathcal{B}_0) \ddot{\Delta}}{\mathcal{C}_0 \mathcal{A}_0^2} + 2\mathcal{B}_0 \hat{b} \left(\frac{\chi T_0^2}{2\mathcal{B}_0^2} \right)_{,1} \Delta \\ + \mathcal{B}_0^2 \left\{ \frac{1}{\mathcal{B}_0^2} \left(\frac{\hat{b} \chi T_0^2}{\mathcal{B}_0} - \chi \omega T_0 \right) \right\}_{,1} \Delta \\ - \frac{2\chi \omega T_0 \mathcal{B}'_0 \Delta}{\mathcal{B}_0} - \frac{1}{\mathcal{B}_0} \left(\hat{b}' - \frac{\mathcal{B}'_0 \hat{b}}{\mathcal{B}_0} \right) \chi T_0^2 \Delta \\ + \frac{\omega \chi T_0^2}{\mathcal{C}_0 \mathcal{A}_0} \left(\frac{\mathcal{A}'_0}{\mathcal{B}_0} - \frac{\hat{a}'}{\mathcal{B}_0^2} - \frac{\hat{a}'}{\mathcal{A}_0} \right) - \frac{2\omega' \chi \hat{a}' \Delta}{\mathcal{C}_0 \mathcal{A}_0 \mathcal{B}_0^2} \\ - 2\chi T_0' \left(\frac{\hat{a}}{\mathcal{A}_0} + \frac{\hat{b}}{\mathcal{B}_0} + \frac{\hat{c}}{\mathcal{C}_0} \right) \Delta + \dot{\bar{q}} \frac{\mathcal{B}_0^2}{\mathcal{A}_0} + \bar{p}' \\ + (\bar{\rho} + \bar{p}) \frac{\mathcal{A}'_0}{\mathcal{A}_0} + (\rho_0 + p_0) \left(\frac{\hat{a}}{\mathcal{A}_0} \right)' \Delta = 0.\end{aligned}\quad (36)$$

The second dynamical equation can be rewritten as

$$\begin{aligned}J_{2\text{eq}} + \dot{\bar{q}} \frac{\mathcal{B}_0^2}{\mathcal{A}_0} + \bar{p}' + (\bar{\rho} + \bar{p}) \frac{\mathcal{A}'_0}{\mathcal{A}_0} \\ + (\rho_0 + p_0) \left(\frac{\hat{a}}{\mathcal{A}_0} \right)' \Delta = 0,\end{aligned}\quad (37)$$

where

$$\begin{aligned}J_{2\text{eq}} = \frac{\omega \chi T_0^2 (1 - \mathcal{B}_0) \ddot{\Delta}}{\mathcal{C}_0 \mathcal{A}_0^2} + 2\mathcal{B}_0 \hat{b} \left(\frac{\chi T_0^2}{2\mathcal{B}_0^2} \right)_{,1} \Delta \\ + \Delta \mathcal{B}_0^2 \left\{ \frac{1}{\mathcal{B}_0^2} \left(\frac{\hat{b} \chi T_0^2}{\mathcal{B}_0} - \chi \omega T_0 \right) \right\}_{,1} \\ - \frac{2\chi \omega T_0 \mathcal{B}'_0 \Delta}{\mathcal{B}_0} - \frac{1}{\mathcal{B}_0} \left(\hat{b}' - \frac{\mathcal{B}'_0 \hat{b}}{\mathcal{B}_0} \right) \chi T_0^2 \Delta \\ + \frac{\omega \chi T_0^2}{\mathcal{C}_0 \mathcal{A}_0} \left(\frac{\mathcal{A}'_0}{\mathcal{B}_0} - \frac{\hat{a}'}{\mathcal{B}_0^2} - \frac{\hat{a}'}{\mathcal{A}_0} \right) - \frac{2\omega' \chi \hat{a}' \Delta}{\mathcal{C}_0 \mathcal{A}_0 \mathcal{B}_0^2} \\ - 2\chi T_0' \left(\frac{\hat{a}}{\mathcal{A}_0} + \frac{\hat{b}}{\mathcal{B}_0} + \frac{\hat{c}}{\mathcal{C}_0} \right) \Delta.\end{aligned}\quad (38)$$

Equation (37) is called general collapse equation for a cylindrical symmetric object in $f(T)$ gravity. To discuss the instability ranges, we have to find the values of $\bar{\rho}$, \bar{p} , \bar{q} , and Δ . For this purpose, we do some mathematics in the following.

After applying perturbation on (19), we obtain the following equation:

$$= \left[\frac{\hat{c}}{2\mathcal{E}_0} (1 + 2\chi T_o) \right] \dot{\Delta} + 8\pi\mathcal{B}_o^2 \bar{q}, \quad (39)$$

$$\left[\left(\frac{\hat{b}}{\mathcal{A}_0\mathcal{B}_0} \right)' + \left(\frac{\hat{c}}{\mathcal{A}_0} \right)' + \left(\frac{1}{\mathcal{E}_0} + \frac{\mathcal{B}'_o}{\mathcal{B}_0} \right) \left(\frac{\hat{c}}{\mathcal{A}_0} \right) \right] \dot{\Delta}$$

which yields

$$\bar{q} = \frac{\left[(\hat{b}/\mathcal{A}_0\mathcal{B}_0)' + (\hat{c}/\mathcal{A}_0)' + (1/\mathcal{E}_0 + \mathcal{B}'_o/\mathcal{B}_0)(\hat{c}/\mathcal{A}_0) - (\hat{c}/2\mathcal{E}_0)(1 + 2\chi T_o) \right] \dot{\Delta}}{8\pi\mathcal{B}_o^2}. \quad (40)$$

Putting the above value of \bar{q} in (35), we get

$$\begin{aligned} & \left\{ \omega\chi T_o + \frac{\chi\omega'(\mathcal{B}_o - 1)}{\mathcal{E}_0\mathcal{B}_o^2} + \frac{\chi T_o'}{\mathcal{E}_0\mathcal{B}_o^2} \left(\hat{b} - \hat{c} + \frac{\hat{c}(1 - \mathcal{B}_o)}{\mathcal{E}_0} - \frac{2\hat{b}(\mathcal{B}_o - 1)}{\mathcal{B}_0} \right) - \frac{\hat{a}}{\mathcal{A}_0} \left(\chi T_o^2 + \frac{2\chi T_o'(\mathcal{B}_o - 1)}{\mathcal{E}_0\mathcal{B}_o^2} \right) \right. \\ & + \mathcal{A}_o^2 \left(\frac{\hat{c}\chi T_o'}{\mathcal{E}_0\mathcal{A}_o^2\mathcal{B}_o^2} \right) + \frac{\chi T_o^2 \hat{a}}{\mathcal{A}_0} \\ & \left. + \frac{2\chi T_o'}{\mathcal{E}_0\mathcal{B}_0} \left(\frac{\hat{a}}{\mathcal{A}_0} - \frac{\hat{a}}{\mathcal{A}_0\mathcal{B}_0} + \frac{\hat{c}\mathcal{A}'_o}{2\mathcal{A}_0\mathcal{B}_0} + \frac{\hat{b}}{2\mathcal{B}_0} + \frac{\hat{c}}{2\mathcal{E}_0} - \frac{\hat{b}}{2\mathcal{B}_o^2} + \frac{\hat{c}\mathcal{A}'_o}{\mathcal{A}_0\mathcal{B}_0} + \frac{\hat{c}\mathcal{B}'_o}{2\mathcal{B}_o^2} + \frac{\omega\mathcal{A}'(1 - \mathcal{B}_o)T_o'}{2\mathcal{A}_0\mathcal{B}_0} \right) \right\} \dot{\Delta} + \dot{\rho}_o + (\rho_o \\ & + p_o) \left(\frac{2\hat{b}}{\mathcal{B}_0} + \hat{c} \right) \dot{\Delta} + \left[\left(\frac{\hat{b}}{\mathcal{A}_0\mathcal{B}_0} \right)' + \left(\frac{\hat{c}}{\mathcal{A}_0} \right)' + \left(\frac{1}{\mathcal{E}_0} + \frac{\mathcal{B}'_o}{\mathcal{B}_0} \right) \left(\frac{\hat{c}}{\mathcal{A}_0} \right) - \frac{\hat{c}}{2\mathcal{E}_0} (1 + 2\chi T_o) \right] \times \frac{\mathcal{A}_o \dot{\Delta}}{8\pi\mathcal{B}_o'^2} \\ & + \frac{\left[(\hat{b}/\mathcal{A}_0\mathcal{B}_0)' + (\hat{c}/\mathcal{A}_0)' + (1/\mathcal{E}_0 + \mathcal{B}'_o/\mathcal{B}_0)(\hat{c}/\mathcal{A}_0) - (\hat{c}/2\mathcal{E}_0)(1 + 2\chi T_o) \right]}{8\pi\mathcal{B}_o^2} \times \mathcal{A}_o \dot{\Delta} \left(\frac{2\mathcal{A}'_o}{\mathcal{A}_0} + \frac{\mathcal{B}'_o}{\mathcal{B}_0} + \frac{\mathcal{E}'_o}{\mathcal{E}_0} \right) = 0. \end{aligned} \quad (41)$$

We can write this equation as

$$\begin{aligned} & \mathcal{A}_o \left(\frac{(\hat{b}/\mathcal{A}_0\mathcal{B}_0)' + (\hat{c}/\mathcal{A}_0)' + (1/\mathcal{E}_0 + \mathcal{B}'_o/\mathcal{B}_0)(\hat{c}/\mathcal{A}_0) - (\hat{c}/2\mathcal{E}_0)(1 + 2\chi T_o)}{8\pi\mathcal{B}_o^2} \right)' \dot{\Delta} \\ & + \frac{((\hat{b}/\mathcal{A}_0\mathcal{B}_0)' + (\hat{c}/\mathcal{A}_0)' + (1/\mathcal{E}_0 + \mathcal{B}'_o/\mathcal{B}_0)(\hat{c}/\mathcal{A}_0) - (\hat{c}/2\mathcal{E}_0)(1 + 2\chi T_o)) \dot{\Delta}}{8\pi\mathcal{B}_o^2} \times \mathcal{A}_o \left(\frac{2\mathcal{A}'_o}{\mathcal{A}_0} + \frac{\mathcal{B}'_o}{\mathcal{B}_0} + \frac{\mathcal{E}'_o}{\mathcal{E}_0} \right) \\ & + (\rho_o + p_o) \left(\frac{2\hat{b}}{\mathcal{B}_0} + \hat{c} \right) \dot{\Delta} + J_{\text{1eq}} \dot{\Delta} + \dot{\rho} = 0, \end{aligned} \quad (42)$$

where

$$\begin{aligned} J_{\text{1eq}} = & \omega\chi T_o + \frac{\chi\omega'(\mathcal{B}_o - 1)}{\mathcal{E}_0\mathcal{B}_o^2} + \frac{\chi T_o'}{\mathcal{E}_0\mathcal{B}_o^2} \left(\hat{b} - \hat{c} \right. \\ & \left. + \frac{\hat{c}(1 - \mathcal{B}_o)}{\mathcal{E}_0} - \frac{2\hat{b}(\mathcal{B}_o - 1)}{\mathcal{B}_0} \right) - \frac{\hat{a}}{\mathcal{A}_0} \left(\chi T_o^2 \right. \\ & \left. + \frac{2\chi T_o'(\mathcal{B}_o - 1)}{\mathcal{E}_0\mathcal{B}_o^2} \right) + \mathcal{A}_o^2 \left(\frac{\hat{c}\chi T_o'}{\mathcal{E}_0\mathcal{A}_o^2\mathcal{B}_o^2} \right) + \frac{\chi T_o^2 \hat{a}}{\mathcal{A}_0} \end{aligned}$$

$$\begin{aligned} & + \frac{2\chi T_o'}{\mathcal{E}_0\mathcal{B}_0} \left(\frac{\hat{a}}{\mathcal{A}_0} - \frac{\hat{a}}{\mathcal{A}_0\mathcal{B}_0} + \frac{\hat{c}\mathcal{A}'_o}{2\mathcal{A}_0\mathcal{B}_0} + \frac{\hat{b}}{2\mathcal{B}_0} \right. \\ & \left. + \frac{\hat{c}}{2\mathcal{E}_0} - \frac{\hat{b}}{2\mathcal{B}_o^2} + \frac{\hat{c}\mathcal{A}'_o}{\mathcal{A}_0\mathcal{B}_0} + \frac{\hat{c}\mathcal{B}'_o}{2\mathcal{B}_o^2} \right. \\ & \left. + \frac{\omega\mathcal{A}'(1 - \mathcal{B}_o)T_o'}{2\mathcal{A}_0\mathcal{B}_0} \right). \end{aligned}$$

(43)

Integrating the above equation with respect to “ t ”, we get

$$\begin{aligned} & \mathcal{A}_0 \left(\frac{(\widehat{b}/\mathcal{A}_0\mathcal{B}_0)' + (\widehat{c}/\mathcal{A}_0)' + (1/\mathcal{C}_0 + \mathcal{B}'_0/\mathcal{B}_0)(\widehat{c}/\mathcal{A}_0) - (\widehat{c}/2\mathcal{C}_0)(1 + 2\chi T_o)}{8\pi\mathcal{B}_0^2} \right)' \Delta \\ & + \frac{((\widehat{b}/\mathcal{A}_0\mathcal{B}_0)' + (\widehat{c}/\mathcal{A}_0)' + (1/\mathcal{C}_0 + \mathcal{B}'_0/\mathcal{B}_0)(\widehat{c}/\mathcal{A}_0) - (\widehat{c}/2\mathcal{C}_0)(1 + 2\chi T_o))\Delta}{8\pi\mathcal{B}_0^2} \times \mathcal{A}_0 \left(\frac{2\mathcal{A}'_0}{\mathcal{A}_0} + \frac{\mathcal{B}'_0}{\mathcal{B}_0} + \frac{\mathcal{C}'_0}{\mathcal{C}_0} \right) \\ & + (\rho_o + p_o) \left(\frac{2\widehat{b}}{\mathcal{B}_0} + \widehat{c} \right) \Delta + J_{1\text{eq}}\Delta + \bar{p} = 0. \end{aligned} \quad (44)$$

The Harrison-Wheeler equation of state is defined by

$$\bar{p} = \Gamma \frac{p_o}{\rho_o + p_o} \bar{\rho}, \quad (45)$$

where Γ is named the adiabatic index. The adiabatic index defines the instability ranges of a self-gravitating

collapsed object. Here we discuss these ranges for cylindrically symmetric self-gravitational fluid in the framework of $f(T)$ gravity both in Newtonian and in post-Newtonian regimes through adiabatic index under collapse equation. Equation (44) can be written as

$$\begin{aligned} \bar{p} = & - \left[\mathcal{A}_0 \left(\frac{(\widehat{b}/\mathcal{A}_0\mathcal{B}_0)' + (\widehat{c}/\mathcal{A}_0)' + (1/\mathcal{C}_0 + \mathcal{B}'_0/\mathcal{B}_0)(\widehat{c}/\mathcal{A}_0) - (\widehat{c}/2\mathcal{C}_0)(1 + 2\chi T_o)}{8\pi\mathcal{B}_0^2} \right)' \Delta \right. \\ & + \frac{((\widehat{b}/\mathcal{A}_0\mathcal{B}_0)' + (\widehat{c}/\mathcal{A}_0)' + (1/\mathcal{C}_0 + \mathcal{B}'_0/\mathcal{B}_0)(\widehat{c}/\mathcal{A}_0) - (\widehat{c}/2\mathcal{C}_0)(1 + 2\chi T_o))\Delta}{8\pi\mathcal{B}_0^2} \times \mathcal{A}_0 \left(\frac{2\mathcal{A}'_0}{\mathcal{A}_0} + \frac{\mathcal{B}'_0}{\mathcal{B}_0} + \frac{\mathcal{C}'_0}{\mathcal{C}_0} \right) + J_{1\text{eq}}\Delta \\ & \left. + (\rho_o + p_o) \left(\frac{2\widehat{b}}{\mathcal{B}_0} + \widehat{c} \right) \Delta \right]. \end{aligned} \quad (46)$$

To find the value of \bar{p} , we insert the value of $\bar{\rho}$ in (45), which gives

$$\begin{aligned} \bar{p} = & \Gamma \frac{-p_o}{\rho_o + p_o} \left[\left(\frac{(\widehat{b}/\mathcal{A}_0\mathcal{B}_0)' + (\widehat{c}/\mathcal{A}_0)' + (1/\mathcal{C}_0 + \mathcal{B}'_0/\mathcal{B}_0)(\widehat{c}/\mathcal{A}_0) - (\widehat{c}/2\mathcal{C}_0)(1 + 2\chi T_o)}{8\pi\mathcal{B}_0^2} \right)' \times \Delta \mathcal{A}_0 \right. \\ & + \frac{((\widehat{b}/\mathcal{A}_0\mathcal{B}_0)' + (\widehat{c}/\mathcal{A}_0)' + (1/\mathcal{C}_0 + \mathcal{B}'_0/\mathcal{B}_0)(\widehat{c}/\mathcal{A}_0) - (\widehat{c}/2\mathcal{C}_0)(1 + 2\chi T_o))\Delta}{8\pi\mathcal{B}_0^2} \times \Delta \mathcal{A}_0 \left(\frac{2\mathcal{A}'_0}{\mathcal{A}_0} + \frac{\mathcal{B}'_0}{\mathcal{B}_0} + \frac{\mathcal{C}'_0}{\mathcal{C}_0} \right) + (\rho_o \\ & \left. + p_o) \left(\frac{2\widehat{b}}{\mathcal{B}_0} + \widehat{c} \right) \Delta + J_{1\text{eq}}\Delta \right]. \end{aligned} \quad (47)$$

In order to determine the value of $\Delta(t)$, we perturbed (21) which is given by

$$\begin{aligned} & \frac{-\widehat{c}}{\mathcal{C}_0\mathcal{A}_0^2} \ddot{\Delta} + \frac{1}{\mathcal{C}_0\mathcal{A}_0\mathcal{B}_0^2} \left(\frac{-2\widehat{b}\mathcal{A}'_0}{\mathcal{B}_0} - \frac{\widehat{a}}{\mathcal{A}_0} \mathcal{A}'_0 - \frac{\widehat{c}}{\mathcal{C}_0} \mathcal{A}'_0 \right. \\ & \left. + \widehat{a}' + \widehat{c}' \mathcal{A}'_0 \right) \Delta = - \frac{2\chi\omega\Delta p_o}{(1 + 2\chi T_o)^2} + \frac{\bar{p}}{1 + 2\chi T_o} \end{aligned}$$

$$- \frac{\omega\chi T_o\Delta}{1 + 2\chi T_o} - \frac{\omega\chi^2 T_o^2\Delta}{(1 + 2\chi T_o)^2}, \quad (48)$$

The solution of this equation can be found through matching of interior and exterior regions which leads to $r = r_\Sigma = \text{constant } r$, on boundary surface.

Under this condition and using value of \bar{p} , (48) yields

$$\ddot{\Delta} - \psi_{\Sigma} \Delta \stackrel{\Sigma}{=} 0, \quad (49)$$

where

$$\psi_{\Sigma} = \frac{2\mathcal{C}_{\Sigma}\chi\omega\mathcal{A}_{\Sigma}^2 p_{\Sigma}}{\hat{c}} - \frac{\mathcal{C}_{\Sigma}\mathcal{A}_{\Sigma}^2 \bar{p}_{\Sigma}}{\hat{c}(1+2\chi T_{\Sigma})} + \frac{\omega\chi\mathcal{C}_{\Sigma}\mathcal{A}_{\Sigma}^2 T_{\Sigma}}{\hat{c}(1+2\chi T_{\Sigma})} + \frac{\omega\chi^2\mathcal{C}_{\Sigma}\mathcal{A}_{\Sigma}^2 T_{\Sigma}^2}{\hat{c}(1+2\chi T_{\Sigma})^2}. \quad (50)$$

In the above equation, all the terms with subscript Σ are as follows:

$$T_{\Sigma} = T_o(r_{\Sigma}),$$

$$\mathcal{C}_{\Sigma} = \mathcal{C}_o(r_{\Sigma}),$$

$$\mathcal{A}_{\Sigma} = \mathcal{A}_o(r_{\Sigma}),$$

$$\mathcal{B}_{\Sigma} = \mathcal{B}_o(r_{\Sigma}),$$

$$p_{\Sigma} = p_o(r_{\Sigma}),$$

$$\bar{p}_{\Sigma} = \bar{p}(r_{\Sigma}) = (\rho_{\Sigma} + p_{\Sigma}) \left(\frac{2\hat{b}}{\mathcal{B}_{\Sigma}} + \hat{c} \right) + \omega\chi T_{\Sigma},$$

$$\rho_{\Sigma} = \rho_o(r_{\Sigma}).$$

(51)

The solution of (49) is

$$\Delta(t) = a_1 \exp^{\sqrt{\psi_{\Sigma}} t} + a_2 \exp^{-\sqrt{\psi_{\Sigma}} t}. \quad (52)$$

This solution exhibits the stability as well as instability configurations and a_1 and a_2 are constants. Here, to discuss the instability analysis, we take only static solution of cylindrical collapsing star which implies $a_2 = 0$, while $a_1 = -1$. We get

$$\Delta = -\exp^{\sqrt{\psi_{\Sigma}} t}, \quad \psi_{\Sigma} > 0 \quad (53)$$

Putting all the correspondent values in general collapse equation, we attain the collapse equation of cylindrically symmetric object in $f(T)$ gravity as

$$\begin{aligned} & \frac{(\hat{b}/\mathcal{A}_o\mathcal{B}_o)' + (\hat{c}/\mathcal{A}_o)' + (1/\mathcal{C}_o + \mathcal{B}'_o/\mathcal{B}_o)(\hat{c}/\mathcal{A}_o) - (\hat{c}/2\mathcal{C}_o)(1+2\chi T_o)}{8\pi\mathcal{B}_o^2} \frac{\mathcal{B}_o^2}{\mathcal{A}_o} \ddot{\Delta} + J_{2\text{eq}} \\ & + \left[-\mathcal{A}_o \left(\frac{(\hat{b}/\mathcal{A}_o\mathcal{B}_o)' + (\hat{c}/\mathcal{A}_o)' + (1/\mathcal{C}_o + \mathcal{B}'_o/\mathcal{B}_o)(\hat{c}/\mathcal{A}_o) - (\hat{c}/2\mathcal{C}_o)(1+2\chi T_o)}{8\pi\mathcal{B}_o^2} \right)' \right. \\ & - \frac{(\hat{b}/\mathcal{A}_o\mathcal{B}_o)' + (\hat{c}/\mathcal{A}_o)' + (1/\mathcal{C}_o + \mathcal{B}'_o/\mathcal{B}_o)(\hat{c}/\mathcal{A}_o) - (\hat{c}/2\mathcal{C}_o)(1+2\chi T_o)}{8\pi\mathcal{B}_o^2} \mathcal{A}_o \times \left(\frac{2\mathcal{A}'_o}{\mathcal{A}_o} + \frac{\mathcal{B}'_o}{\mathcal{B}_o} + \frac{\mathcal{C}'_o}{\mathcal{C}_o} \right) - (\rho_o \\ & + p_o) \left(\frac{2\hat{b}}{\mathcal{B}_o} + \hat{c} \right) - 2J_{1\text{eq}} - \frac{\Gamma p_o \mathcal{A}_o}{\rho_o + p_o} \\ & \times \left. \left[\left(\frac{(\hat{b}/\mathcal{A}_o\mathcal{B}_o)' + (\hat{c}/\mathcal{A}_o)' + (1/\mathcal{C}_o + \mathcal{B}'_o/\mathcal{B}_o)(\hat{c}/\mathcal{A}_o) - (\hat{c}/2\mathcal{C}_o)(1+2\chi T_o)}{8\pi\mathcal{B}_o^2} \right)' \right. \right. \\ & - \left. \left. \frac{(\hat{b}/\mathcal{A}_o\mathcal{B}_o)' + (\hat{c}/\mathcal{A}_o)' + (1/\mathcal{C}_o + \mathcal{B}'_o/\mathcal{B}_o)(\hat{c}/\mathcal{A}_o) - (\hat{c}/2\mathcal{C}_o)(1+2\chi T_o)}{8\pi\mathcal{B}_o^2} \mathcal{A}_o \times \left(\frac{2\mathcal{A}'_o}{\mathcal{A}_o} + \frac{\mathcal{B}'_o}{\mathcal{B}_o} + \frac{\mathcal{C}'_o}{\mathcal{C}_o} \right) - (\rho_o \right. \right. \\ & \left. \left. + p_o) \left(\frac{2\hat{b}}{\mathcal{B}_o} + \hat{c} \right) \right] \right] \left(\frac{\mathcal{A}'_o}{\mathcal{A}_o} \right) \Delta + \left[\frac{-\Gamma p_o}{\rho_o + p_o} \right. \\ & \times \left. \left[\mathcal{A}_o \left(\frac{(\hat{b}/\mathcal{A}_o\mathcal{B}_o)' + (\hat{c}/\mathcal{A}_o)' + (1/\mathcal{C}_o + \mathcal{B}'_o/\mathcal{B}_o)(\hat{c}/\mathcal{A}_o) - (\hat{c}/2\mathcal{C}_o)(1+2\chi T_o)}{8\pi\mathcal{B}_o^2} \right)' + J_{1\text{eq}} \right. \right. \\ & \left. \left. + \frac{(\hat{b}/\mathcal{A}_o\mathcal{B}_o)' + (\hat{c}/\mathcal{A}_o)' + (1/\mathcal{C}_o + \mathcal{B}'_o/\mathcal{B}_o)(\hat{c}/\mathcal{A}_o) - (\hat{c}/2\mathcal{C}_o)(1+2\chi T_o)}{8\pi\mathcal{B}_o^2} \times \mathcal{A}_o \left(\frac{2\mathcal{A}'_o}{\mathcal{A}_o} + \frac{\mathcal{B}'_o}{\mathcal{B}_o} + \frac{\mathcal{C}'_o}{\mathcal{C}_o} \right) + (\rho_o \right. \right. \end{aligned}$$

$$+ p_o) \left(\frac{2\hat{b}}{\mathcal{B}_0} + \hat{c} \right) \left. \right] \Delta + (\rho_o + p_o) \times \left(\frac{\hat{a}}{\mathcal{A}_0} \right)' \Delta = 0, \quad (54)$$

where Δ is given in (53).

4. Instability Ranges

Now we consider the collapse equation (54) with Newtonian and post-Newtonian regime constraints to figure out the instability ranges of self-gravitating object in $f(T)$ gravity.

4.1. Newtonian Order. The constraints for Newtonian regime are given as follows:

$$\begin{aligned} \mathcal{A}_0 &= 1 = \mathcal{B}_0, \\ \frac{p_o}{\rho_o} &< 1, \\ &\Downarrow \\ \mathcal{A}'_0 &= 0 = \mathcal{B}'_0, \\ \frac{p_o/\rho_o}{1 + p_o/\rho_o} &\longrightarrow 0. \end{aligned} \quad (55)$$

Using these conditions in (54), the collapse equation turns out as

$$\begin{aligned} \Gamma p'_o (2\hat{b} + \hat{c})' &= (\rho_o + p_o) \hat{a}' + \frac{1}{\Delta} J_{2\text{eq}}^N \\ &+ \left(\hat{b}' + \hat{c}' + \frac{\hat{c}}{\mathcal{E}_0} - \frac{\hat{c}\xi T_0}{\mathcal{E}_0} \right) \psi_\Sigma, \end{aligned} \quad (56)$$

which indicates the hydrostatic state of a cylindrically symmetric self-gravitating fluid where $J_{2\text{eq}}^N$ represents those terms in $J_{2\text{eq}}$ which comes through Newtonian approximation. This equation contains the matter, metric, and torsion scalar components which take part to develop the instability ranges. It is noted that we take adiabatic index with positive sign all over the scenario to preserve the variation between gravitational forces, gradient of heat, and pressure components. This system will be unstable if

$$\Gamma < \frac{(\rho_o + p_o) \hat{a}' - e^{-\sqrt{V_\Sigma} t} J_{2\text{eq}}^N + (\hat{b}' + \hat{c}' + \hat{c}/\mathcal{E}_0 - \hat{c}\xi T_0/\mathcal{E}_0) \psi_\Sigma}{p'_o (2\hat{b} + \hat{c})'}. \quad (57)$$

The left hand side of inequality remains positive while the system remains in instable state till this inequality holds. Now we discuss the following three cases.

Case 1. If the term $(\rho_o + p_o) \hat{a}' + (1/\Delta) J_{2\text{eq}}^N + (\hat{b}' + \hat{c}' + \hat{c}/\mathcal{E}_0 - \hat{c}\xi T_0/\mathcal{E}_0) \psi_\Sigma$ is balanced by the term $p'_o (2\hat{b} + \hat{c})'$ in (56), then we obtain $\Gamma = 1$. This leads to the hydrostatic equilibrium

state for this particular case for cylindrically symmetric self-gravitating object.

Case 2. Now if $(\rho_o + p_o) \hat{a}' + (1/\Delta) J_{2\text{eq}}^N + (\hat{b}' + \hat{c}' + \hat{c}/\mathcal{E}_0 - \hat{c}\xi T_0/\mathcal{E}_0) \psi_\Sigma$ is lesser than $p'_o (2\hat{b} + \hat{c})'$, it leads to $\Gamma < 1$ through (56). The constraint on adiabatic index, $0 < \Gamma < 1$, contributes to the instable state of the fluid without collapse.

Case 3. Equation (56) implies that if $p'_o (2\hat{b} + \hat{c})'$ is lesser than $(\rho_o + p_o) \hat{a}' + (1/\Delta) J_{2\text{eq}}^N + (\hat{b}' + \hat{c}' + \hat{c}/\mathcal{E}_0 - \hat{c}\xi T_0/\mathcal{E}_0) \psi_\Sigma$, we get $\Gamma > 1$ which establishes instability ranges for collapse of the self-gravitating cylindrically symmetric object in the framework of $f(T)$ gravity.

It is noted that we may recover the general relativity in Newtonian limit for instability range as $\Gamma < 4/3$.

4.2. Post-Newtonian Order. In the post-Newtonian order, we deal with the dynamics of cylindrical symmetric star as $\mathcal{A}_0 = 1 - m_o/r$, $\mathcal{B}_0 = 1 + m_o/r$. We have taken $\{m/r_0\}$ terms up to first order and neglected all the higher terms. Using these constraints, we obtain some terms of collapse equation (54) as

$$\begin{aligned} \frac{1}{\mathcal{A}_0 \mathcal{B}_0} &= 1, \\ \frac{\mathcal{A}'_0}{\mathcal{A}_0} &= -\frac{m'_o}{r} + \frac{m_o}{r}, \\ \frac{\mathcal{B}'_0}{\mathcal{B}_0} &= +\frac{m'_o}{r} - \frac{m_o}{r}, \\ \mathcal{E}_0 &= r, \\ \frac{\mathcal{E}'_0}{\mathcal{E}_0} &= \frac{1}{r}, \\ \frac{A_0}{B_0^2} &= 1 - \frac{3m_o}{r}, \\ \frac{B_0^2}{A_0} &= 1 + \frac{3m_o}{r}, \\ \frac{1}{\mathcal{A}_0} &= 1 + \frac{m_o}{r}, \\ \frac{1}{\mathcal{B}_0} &= 1 - \frac{m_o}{r}. \end{aligned} \quad (58)$$

Using these values in collapse equation, we obtain an equation representing hydrodynamical state for post-Newtonian order like Newtonian order. The cylindrical system becomes

unstable in post-Newtonian orders if the adiabatic index satisfies the following inequality:

$$\Gamma < \frac{J_{1pN}}{J_{2pN}}, \quad (59)$$

where J_{1pN} and J_{2pN} are the expressions from (53) under post-Newtonian constraints. Following the Newtonian order cases, we can construct the possibilities of hydrodynamical equilibrium as well as instable states.

5. Conclusion

The collapse process happens when, due to some internal or external disturbance, the matter of object becomes unbalanced and to maintain its equilibrium it collapses down and leads to different structures, i.e., white dwarfs, black holes, and stellar groups. Dynamical instability ranges are used for spherically symmetric objects such as galactic halos and globular clusters, while cylindrical symmetry and plates are associated with the postshocked clouds at stellar scale. To study the unstable behavior of spherically symmetric collapsing object due to its own gravity, the adiabatic index is used whose numerical range is less than $4/3$ in GR. Chandrasekhar [18] gave the direction to study and explore the dynamical instability in demonstrating the development and shaping of stellar objects that must be stable against fluctuations. During these processes, the self-gravitating fluid happens to undergo many phases of dynamical activities that remain in hydrostatic equilibrium for a short span.

After this equilibrium state, the system is changed from its initial static phase to perturbed and oscillating phase. It is necessary to study the evolution of the system immediately after its departure from the equilibrium state. In this work, we have analyzed the instability ranges of self-gravitating cylindrically symmetric collapsing object in $f(T)$ gravity. We have taken isotropic matter distribution for the interior metric and have found the important results for interior and exterior regimes. For this purpose, we have calculated physical quantities like torsion tensor and superpotential tensor. The selection of tetrad field is crucial for the framework of $f(T)$ gravity. We have taken nondiagonal tetrad for the interior space-time. Torsion scalar has been formulated with the help of tetrad field, torsion, and superpotential tensor in $f(T)$ gravity.

We have calculated $f(T)$ field equations along with dynamical equations by using Bianchi identities. A power-law model up to quadratic torsion scalar term has been taken to examine the dynamics of collapsing object. Harrison-Wheeler equation (45) uses the relationship between energy density and pressure and the adiabatic index to analyze the dynamical instability ranges of the collapsing object in $f(T)$ gravity. It can be noticed that Newtonian and post-Newtonian conditions, defined in (57) and (59), are used in collapse equations to find the corresponding instability ranges. It is also mentioned that matter inside the cylinder must satisfy some energy conditions. As discussed in [33], the obtained solutions satisfied some energy conditions.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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