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Research Article

Apparent Horizon and Gravitational Thermodynamics of Universe in the Eddington-Born-Infeld Theory

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The thermodynamics of Universe in the Eddington-Born-Infeld (EBI) theory was restudied by utilizing the holographic-style gravitational equations that dominate the dynamics of the cosmical apparent horizon Υ_A and the evolution of Universe. We started in rewriting the EBI action of the Palatini approach into the Bigravity-type action with an extra metric $q_{\mu\nu}$. With the help of the holographic-style dynamical equations, we discussed the property of the cosmical apparent horizon Υ_A including timelike, spacelike, and null characters, which depends on the value of the parameter of state w_m in EBI Universe. The unified first law for the gravitational thermodynamics and the total energy differential for the open system enveloped by Υ_A in EBI Universe were obtained. Finally, applying the positive-heat-out sign convention, we derived the generalized second law of gravitational thermodynamics in EBI Universe.

1. Introduction

Gravitational thermodynamics is quite an interesting question, which has attracted much attention. Recently, many studies have covered both the first and second laws of gravitational thermodynamics for the Friedmann-Robertson-Walker (FRW) Universe with a generic spatial curvature. The inspired work is the first law of thermodynamics for Universe by Cai and Kim [1], which is a part of the effort to seek the connections between thermodynamics and gravity [2] after discovering the black-hole thermodynamics [3, 4]. In [1, 5], Akbar and Cai reversed the formulation by rewriting the Friedmann equations into the heat balance equation and the unified first law of thermodynamics at the cosmical apparent horizon, for General Relativity (GR), Gauss-Bonnet, and Lovelock gravity. The results in [5] were soon generalized to other theories of gravity, such as the scalar-tensor gravity [6], $f(R)$ gravity [7], braneworld scenarios [8–10], generic $f(R,\phi,\nabla_\mu\phi\nabla^\mu\phi)$ gravity [11], and Horava-Lifshitz gravity [12, 13], to construct the effective total energy differentials by the corresponding modified Friedmann equations.

Inspired by the gravitational thermodynamics in these gravitational theories [5–14] and characteristics of the EBI action, we focused on generalizing the results to the EBI gravity. The Eddington-Born-Infeld action (EBI) was proposed in [15], which could mimic the presence of dark energy and dark matter in the expansion of Universe [15–17] so that EBI gravity can be regarded as a candidate for nonparticulate dark matter and dark energy, and it could also modify the Newton-Poisson equation that leads to flat rotation curves for galaxies. Generally, the EBI action is a Palatini-type action where the metric $g_{\mu\nu}$ is not associated with the connection $C^{\lambda}_{\mu\nu}$. However, by defining an extra metric $q_{\mu\nu}$ to satisfy the condition in [15, 18], the EBI action can be rewritten as the Bigravity-type action.

In this paper, we derived the holographic-style dynamical equations and discussed the properties of the cosmical apparent horizon Υ_A in EBI Universe, which rely on the contents inside the cosmical apparent horizon including the matter and the dark energy provided by the cosmological constant Λ and the spacetime self-coupling. Furthermore, we applied the Misner-Sharp energy, the Cai-Kim temperature $\widehat{T}_A$, and the Hawking-Bekenstein entropy S_A to obtain the unified first law for the gravitational thermodynamics and the total energy differential for the open system enveloped by Υ_A

in EBI Universe. Finally, we derived the generalized second law of the nondecreasing entropy $S_{eff}^{(A)}$ enclosed by Υ_A in EBI Universe.

This paper is organized as follows. In Section 2, we reviewed the cosmical apparent horizon and derived the holographic-style dynamical equations in the EBI theory. Then we discussed the properties of the cosmical apparent horizon. In Section 3, the unified first law of gravitational thermodynamics and the Clausius equation on Υ_A for an isochoric process in EBI Universe were discussed. And we derived the total energy differential enclosed by Υ_A in EBI Universe. In Section 4, the generalized second law of gravitational thermodynamics in the EBI Universe was derived. Conclusions and discussion are given in Section 5.

2. Dynamics of the Cosmical Apparent Horizon in Eddington-Born-Infeld Gravity

2.1. Apparent Horizon. Physically, apparent horizons constitute the observable boundary which is the largest boundary of Universe in an instant. Mathematically, apparent horizons are many hypersurfaces where the outward expansion rate $\theta_{(\ell)}$ or the inward expansion rate $\theta_{(n)}$ is equal to zero. In general, the first kind of apparent horizons where $\theta_{(\ell)} = 0$ and $\theta_{(n)} \neq 0$ usually locates near the black holes, and the another kind of apparent horizon where $\theta_{(n)} = 0$ and $\theta_{(\ell)} \neq 0$ appears in the vicinity of the expanding boundary of Universe, called the cosmical apparent horizons. In this paper, we only discussed the cosmical apparent horizon via dynamic equations of Universe and thermodynamic methods.

In order to calculate the apparent horizon of the cosmology, we use the FRW metric to describe the spatially homogeneous and isotropic Universe [1, 19]

$$ds^2 = -dt^2 + \frac{a(t)^2}{1-kr^2}dr^2 + a(t)^2 r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right) \quad (1)$$

where $a(t)$ is the scale factor of the evolution of Universe and the index k denotes the normalized spatial curvature, with $k = \{+1, 0, -1\}$ corresponding to closed, flat, and open Universes, respectively. Using the spherical symmetry, the metric can be rewritten as

$$ds^2 = h_{\alpha\beta}dx^\alpha dx^\beta + \Upsilon^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right) \quad (2)$$

where $h_{\alpha\beta} := \mathrm{diag}[-1, a(t)^2/(1-kr^2)]$ represents the transverse 2-metric spanned by $(x^0 = t, x^1 = r)$ and $\Upsilon := a(t)r$ stands for the astronomical circumference/areal radius. Based on the FRW metric, one can structure the following null tetrad adapted to the spherical symmetry and the null radial flow:

$$\begin{aligned}
\ell^\mu &= \frac{1}{\sqrt{2}}\left(1, \frac{\sqrt{1-kr^2}}{a}, 0, 0\right)\\
n^\mu &= \frac{1}{\sqrt{2}}\left(-1, \frac{\sqrt{1-kr^2}}{a}, 0, 0\right)\\
m^\mu &= \frac{1}{\sqrt{2}\Upsilon}\left(0, 0, 1, \frac{i}{\sin\theta}\right)\\
\overline{m}^\mu &= \frac{1}{\sqrt{2}\Upsilon}\left(0, 0, 1, -\frac{i}{\sin\theta}\right),
\end{aligned} \quad (3)$$

corresponding to the metric signature $(-,+,+,+)$. By calculating the Newman-Penrose spin coefficients $\rho_{NP} := -m^\mu \overline{m}^\nu \nabla_\nu \ell_\mu$ and $\mu_{NP} := \overline{m}^\mu m^\nu \nabla_\nu n_\mu$, the outward expansion rate $\theta_{(\ell)} = -(\rho_{NP} + \overline{\rho}_{NP})$ and the inward expansion rate $\theta_{(n)} = -(\mu_{NP} + \overline{\mu}_{NP})$ are, respectively, given by

$$\begin{aligned}
\theta_{(\ell)} &= \sqrt{2}\left[H + \Upsilon^{-1}\sqrt{1-\frac{k\Upsilon^2}{a^2}}\right]\\
\theta_{(n)} &= \sqrt{2}\left[-H + \Upsilon^{-1}\sqrt{1-\frac{k\Upsilon^2}{a^2}}\right],
\end{aligned} \quad (4)$$

where $H := \dot{a}/a$ is the Hubble parameter of cosmic spatial expansion. The overdot denotes the derivative with respect to the comoving time t.

For the expanding Universe ($H > 0$), the cosmical apparent horizon is given by

$$\Upsilon_A = \frac{1}{\sqrt{H^2 + k/a^2}}, \quad (5)$$

derived from $\theta_{(n)} = 0$ and $\theta_{(\ell)} > 0$ corresponding to the unique marginally inner trapped horizon where $\partial_\mu \Upsilon$ becomes a null vector with $g^{\mu\nu}\partial_\mu \Upsilon \partial_\nu \Upsilon = 0$ [20]. Then one derives the temporal derivative of (5)

$$\dot{\Upsilon}_A = -H\Upsilon_A^3 \left(\dot{H} - \frac{k}{a^2}\right) \quad (6)$$

that is a kinematic equation of the cosmical apparent horizon.

2.2. The Holographic-Style Dynamical Equations in Eddington-Born-Infeld Universe. The action of Eddington-Born-Infeld theory is given by [15–18]

$$\begin{aligned}
S_{EBI}\left(g_{\mu\nu}, C^\mu_{\nu\rho}, \mathscr{L}_m\right) &= \frac{1}{16\pi G}\int d^4x \left\{\sqrt{-g}(R - 2\Lambda)\right.\\
&\left. + \frac{2}{\alpha\ell^2}\sqrt{-\det\left(g_{\mu\nu} - \ell^2 K_{\mu\nu}(C)\right)}\right\} + \int d^4x \sqrt{-g}\mathscr{L}_m,
\end{aligned} \quad (7)$$

where R is the Ricci scalar for the metric $g_{\mu\nu}$ and g represents the determinant of $g_{\mu\nu}$. $K_{\mu\nu}$ is two-order Riemann curvature tensor dependent on the connection $C^\mu_{\nu\rho}$, provided by the Palatini approach. Λ is the cosmological constant and α is an arbitrary constant. G is the gravitational constant and $\mathscr{L}_m$ is the Lagrangian density of matter.

Applying the Bigravity method [18] to replace the connection $C^{\mu}_{\nu\rho}$ by the extra metric $q_{\mu\nu}$ in the EBI theory, the action (7) can be rewritten into the Bigravity-type action

$$S_{EBI} = \frac{1}{16\pi G}\int d^4x\left\{\sqrt{-g}\left(R-2\Lambda\right)+\sqrt{-q}\left(K-2\lambda\right) - \frac{1}{\ell^2}\sqrt{-q}\left(q^{\alpha\beta}g_{\alpha\beta}\right)\right\} + \int d^4x\sqrt{-g}\mathscr{L}_m, \tag{8}$$

where

$$q_{\mu\nu} = -\frac{1}{\alpha}\left(g_{\mu\nu}-\ell^2 K_{\mu\nu}\right). \tag{9}$$

$K_{\mu\nu}$ is the Ricci tensor for the extra metric $q_{\mu\nu}$ and K is the Ricci scalar for the extra metric $q_{\mu\nu}$. λ is a constant ($\lambda \equiv \alpha/\ell^2$) corresponding to $q_{\mu\nu}$ and q is the determinant of $q_{\mu\nu}$. Here, both $g_{\mu\nu}$ and $q_{\mu\nu}$ are innate metrics of spacetime and they are mutually independent. Hence, $(1/\ell^2)\sqrt{-q}(q^{\alpha\beta}g_{\alpha\beta})$ can be regarded as the term from self-coupling of spacetime.

Varying the Bigravity-type action (8) with respect to the metric $g_{\mu\nu}$, we get the field equations [15]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT^{(m)}_{\mu\nu} - \Lambda g_{\mu\nu} - \frac{1}{\ell^2}\frac{\sqrt{-q}}{\sqrt{-g}}g_{\mu\alpha}q^{\alpha\beta}g_{\beta\nu}, \tag{10}$$

where $-(1/\ell^2)(\sqrt{-q}/\sqrt{-g})g_{\mu\alpha}q^{\alpha\beta}g_{\beta\nu}$ is the energy-momentum tensor of the spacetime self-coupling.

The matter content of Universe is construed as the perfect fluid whose the energy-momentum tensor is

$$T^{\mu(m)}_{\nu} = \mathrm{diag}\left[-\rho_m, p_m, p_m, p_m\right] \quad \mathit{with}\ \frac{p_m}{\rho_m} =: w_m, \tag{11}$$

where w_m refers to the Equation-of-State (EoS) parameter of the perfect fluid. In order to study the cosmological property of the EBI Universe, we made $g_{\mu\nu}$ be the FRW metric and assumed the extra metric $q_{\mu\nu}$ [15, 21] as

$$ds_q^2 = -Udt^2 + \frac{a(t)^2 V}{1-kr^2}dr^2 + a(t)^2 Vr^2 d\theta^2 + a(t)^2 Vr^2\sin^2\theta d\varphi^2, \tag{12}$$

where U and V are two undetermined positive functions independent with t.

Depending on the field equation and the two metrics, we get the first Friedmann equation

$$\frac{\dot a^2}{a^2} + \frac{k}{a^2} - \frac{\Lambda}{3} = \frac{8\pi G\rho_m}{3} + \frac{1}{3\ell^2}\sqrt{\frac{V}{U}}V \tag{13}$$

and the second Friedmann equation

$$\frac{\dot a^2}{a^2} + \frac{k}{a^2} + 2\frac{\ddot a}{a} - \Lambda = -8\pi Gp_m + \frac{1}{\ell^2}\sqrt{\frac{V}{U}}U. \tag{14}$$

Eq. (14) can be rewritten into

$$2\dot H + 3H^2 + \frac{k}{a^2} - \Lambda = -8\pi Gp_m + \frac{1}{\ell^2}\sqrt{\frac{V}{U}}U, \tag{15}$$

which is equivalent to

$$\Upsilon_A^{-3}\left(\dot\Upsilon_A - \frac{3}{2}H\Upsilon_A\right) = \left[4\pi Gp_m - \frac{\Lambda}{2} - \frac{1}{2\ell^2}\sqrt{\frac{V}{U}}U\right]H. \tag{16}$$

With the help of (13) and (15), we obtain

$$\dot H - \frac{k}{a^2} = -4\pi G\left(\rho_m + p_m\right) - \frac{1}{2\ell^2}\sqrt{\frac{V}{U}}\left(V-U\right). \tag{17}$$

Based on (5) and (13), we get

$$\Upsilon_A^{-2} = \frac{8\pi G}{3}\rho_m + \frac{\Lambda}{3} + \frac{1}{3\ell^2}\sqrt{\frac{V}{U}}V \tag{18}$$

and substituting (17) into (6), we get

$$\dot\Upsilon_A = H\Upsilon_A^3\left[4\pi G\left(\rho_m + p_m\right) + \frac{1}{2\ell^2}\sqrt{\frac{V}{U}}\left(V-U\right)\right]. \tag{19}$$

Eq. (16), (18), and (19) are the holographic-style dynamical equations of the cosmical apparent horizon [14], which means the evolution of Universe has a relation with the cosmical apparent horizon. If one takes $U = V = 1$ and $\Lambda = 0$, the holographic-style dynamical equations will return to the condition of the Einstein theory.

Furthermore, from (13) and (14), we can obtain the acceleration equation of the EBI Universe

$$\begin{aligned}\frac{\ddot a}{a} &= -\frac{4\pi G}{3}\left(\rho_m + 3p_m\right) + \frac{\Lambda}{3} - \frac{1}{2\ell^2}\sqrt{\frac{V}{U}}\left(\frac{1}{3}V - U\right) \\ &= -4\pi G\rho_m\left[w_m + \frac{1}{3} - \frac{\Lambda}{12\pi G\rho_m} + \frac{1}{8\pi G\rho_m\ell^2}\sqrt{\frac{V}{U}}\left(\frac{1}{3}V - U\right)\right].\end{aligned} \tag{20}$$

2.3. The Characters of the Cosmical Apparent Horizon. In general, the cosmical apparent horizon is not null surface, which is different from the event and particle horizon. The equation of the cosmical apparent horizon in comoving coordinates is [22]

$$\mathscr{F}(t,r) = a(t)r - \frac{1}{\sqrt{H^2 + k/a^2}} = 0. \tag{21}$$

Its normal has components

$$\begin{aligned}N_\mu &= \nabla_\mu\mathscr{F}\Big|_{AH} \\ &= \left\{\left[\dot ar + \frac{H\left(\dot H - k/a^2\right)}{\left(H^2 + k/a^2\right)^{3/2}}\right]\delta_{\mu0} + a\delta_{\mu1}\right\}\Bigg|_{AH} \\ &= H\Upsilon_A\left[1 + \left(\dot H - \frac{k}{a^2}\right)\Upsilon_A^2\right]\delta_{\mu0} + a\delta_{\mu1} \\ &= H\Upsilon_A^3\frac{\ddot a}{a}\delta_{\mu0} + a\delta_{\mu1}.\end{aligned} \tag{22}$$

The norm squared of the normal vector is

$$\begin{aligned} N_a N^a &= 1 - kr_A^2 - H^2\Upsilon_A^6\left(\frac{\ddot{a}}{a}\right)^2 \\ &= H^2\Upsilon_A^2\left[1 - \Upsilon_A^4\left(\frac{\ddot{a}}{a}\right)^2\right] \\ &= H^2\Upsilon_A^6\left(\Upsilon_A^{-2} - \frac{\ddot{a}}{a}\right)\left(\Upsilon_A^{-2} + \frac{\ddot{a}}{a}\right), \end{aligned} \tag{23}$$

where $r_A = \Upsilon_A/a$. Substituting (20) and (18), we get

$$\begin{aligned} N_a N^a &= \mathscr{H}(w_m) = -H^2\Upsilon_A^6(4\pi G\rho_m)^2\left[w_m - \frac{1}{3}\right. \\ &\left. - \frac{\Lambda}{6\pi G\rho_m} - \frac{1}{8\pi G\rho_m\ell^2}\sqrt{\frac{V}{U}}\left(\frac{1}{3}V + U\right)\right]\left[w_m + 1\right. \\ &\left. + \frac{1}{8\pi G\rho_m\ell^2}\sqrt{\frac{V}{U}}(V - U)\right], \end{aligned} \tag{24}$$

where we consider that $N_a N^a$ is only the quadratic function $\mathscr{H}(w_m)$ representing the inner product of the normal vector of the cosmical apparent horizon. The quadratic function $\mathscr{H}(w_m)$ has two zero points, $w_m = 1/3 + \Lambda/6\pi G\rho_m + (1/8\pi G\ell^2\rho_m)\sqrt{V/U}((1/3)V + U)$ and $w_m = -[1 + (1/8\pi G\ell^2\rho_m)\sqrt{V/U}(V - U)]$.

Considering the properties of the quadratic function $\mathscr{H}(w_m)$, we get three results as follows (considering the condition that $\Lambda > -(1/\ell^2)\sqrt{V/U}V - 8\pi G\rho_m$).

(A) When $w_m = 1/3 + \Lambda/6\pi G\rho_m + (1/8\pi G\ell^2\rho_m)\sqrt{V/U}((1/3)V + U)$ or $w_m = -[1 + (1/8\pi G\ell^2\rho_m)\sqrt{V/U}(V - U)]$, $N_a N^a = 0$ that shows the normal vector N^a is a null vector and the apparent horizon Υ_A is a null surface. It coincides with the cosmological event horizon $\Upsilon_E = a\int_t^\infty a^{-1}d\hat{t}$, which is a future-pointed null causal boundary [19, 22]. And it shares the signature of isolated black-hole horizons [23].

(B) When $-[1 + (1/8\pi G\ell^2\rho_m)\sqrt{V/U}(V - U)] < w_m < [1/3 + \Lambda/6\pi G\rho_m + (1/8\pi G\ell^2\rho_m)\sqrt{V/U}((1/3)V + U)]$, $N_a N^a > 0$ that shows N^a is a spacelike vector and Υ_A is the timelike surface. Υ_A has the signature $(-,+,+)$ that shares the signature of a quasi-local timelike membrane in black-hole physics [20, 24].

(C) When $[1/3 + \Lambda/6\pi G\rho_m + (1/8\pi G\ell^2\rho_m)\sqrt{V/U}((1/3)V + U)] < w_m$ or $w_m < -[1 + (1/8\pi G\ell^2\rho_m)\sqrt{V/U}(V - U)]$, $N_a N^a < 0$ that shows N^a is a timelike vector and Υ_A is the spacelike surface. Its signature is $(+,+,+)$ that is same as the signature of the dynamical black-hole horizons [25].

As we know, the present Universe is an accelerated expanding Universe that means the matter outside the cosmical apparent horizon may enter into the cosmical apparent horizon. Hence we considered that the timelike cosmical apparent horizon is reasonable and the range of the EoS parameter $-[1 + (1/8\pi G\ell^2\rho_m)\sqrt{V/U}(V - U)] < w_m < [1/3 + \Lambda/6\pi G\rho_m + (1/8\pi G\ell^2\rho_m)\sqrt{V/U}((1/3)V + U)]$ is significative, which is similar to the range of the EoS parameter $(-1 < w < 1/3)$ in Einstein Universe [14].

3. Thermodynamics of the Holographic-Style Dynamical Equations in the Eddington-Born-Infeld Universe

Based on the holographic-style dynamical equations (18), (19), and (16) in Section 2, we continue to investigate the thermodynamics about the cosmical apparent horizon. Firstly, we define the effective energy within a sphere of radius Υ (surface area $A = 4\pi\Upsilon^2$ and volume $\widehat{V} = (4/3)\pi\Upsilon^3$): $E_{eff} = \rho_{eff}\widehat{V}$, which is the entire energy enveloped by the cosmical apparent horizon Υ_A (take $\widehat{V}$ to represent the volume in order to distinguish the function V).

3.1. Unified First Law of Thermodynamics. Applying the Misner-Sharp mass/energy $E_{MS} := (\Upsilon/2G)(1 - h^{\alpha\beta}\partial_\alpha\Upsilon\partial_\beta\Upsilon)$ [26, 27] to be the effective energy E_{eff} and substituting $h_{\alpha\beta} = \mathrm{diag}[-1, a^2/(1 - kr^2)]$, one obtains

$$dE = -\frac{\dot{\Upsilon}_A}{G}\frac{\Upsilon^3}{\Upsilon_A^3}dt + \frac{3}{2G}\frac{\Upsilon^2}{\Upsilon_A^2}d\Upsilon \tag{25}$$

and

$$dE = -\frac{1}{G}\frac{\Upsilon^3}{\Upsilon_A^3}\left(\dot{\Upsilon}_A - \frac{3}{2}H\Upsilon_A\right)dt + \frac{3}{2G}\frac{\Upsilon^2}{\Upsilon_A^2}a\,dr. \tag{26}$$

For the EBI Universe, substituting (16) and (18) into (25), the total energy differential in the (t, Υ) coordinates is obtained

$$\begin{aligned} dE &= A\left[\rho_m + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}V + \frac{\Lambda}{8\pi G}\right]d\Upsilon \\ &- A\left[(\rho_m + p_m) + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}(V - U)\right] \\ &\cdot H\Upsilon dt, \end{aligned} \tag{27}$$

where $A = 4\pi\Upsilon^2$. Similarly, we can obtain

$$\begin{aligned} dE &= A\left[\rho_m + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}V + \frac{\Lambda}{8\pi G}\right]a\,dr \\ &- A\left[p_m - \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}U - \frac{\Lambda}{8\pi G}\right]\cdot H\Upsilon dt, \end{aligned} \tag{28}$$

which is the total energy differential in the (t, r) coordinates. From the above two equations, we can know that the EoS parameter from spacetime self-coupling w_{ac} is negative, because of the energy density term "$(1/8\pi G\ell^2)\sqrt{V/U}V$" and the intensity of pressure "$-(1/8\pi G\ell^2)\sqrt{V/U}U$".

The unified first law of (equilibrium) thermodynamics is given by

$$dE = A\mathbf{\Psi} + Wd\widehat{V}, \tag{29}$$

proposed by Hayward [28]. W is the work density, given by

$$W := -\frac{1}{2}T_{(m)}^{\alpha\beta}h_{\alpha\beta}, \tag{30}$$

where $h_{\alpha\beta} = \text{diag}[-1, a(t)^2/(1-kr^2)]$. $\mathbf{\Psi}$ is the energy supply covector, $\mathbf{\Psi} = \mathbf{\Psi}_\alpha dx^\alpha$, where

$$\mathbf{\Psi}_\alpha := T^{\beta}_{\alpha(m)}\partial_\beta \Upsilon + W\partial_\alpha \Upsilon. \quad (31)$$

Here, W and $\mathbf{\Psi}_\alpha$ are invariant. Moreover, the definitions of W and $\mathbf{\Psi}_\alpha$ are valid for all spherically symmetric spacetimes and FRW spacetime.

In the EBI theory, the field equations can be rewritten into

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T^{(eff)}_{\mu\nu}, \quad (32)$$

where we define an effective energy-momentum tensor

$$\begin{aligned} T^{(eff)}_{\mu\nu} &= T^{(m)}_{\mu\nu} - \frac{\Lambda}{8\pi G}g_{\mu\nu} - \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}UV \\ &\cdot g_{\mu\alpha}q^{\alpha\beta}g_{\beta\nu}. \end{aligned} \quad (33)$$

From the above equation, we can consider that the effective energy-momentum tensor includes the part of the dark energy corresponding to the terms with Λ and (U, V). Then we can generalize Hayward's unified first law of (equilibrium) thermodynamics into the EBI Universe by taking $T^{(eff)}_{\mu\nu}$ to replace $T^{(m)}_{\mu\nu}$. Imitating the definitions of W and $\mathbf{\Psi}_\alpha$ [28], we define

$$dE = A\widetilde{\mathbf{\Psi}} + \widetilde{W}d\widehat{V} \quad (34)$$

and

$$\widetilde{\mathbf{\Psi}} = \widetilde{\mathbf{\Psi}}_\alpha dx^\alpha, \quad (35)$$

where

$$\widetilde{W} := -\frac{1}{2}T^{\alpha\beta}_{(eff)}h_{\alpha\beta} \quad (36)$$

and

$$\widetilde{\mathbf{\Psi}}_\alpha := T^{\beta}_{\alpha(eff)}\partial_\beta\Upsilon + \widetilde{W}\partial_\alpha\Upsilon. \quad (37)$$

We consider that the FRW metric $g_{\mu\nu}$ is physically subsistent, which is used to raise or descend the index here, and another metric $q_{\mu\nu}$ is an extra metric provided by the primordial mechanism of Universe. Based on this, we get

$$\begin{aligned} T^{\beta}_{\alpha(eff)} &= g^{\mu\beta}T^{(eff)}_{\mu\alpha} \\ &= g^{\mu\beta}T^{(m)}_{\mu\alpha} - \frac{\Lambda}{8\pi G}\delta^\beta_\alpha - \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}UVq^{\beta\mu}g_{\mu\alpha} \end{aligned} \quad (38)$$

and

$$\begin{aligned} T^{\alpha\beta}_{(eff)} &= g^{\mu\alpha}g^{\nu\beta}T^{(eff)}_{\mu\nu} \\ &= g^{\mu\alpha}g^{\nu\beta}T^{(m)}_{\mu\nu} - \frac{\Lambda}{8\pi G}g^{\alpha\beta} - \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}UVq^{\alpha\beta}. \end{aligned} \quad (39)$$

Substituting $h_{\alpha\beta} = \text{diag}[-1, a(t)^2/(1-kr^2)]$, we obtain

$$\begin{aligned} \widetilde{W} &= -\frac{1}{2}\left[T^{00}_{(eff)}h_{00} + T^{11}_{(eff)}h_{11}\right] \\ &= \frac{1}{2}\left(\rho_m - p_m\right) + \frac{\Lambda}{8\pi G} + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}\left(\frac{U+V}{2}\right); \end{aligned} \quad (40)$$

$$\widetilde{\mathbf{\Psi}}_t = -\frac{1}{2}\left[\left(\rho_m + p_m\right) + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}(V-U)\right]H\Upsilon; \quad (41)$$

$$\widetilde{\mathbf{\Psi}}_r = \frac{1}{2}\left[\left(\rho_m + p_m\right) + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}(V-U)\right]a. \quad (42)$$

Substituting $\widetilde{\mathbf{\Psi}}_t$, $\widetilde{\mathbf{\Psi}}_r$, and $\widetilde{W}$ into (34), we get

$$\begin{aligned} dE &= A\widetilde{\mathbf{\Psi}} + \widetilde{W}d\widehat{V} = A\left[\widetilde{\mathbf{\Psi}}_t dt + \widetilde{\mathbf{\Psi}}_r dr + \widetilde{W}d\Upsilon\right] \\ &= A\left[-p_m + \frac{\Lambda}{8\pi G} + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}U\right]H\Upsilon dt \\ &+ A\left[\rho_m + \frac{\Lambda}{8\pi G} + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}V\right]adr, \end{aligned} \quad (43)$$

which is the expression of dE in (t, r) coordinates.

Naturally, because of the invariance of $\widetilde{W}$ and $\widetilde{\mathbf{\Psi}}$, we can rewrite these in the (t, Υ) coordinates, given by

$$\begin{aligned} dE &= A\widetilde{\mathbf{\Psi}} + \widetilde{W}d\widehat{V} = A\left[\widetilde{\mathbf{\Psi}}'_t dt + \widetilde{\mathbf{\Psi}}'_\Upsilon d\Upsilon + \widetilde{W}d\Upsilon\right] \\ &= -A\left[\left(\rho_m + p_m\right) + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}(V-U)\right]H\Upsilon dt \\ &+ A\left[\rho_m + \frac{\Lambda}{8\pi G} + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}V\right]d\Upsilon, \end{aligned} \quad (44)$$

where

$$\widetilde{\mathbf{\Psi}}'_t = -\left[\left(\rho_m + p_m\right) + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}(V-U)\right]H\Upsilon \quad (45)$$

and

$$\widetilde{\mathbf{\Psi}}'_\Upsilon = \frac{1}{2}\left[\left(\rho_m + p_m\right) + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}(V-U)\right]. \quad (46)$$

It illustrates our hypothesis of replacing $T^{(m)}_{\mu\nu}$ by $T^{(eff)}_{\mu\nu}$ is reasonable that (43) and (44) are, respectively, identical to (28) and (27). The unified first law for gravitational thermodynamics of Universe is totally different from the first law in the black-hole thermodynamics [14]. Eq. (43) and (44) are both called "the unified first law" for the EBI Universe's gravitational thermodynamics.

3.2. Clausius Equation on the Cosmical Apparent Horizon for an Isochoric Process. Having obtained the unified first law $dE = A\widetilde{\mathbf{\Psi}} + \widetilde{W}d\widehat{V}$ in EBI Universe, we are interested in the region enclosed by the cosmical apparent horizon Υ_A.

Eq. (19) leads to

$$\begin{aligned}\frac{\dot{\Upsilon}_A}{G}dt &= A_A\left[(\rho_m + p_m) + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}(V-U)\right]H\Upsilon_A \\ &\quad\cdot dt,\end{aligned} \tag{47}$$

where $A_A = 4\pi\Upsilon_A^2$. The left-hand side of (47) can be manipulated into [14]

$$\begin{aligned}\frac{\dot{\Upsilon}_A}{G}dt &= \frac{1}{2\pi\Upsilon_A}\cdot\left(\frac{2\pi\Upsilon_A\dot{\Upsilon}_A}{G}\cdot dt\right) \\ &= \frac{1}{2\pi\Upsilon_A}d\left(\frac{\pi\Upsilon_A^2}{G}\right).\end{aligned} \tag{48}$$

One applies the geometrically defined Hawking-Bekenstein entropy [4, 29] (in the units $\hbar = c = k[\textit{Boltzmann constant}] = 1$)

$$S_A = \frac{\pi\Upsilon_A^2}{G} = \frac{A_A}{4G}, \tag{49}$$

and the Cai-Kim temperature [1, 30]

$$\widehat{T}_A \equiv \frac{1}{2\pi\Upsilon_A}, \tag{50}$$

at the cosmical apparent horizon to simplify (48), given by $(\dot{\Upsilon}_A/G)dt = \widehat{T}_A dS_A$ (take "$\widehat{T}$" on behalf of temperature not only the Cai-Kim temperature). With the help of (45), we get $-[(\rho_m + p_m) + (1/8\pi G\ell^2)\sqrt{V/U}(V-U)]H\Upsilon_A = \widetilde{\mathbf{\Psi}}'_t|_{(\Upsilon=\Upsilon_A)} \equiv \widetilde{\mathbf{\Psi}}'_{tA}$. From (44) and (47), we obtain

$$\delta Q_A = \widehat{T}_A dS_A = -A_A\widetilde{\mathbf{\Psi}}'_{tA} = -dE_A\big|_{d\Upsilon=0}, \tag{51}$$

where dE_A is the specific condition of (44) when $\Upsilon = \Upsilon_A$. Eq. (51) is actually the Clausius equation for equilibrium and reversible thermodynamic processes as same as the situation in GR [14]. After considering the EBI theory, the Clausius equation is generalized to include dark energy from the cosmological constant Λ and the effect the self-coupling of spacetime (U, V), which may explain the problems about the cosmic expansion.

Finally, for the open system enveloped by Υ_A, we substitute the unified first law (44) and the Clausius equation (51) into the total energy differential

$$dE_A = -\widehat{T}_A dS_A + \left[\rho_m + \frac{\Lambda}{8\pi G} + \frac{1}{8\pi G\ell^2}\sqrt{\frac{V}{U}}V\right]d\widehat{V}_A, \tag{52}$$

where the "-" sign shows the positive-heat-out sign convention that means heat emitted by the open system takes positive values ($\delta Q_A = \delta Q_A^{(out)} > 0$) rather than the traditional positive-heat-in thermodynamic sign convention [14].

Comparing (52) with the total energy differential of the Einstein gravity, there are two extra terms $(\Lambda/8\pi G)d\widehat{V}_A$ and $(1/8\pi G\ell^2)\sqrt{V/U}Vd\widehat{V}_A$, which originate from dark energy of the cosmological constant and the spacetime self-coupling, respectively. It means that there are the matter's transition and the dark energy's fluxion on both sides of the cosmical apparent horizon during the expansion of Universe.

4. Generalized Second Laws of Thermodynamics in Eddington-Born-Infeld Universe

With the help of the first holographic-style dynamical equation (18), the effective energy ($E_{MS} = \Upsilon^3/2G\Upsilon_A^2$) can be rewritten into

$$E_{MS} = \rho_{eff}\left(\frac{4}{3}\pi\Upsilon^3\right) = \rho_{eff}\widehat{V}, \tag{53}$$

where $\rho_{eff} = \rho_m + \Lambda/8\pi G + (1/8\pi G\ell^2)\sqrt{V/U}V$. Then we can rewrite the Friedmann equations into

$$\frac{\dot{a}}{a} + \frac{k}{a} = \frac{8\pi G\rho_{eff}}{3} \tag{54}$$

and

$$\frac{\dot{a}}{a} + \frac{k}{a} + 2\frac{\ddot{a}}{a} = -8\pi G p_{eff}, \tag{55}$$

where we define that $p_{eff} = p_m - \Lambda/8\pi G - (1/8\pi G\ell^2)\sqrt{V/U}U$. Based on two above equations, one can obtain the continuity equation in the EBI theory

$$\dot{\rho}_{eff} + 3\frac{\dot{a}}{a}\left(\rho_{eff} + p_{eff}\right) = 0. \tag{56}$$

In many papers [31–33], the entropy S_m of the cosmic energy-matter content with temperature $\widehat{T}_m$ is always determined by the traditional Gibbs equation $dE = \widehat{T}_m dS_m - p_m d\widehat{V}$. In order to generalize to the EBI theory, we keep the same form and redefine it into the positive-heat-out sign convention for consistency with the horizon entropy S_A [14], given by

$$dE_{eff} = -\widehat{T}_{eff}dS_{eff} - p_{eff}d\widehat{V}. \tag{57}$$

From $dE_{eff} = \rho_{eff}d\widehat{V} + \widehat{V}d\rho_{eff}$, we obtain

$$\widehat{T}_{eff}dS_{eff} = -\widehat{V}d\rho_{eff} - \left(\rho_{eff} + p_{eff}\right)d\widehat{V}. \tag{58}$$

Based on the continuity equation (56), we obtain

$$d\rho_{eff} = -3H\left(\rho_{eff} + p_{eff}\right)dt. \tag{59}$$

When $\Upsilon = \Upsilon_A$, one can get

$$\widehat{T}_{eff}dS_{eff}^{(A)} = A_A\left(\rho_{eff} + p_{eff}\right)\cdot\left(H\Upsilon_A - \dot{\Upsilon}_A\right)dt, \tag{60}$$

where $A_A = 3/2G\rho_{eff}$. With the help of (19), (60) yields

$$\dot{S}_{eff}^{(A)} = -\frac{9}{4G}\cdot\frac{H\Upsilon_A}{\widehat{T}_{eff}} \cdot \frac{1}{\rho_{eff}^2}\left(\rho_{eff}+p_{eff}\right)\left(\frac{1}{3}\rho_{eff}+p_{eff}\right) \tag{61}$$

that is the evolution of the effective inner entropy enclosed by Υ_A.

In order to discuss $\dot{S}_{eff}^{(A)}$ more concretely, we assume that $\rho_{aux} \equiv \rho_m + (1/8\pi G\ell^2)\sqrt{V/U}V$ and $p_{aux} \equiv p_m - (1/8\pi G\ell^2)\sqrt{V/U}U$, which are two auxiliary parameters. Following $p_m = w_m\rho_m$, we set

$$p_{eff} = \varepsilon_{eff}\rho_{eff} \tag{62}$$

and

$$p_{aux} = \sigma_{aux}\rho_{aux}, \tag{63}$$

where ε_{eff} and σ_{aux} are the parameters of state like w_m. And (61) can be simplified as

$$\dot{S}_{eff}^{(A)} = -\frac{9}{4G}\cdot\frac{H\Upsilon_A}{\widehat{T}_{eff}}\cdot\left(\varepsilon_{eff}+1\right)\left(\varepsilon_{eff}+\frac{1}{3}\right). \tag{64}$$

Based on the above assumptions, we can obtain

$$\sigma_{aux} = w_m + \frac{\sqrt{V/U}\left(w_m V + U\right)}{8\pi G\ell^2\rho_m + \sqrt{V/U}V} \tag{65}$$

and

$$\varepsilon_{eff} = \sigma_{aux} - \left(\sigma_{aux}+1\right)\frac{\Lambda}{8\pi G\rho_{aux}+\Lambda}. \tag{66}$$

Then we obtain

$$\varepsilon_{eff} = \frac{8\pi G\ell^2\rho_m}{8\pi G\ell^2\rho_m + \sqrt{V/U}V + \Lambda\ell^2}\cdot w_m - \frac{\sqrt{V/U}U + \Lambda\ell^2}{8\pi G\ell^2\rho_m + \sqrt{V/U}V + \Lambda\ell^2}. \tag{67}$$

As a result, $\dot{S}_{eff}^{(A)}$ can be rewritten into

$$\begin{aligned}\dot{S}_{eff}^{(A)} &= -\frac{9}{4G}\cdot\frac{H\Upsilon_A}{\widehat{T}_{eff}}\cdot\left(\frac{8\pi G\ell^2\rho_m}{8\pi G\ell^2\rho_m + \sqrt{V/U}V + \Lambda\ell^2}\right)^2 \\ &\cdot\left[w_m + 1 + \frac{\sqrt{V/U}\left(V-U\right)}{8\pi G\ell^2\rho_m}\right] \\ &\cdot\left[w_m + \frac{1}{3} + \frac{\sqrt{V/U}\left((1/3)V-U\right)}{8\pi G\ell^2\rho_m} - \frac{\Lambda}{12\pi G\rho_m}\right].\end{aligned} \tag{68}$$

Physically, temperature is positive ($\widehat{T}_{eff} > 0$) and the present Universe is expanding ($H > 0$).

If Λ satisfies the precondition ($\Lambda > -(1/\ell^2)\sqrt{V/U}V - 8\pi G\rho_m$) shown in Section 2, we can exactly obtain that $[1 + \sqrt{V/U}(V-U)/8\pi G\ell^2\rho_m] > [1/3 + \sqrt{V/U}((1/3)V - U)/8\pi G\ell^2\rho_m - \Lambda/12\pi G\rho_m]$. And (68) illustrates that

(A) when $-[1+\sqrt{V/U}(V-U)/8\pi G\ell^2\rho_m] < w_m < -[1/3+\sqrt{V/U}((1/3)V-U)/8\pi G\ell^2\rho_m - \Lambda/12\pi G\rho_m]$, $\dot{S}_{eff}^{(A)} > 0$;

(B) when $-[1+\sqrt{V/U}(V-U)/8\pi G\ell^2\rho_m] > w_m$ or $-[1/3+\sqrt{V/U}((1/3)V - U)/8\pi G\ell^2\rho_m - \Lambda/12\pi G\rho_m] < w_m$, $\dot{S}_{eff}^{(A)} < 0$;

(C) when $w_m = -[1+\sqrt{V/U}(V-U)/8\pi G\ell^2\rho_m]$ or $-[1/3+\sqrt{V/U}((1/3)V-U)/8\pi G\ell^2\rho_m - \Lambda/12\pi G\rho_m]$, $\dot{S}_{eff}^{(A)} = 0$.

On the other hand, from the acceleration equation of the EBI Universe (20), we know that when $w_m < -[1/3 - \Lambda/12\pi G\rho_m + \sqrt{V/U}((1/3)V - U)/8\pi G\ell^2\rho_m]$, the present Universe is an accelerated expanding Universe. Hence, if $V < 3U - 8\pi G\ell^2\rho_m\sqrt{U/V}$, w_m has the possibility to be a positive number to produce an accelerated expanding Universe, which is different from the result of Einstein Universe [14].

In a word, the physical effective entropy $S_{eff}^{(A)}$ inside the cosmical apparent horizon satisfies $\dot{S}_{eff}^{(A)} > 0$ for the stage of accelerated expansion ($\ddot{a} > 0$) when $-[1 + \sqrt{V/U}(V-U)/8\pi G\ell^2\rho_m] < w_m < -[1/3 + \sqrt{V/U}((1/3)V - U)/8\pi G\ell^2\rho_m - \Lambda/12\pi G\rho_m]$. Noteworthily, $S_{eff}^{(A)}$ is composed of the matter's entropy and the dark energy's entropy.

5. Conclusions and Discussion

In this paper, we obtained the gravitational dynamics in the EBI Universe. Firstly, we derived the holographic-style dynamical equations. Because of the present accelerated expanding Universe that means the outer matter can enter into the cosmical apparent horizon, we considered the timelike cosmical apparent horizon is reasonable and $-[1 + (1/8\pi G\ell^2\rho_m)\sqrt{V/U}(V-U)] < w_m < [1/3 + \Lambda/6\pi G\rho_m + (1/8\pi G\ell^2\rho_m)\sqrt{V/U}((1/3)V + U)]$ is a rational range of the matter's EoS parameter.

Secondly, based on the holographic-style dynamical equations, we obtained two forms of the total energy differential in the (t,Υ) coordinates and (t,r) coordinates. And we proved that these two forms of the total energy differential can be derived from the unified first laws of the gravitational dynamics $dE = A\mathbf{\Psi}+\widetilde{W}d\widetilde{V}$ by redefining the effective energy-momentum tensor $T_{\mu\nu}^{(eff)}$ in Hayward's approach [28].

Thirdly, we derived the total energy differential for the open system enveloped by Υ_A, $dE_A = -\widehat{T}_A dS_A + [\rho_m + \Lambda/8\pi G + (1/8\pi G\ell^2)\sqrt{V/U}V]d\widehat{V}_A$, where the two extra terms $(\Lambda/8\pi G)d\widehat{V}_A$ and $(1/8\pi G\ell^2)\sqrt{V/U}Vd\widehat{V}_A$ are, respectively, corresponding to the dark energy and the spacetime self-coupling in EBI Universe. It illustrates that not only the matter's transition but also the dark energy's fluxion arises on the cosmical apparent horizon Υ_A with the expansion of Universe.

Finally, we discussed the properties of the effective entropy $S_{eff}^{(A)}$ enclosed by the cosmical apparent horizon Υ_A in EBI Universe. The results show that when $-[1 + \sqrt{V/U}(V - U)/8\pi G\ell^2\rho_m] < w_m < -[1/3 + \sqrt{V/U}((1/3)V - U)/8\pi G\ell^2\rho_m - \Lambda/12\pi G\rho_m]$ and $\ddot{a} > 0$, the generalized second law of the nondecreasing entropy $S_{eff}^{(A)}$ is obtained. If V satisfies the condition $V < 3U - 8\pi G\ell^2\rho_m\sqrt{U/V}$, w_m can be a positive number to generate an accelerated expanding Universe, which is different from the results in Einstein Universe.

In addition to all the above we would like to point out that the method of the theories we mentioned in our paper satisfying the equilibrium thermodynamics does not mean this method fits all possible theories. Actually, nonequilibrium thermodynamics might be also a good way in finding new gravitational theories [34–36], which would be a further task for us to study.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

[1] R. G. Cai and S. P. Kim, "First law of thermodynamics and Friedmann equations of Friedmann-Robertson-Walker universe," *Journal of High Energy Physics*, vol. 2005, no. 02, p. 050, 2005.

[2] T. Padmanabhan, "Thermodynamical aspects of gravity: new insights," *Reports on Progress in Physics*, vol. 73, no. 4, Article ID 046901, 2010.

[3] S. W. Hawking, "Black hole explosions?" *Nature*, vol. 248, no. 5443, pp. 30-31, 1974.

[4] J. D. Bekenstein, "Black holes and entropy," *Physical Review D*, vol. 7, pp. 2333–2346, 1973.

[5] M. Akbar and R.-G. Cai, "Thermodynamic behavior of the friedmann equation at the apparent horizon of the frw universe," *Physical Review D*, vol. 75, Article ID 084003, 2007.

[6] R.-G. Cai and L.-M. Cao, "Unified first law and the thermodynamics of the apparent horizon in the frw universe," *Physical Review D*, vol. 75, Article ID 064008, 2007.

[7] M. Akbar and R.-G. Cai, "Thermodynamic behavior of field equations for $f(R)$ gravity," *Physics Letters B*, vol. 648, no. 2, pp. 243–248, 2007.

[8] R. G. Cai and L. M. Cao, "Thermodynamics of apparent horizon in brane world scenario," *Nuclear Physics B*, vol. 785, no. 1, pp. 135–148, 2007.

[9] A. Sheykhi, B. Wang, and R.-G. Cai, "Thermodynamical properties of apparent horizon in warped DGP braneworld," *Nuclear Physics B*, vol. 779, no. 1, pp. 1–12, 2007.

[10] A. Sheykhi, B. Wang, and R.-G. Cai, "Deep connection between thermodynamics and gravity in Gauss-Bonnet braneworlds," *Physical Review D*, vol. 76, no. 2, Article ID 023515, 2007.

[11] K. Bamba, C.-Q. Geng, and S. Tsujikawa, "Equilibrium thermodynamics in modified gravitational theories," *Physics Letters B*, vol. 688, no. 1, pp. 101–109, 2010.

[12] R.-G. Cai and N. Ohta, "Horizon thermodynamics and gravitational field equations in Hŏrava-Lifshitzgravity," *Physical Review D*, vol. 81, no. 8, Article ID 084061, 2010.

[13] Q.-J. Cao, Y.-X. Chen, and K.-N. Shao, "Clausius relation and Friedmann equation in FRW universe model," *Journal of Cosmology and Astroparticle Physics*, vol. 2010, no. 5, p. 030, 2010.

[14] D. W. Tian and I. Booth, "Apparent horizon and gravitational thermodynamics of the universe: solutions to the temperature and entropy confusions and extensions to modified gravity," *Physical Review D*, vol. 92, Article ID 024001, 2015.

[15] M. Banados, "Eddington-Born-Infeld action for dark matter and dark energy," *Physical Review D*, vol. 77, Article ID 123534, 2008.

[16] A. De Felice, B. Gumjudpai, and S. Jhingan, "Cosmological constraints for an Eddington-Born-Infeld field," *Physical Review D*, vol. 86, no. 4, Article ID 043525, 2012.

[17] M. Bañados, P. G. Ferreira, and C. Skordis, "Eddington-born-infeld gravity and the large scale structure of the universe," *Physical Review D*, vol. 79, no. 6, Article ID 063511, 2009.

[18] M. Bañados, A. Gomberoff, D. C. Rodrigues, and C. Skordis, "Note on bigravity and dark matter," *Physical Review D*, vol. 79, no. 6, Article ID 063515, 2009.

[19] D. Bak and S.-J. Rey, "Cosmic holography," *Classical and Quantum Gravity*, vol. 17, no. 15, p. L83, 2000.

[20] S. A. Hayward, "General laws of black-hole dynamics," *Physical Review D*, vol. 49, pp. 6467–6474, 1994.

[21] M. Bañados and P. G. Ferreira, "Eddington's theory of gravity and its progeny," *Physical Review Letters*, vol. 105, no. 1-2, Article ID 011101, 2010.

[22] V. Faraoni, "Cosmological apparent and trapping horizons," *Physical Review D*, vol. 84, no. 2, Article ID 024003, 2011.

[23] A. Ashtekar, S. Fairhurst, and B. Krishnan, "Isolated horizons: hamiltonian evolution and the first law," *Physical Review D*, vol. 62, Article ID 104025, 2000.

[24] I. Booth, "Black-hole boundaries," *Canadian Journal of Physics*, vol. 83, no. 11, pp. 1073–1099, 2005.

[25] A. Ashtekar and B. Krishnan, "Dynamical horizons: energy, angular momentum, fluxes, and balance laws," *Physical Review Letters*, vol. 89, no. 26, 261101, 4 pages, 2002.

[26] C. W. Misner and D. H. Sharp, "Relativistic equations for adiabatic, spherically symmetric gravitational collapse," *Physical Review A*, vol. 136, no. 2, pp. B571–B576, 1964.

[27] S. A. Hayward, "Gravitational energy in spherical symmetry," *Physical Review D*, vol. 53, no. 4, pp. 1938–1949, 1996.

[28] S. A. Hayward, "Unified first law of black-hole dynamics and relativistic thermodynamics," *Classical and Quantum Gravity*, vol. 15, no. 10, pp. 3147–3162, 1998.

[29] J. M. Bardeen, B. Carter, and S. W. Hawking, "The four laws of black hole mechanics," *Communications in Mathematical Physics*, vol. 31, pp. 161–170, 1973.

[30] R.-G. Cai, L.-M. Cao, and Y.-P. Hu, "Hawking radiation of an apparent horizon in a FRW universe," *Classical and Quantum Gravity*, vol. 26, no. 15, Article ID 155018, 2009.

[31] R. Brustein, "Generalized second law in cosmology from causal boundary entropy," *Physical Review Letters*, vol. 84, no. 10, pp. 2072–2075, 2000.

[32] M. R. Setare, "Generalized second law of thermodynamics in quintom dominated universe," *Physics Letters B*, vol. 641, no. 2, pp. 130–133, 2006.

[33] A. Abdolmaleki, T. Najafi, and K. Karami, "Generalized second law of thermodynamics in scalar-tensor gravity," *Physical Review D*, vol. 89, no. 10, Article ID 104041, 2014.

[34] C. Eling, R. Guedens, and T. Jacobson, "Nonequilibrium thermodynamics of spacetime," *Physical Review Letters*, vol. 96, no. 12, Article ID 121301, 2006.

[35] G. Chirco, C. Eling, and S. Liberati, "Reversible and irreversible spacetime thermodynamics for general Brans-Dicke theories," *Physical Review D*, vol. 83, no. 2, Article ID 024032, 2011.

[36] G. Chirco and S. Liberati, "Nonequilibrium thermodynamics of spacetime: The role of gravitational dissipation," *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 81, no. 2, Article ID 024016, 2010.