

## Research Article

# A Cylindrically Symmetric and Static Anisotropic Fluid Spacetime and the Naked Singularity

Faizuddin Ahmed <sup>1</sup> and Farook Rahaman<sup>2</sup>

<sup>1</sup>Ajmal College of Arts and Science, Dhubri, Assam 783324, India

<sup>2</sup>Department of Mathematics, Jadavpur University, Kolkata, West Bengal 700032, India

Correspondence should be addressed to Faizuddin Ahmed; faizuddinahmed15@gmail.com

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A cylindrically symmetric and static solution of Einstein's field equations was presented. The spacetime is conformally flat and regular everywhere except on the symmetry axis where it possesses a naked curvature singularity. The matter-energy source anisotropic fluids violate the weak energy condition (WEC) and diverge on the symmetry axis. We discuss geodesics motion of free test-particles near to the singularity, geodesic expansion in the metric to understand the nature of singularity which is naked or covered, and finally the C-energy of the spacetime.

## 1. Introduction

Investigation of the nature of singularities in gravitational collapse solution of Einstein's field equations in different systems is of particular interest in general relativity. The study of gravitational collapse in spherically symmetric spacetime has led to many examples of naked singularities (e.g., [1, 2], see also [3, 4] and references therein). In addition, cylindrical symmetric system has great interest because of one degree of freedom for gravitational waves. Therefore, gravitational collapse in cylindrical symmetric system is the simplest one which contains both collapsing body and gravitational waves. A small sample of gravitational collapse model in this system with naked singularities is [5–18]. Some other nonspherical gravitational collapse model also possesses a naked singularity (e.g., [19–21]). Long back, Thorne proposed a hoop conjecture [22] concerning the formation of black holes in cylindrical symmetric system. However, the general proof of this conjecture has not yet been known.

In this article, a cylindrical symmetric and static solution of the field equations possessing a naked curvature singularity on the symmetry axis, satisfying the strong curvature condition, will be presented. Here we have relaxed the Darmois conditions, and the presented solution is a special case of

the very well-known static, cylindrically symmetric, and conformally flat solutions [23] (see also, [24]).

## 2. Conformally Flat Nonvacuum Spacetime

Consider the following static and cylindrical symmetric spacetime ( $x^0 = t$ ,  $x^1 = r$ ,  $x^2 = \phi$ ,  $x^3 = z$ ) given by

$$ds^2 = -\sinh^2 r dt^2 + \cosh^2 r dr^2 + \sinh^2 r d\phi^2 + \sinh^2 r dz^2, \quad (1)$$

where we have chosen the speed of light  $c = 1$ , Planck's constant  $h = 1$ , and the gravitational constant  $G = 1$  such that individual term in the line element has same dimension. The ranges of the coordinates are

$$\begin{aligned} -\infty < t < \infty, \\ 0 \leq r < \infty, \\ \phi \in [0, 2\pi), \\ -\infty < z < \infty. \end{aligned} \quad (2)$$

The metric has signature +2 and its determinant

$$\det g = -\sinh^6 r \cosh^2 r \quad (3)$$

degenerates on the symmetry axis at  $r = 0$ . The presented spacetime (1) is a nonvacuum solution of the field equations and the nonzero components of the Einstein tensor are

$$G_0^0 = \frac{1}{3}G_1^1 = G_2^2 = G_3^3 = \text{csch}^2 r. \quad (4)$$

The Ricci scalar (curvature scalar)  $R$  of the metric (1) is

$$R_\mu^\mu = R = -6 \text{csch}^2 r. \quad (5)$$

And Kretschmann's curvature scalar is

$$K = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = 12 \text{csch}^4 r. \quad (6)$$

From above, it is clear that Kretschmann's curvature scalar ( $K$ ) diverges on the symmetry axis at  $r = 0$  and vanishes rapidly at spatial infinity  $r \rightarrow \infty$  along the radial direction. Since the curvature singularity occurs without an event horizon, the presented solution does not represent a black hole. Therefore, the singularity which is formed due to scalar curvature in a region noncovered by an event horizon is naked.

The presented spacetime (1) satisfies the following conditions:

(i) *The Existence of an Axially Symmetric Axis* [25–27]. The spacetime that has an axially symmetric axis is assured by the condition,

$$\mathbf{X} = \|\partial_\phi\|^2 = |g(\partial_\phi, \partial_\phi)| = |g_{\phi\phi}| = |\sinh^2 r| \rightarrow 0, \quad (7)$$

as  $r \rightarrow 0^+$ , where we have chosen the radial coordinate such that the axis is located at  $r = 0$ . Here  $\partial_\phi$  is a spacelike Killing vector fields, the generator of axial symmetry along the cylinder whose orbit is closed.

(ii) *The Elementary Flatness on the Axis* [26–29]. This condition implies that the spacetime is locally flat on the axis, which can be expressed as

$$\frac{(\nabla_\mu \mathbf{X})(\nabla^\mu \mathbf{X})}{4\mathbf{X}} \rightarrow 1, \quad (8)$$

as  $r \rightarrow 0^+$ . These two conditions ensure the cylindrical symmetry of a spacetime, and  $\phi$  denotes the axial coordinate with the hypersurfaces  $\phi = 0$  and  $\phi = 2\pi$  being identical.

(iii) *No Closed Timelike Curves (CTCs)*. We shall consider that there is no possibility of occurring closed timelike curves and simply require that

$$g_{\phi\phi} > 0 \quad (9)$$

holds in all the region of the spacetime considered.

The Einstein field equations (taking cosmological constant  $\Lambda = 0$ ) are given by

$$G^{\mu\nu} = \kappa T^{\mu\nu}, \quad \mu, \nu = 0, 1, 2, 3, \quad (10)$$

where  $\kappa = 8\pi$  and  $T^{\mu\nu}$  is the energy-momentum tensor.

For the presented metric, we consider the following energy-momentum tensor:

$$T_{\mu\nu} = (\rho + p_\phi)U_\mu U_\nu + p_\phi g_{\mu\nu} + (p_r - p_\phi)\eta_\mu \eta_\nu + (p_z - p_\phi)\zeta_\mu \zeta_\nu, \quad (11)$$

where  $\rho$  is the energy density and  $p_r$ ,  $p_\phi$ , and  $p_z$  are the principal stresses. Here  $U^\mu$  is the timelike unit four-velocity vector;  $\eta_\mu$  and  $\zeta_\mu$  are the spacelike four vectors along the spatial directions  $r$  and  $z$ , respectively, satisfying

$$\begin{aligned} U^\mu U_\mu &= -1, \\ \eta_\mu \eta^\mu &= 1 = \zeta_\mu \zeta^\mu, \\ \eta_\mu U^\mu &= \zeta_\mu U^\mu = \eta_\mu \zeta^\mu = 0. \end{aligned} \quad (12)$$

For static fluid distributions, from the spacetime (1), we have

$$\begin{aligned} U_\mu &= -\sinh r \delta_\mu^0, \\ \eta_\mu &= \cosh r \delta_\mu^1, \\ \zeta_\mu &= \sinh r \delta_\mu^3. \end{aligned} \quad (13)$$

Therefore, the nonzero components of the energy-momentum tensor from (11) using (13) are

$$\begin{aligned} T_0^0 &= -\rho, \\ T_1^1 &= p_r, \\ T_2^2 &= p_\phi, \\ T_3^3 &= p_z, \end{aligned} \quad (14)$$

and the trace of the energy-momentum tensor is

$$T_\mu^\mu = T = -\rho + p_r + p_\phi + p_z. \quad (15)$$

Equating the nonzero components of the field equations (10) using (4) and (14), one will get

$$\begin{aligned} -\kappa\rho &= \text{csch}^2 r, \\ \kappa p_r &= 3 \text{csch}^2 r, \\ \kappa p_\phi &= \kappa p_z = \text{csch}^2 r, \end{aligned} \quad (16)$$

where  $\kappa = 8\pi$ .

From above, it is clear that the energy density of anisotropic fluids ( $p_\phi = p_z = p_t$  tangential stress) violates the weak energy condition (WEC) [30]. In addition, the physical parameters  $\rho$ ,  $p_r$ , and  $p_t$  are singular (diverge) on the

symmetry axis at  $r = 0$  and vanish rapidly at spatial infinity  $r \rightarrow \infty$ .

The presented cylindrically symmetric spacetime (1) is conformally flat since the Weyl tensor vanishes, that is,  $C_{\mu\nu\rho\sigma} = 0$ , and static. The general integration of the conformally flat condition, for static and cylindrically symmetric case with anisotropic fluids, was obtained by Herrera et al. [23]. The authors there show that any conformally flat and cylindrically symmetric static source cannot be matched through Darmois conditions to the Levi-Civita spacetime, satisfying the regularity conditions. Furthermore, all static, cylindrical symmetric solutions (conformally flat or not) for anisotropic fluids have been found in [24]. In the present work, we have relaxed the Darmois conditions and the solution is a special case of the solutions [23] (Section 4), and also a member of the general solutions exhibited in [24] (Section 9.2).

We considered static fluid distributions; there are only three kinematic variables, namely, the *expansion*  $\Theta$ , the *acceleration* vector  $\dot{U}^\mu$ , and the *shear tensor*  $\sigma_{\mu\nu}$  associated with the fluid four-velocity vector which are defined by

$$\begin{aligned}\Theta &= U^\mu_{;\mu}, \\ a^\mu &= \dot{U}^\mu = U^{\mu;\nu}U_\nu, \\ \sigma_{\mu\nu} &= U_{(\mu;\nu)} + \dot{U}_{(\mu}U_{\nu)} - \frac{1}{3}\Theta h_{\mu\nu},\end{aligned}\quad (17)$$

where  $h_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu$  is the projection tensor. For the metric (1), these parameters have the following expressions:

$$\begin{aligned}\Theta &= 0 = \sigma_{\mu\nu}, \\ a^\mu &= \text{csch } r \text{ sech } r \delta_r^\mu.\end{aligned}\quad (18)$$

The magnitude of the acceleration of four vectors defined by  $a^2 = a^\mu a_\mu$  is  $a = \text{csch } r$ , which vanishes at spatial infinity  $r \rightarrow \infty$ . The presented cylindrically symmetric and static solution therefore has acceleration and is nonexpanding and shear-free.

Below, we discuss the geodesic motion of free test-particles, nature of singularities and its strength, and the  $C$ -energy of the presented spacetime.

**2.1. Geodesics in the Neighborhood of the Singularity.** The Lagrangian for metric (1) is given by

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu, \\ &= \frac{1}{2}\left[-g_{tt}\dot{t}^2 + g_{rr}\dot{r}^2 + g_{\phi\phi}\dot{\phi}^2 + g_{zz}\dot{z}^2\right],\end{aligned}\quad (19)$$

where dot stands for derivative with respect to an affine parameter. From (1) and (19), it is clear that there exist three

constants of motion corresponding to the cyclic coordinates  $t$ ,  $\phi$ , and  $z$ . These constants of motion are

$$\begin{aligned}p_t &= -E = -g_{tt}\dot{t}, \\ p_\phi &= g_{\phi\phi}\dot{\phi} = L, \\ p_z &= g_{zz}\dot{z}.\end{aligned}\quad (20)$$

The normalization condition is

$$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = \delta, \quad (21)$$

where  $\delta = 0$  for null geodesics and  $\delta = -1$  for timelike.

For null geodesics in the  $z = \text{const}$ -planes, we have from the normalization condition

$$\begin{aligned}-g_{tt}\dot{t}^2 + g_{rr}\dot{r}^2 + g_{\phi\phi}\dot{\phi}^2 &= 0 \implies \\ \dot{r}^2 &= \frac{(E^2 - L^2)}{g_{rr}g_{\phi\phi}} \implies\end{aligned}\quad (22)$$

$$\sinh r \cosh r \dot{r} = \sqrt{E^2 - L^2}.$$

The solution of the above equation is

$$r(\lambda) = \sinh^{-1}\left[\left\{2\sqrt{E^2 - L^2}\lambda\right\}^{1/2}\right], \quad E > L, \quad (23)$$

where the constant of integration is taken zero.

For the radial null geodesics  $\dot{z} = 0 = \dot{\phi}$ , we have from (21)

$$\begin{aligned}-g_{tt}\dot{t}^2 + g_{rr}\dot{r}^2 &= 0 \implies \\ \dot{r}^2 &= \frac{E^2}{g_{rr}g_{\phi\phi}}.\end{aligned}\quad (24)$$

The solution of (24) is

$$r(\lambda) = \sinh^{-1}\left[\sqrt{2E\lambda}\right]. \quad (25)$$

Therefore, the first derivative of  $t$  with its solution from (20) for the null geodesics is

$$\begin{aligned}\dot{t} &= \frac{E}{g_{\phi\phi}} = \frac{E}{\sinh^2 r} = \frac{E}{2\sqrt{E^2 - L^2}\lambda}, \\ t(\lambda) &= \frac{E}{2\sqrt{E^2 - L^2}} \ln \lambda.\end{aligned}\quad (26)$$

Therefore, from (26) it is clear that time coordinate  $t$  for the null geodesics is incomplete; that is, the null geodesics coming from infinity hit the singularity  $r = 0$  (see Figure 1).

**2.2. Geodesic Expansion in the Metric.** Consider a congruence of radial null geodesics in the presented spacetime, having the tangent vectors  $K^\mu(r) = (K^t, K^r, 0, 0)$ , where  $K^t = dt/d\lambda$ , and  $K^r = dr/d\lambda$ . The geodesic expansion is given by [31]

$$\Theta = K^\mu_{;\mu} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^i} (\sqrt{-g} K^i). \quad (27)$$

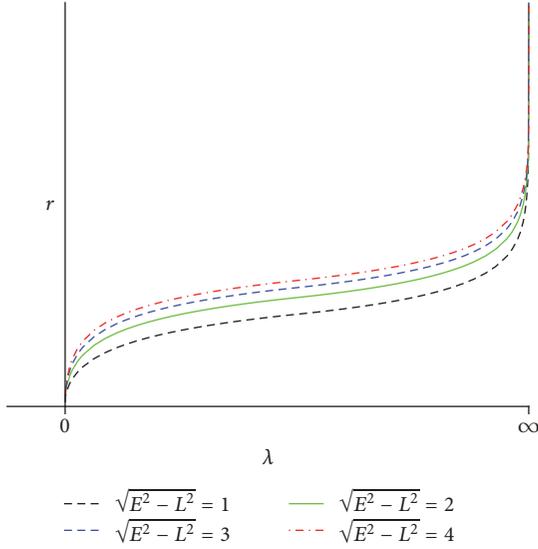


FIGURE 1: Null geodesics for different values of parameter.

For the presented metric, one will get from above

$$\Theta = \frac{\partial K^r}{\partial r} + \frac{K^r}{\sqrt{-g}} (\sqrt{-g})', \quad (28)$$

where prime denotes derivative with respect to  $r$ . We proceed by noting that

$$\begin{aligned} \frac{dK^r}{d\lambda} &= \frac{\partial K^r}{\partial r} \frac{\partial r}{\partial \lambda}, \\ \frac{dK^t}{d\lambda} &= \frac{\partial K^t}{\partial r} \frac{\partial r}{\partial \lambda}. \end{aligned} \quad (29)$$

Therefore, we get the final expression of the geodesic expansion (28)

$$\begin{aligned} \Theta(r) &= \frac{1}{K^r} \frac{dK^r}{d\lambda} + \frac{K^r}{\sqrt{-g}} (\sqrt{-g})' \\ &= \frac{1}{K^r} \left[ -\Gamma_{rr}^r (K^r)^2 - \Gamma_{tt}^r (K^t)^2 \right] + \frac{K^r}{\sqrt{-g}} (\sqrt{-g})' \\ &= -\Gamma_{rr}^r K^r - \frac{(K^t)^2}{K^r} \Gamma_{tt}^r + \frac{K^r}{\sqrt{-g}} (\sqrt{-g})' = \frac{2E}{\sinh^2 r} \\ &> 0, \end{aligned} \quad (30)$$

which is positive, a fact which indicates the nature of singularity that is formed because scalar curvature is naked and would be observable for far away observers.

According to Clarke and Krolak [32], a sufficient condition for the naked singularity to be strong in the sense of Tipler et al. [33, 34] is that

$$\lim_{\lambda \rightarrow 0} \lambda^2 R_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} \neq 0 (> 0). \quad (31)$$

Here  $dx^\mu/d\lambda$  is defined as the tangent vector fields to the radial null geodesics and  $R_{\mu\nu}$  is the Ricci tensor. The weaker

condition (known as the *limiting focusing condition* [35]) is defined by

$$\lim_{\lambda \rightarrow 0} \lambda R_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} \neq 0. \quad (32)$$

For the presented metric (1), we have from condition (31)

$$\begin{aligned} \lim_{\lambda \rightarrow 0} \lambda^2 \left[ R_{tt} \left( \frac{dt}{d\lambda} \right)^2 + R_{rr} \left( \frac{dr}{d\lambda} \right)^2 + R_{\phi\phi} \left( \frac{d\phi}{d\lambda} \right)^2 \right. \\ \left. + R_{zz} \left( \frac{dz}{d\lambda} \right)^2 \right] &= \lim_{\lambda \rightarrow 0} 2\lambda^2 \left( \frac{dt}{d\lambda} \right)^2 = \lim_{\lambda \rightarrow 0} \frac{2E^2 \lambda^2}{\sinh^4 r} \\ &= \lim_{\lambda \rightarrow 0} \left( \frac{E^2 \lambda^2}{2E^2 \lambda^2} \right) > 0, \end{aligned} \quad (33)$$

where  $R_{rr} = 0$ ,  $\dot{\phi} = 0 = \dot{z}$ . Similarly, if one can calculate the *limiting focusing condition* (32), then it becomes zero. Thus the naked singularity (NS) which is formed due to scalar curvature on the axis  $r = 0$  satisfies the *strong curvature condition* only. Therefore, the analytical extension of the spacetime through the singularity is not possible.

**2.3. The C-Energy of the Spacetime.** A cylindrical symmetric spacetime is defined locally by the existence of two commuting, spacelike Killing vectors, where the orbit of one Killing vector is closed and the other one is open. For such a spacetime, there exist coordinates  $(\phi, z)$  such that the Killing vectors are  $(\xi_{(\phi)}, \xi_{(z)}) = (\partial_\phi, \partial_z)$ . The metric tensor  $g_{\mu\nu}$  therefore is independent of  $\phi$  and  $z$ . The norm of these Killing vectors is invariant, namely, the circumferential radius

$$\mathbf{r} = \sqrt{|\xi_{(\phi)\mu} \xi_{(\phi)\nu} g^{\mu\nu}|}, \quad (34)$$

and the specific length

$$l = \sqrt{|\xi_{(z)\mu} \xi_{(z)\nu} g^{\mu\nu}|}. \quad (35)$$

The gravitational energy per unit specific length in a cylindrical symmetric system (also known as C-energy) as defined by Thorne [5] is

$$\mathbf{U} = \frac{1}{8} \left( 1 - \frac{1}{4\pi^2} l^{-2} \nabla^\mu \mathbf{A} \nabla_\mu \mathbf{A} \right), \quad (36)$$

where  $\mathbf{A} = \mathbf{r}l$  is the areal radius and  $\mathbf{U}$  is the C-energy scalar.

For the presented metric,  $\mathbf{r} = \sinh r$  and  $l = \sinh r$  so that area of the two-dimensional cylindrical surface  $\mathbf{A} = \sinh^2 r$ . Hence the C-energy scalar  $\mathbf{U}$  is

$$\mathbf{U} = \frac{1}{8} \left( 1 - \frac{1}{\pi^2} \right) < \frac{1}{8}. \quad (37)$$

The C-energy is defined in terms of the C-energy flux vector  $P^\mu$  which satisfies the conservation law  $\nabla_\mu P^\mu = 0$ . The C-energy flux vector  $P^\mu$  is defined by

$$P^\mu = 2\pi \epsilon^{\mu\nu\rho\sigma} \mathbf{U}_{,\nu} \xi_{(z)\rho} \xi_{(\phi)\sigma}, \quad (38)$$

where  $e^{\mu\nu\rho\sigma}$  is the Levi-Civita skew tensor, and we find the  $C$ -energy flux vector  $P^\mu = 0$ . The  $C$ -energy density measured by an observer whose worldline has a unit tangent  $U^\mu$  is

$$E = -P^\mu U_\mu = 0, \quad (39)$$

and the  $C$ -energy flux which the observer sees flowing in a direction  $n^\mu$  (tangent vector to the singularity) orthogonal to his/her worldline is

$$F = P^\mu n_\mu = 0. \quad (40)$$

### 3. Conclusions

A cylindrical symmetric solution of Einstein's field equations possessing a naked curvature singularity on the symmetry axis was presented. The spacetime is conformally flat and static, and the matter-energy content anisotropic fluids violate the weak energy condition (WEC). Furthermore, the different physical parameters, the energy density, and pressures are singular on the symmetry axis and vanish rapidly at spatial infinity along the radial direction. All cylindrical symmetric and conformally flat static solutions, satisfying the regularity conditions, are well known. In this work, we relaxed the Darmois conditions, and the presented solution is a special case of the known solutions [23, 24]. Finally, we discussed geodesic expansion in the metric to understand the nature of singularity and found that the curvature singularity is naked which satisfies the strong curvature condition. Finally, the  $C$ -energy of the cylindrical spacetime and the  $C$ -energy flux vector were discussed.

### Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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