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*Research Article*

# Reissner-Nordström Black Holes Statistical Ensembles and First-Order Thermodynamic Phase Transition

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We apply Debbasch proposal to obtain mean metric of coarse graining (statistical ensemble) of quantum perturbed Reissner-Nordstöm black hole (*RNBH*). Then we seek its thermodynamic phase transition behavior. Our calculations predict first-order phase transition which can take Bose-Einstein's condensation behavior.

## 1. Introduction

Every observation in any arbitrary system is necessarily finite which deals with a finite number of measured quantities with a finite precision. A given system is therefore generally susceptible of different, equally valid descriptions and building the bridges between those different descriptions is the task of statistical physics (see introduction in [1] for more discussion). Nonlinearity property of Einstein's metric equations causes their averaging to be nontrivial. Various possible ways of averaging the geometry of space time have already been proposed by [2–8], but none of them seems fully satisfactory (see section 7 in [1] for full discussion). Debbasch used an alternative way to averaging Einstein's metric equation in [1]. To do so, he chose a general framework where the mean metric still obeys the equations of general theory of relativity. In his approach averaging and/or coarse graining a gravitational field changes the matter content of space time called "apparent matter" which in cosmological context is related to the dark energy (see [9–12]). So general relativity mean field theory can propose a physical meaning for unknown cosmological dark energy/matter via the "apparent matter". In the Debbasch approach, statistical ensemble of metric is ensembles of histories and not ensembles of states. This is different basically with ordinary statistical mechanics of classical and/or quantum particles. From the latter point of view, it has been known for a long time that black holes in asymptotically flat space times do not admit stable equilibrium states in the canonical ensemble (see introduction in [13]). But from the former point of view the Debbasch gives in [1] general proposal to obtain a mean field theory for the general theory of relativity. In his model members of the ensembles will be labeled by the symbol $\omega \in \Omega$ where $\Omega$ is an arbitrary probability space [14]. To each $\omega$, there are corresponding metric tensor $g(\omega)$, compatible connections $\Gamma(\omega)$, and the Einstein metric equation (see [1] and section 2 in [10]). All members of the ensemble correspond to the same macroscopic history of the space time manifold, in particular to a given same mean metric $\overline{g}_{\mu\nu}(x) = \langle g_{\mu\nu}(x,\omega)\rangle$ and corresponding mean connection $\overline{\Gamma}^{\mu}_{\nu\eta} = \langle \Gamma^{\mu}_{\nu\eta}(g, \partial g, \omega)\rangle$. As application of his model Debbasch and coworkers considered statistical ensemble of Schwarzschild black holes as nonvacuum solutions of mean Einstein metric equation by using Kerr-Schild coordinates $R = r-\omega$. They calculated nonvanishing temperature of mean metric where single Schwarzschild black hole is well known which has nonvanishing temperature as a vacuum solution of the Einstein equation. They discussed their results with special emphasis on their connections with the context of astrophysical observations [12]. Extreme RNBH with $m = 1$ has vanishing temperature (see next section) and regular Kerr-Schild coordinates $R = r - \omega$ are not applicable to obtain mean metric similar to the Schwarzschild one because the coarse graining space time turns out not to be a black hole [9]. Hence Chevalier and Debbasch used analytic continuation of the Kerr-Schild coordinates as $R = r - i\omega$ to obtain mean metric of extreme classical black hole in [11]. According to the Debbasch approach we are free to choose types of coarse graining and/or ensemble space to obtain mean metric of the space times ensemble under consideration. We should point that topology of ensemble space times must be similar to topology of their mean metric (see [9]) which restrict us to choose an analytic continuation of Kerr-Schild coordinates for extreme RNBH. In short, with Debbasch proposal the averaging process does not change topology between ensemble of the curved space times and the corresponding mean space time. Precisely, the averaging process modifies the horizon radius and changes the energy-momentum tensor of space time but not total energy or mass of the black holes ensemble. Really the averaging process just redistributes without any change in the total mass which means that the total energy of the black holes dose not change by the coarse graining proposal.

Similar to study of thermodynamic behavior of single RNBH [15], we seek thermodynamic aspect of mean metric of nonextreme RNBHs ensemble in this work, by applying the Debbasch approach to evaluate the mean and/or coarse graining metric. Organization of the paper is as follows.

In Section 2, we calculate mean metric of ensemble of RNBHs. In Section 3 we obtain locations of mean metric horizons. In Section 4 we calculate interior and exterior horizons entropy, temperature, heat capacity, Gibbs free energy, and pressure of RNBHs mean metric. In Section 5 we calculate interior and exterior horizons luminosity and corresponding mass loss equation of quantum perturbed RN mean metric. Section 6 denotes concluding remark and discussion.

## 2. RNBHs Ensemble and Mean Metric

Exterior metric tensor of a single charged, nonrotating, spherically symmetric body is given by

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right) dt^2 - \frac{dr^2}{(1 - 2M/r + e^2/r^2)} - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{1}$$

This is metric solution of Einstein-Maxwell equation and is called RNBH in which $M$ and $e$ are corresponding ADM mass and electric charge defined in units where $c = G = 1$. Equating $g^{\mu\nu}\partial_\mu r \partial_\nu r = 0$ for arbitrary spherically symmetric hypersurface $r = constant$, one can obtain apparent (exterior) horizon radius as $r_+ = M + \sqrt{M^2 - e^2}$ and Cauchy (interior) horizon radius as $r_- = M - \sqrt{M^2 - e^2}$ which appear only for $0 \leq (e/M)^2 \leq 1$. One can obtain mass independent relation between $r_+$ and $r_-$ as $r_- = e^2/r_+$. With particular choice $e = M$ (called extreme and/or Lukewarm RNBH) these horizons coincide as $r_- = r_+ = M$. Clearly the RNBH metric solution (1) leads to Schwarzschild one by setting $e = 0$ for which we will have $r_+ = 2M$ and $r_- = 0$. Temperature of a single RNBH can be obtained for interior and exterior horizons as $T_\pm = (1/8\pi r_\pm)(\partial r_\pm/\partial M)_e^{-1} = \pm\sqrt{M^2 - e^2}/8\pi(M \pm \sqrt{M^2 - e^2})^2$ [1] which reduce to a zero value for extreme (Lukewarm) RNBH because of $M = e$. They show positive (negative) temperature for exterior (interior) horizons. Negative temperatures of systems have physical meaning and happen under particular conditions. More authors studied conditions where the physical systems are taken to have negative temperatures. See [16] for temperatures of interior and exterior horizons of Kerr-Newman black hole. One can see [17–19] for negative temperature of nongravitational systems. In the nature, materials are obtained which have interesting properties like negative refraction index and reversibility of the Doppler's effect, and so the phase and group velocity (velocity of energy propagation) have opposite singes. In these systems temperature will have negative values (see [17] and references therein). Such systems are called dual system (left-handed) of direct counterpart (right-handed conventional materials). Absolute temperature is usually bounded to be positive but its violation is shown in [18] by Braun et al. They showed, under special conditions, however negative temperatures where high energy states are more occupied than low energy states. Such states have been demonstrated in localized systems with finite, discrete spectra. They used the Bose-Hubbard Hamiltonian and obtained attractively interacting ensemble of ultra-cold bosons at negative temperature which are stable against collapse for arbitrary atom number. Furman et al. studied in [19] behavior of quantum discord of dipole-dipole interacting spins in an external magnetic field in the whole temperature range $-\infty < T < \infty$. They obtained that negative temperatures, which are introduced to describe inversions in the population in a finite level system, provide more favorable conditions for emergence of quantum correlations including entanglement. At negative temperature the correlations become more intense and discord exists between remove spins being in separated states. According to the documentation and looking to diagrams of the present work, one can be convinced that a quantum perturbed mean metric of coarse graining RNBHs will be exhibited finally with a first-order phase transition and Bose-Einstein condensation state microscopically. According to the Debbasch approach [1] ensemble of the nonextreme RNBHs is collection of coarse graining RNBHs indexed by a 3-dimensional real parameter $\vec{\omega} \in \vec{\Omega}$ where $\vec{\Omega}$ is the three balls of radius $\vec{a}$ as follows:

$$\vec{\Omega} = \left\{\vec{\omega} \in \mathbb{R}^3;\ \omega^2 \leq a^2\right\}. \tag{2}$$

The metric solution (1) is convenient to be rewritten with Kerr-Schild coordinates $(\tau, r, \theta, \varphi)$ by transforming

$$dt = d\tau + \frac{h(r)\,dr}{1 - h(r)} \tag{3}$$

as follows (see [10–12]):

$$ds^2 = d\tau^2 - d\vec{r}\cdot d\vec{r} - h(r)\left(d\tau - \frac{\vec{r}\cdot d\vec{r}}{r}\right)^2 \tag{4}$$

where

$$h(r) = \frac{2M}{r} - \frac{e^2}{r^2} \tag{5}$$

and $r = |\vec{r}|$ is the Euclidean norm of the vector $\vec{r}$. It should be pointed that all metric solutions of Einstein's field equation will have simple form by using Kerr-Schild coordinates. They are decomposed into the well-known flat Minkowski background metric $\eta_{\mu\nu}$ and null vector fields $K_\mu$ as $g_{\mu\nu} = \eta_{\mu\nu} - 2h(x^\mu)K_\mu K_\nu$ where $K_\mu K^\mu = 0 = g_{\mu\nu}K^\mu K^\nu = \eta_{\mu\nu}K^\mu K^\nu$ and $h(x^\mu)$ is a scalar function (see [20] and references therein). Now, we must choose a probability measure. Hence we follow the assumption presented in [11] and choose uniform probability measure $d\mathsf{p}_\omega$ in which $\mathsf{p}$ is probability density of this measure with respect to Lebesgue measure $d^3\omega$ as $\mathsf{p}(\omega) = 1/V_a$ with $V_a = (4/3)\pi a^3$. Applying the Kerr-Schild radial coordinate (in case of extreme RNBH where $M = e$ we must use analytic continuation of the Kerr-Schild coordinates as $R = r - i\omega$ (see discussion given in the introduction)) $\vec{R}(\vec{r},\vec{\omega}) = \vec{r} - \vec{\omega}$, we extend single RNBH metric (4) to obtain metric of coarse graining and/or statistical ensemble of RNBHs as follows:

$$ds^2 = d\tau^2 - d\vec{r}\cdot d\vec{r} - h(R)\left(d\tau - \frac{\vec{R}.d\vec{r}}{R}\right)^2 \tag{6}$$

where $h(R) = 2M/R - e^2/R^2$ and $R = \sqrt{\vec{R}\cdot\vec{R}}$. Using perturbation series expansion method and averaging the metric (6) against $\vec{\omega}$ we obtain mean metric of (6) such that (see [21] for details of calculations)

$$\begin{aligned}\left\langle ds^2\right\rangle_\omega = {} & b_1(r)\,d\tau^2 + b_2(r)\,d\vec{r}\cdot d\vec{r} + b_3(r)\,dr^2 \\ & + b_4(r)\,dr\,d\tau\end{aligned} \tag{7}$$

where $|e| < M$, $d\vec{r}\cdot d\vec{r} = dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$,

$$b_1(r) = 1 - \frac{2M}{r} + \frac{e^2}{r^2}\left(1 + \frac{a^2}{5r^2}\right), \tag{8}$$

$$b_2(r) = -1 - \frac{2a^2M}{5r^3} + \frac{a^2e^2}{5r^4}, \tag{9}$$

$$b_3(r) = -\frac{2M}{r}\left(1 - \frac{3a^2}{5r^2}\right) + \frac{e^2}{r^2}\left(1 - \frac{2a^2}{5r^2}\right), \tag{10}$$

and

$$b_4(r) = \frac{4M}{r}\left(1 - \frac{a^2}{5r^2}\right) - \frac{2e^2}{r^2}. \tag{11}$$

It is simple to show that the mean metric (7) reduces to a single RNBH metric (4) by setting $a = 0$. We can rewrite the mean metric (7) in the static frame by defining the Schwarzschild coordinates. To do so, we first choose a suitable local frame with coordinates $(t, \rho, \theta, \varphi)$ as

$$\rho(r) = r\sqrt{-b_2(r)} \tag{12}$$

and

$$d\tau = dt - \alpha(\rho)\,d\rho \tag{13}$$

where

$$\alpha(\rho) = \frac{b_4(r)}{2b_1(r)}\left(\frac{\partial\rho}{\partial r}\right)^{-1}. \tag{14}$$

In the latter case the mean metric (7) reads

$$\begin{aligned}\left\langle ds^2\right\rangle = {} & F(\rho)\,d\tau^2 - f(\rho)\,d\rho^2 \\ & - \rho^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\end{aligned} \tag{15}$$

where we defined

$$F(\rho) = 1 - \frac{2M}{r(\rho)} + \frac{e^2}{r^2(\rho)}\left(1 + \frac{a^2}{5r^2(\rho)}\right) \tag{16}$$

and

$$f(\rho) = \frac{1}{F(\rho)}\left(1 - \frac{e^2a^2}{5r^4(\rho)}\right). \tag{17}$$

We now seek location of mean metric horizons.

## 3. Horizons Location for Mean Metric

One can obtain event horizon location of the mean metric (15) by solving $F(\rho_{EH}) = 0$ and location of apparent (interior and exterior) horizons by solving null condition $g^{\mu\nu}\partial_\mu\rho\partial_\nu\rho = 0$ which leads to the equation $F(\rho_{AH}) = 0$ such that

$$1 - \frac{2M}{r_H} + \frac{e^2}{r_H^2} + \frac{e^2a^2}{5r_H^4} = 0. \tag{18}$$

The above equation has not exactly analytic solution for $a \neq 0$ but for small $a$ we can use perturbation series expansion to evaluate the event horizon location. To do so we first define $\epsilon = a/r_H$ for which the horizon equation (18) can be written as $r_H^2 - 2Mr_H + e^2(1 + \epsilon^2) = 0$. The latter equation has a real solution as $r_H = M\{1 + \sqrt{1 - (e^2/M^2)(1 + \epsilon^2/5)}\}$ for $(e^2/M^2)(1 + \epsilon^2/5) < 1$. We know that for a single RN black hole $e/M < 1$ and so the condition $(e^2/M^2)(1 + \epsilon^2/5) < 1$ reads $\epsilon < 1 (a < r_H)$ for which horizon of the ensemble of statistical RN black holes is not destructed by raising $0 < \epsilon < 1$ if we want to apply perturbation series expansion method to obtain asymptotically behavior of the event horizon solution versus the parameters $(a, e, M)$. Thus we must obtain perturbation series expansion form of the event horizon but for $a < r_H$ as follows. Inserting

$$r_H^\pm = r_0^\pm + ar_1^\pm + a^2r_2^\pm + O\left(a^3\right) \tag{19}$$

and solving (18) as order by order, we obtain

$$r_0^\pm = M \pm \sqrt{M^2 - e^2},$$

$$r_1^\pm = 0,$$

$$r_2^{\pm} = \frac{\mp e^2}{10\sqrt{M^2 - e^2}\left(M \pm \sqrt{M^2 - e^2}\right)^2} \tag{20}$$

where $r_H^+$ and $r_H^-$ denote apparent exterior and Cauchy (interior) horizon radiuses of the mean metric (7), respectively. Inserting (9) and (19), one can obtain perturbation series expansion of (12) which up to terms in order of $O(a^3)$ becomes

$$\rho_H = \rho_0^{\pm} + a^2 \rho_2^{\pm} \tag{21}$$

where we defined

$$\begin{aligned} \rho_0^{\pm} &= r_0^{\pm}, \\ \rho_2^{\pm} &= r_2^{\pm} + \frac{2Mr_0^{\pm} - e^2}{10 r_0^{\pm 3}}. \end{aligned} \tag{22}$$

Area equation of apparent horizon hypersurface of the spherically symmetric static mean metric (15) is defined by $A = 4\pi\rho_H^2$ which up to terms in order of $O(a^3)$ reads

$$A^{\pm} = A_0^{\pm} + a^2 A_2^{\pm} \tag{23}$$

where we defined

$$\begin{aligned} A_0^{\pm} &= 4\pi \left(r_0^{\pm}\right)^2, \\ A_2^{\pm} &= 8\pi \left[ r_0^{\pm} r_2^{\pm} + \frac{1}{10 r_0^{\pm}} \left( 2M - \frac{e^2}{r_0^{\pm}} \right) \right]. \end{aligned} \tag{24}$$

According to Bekenstein-Hawking entropy theorem we have the result that $A_+ (A_-)$ given by (23) will be entropy function of exterior (interior) horizon of the mean metric (15). Black holes containing multiple horizons have several corresponding temperatures. Such a black hole will be in-equilibrium thermally throughout the space time where the temperature has a gradient between the horizons. Thermal equilibrium is possible only if horizon radiuses and so the corresponding temperatures become equal (see, for instance, [22, 23]). The latter situations happen for an extreme RNBH where $M = e$ and so $r_H^+ = r_H^-$. We now calculate thermodynamic characteristics of interior and exterior horizons of the nonextreme mean metric of RNBHs statistical ensemble.

## 4. Mean Metric Thermodynamics

In the next section we will consider massless, chargeless quantum scalar field effects on luminosity of the quantum perturbed coarse graining RNBHs where its electric charge becomes invariant quantity. Hence it is useful to define dimensionless black hole mass $m = M/e$ and ensemble factor $\delta = a/e$ in what follows. In the latter case exterior horizon entropy of mean metric (15) can be obtained up to terms in order of $O(\delta^3)$ as follows:

$$\begin{aligned} S_+ (m, \delta) &= \left(m + \sqrt{m^2 - 1}\right)^2 \\ &\quad + \frac{\delta^2}{5} \left[ \frac{2\left(m^2 - 1\right)^{3/2} + m\left(2m^2 - 3\right)}{\sqrt{m^2 - 1}\left(m + \sqrt{m^2 - 1}\right)^2} \right] \end{aligned} \tag{25}$$

and its interior horizon entropy becomes

$$\begin{aligned} S_- (m, \delta) &= \left(m - \sqrt{m^2 - 1}\right)^2 \\ &\quad + \frac{\delta^2}{5} \left[ \frac{2\left(m^2 - 1\right)^{3/2} - m\left(2m^2 - 3\right)}{\sqrt{m^2 - 1}\left(m - \sqrt{m^2 - 1}\right)^2} \right] \end{aligned} \tag{26}$$

where $0 < \delta < 1$ and

$$S_{\pm} = \frac{A_{\pm}}{4\pi e^2} > 0. \tag{27}$$

Diagrams of entropies (25) and (26) are plotted versus $m$ in Figure 4. They show that $S_{\pm} > 0$ for a single RNBH ($\delta = 0$) in limits $m \longrightarrow 1$ but for an ensemble of RNBHs for which we use $\delta = 0.9$, they reach infinity $S_{\pm} \longrightarrow \mp\infty$. In fact for physical systems the entropy itself must be positive function but its variations may reach some negative values. Hence we define difference between interior horizon entropy and exterior horizon entropy as

$$\begin{aligned} \Delta S = S_+ - S_- &= 4m\sqrt{m^2 - 1} \\ &\quad + \frac{2\delta^2}{5} \left[ \frac{m\left(2m^2 - 1\right)\left(2m^2 - 3\right)}{\sqrt{m^2 - 1}} \right. \\ &\quad \left. - 4m\left(m^2 - 1\right)^{3/2} \right] \end{aligned} \tag{28}$$

and total entropy such as follows:

$$S_{tot} = S_+ + S_- = 4m^2 - 2 + \frac{4\delta^2}{5}\left(4m^4 - 6m^2 + 1\right). \tag{29}$$

Diagrams of $\Delta S$ and $S_{tot}$ are plotted in Figure 3. Fortunately these diagrams show that, for a single RNBH where $\delta = 0$, we will have $\Delta S > 0$ by decreasing $m \longrightarrow 1$ and $S_{tot} > 0$ but for ensemble of RNBHs with $\delta = 0.9$ we have $\Delta S < 0$ while $S_{tot} > 0$. Hence $\Delta S$ and $S_{tot}$ should be considered as physical entropies of coarse graining RNBHs. Decrease of entropy causes some negative temperatures (see Figure 2) in thermodynamic systems containing bounded energy levels. In the latter case there is a critical temperature for which the system exhibits a phase transition reaching Bose-Einstein condensation state microscopically. In thermodynamics, increase of entropy $\Delta S > 0$ means an increase of disorder or randomness in natural systems. It measures heat transfer of the system for which heat flows naturally from a warmer to a cooler substance. Decrease of entropy $\Delta S < 0$ means an increase of orderliness or organization of microstates of a system. To do so the substance of a system must loose heat in the transfer process. Individual systems can experience negative entropy, but overall, natural processes in the universe trend toward positive entropy. Negative entropy was first introduced for living things by Ervin Schrödinger in 1944 as the reverse concept of entropy, to describe the order that can emerge from chaos [24]. The heat generated by computations in the information theory is other applications for negative entropy concept (see [25–28] for more discussions). However we consider $\Delta S$ and $S_{tot}$ to be physical entropies of RNBHs statistical ensemble containing two horizons which is in accord with positivity condition of the Bekenstein-Hawking entropy theorem. Our coarse graining RNBHs can be considered as a two-level thermodynamical system with upper bound finite energy $M$ because it has two dual (interior and exterior) horizons. We now calculate exterior (interior) horizon temperature $T_+(T_-)$ of the RNBHs mean metric (15) as follows:

$$T_{\pm}^{*} = (4\pi e) T_{\pm} = \frac{1}{(\partial S_{\pm}/\partial m)_{\delta}} = \pm\frac{1}{2}\frac{\sqrt{m^2-1}}{\left(m \pm \sqrt{m^2-1}\right)^2} + \frac{\delta^2}{60}\left[\frac{4m-2m^3-2m^5 \mp \sqrt{m^2-1}\left(2m^4+3m^2-3\right)}{\left(m^2-1\right)^{5/6}\left(m \pm \sqrt{m^2-1}\right)^6}\right] \tag{30}$$

Their diagrams are plotted against $m$ in Figure 2 for $\delta = 0; 0.9$. For $m \gg 1$ we see that $T_-^*(T_+^*)$ has some negative (positive) values and their sign is changed when $m \longrightarrow 1$. We also plotted diagram for $T_{\pm}^*$ versus $\Delta T^* = T_+^* - T_-^*$ in Figure 2. They show that $T_-^* < 0$ for $\Delta T^* > 0$ reaching zero value at $\Delta T^* = 0$ for $\delta = 0, 0.9$. While $T_+^* > 0(T_+^* < 0)$, $\Delta T^* \longrightarrow 0^+$ for $\delta = 0(0.9)$ after that to obtain a finite positive maximum value. This maximum has smaller value for $\delta = 0.9$ with respect to situations where we choose $\delta = 0$. In ordinary statistical physics, negative temperatures are taken into account when the system has upper bound (maximum finite) energy for which entropy is continuously increasing but the energy and temperature decrease and vice versa. In the latter case the system reaches Bose-Einstein condensation state microscopically. Energy upper bound of our system is its total mass $M$ for which we have $m > 1$. Regarding quantum matter effects on mean metric we will show in Section 5 that mass of mean metric decreases finally as $m_{final} = 1$ (see Figure 1). Bose-Einstein condensation state needs a phase transition which happens when sign of heat capacity is changed. Hence we now calculate interior and exterior horizon of mean metric heat capacity $C_{\pm}^*$ which up to terms in order of $O(\delta^3)$, at constant electric charge $e$ and ensemble radius $a$, become

$$\begin{aligned} C_{\pm}{}^{*} &= \frac{C_{\delta}^{\pm}}{4\pi e^2} = \left(T_{\pm}\frac{\partial S_{\pm}}{\partial T_{\pm}}\right)_{\delta} = \left(\frac{\partial T_{\pm}^*}{\partial m}\right)_{\delta}^{-1} \\ &= -\frac{2\sqrt{m^2-1}\left(m \pm \sqrt{m^2-1}\right)^2}{2\sqrt{m^2-1} \mp m} + \frac{2\delta^2}{45}\left[\frac{2m\sqrt{m^2-1}\left(4m^4+12m^2-15\right) \pm 8m^6 \pm 20m^4 \mp 49m^2 \pm 21}{\left(m^2-2 \pm m\sqrt{m^2-1}\right)^2\left(m^2-1\right)^{5/6}}\right]. \end{aligned} \tag{31}$$

Their diagrams are plotted against $m$ in Figure 5. They show that sign of $C_+^*$ is changed at $m_c = 1.15(1.2)$ for $\delta = 0(0.9)$ but sign of $C_-^*$ is changed at $m = 1$ for $\delta = 0, 0.9$. We plot also diagrams of $C_{\pm}^*$ versus $\Delta T^*$ in Figure 5. They show a changing of sign for $C_+^*$ when $\Delta T^* \longrightarrow 0$ and $\delta = 0, 0.9$ but not for $C_-^*$. In case $\delta = 0.9$ we see $C_-^* < 0$ for $\Delta T^* > 0$ but its absolute value exhibits a minimum value. When $\Delta T^* \longrightarrow 0$ we see $C_-^*$ which decreases monotonically to negative infinite value for $\delta = 0$. Changing of sign of exterior horizon heat capacity means that a phase transition happens when the quantum perturbed RNBHs ensemble reaches its stable state with minimum mass $m_{final} = 1$. To determine order kind of this phase transition we should study behavior of the corresponding Gibbs free energy as follows.

Exterior and interior horizon Gibbs free energies are defined by

$$G_{\pm} = M - T_{\pm}A_{\pm} - \Phi_{\pm} \tag{32}$$

where entropy $A_{\pm}$ is given by (24) and electric potential $\Phi_{\pm}$ is defined by

$$\Phi_{\pm} = -T_{\pm}\left(\frac{\partial A_{\pm}}{\partial e}\right)_{a,M}. \tag{33}$$

Inserting $M = em$, $A_{\pm} = 4\pi e^2 S_{\pm}$, and (30), the above Gibbs energy equation reads

$$G_{\pm}^* = \frac{G_{\pm}}{e} = m - T_{\pm}^* S_{\pm} - 8\pi S_{\pm} - 4\pi e\frac{\partial S_{\pm}}{\partial e} \tag{34}$$

in which we have

$$\begin{aligned} e\frac{\partial S_{\pm}}{\partial e} = {} & \mp\frac{2m\left(m \pm \sqrt{m^2-1}\right)^2}{\sqrt{m^2-1}} \\ & - \frac{\delta^2}{15\left(m^2-1\right)^{11/6}\left(m \pm \sqrt{m^2-1}\right)^2} \times \left[28m^7\right. \\ & - 75m^5 + 74m^3 - 27m \\ & \left.\pm \sqrt{m^2-1}\left(26m^6 - 61m^4 + 47m^2 - 12\right)\right]. \end{aligned} \tag{35}$$

We plot diagrams of the above equations against $m$ in Figure 6. They show that $G_-^*$ has minimum zero value at $m = 1$ but $G_+^*$ raises to $+\infty$ by decreasing $m \longrightarrow 1$ for $\delta = 0.9$. In case $\delta = 0$, we see $G_{\pm}^* \longrightarrow \pm\infty$ when $m \longrightarrow 1$. Furthermore we plot diagrams of $G_{\pm}^*$ versus $\Delta T^*$ in Figure 6. We see $G_-^* \longrightarrow -\infty$ when $\Delta T^* \longrightarrow 0$ for $\delta = 0$ but $G_-^* \longrightarrow 0^+$ for $\delta = 0$. $G_+^*$ decreases to a positive minimum value by decreasing $\Delta T^* \longrightarrow 0$ and then reaches positive infinite value.

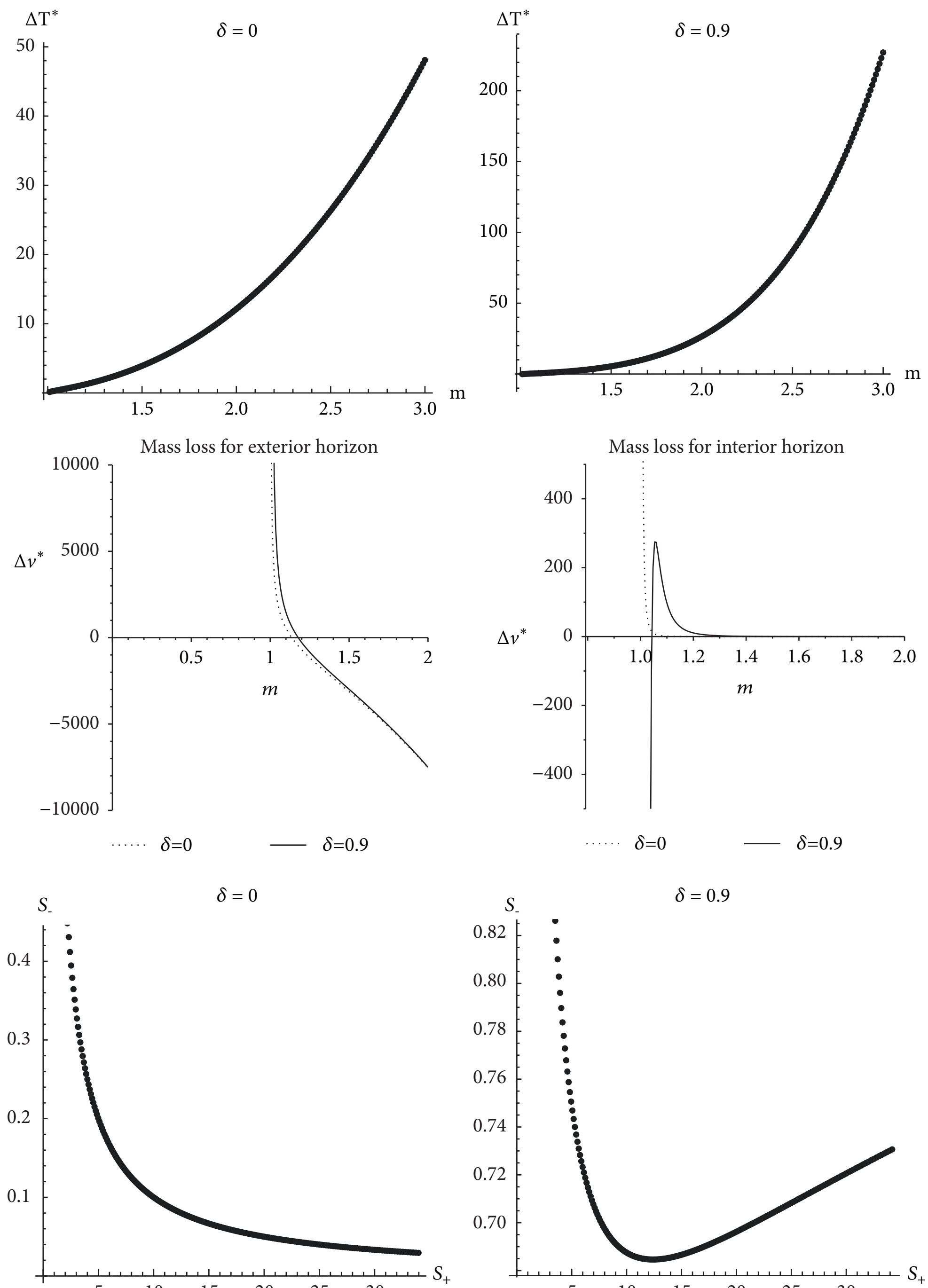


Figure 1: Diagram of mass loss $m(v)$, interior (exterior) horizon entropy $S_-(S_+)$, and difference of temperatures between interior and exterior horizons $\Delta T^*(m)$ is plotted against for single RNBH $\delta = 0$ and mean metric of coarse graining RNBHs $\delta = 0.9$.

The latter behavior shows changing the sign of first derivative of $G_+^*$ when decreases $m$ and/or $\Delta T^*$ which means that the phase transition is first order.

One of other suitable quantities which should be calculated is pressure of black hole microparticles which coincide with the interior horizon as follows. If a quantum particle is collapsed inside of the interior (exterior) horizon then its de Broglie wave length must be at least $\lambda_- \approx 2\rho_-(\lambda_+ \approx 2\rho_+)$. We use de Broglie quantization condition on quantum particles as $p_\pm = h/\lambda_\pm$ where $h$ is Planck constant and $p_\pm$ is momentum of in-falling quantum particles inside of the horizons. In Plank units where $c = h = G = 1$ we can write

$$\Delta p = p_- - p_+ = \frac{1}{2}\left(\frac{1}{\rho_-} - \frac{1}{\rho_+}\right) \tag{36}$$

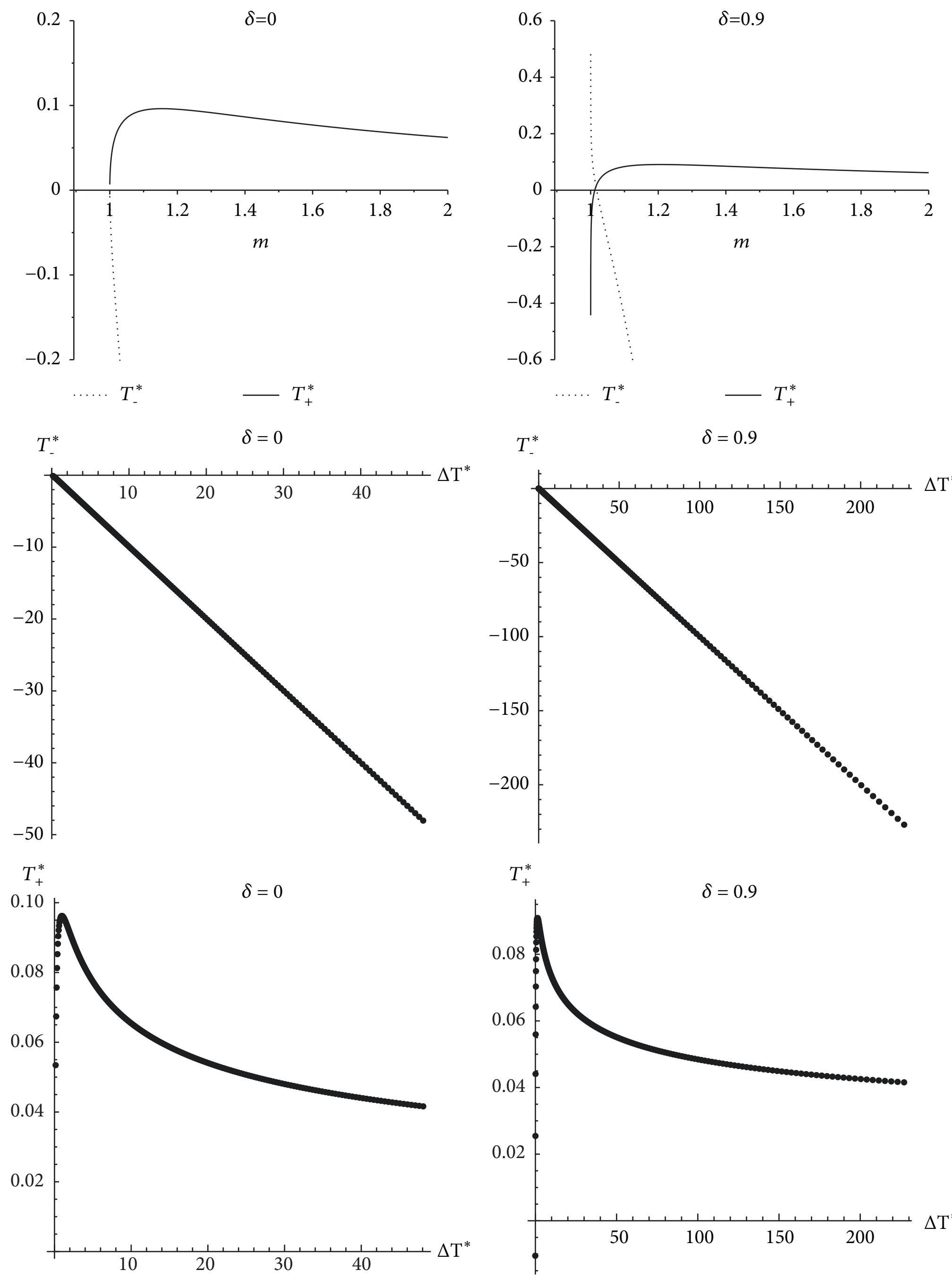


FIGURE 2: Diagram of $T_{\pm}^*$ is plotted against $m$ and $\Delta T^*$ for $\delta = 0$ and $\delta = 0.9$.

in which $\Delta p$ is difference of momentum of quantum particles which move from exterior horizon $\rho_+$ to interior $\rho_-$ horizon. For $c = 1$ they move for durations $\Delta t = \rho_+ - \rho_-$. We now use the latter assumptions to rewrite Newton's second law as

$$F = \frac{\Delta p}{\Delta t} = \frac{1}{2\rho_+\rho_-}. \tag{37}$$

$F$ is dimensionless force which affects interior horizon surface. When the system becomes stable mechanically, then $F$ must be balanced by the electric force of the system defined by $F_E = e((\Phi_- - \Phi_+)/(\rho_+ - \rho_-))$. Spherically symmetric condition of the system causes choosing some radial motions for quantum particles located inside of the statistical ensemble of RNBHs. However one can define pressure of moving charged quantum particle on the interior horizon as

$$P_- = \frac{F}{4\pi\rho_-^2} = \frac{1}{8\pi\rho_+\rho_-^3} \tag{38}$$

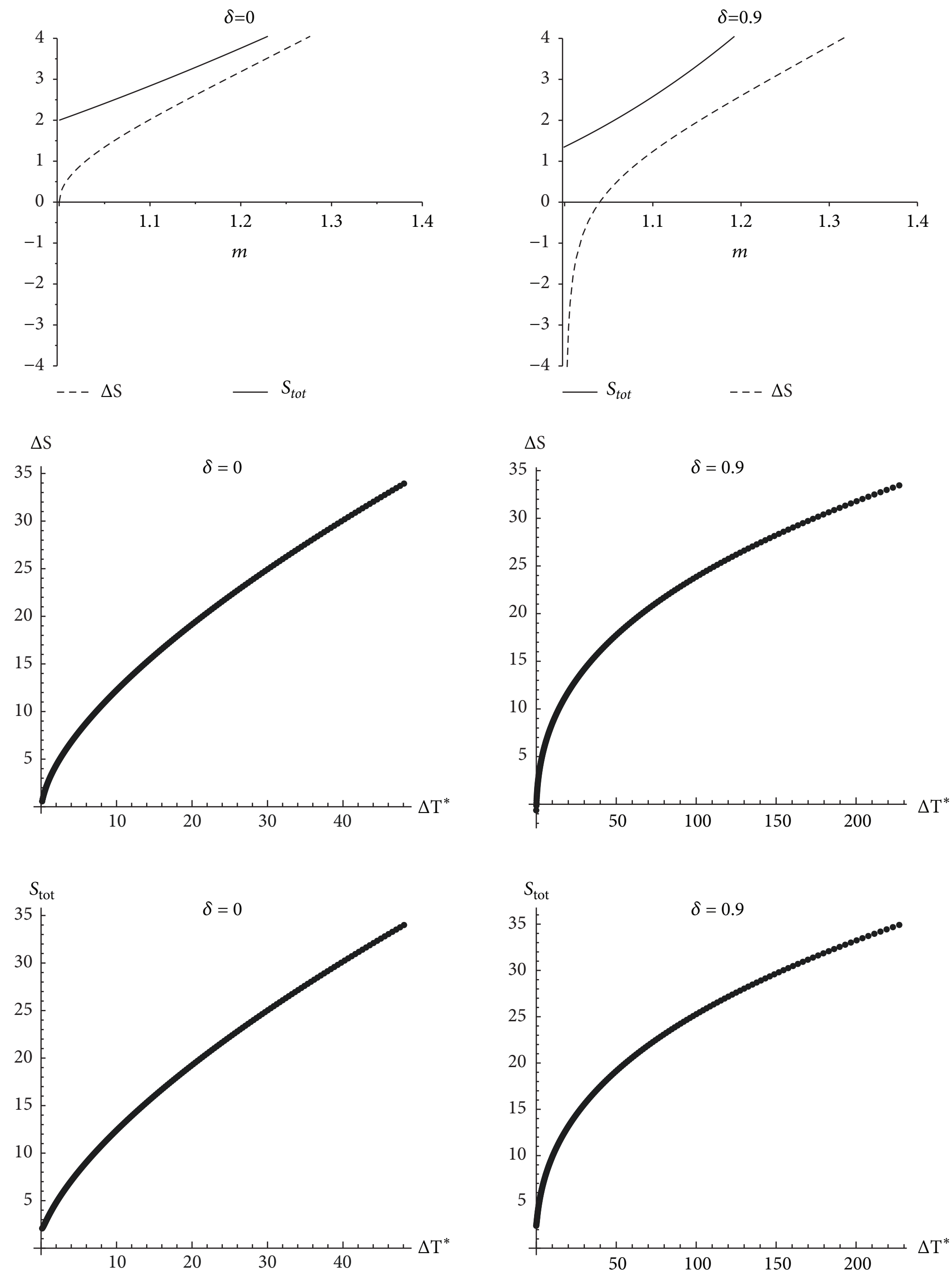


FIGURE 3: Diagram of $\Delta S$ and $S_{tot}$ is plotted against $m$ and $\Delta T^*$ for $\delta = 0$ and $\delta = 0.9$.

which by inserting (22) and using some simple calculations reads

$$P_-^*(m) = 8\pi e^4 P_- = \left(m - \sqrt{m^2-1}\right)^{-2} \left\{1 + \frac{\delta^2}{5} \times \left[\frac{24m^5 - 26m^3 + 2m^2 - 6m - 2}{\left(m - \sqrt{m^2-1}\right)^3} - \frac{\left(24m^6 - 38m^4 + 2m^3 + 16m^2 - 3m - 2\right)}{\sqrt{m^2-1}\left(m - \sqrt{m^2-1}\right)^3}\right]\right\}. \tag{39}$$

We plot diagram of the above pressure in Figure 8. They show that $P_-^* > 0 (< 0)$ in case $\delta = 0(0.9)$ for all values of $\Delta T^* > 0$. Diagrams show that $P_-^*$ is vanishing when $\Delta T^* \longrightarrow 0$. Also we plot diagram for $P_-^*$ versus $m$. It shows

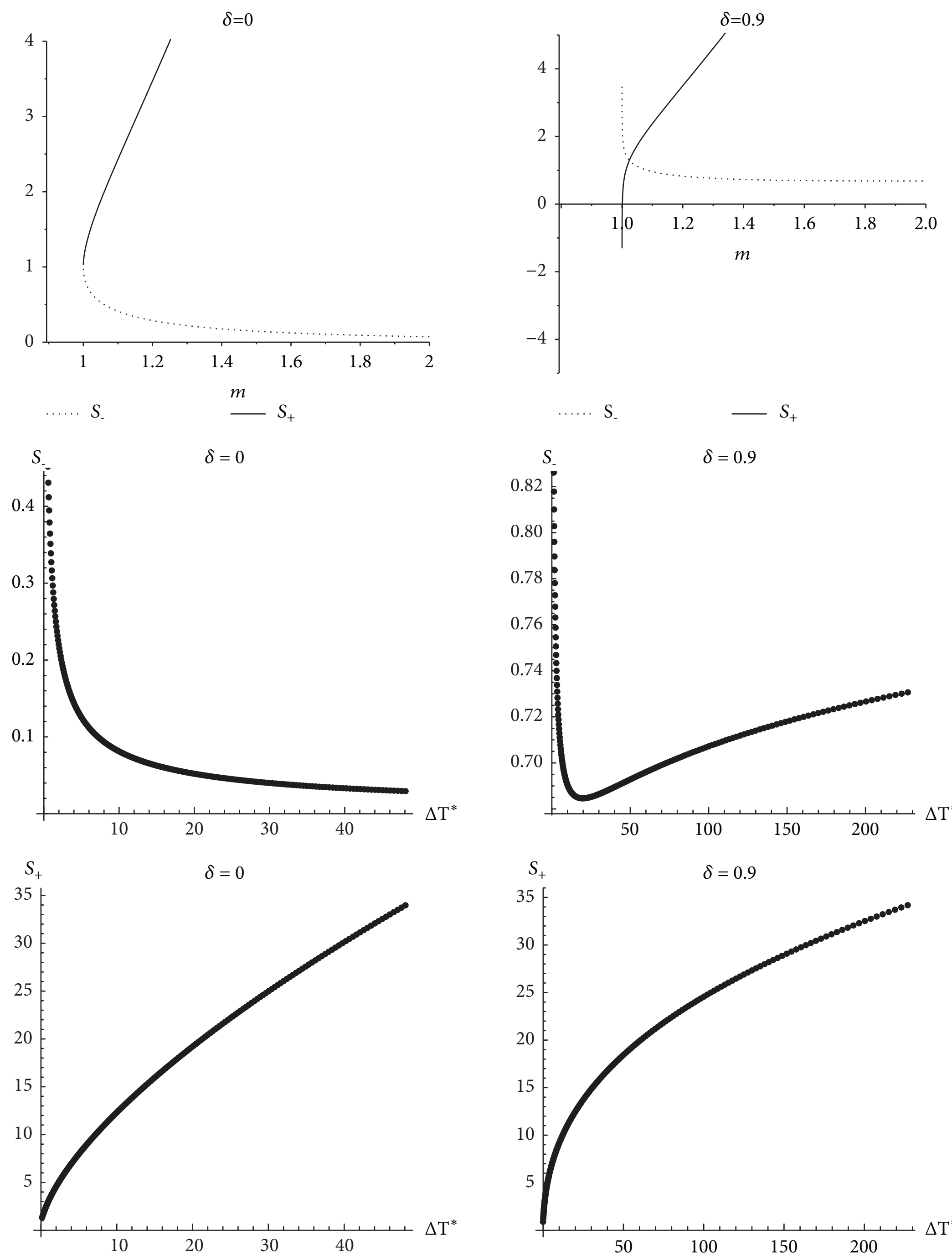


Figure 4: Diagram of $S_{\pm}$ is plotted against $m$ and $\Delta T^{*}$ for $\delta = 0$ and $\delta = 0.9$.

$P_{-}^{*} \longrightarrow 0^{+}$ for $\delta = 0$. In case where $\delta = 0.9$ one can see $P_{-}^{*} < 0$ when $m \longrightarrow 1$ for $m > 1$ but $P_{-}^{*} \longrightarrow +\infty$. The latter results predict dark matter behavior of the interior horizon matter counterpart where for positive mass $m > 1$ there is some "negative" pressure. How can mass of mean metric RNBHs decrease? Dynamically this is possible if we consider corrections of quantum matter field interacting with the mean metric of RNBHs as follows. This makes the mean metric of RNBHs unstable quantum mechanically. In the next section we assume interaction of the mean metric of RNBHs statistical ensemble with massless, chargeless quantum scalar field for which $e$ will be invariant of the system and so there is not any electromagnetic radiation. In other words there will be only mass interaction between quantum scalar field and ensemble of the RNBHs. They reduce usually to the well-known Hawking thermal radiation of the quantum perturbed mean metric which is causing mass loss of the mean RNBHs. For such a quantum mechanically unstable mean metric we now calculate its luminosity, mass loss process, and switching off effect.

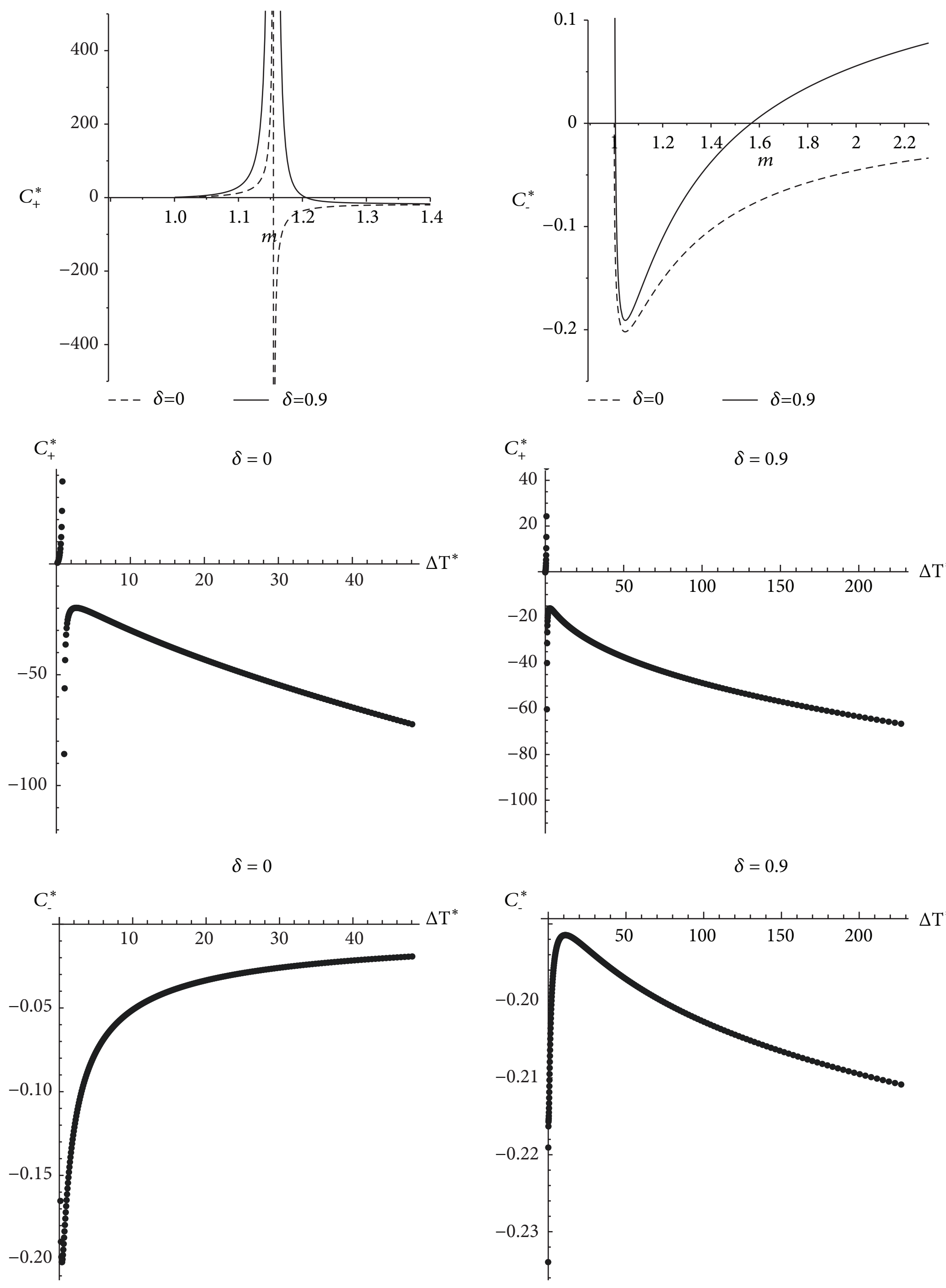


Figure 5: Diagram of interior and exterior horizons heat capacities $C^*_{\pm}$ is plotted against $m$ and $\Delta T^*$ for $\delta = 0$ and $\delta = 0.9$.

## 5. Mean Quantum RNBH Mass Loss

We applied massless, chargeless quantum scalar field Hawking thermal radiation effects on single quantum unstable RNBH and calculated time dependence mass loss function in [15]. We obtained that the evaporating quantum perturbed RNBH exhibits switching off effect before its mass disappears completely. It should be pointed that electric charge of the black hole is invariant of the system because there is no electromagnetic interaction between its electric charge and chargeless quantum matter scalar field. Thus mass of the RNBH decreases to reach nonvanishing remnant stable mini Lukewarm black hole with $m_{final} = 1$. In other words its luminosity is eliminated while its mass is not eliminated completely (see figures 9, 10, and 11 given in [15]). Here we study mass loss and switching off effect of quantum perturbed mean metric (15). This is a dynamical approach to describe that how mean metric of RNBHs statistical ensemble exhibits a phase

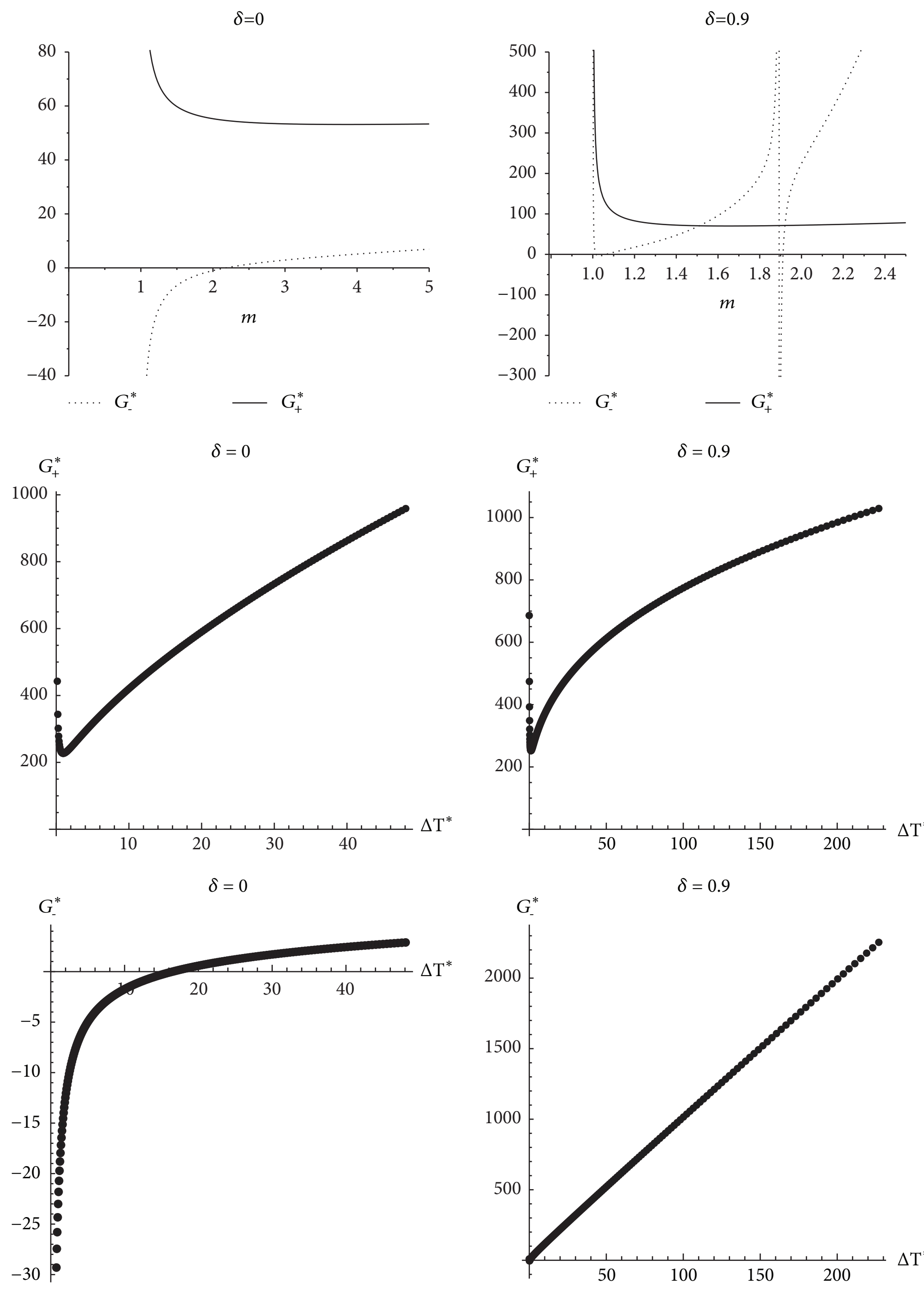


Figure 6: Diagram of interior and exterior horizons Gibbs free energies $G^*_{\pm}$ is plotted against $m$ and $\Delta T^*$ for $\delta = 0$ and $\delta = 0.9$.

transition leading to a possible Bose-Einstein condensation state microscopically. Line element of the evaporating mean metric (15) can be written near the exterior horizon as Vaida form (see, for instance, [29]):

$$ds^2 \simeq \left(1 - \frac{r_+(v)}{r}\right) dv^2 - 2dvdr - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right) \quad (40)$$

with the associated stress energy tensor

$$\left\langle \widehat{T}^{quant}_{\mu\nu} \right\rangle_{ren} = \frac{1}{4\pi r^2} \frac{dr_+(v)}{dv} \delta_{\mu v} \delta_{\nu v} \quad (41)$$

where $(v, r)$ is advance Eddington-Finkelstein coordinates system. Subscript *ren* denotes the word *Renormalization*, and $\langle\rangle$ denotes expectation value of quantum matter scalar field stress tensor operator evaluated in its vacuum state. The

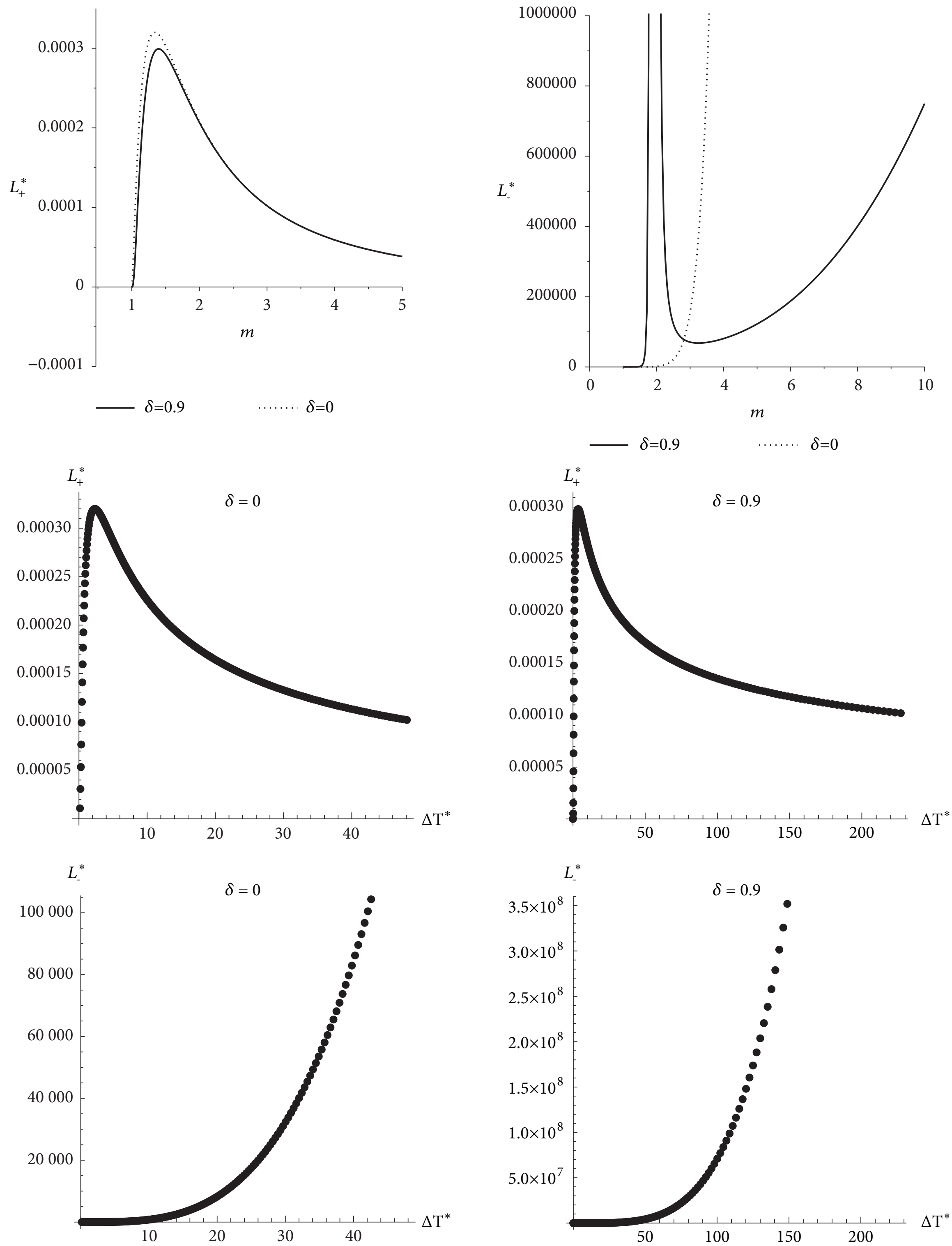


FIGURE 7: Diagram of exterior and interior horizons luminosity $L^*_\pm$ is plotted against $m$ and $\Delta T^*$ for $\delta = 0$ and 0.9.

black hole luminosity is defined by the following equation from point of view of distant observer located in $r$.

$$L(r, v) = 4\pi r^2 \left\langle \widehat{T}^r_v \right\rangle^{quant}_{ren}. \tag{42}$$

Applying (40) and (41), (42) becomes

$$L = -\frac{1}{2}\frac{dr_+(v)}{dv} \tag{43}$$

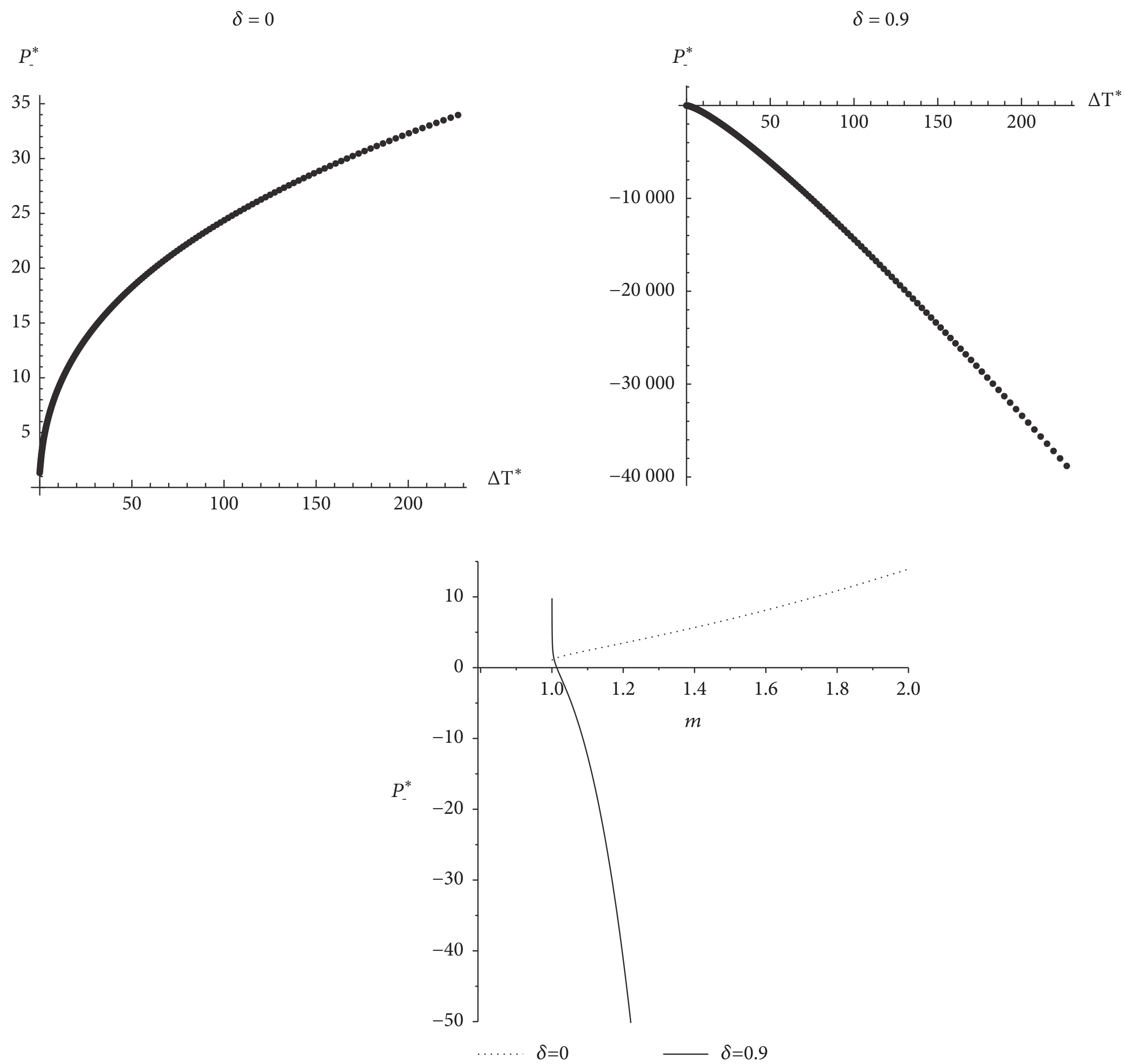


Figure 8: Diagram of interior horizon pressure $P_-^*$ is plotted against $m$ and $\Delta T^*$ for $\delta = 0$ and $\delta = 0.9$.

where negative sign describes inward flux of negative energy across the horizon. This causes the mean metric horizon of RNBHs statistical ensemble to shrink. In the latter case quantum particles of matter content of the black hole are in high energy state and so one can assume that the quantum black hole behaves as a black body radiation of which luminosity is defined by well-known Stefan-Boltzman law as follows:

$$L = \sigma_{SB} A T^4 \tag{44}$$

where $A$ is surface area of the black body and $T$ is its temperature. $\sigma_{SB} = 5.67 \times 10^{-8} (J/(m^2 \times {}^\circ K^4 \times Sec))$ is Stefan-Boltzman coupling constant of which dimensions become as $(length)^2$ in units $G = c = 1$. If (44) satisfies (43), then we can obtain mass loss equation of the mean metric of RNBHs statistical ensemble such that

$$\frac{dr_+(v)}{dv} = -2\sigma_{SB} \xi A_+(v) T_+^4(v) \tag{45}$$

where the normalization constant $\xi$ depends linearity on the number of massless, chargeless quantum matter fields and will control the rate of evaporation. Inserting (27) one can show that the luminosity (44) for RNBHs mean metric (15) becomes

$$L_+^*(m) = \frac{(4\pi)^3 e^2}{\sigma_{SB}} L = S_+(m) T^{*4}_+(m) \tag{46}$$

where $S_+(m)$ and $T_+^*(m)$ should be inserted from (25) and (30), respectively. Applying (19), (20), (21), (22), (27), and some simple calculations, we can show that the mean mass loss equation (45) for RNBHs mean metric (15) reads

$$\Delta v^* = v^*(m) - v_\infty^* = -\frac{1}{2} \int_m^1 \left(\frac{dr_+}{d\rho_+}\right) \frac{dm}{S_+^{3/2} T_+^{*5}} \tag{47}$$

where we used (19), (20), (21), (22), (27), $\delta = a/e$, $m = M/e$, and $\rho_+ = e\sqrt{S_+}$ to calculate $dr_+/d\rho_+$ which up to terms in order of $O(\delta^3)$ become

$$\frac{dr_+}{d\rho_+} = 1 + \frac{\delta^2}{5}\left(\frac{2}{\left(m+\sqrt{m^2-1}\right)^3} - \frac{3}{\left(m+\sqrt{m^2-1}\right)^4}\right). \quad (48)$$

$v^*_\infty = v^*(1)$ given in (47) is integral constant for which evaporating mean mass of RNBHs statistical ensemble reaches its final value as $m_{final} = 1$. Also we defined dimensionless advance Eddington-Finkelstein time coordinate $v^*$ as follows:

$$\frac{v^*}{v} = \frac{2\xi\sigma_{SB}}{(4\pi e)^3}. \quad (49)$$

When exterior horizon of quantum evaporating RNBHs mean metric reduces to scale of its interior horizon as $r_+(v) \longrightarrow r_-$ then one can use similar equations for luminosity and mass loss equations (46) and (47) for interior horizon as follows:

$$L^*_-(m) = \frac{(4\pi)^3 e^2}{\sigma_{SB}} L = S_-(m)\, T^{*4}_-(m) \quad (50)$$

$$\Delta v^* = v^*(m) - v^*_\infty = -\frac{1}{2}\int_m^1 \left(\frac{dr_-}{d\rho_-}\right)\left(\frac{dS_-}{dm}\right)\frac{dm}{S_-^{3/2} T_-^{*4}} \quad (51)$$

where

$$\frac{dr_-}{d\rho_-} = 1 + \frac{\delta^2}{5}\left(\frac{2}{\left(m-\sqrt{m^2-1}\right)^3} - \frac{3}{\left(m-\sqrt{m^2-1}\right)^4}\right). \quad (52)$$

Diagrams of the luminosities (46) and (50) and the evaporating mean RNBHs mass loss equations (47) and (51) are plotted versus mass parameter $m$ in Figure 7. They show that evaporating quantum unstable mean mass of RNBHs final state reaches remnant stable cold mini Lukewarm RNBH with final mass $m_{final} = 1$ where its causal singularity is still covered by its shrunken horizon and its luminosity reaches zero value. We see that invariant conditions on the black hole electric charge $e$ causes the Penrose cosmic censorship hypothesis to be valid while the black hole metric is evaporated where the casual singularity of mean metric (15) defined by $\rho = 0$ is still covered by their smallest scale horizons hypersurface with no naked singularity.

## 6. Summary and Discussion

According to the Debbasch approach we calculated mean metric of RNBHs statistical ensemble to obtain locations of interior and exterior horizons. We calculated corresponding entropy, temperature, heat capacity, Gibbs free energy, and pressure. At last section of the paper we considered interaction of massless, chargeless quantum scalar matter field on quantum perturbed mean metric of coarse graining RNBHs. Our mathematical calculations predict evaporation of the mean metric which reduces to a remnant stable mini black hole metric with nonvanishing mass. Before the evaporation reaches its final state, the mean metric exhibits a first-order phase transition and Bose-Einstein condensation state happens microscopically. Our results approve outputs of the published work [15] qualitatively in which the author studied thermodynamic behavior of a single RN black hole.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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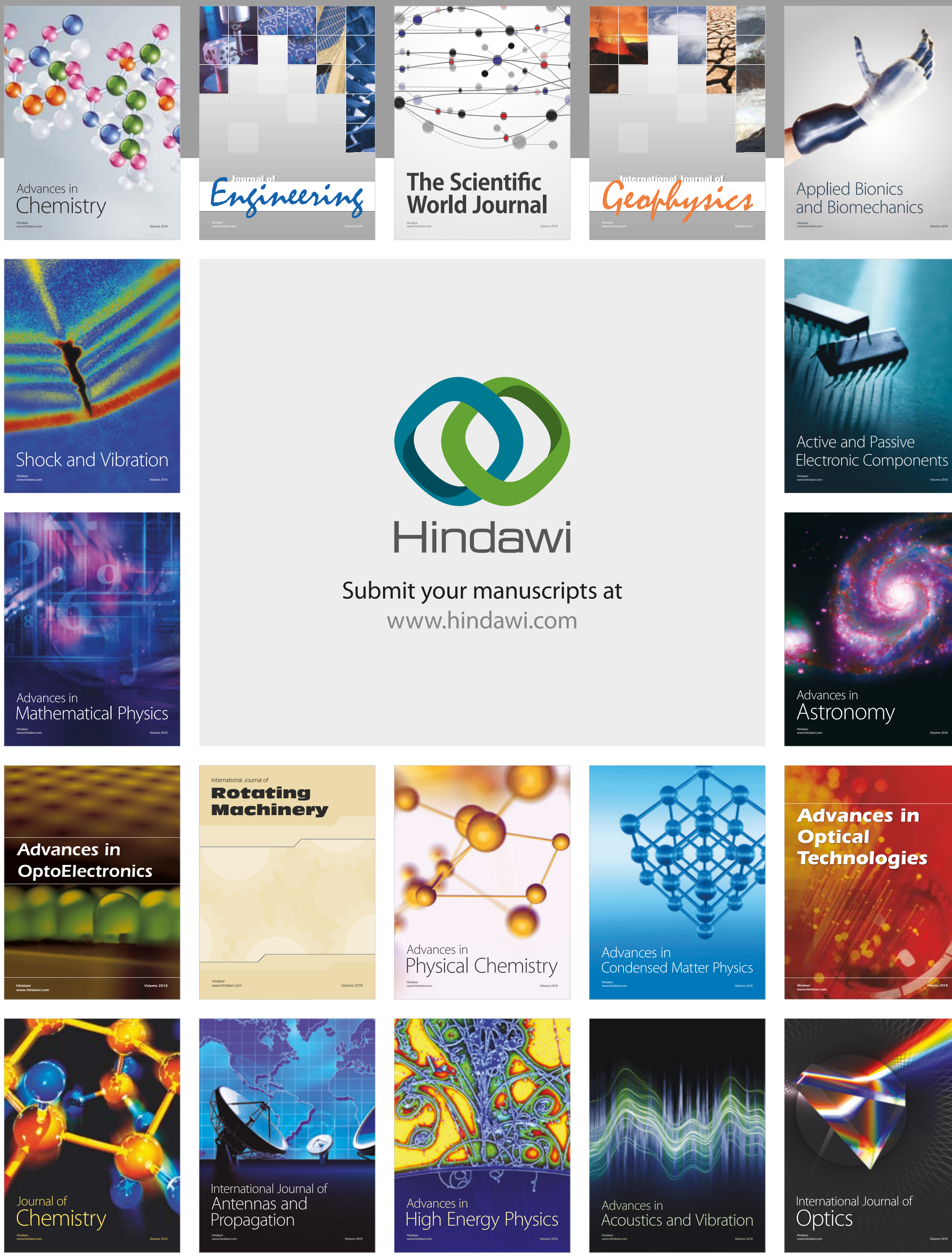
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