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Research Article

Cosmological Analysis of Modified Holographic Ricci Dark Energy in Chern-Simons Modified Gravity

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In this paper, we study the cosmological analysis of the modified holographic Ricci dark energy model and reconstruct different scalar field models in the context of Chern-Simons modified gravity. We investigate the deceleration parameter, which shows that the universe is in the accelerating expansion phase. The equation of state parameter in this case also favors the fact that dark energy is the dominant component of universe, which is responsible for the accelerated expansion. A number of scalar fields, such as quintessence, tachyon, K-essence, and dilaton models, are reconstructed using modified holographic Ricci dark energy model in the context of dynamical CS modified gravity. The quintessence and K-essence models represent exponentially increasing behaviors, while tachyon model shows decreasing behavior. Unfortunately, the dilaton model has no numerical solution for modified holographic Ricci dark energy model in the framework of dynamical Chern-Simons modified gravity.

1. Introduction

A large number of evidences have been provided in the favor of accelerated expansion of the universe by Type Ia supernovae [1, 2], Cosmic Microwave Background (CMB) [3], weak lensing [4], Large Scale Structures [5, 6], and integrated Sachs-Wolfe effect [7, 8]. It is postulated that there exists a component in the universe which has negative pressure and is responsible for the accelerated expansion of the universe, called dark energy (DE). The most familiar candidate of DE model is cosmological constant Λ which satisfies the cosmological observations [9, 10] but fails to resolve the fine tuning problem and cosmic coincidence [11]. In literature, there are many DE models such as an evolving canonical scalar field [12, 13] quintessence, the phantom energy [14, 15], and quintom energy [16–18].

In recent studies, to understand the nature of universe, a new DE model has been constructed in the context of quantum gravity on holographic principle named holographic dark energy (HDE) model [19–22]. This principle is extensively used to study the quantum behavior of black holes. The energy density of HDE is defined as $\rho = 3c^2 M_{pl}^2 L^{-2}$, where c is constant, M_{pl} is Plank mass, and L is supposed to be size of the universe. For Hubble radius H^{-1}, this HDE model's density is very similar to the observational results. Gao et al. [23], motivated by the holographic principle, introduced a new DE model, which was inversely proportional to Ricci scalar curvature called Ricci dark energy (RDE). Their investigation shows that RDE model solves the causality problem and the evolution of density perturbations of matter power spectra and CMB anisotropy is not much affected by such modification. Granda and Oliveros [24] introduced a new infrared cut-off for HDE model and reconstructed the potentials and fields for different DE models such as the quintessence, tachyon, K-essence, and dilaton for FRW universe. Karami and Fehri [25] using Granda and Oliveros cut-off studied the nonflat FRW universe to find the DE density, deceleration parameter, and equation of state (EoS).

Jackiw and Pi [26] introduced Chern-Simons (CS) modified gravity, in which the Einstein-Hilbert action is modified as the sum of parity-violating CS term and scalar field. Silva and Santos [27] analyzed the RDE of FRW universe and found it to be similar to GCG in the context of CS modified gravity. Jamil and Sarfraz [28] did the same for amended FRW universe and presented their results graphically. Jawad and Sohail [29] considering modified QCD ghost dark energy model investigated the dynamics of scalar field and potentials of various scalar field models in the framework of dynamical CS modified gravity. Jamil and Sarfraz [30] considering HDE model found the accelerated expansion behavior of the universe under certain restrictions on the parameter α. We studied the correspondence between quintessence, K-essence, tachyon, and dilaton field models and holographic dark energy model. Pasqua et al. [31] investigated the HDE, modified holographic Ricci dark energy (MHRDE), and another model, which is a combination of higher-order derivatives of the Hubble parameter in the framework of CS modified gravity.

In this paper, working on same lines using the MHRDE model, we explored the energy density, deceleration parameter, EoS parameter, and correspondence between different models. The paper is organized in following order. The basic formalism of CS modified gravity is discussed in Section 2. Section 3 is devoted for the investigation of energy density, deceleration parameter, and EoS parameter. The correspondence between scalar field models such as quintessence, tachyon, K-essence, and dilaton model is given in Section 4. Summery and concluding remarks are in the last section.

2. ABC of Chern-Simons Modified Gravity

Jackiw and Pi [26] modify the 4-dimensional GR theory introducing a Chern-Simons term in Einstein-Hilbert action given by [32, 33]

$$\begin{aligned} S = \int d^4x\sqrt{-g}\Big[\chi R + \frac{\zeta}{4}\Theta\, {}^{*}RR \\ - \frac{\eta}{2}\left(g^{\mu\nu}\nabla_\mu\Theta\nabla_\nu\Theta + 2V[\Theta]\right)\Big] + S_{mat}, \end{aligned} \tag{1}$$

where the terms used in this relation are defined as $\chi = (16\pi G)^{-1}$, R is the usual Ricci scalar, the term ${}^{*}RR$ is defined as ${}^{*}RR = {}^{*}R^{a}{}_{b}{}^{cd}R^{b}{}_{acd}$, is topological invariant, called Pontryagin term where ${}^{*}R^{a}{}_{b}{}^{cd}$ is the dual of Riemann tensor $R^{b}{}_{acd}$ can be expressed as ${}^{*}R^{a}{}_{b}{}^{cd} = (1/2)\epsilon^{cdef}R^{a}{}_{bef}$, ∇_μ is titled as covariant derivative, the dimensionless parameters ζ and η are treated here as coupling constants, and $V[\Theta]$ is potential and in the context of string theory it is assumed that $V[\Theta] = 0$. The most important term Θ, called CS coupling field, works as a deformation function of the space-time which is always other than constant (if Θ is constant function, then the CS theory reduces to GR).

The variation of Einstein-Hilbert action S corresponding to metric tensor $g_{\mu\nu}$ and scalar field Θ resulted into a set of field equations of CS modified gravity given by

$$G_{\mu\nu} + lC_{\mu\nu} = \chi T_{\mu\nu}, \tag{2}$$

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\Theta = -\frac{\zeta}{4}{}^{*}RR, \tag{3}$$

where $G_{\mu\nu}$, $C_{\mu\nu}$, l, and $T_{\mu\nu}$ are called Einstein tensor, Cotton tensor (C-tensor), coupling constant, and energy-momentum tensor, respectively. The energy-momentum tensor is a combination of matter part $T^{m}_{\mu\nu}$ and the external field part $T^{\Theta}_{\mu\nu}$. The mathematical expressions for these terms are as follows:

$$\begin{aligned} C^{\mu\nu} = -\frac{1}{2\sqrt{-g}}\left[\upsilon_\sigma\epsilon^{\sigma\mu\zeta\eta}\nabla_\zeta R^{\nu}_{\eta} + \frac{1}{2}\upsilon_{\sigma\tau}\epsilon^{\sigma\nu\zeta\eta}R^{\tau\mu}_{\zeta\eta}\right] \\ + (\mu \longleftrightarrow \nu), \end{aligned} \tag{4}$$

$$T^{m}_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \tag{5}$$

$$T^{\Theta}_{\mu\nu} = \zeta(\partial_\mu\Theta)(\partial_\nu\Theta) - \frac{\zeta}{2}g_{\mu\nu}(\partial^\lambda\Theta)(\partial_\lambda\Theta). \tag{6}$$

Here ρ is energy density, p is pressure, and $u_\mu = (1,0,0,0)$ denotes standard time-like 4-velocity, respectively. The terms $\upsilon_\sigma \equiv \nabla_\sigma\Theta$ and $\upsilon_{\sigma\tau} \equiv \nabla_\sigma\nabla_\tau\Theta$. On the basis of choice of ($\zeta \neq 0$ and $\chi \neq 0$) and ($\chi \neq 0$ and $\zeta = 0$), this theory is divided into two distinct theories named dynamical and nondynamical Chern-Simons modified gravity, respectively.

3. Modified Holographic Ricci Dark Energy Model

In this paper, we studied FRW universe in the framework of dynamical CS modified gravity. By the 00-component of field equation for FRW universe using (2), we get

$$H^2 = \frac{1}{3}\rho_D + \frac{1}{6}\dot{\Theta}^2. \tag{7}$$

Here $H = \dot{a}/a$ is called Hubble parameter and $\dot{a}$ is time derivative of scale factor $a(t)$. The Pontryagin term ${}^{*}RR$ vanishes for FRW metric identically, so (3) takes the form

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\Theta = g^{\mu\nu}\left[\partial_\mu\partial_\nu\Theta - \Gamma^{\rho}_{\mu\nu}\partial_\rho\Theta\right] = 0. \tag{8}$$

Keeping in view the fact that Θ is a function of space-time, we consider $\Theta = \Theta(t)$; (8) turns out to be

$$\dot{\Theta} = Ca^{-3}. \tag{9}$$

C is constant of integration other than zero (as if this C is zero then the function Θ becomes constant which reduces the CS theory to GR).

Using holographic principle, Hooft [34] proposed very simple and convenient model to investigate the issues raised in DE, named as HDE model. This model is used in different scenarios such as Hubble radius and cosmological conformal time of particle horizon [35, 36]. An interesting holographic RDE model defined as $L = |R|^{-1/2}$, where R is Ricci curvature scalar, was proposed by Gao et al [37].

In this paper, we use HDE model suggested by Granda and Oliveros in [22] defined as

$$\rho_{MHRDE} = \frac{2}{\alpha - \beta}\left(\dot{H} + \frac{3\alpha}{2}H^2\right), \tag{10}$$

and here α and β are constants. It is worth mentioning here, in limiting case ($\alpha = 4/3, \beta = 1$), that the Granda and Oliveros IR cut-off reduces to Gao et al.'s IR cut-off. Using (9) and (10) in (7), we arrive at

$$H^2 = \frac{2}{3(\alpha - \beta)}\left(\dot{H} + \frac{3\alpha}{2}H^2\right) + \frac{1}{6}C^2a^{-6} \tag{11}$$

and solving the differential equation, the scale factor $a(t)$ is explored as

$$a(t) = \left[\frac{12C_1}{C(\alpha-4)} - \frac{3C(\alpha-4)}{4}t^2\right]^{1/6}. \quad (12)$$

To avoid the singular solution, it is provided that $\alpha \neq 4$. The Hubble parameter $H(t) = \dot{a}/a$ can be evaluated as

$$H(t) = -\frac{Ct(\alpha-4)}{4\left(12C_1/C(\alpha-4) - (3/4)Ct^2(\alpha-4)\right)}. \quad (13)$$

Since the scale factor has been explored by assuming $\beta = 4$ in (10), it takes the form

$$\rho_{MHRDE} = \frac{2}{\alpha-4}\left(\dot{H} + \frac{3\alpha}{2}H^2\right), \quad (14)$$

and using corresponding values of $H(t)$ and $\dot{H}(t)$, the expression for MHRDE density turned out to be

$$\rho_{MHRDE} = \frac{(\alpha-4)\left(C^4t^2(\alpha-4)^2(\alpha-2) - 32C^2C_1\right)}{3\left(C^2t^2(\alpha-4)^2 - 16C_1\right)^2}. \quad (15)$$

In terms of redshift parameter, the density is given as

$$\rho(z) = \frac{1}{4}(1+z)^6\left[-C(\alpha-2) + 12C_1(1+z)^6\right]. \quad (16)$$

The energy density of this model is increasing for all values of $C, \alpha < 2$, and $C_1 > 0$. We plot a graph for different values of these parameters.

The graphical behavior of the density is exponentially increasing after $z > 0$ in the context of CS gravity using this MHRDE model.

3.1. Deceleration Parameter. The rate of expansion of the universe remained unchanged at constant values of $\dot{a}(t)$ and deceleration term q along with condition imposed on scale factor $a(t)$, that is, $a(t) \propto t$, where t is cosmic time. The Hubble parameter H remains constant and deceleration term $q = -1$, when de Sitter and steady-state universes are under consideration. Furthermore, the deceleration parameter varies with time for some universes available in literature. Using the variational values of H and q, we can classify all the defined universe models whether they are in expansion or contraction mode and acceleration or deceleration mode:

(1) $H > 0$, $q > 0$, expanding and decelerating
(2) $H > 0$, $q = 0$, expanding, zero deceleration
(3) $H < 0$, $q = 0$, contracting, zero deceleration
(4) $H < 0$, $q > 0$, contracting and decelerating
(5) $H < 0$, $q < 0$, contracting and accelerating
(6) $H = 0$, $q = 0$, static.

The deceleration parameter q in terms of Hubble parameter H is defined as

$$q(t) = -1 - \frac{\dot{H}}{H^2}, \quad (17)$$

where $\dot{H}$ represents the derivative of H with respect to t, termed as

$$\dot{H}(t) = -\frac{C^2(\alpha-4)^2\left(C^2t^2(\alpha-4)^2 + 16C_1\right)}{3\left(C^2t^2(\alpha-4)^2 - 16C_1\right)^2}. \quad (18)$$

Substituting these values in (17), we find deceleration parameter q:

$$q(t) = 2 + \frac{48C_1}{(\alpha-4)^2C^2t^2}. \quad (19)$$

Representing in the form of redshift, it becomes

$$q(z) = \frac{2\left[C(\alpha-4) - 30C_1(1+z)^6\right]}{C(\alpha-4) - 12C_1(1+z)^6} \quad (20)$$

It is obvious that the deceleration parameter depends on C, C_1, α, and redshift parameter z. At present epoch $z = 0$, the deceleration parameter $q < 0$ for each case given below:

$$\begin{aligned}
&(1) \quad \alpha < 4,\\
&\qquad C_1 < 0,\\
&\qquad \frac{12C_1}{\alpha-4} < C < \frac{30C_1}{\alpha-4},\\
&(2) \quad \alpha > 4,\\
&\qquad C_1 < 0,\\
&\qquad \frac{30C_1}{\alpha-4} < C < \frac{12C_1}{\alpha-4},\\
&(3) \quad \alpha < 4,\\
&\qquad C_1 > 0,\\
&\qquad \frac{30C_1}{\alpha-4} < C < \frac{12C_1}{\alpha-4},\\
&(4) \quad \alpha > 4,\\
&\qquad C_1 > 0,\\
&\qquad \frac{12C_1}{\alpha-4} < C < \frac{30C_1}{\alpha-4}.
\end{aligned} \quad (21)$$

For all these conditions, the model under consideration in CS gravity advocates that the universe is in accelerated expansion phase.

3.2. Equation of State Parameter. The nature of component which is dominating universe can be studied with the EoS parameter ω. In fact, it illustrates the era of dominance of universe by certain component. For example, $\omega = 0, 1/3$, and 1 predict that the universe is under dust, radiation, and stiff fluid influence, respectively. Meanwhile, $\omega = -1/3, -1$, and $\omega < -1$ stand for quintessence DE, ΛCDM, and Phantom eras, respectively. Now, differentiating (15) with respect to time t,

$$\dot{\rho}(t) = -\frac{2C^4 t(\alpha-4)^3\left(C^2t^2(\alpha-4)^2(\alpha-2)+16C_1(\alpha-6)\right)}{3\left(C^2t^2(\alpha-4)^2-16C_1\right)^3}. \quad (22)$$

The conservation equation in CS modified gravity is given by [38]

$$\dot{\rho}+3H(\rho+p)=0. \quad (23)$$

The expression for EoS parameter ω can be explored using (23) such that

$$\omega(t) = -1-\frac{\dot{\rho}(t)}{3H(t)\rho(t)}. \quad (24)$$

Making use of (13), (18), and (22) in (24), we found analytic solution of EoS parameter as given below:

$$\omega(t) = -1-\frac{-2C^2t^2(\alpha-4)^2(\alpha-2)-32C_1(\alpha-6)}{C^2t^2(\alpha-4)^2(\alpha-2)-32C_1}. \quad (25)$$

The EoS parameter ω in terms of redshift parameter z looks like

$$\omega(z) = \frac{C(\alpha-2)-36C_1(1+z)^6}{C(\alpha-2)-12C_1(1+z)^6}. \quad (26)$$

Obviously, EoS parameter ω is a function of variable redshift parameter z depending on C, C_1, and α. At the present epoch $z=0$ the EoS is $\omega<-1$ for each of the cases described below:

$$\begin{aligned}
&(1)\quad C<0,\\
&\qquad C_1<0,\\
&\qquad \frac{12C_1+2C}{C}<\alpha<\frac{24C_1+2C}{C},\\
&(2)\quad C<0,\\
&\qquad C_1>0,\\
&\qquad \frac{24C_1+2C}{C}<\alpha<\frac{12C_1+2C}{C},\\
&(3)\quad C>0,\\
&\qquad C_1<0,\\
&\qquad \frac{24C_1+2C}{C}<\alpha<\frac{12C_1+2C}{C},\\
&(4)\quad C>0,\\
&\qquad C_1>0,\\
&\qquad \frac{12C_1+2C}{C}<\alpha<\frac{24C_1+2C}{C}.
\end{aligned} \quad (27)$$

For different values of these parameters, the EoS $\omega<-1$ favors the fact that the universe is dominated by DE.

4. Study of MHRDE Model Using Scalar Field Models

In this section, we discuss different scalar field models like quintessence, tachyon, K-essence, and dilaton models in the framework of CS modified gravity. To study the behavior of quantum gravity, we explore the potential and scalar field.

4.1. Quintessence Model. A DE model is developed to explain the late-time cosmic acceleration called quintessence, which is a simplest scalar field that has no theoretical problem like ghosts and Laplacian instabilities appearance [11]. This model is useful to settle down the issue of fine-tuning in cosmology considering the time-dependent EoS. Using this model, we can explain the cosmic acceleration having negative pressure when potential energy dominates the kinetic energy. The energy and pressure densities are defined as

$$\begin{aligned}
\rho_Q &= \frac{1}{2}\dot{\phi}^2+V(\phi),\\
p_Q &= \frac{1}{2}\dot{\phi}^2-V(\phi),
\end{aligned} \quad (28)$$

where the scalar field ϕ is differentiated with respect to t. The EoS parameter for the quintessence is

$$\omega_\phi = \frac{(1/2)\dot{\phi}^2-V(\phi)}{(1/2)\dot{\phi}^2+V(\phi)}. \quad (29)$$

The comparison of EoS ω_ϕ formulated for quintessence model given in (29) and EoS ω calculated for MHRDE modal given in (25) turned as

$$\frac{(1/2)\dot{\phi}^2-V(\phi)}{(1/2)\dot{\phi}^2+V(\phi)} = \frac{C^2t^2(\alpha-4)^2(\alpha-2)+32C_1(\alpha-5)}{C^2t^2(\alpha-4)^2(\alpha-2)-32C_1}. \quad (30)$$

Now, equating density of quintessence model given in (28) and density evaluated from MHRDE model represented in (15) are expressed as

$$\frac{1}{2}\dot{\phi}^2+V(\phi) = \frac{(\alpha-4)\left(C^4t^2(\alpha-4)^2(\alpha-2)-32C^2C_1\right)}{3\left(C^2t^2(\alpha-4)^2-16C_1\right)^2}. \quad (31)$$

Using (30) and (31), we arrive at

$$\dot{\phi}^2 = \frac{2C^2(\alpha-4)\left(C^2t^2(\alpha-4)^2(\alpha-2)+16C_1(\alpha-6)\right)}{3\left(C^2t^2(\alpha-4)^2-16C_1\right)^2} \quad (32)$$

and integrating the last equation with respect to t,

$$\phi(t) = \sqrt{\frac{2}{3}}\left[-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}Ct(\alpha-4)^{3/2}}{\sqrt{C^2t^2(\alpha-4)^2(\alpha-2)+16C_1(\alpha-6)}}\right)\right.$$
$$\left.+\sqrt{\frac{\alpha-2}{\alpha-4}}\ln\left[C\left(Ct\left(\alpha^2-2\alpha+8\right)+\sqrt{\alpha-2}\times\sqrt{C^2t^2(\alpha-4)^2(\alpha-2)+16C_1(\alpha-6)}\right)\right]\right]. \tag{33}$$

In terms of redshift parameter z, it turns out to be

$$\phi(z) = -\frac{2}{3(\alpha-4)}\tanh^{-1}\left[\frac{(\alpha-4)^{3/2}\sqrt{4C/(1+z)^6+48C_1/(4-\alpha)}}{\sqrt{8-2\alpha}\sqrt{(\alpha-4)\left(-c(\alpha-2)+24(1+z)^6C_1\right)/(1+z)^6}}\right]+(\alpha-4)\sqrt{\frac{2(\alpha-2)}{3}}$$
$$\cdot\log\left[\frac{2C}{\sqrt{3}}\left(\sqrt{\alpha-2}\sqrt{\frac{(\alpha-4)\left(-c(\alpha-2)+24(1+z)^6C_1\right)}{(1+z)^6}}+\frac{\sqrt{c}\left(8-6\alpha+\alpha^2\right)\sqrt{4/(1+z)^6+48C_1/C(4-\alpha)}}{2\sqrt{4-\alpha}}\right)\right]. \tag{34}$$

Again using (30) and (31), the potential for quintessence model can be explored as

$$V(t) = -\frac{16C^2C_1(\alpha-4)^2}{3\left(C^2t^2(\alpha-4)^2-16C_1\right)^2}. \tag{35}$$

And making it convenient to discuss, we change into redshift parameter

$$V(z) = -3C_1(1+z)^{12}. \tag{36}$$

It is obvious that the potential of quintessence model depends on the values of constant of integration C_1 only. It shows the increasing behavior for all $C_1 < 0$ and decreasing behavior for $C_1 > 0$. It is interesting that the parameters C and α do not appear in the final expression of potential in the framework of CS Modified gravity.

4.2. Tachyon Model. Much attention has been given to tachyon field models in the last few decades in string theory and cosmology [39–45]. In fact, isotropic cosmological models whose radius depends on time and their potential can be constructed using minimally coupled scalar field model [46]. The same procedure for the correspondence between minimally coupled scalar field models and tachyon can be utilized to study the similar cosmological evolution [45]. The energy and pressure densities for the tachyon fields model are expressed as

$$\rho = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}},$$
$$p = -V(\phi)\sqrt{1-\dot{\phi}^2}. \tag{37}$$

Since $p = \rho\omega$, using the above expressions, the EoS parameter can be evaluated as

$$\omega = \dot{\phi}^2 - 1 \tag{38}$$

The comparison of (25) with (38) gives kinetic energy $\phi(t)$ such that

$$\phi(t) = \sqrt{\frac{C_1(6-\alpha)}{C^2(\alpha-4)^2(\alpha-2)}}E\left[\arcsin\left(t\sqrt{\frac{C^2(\alpha-4)^2(\alpha-2)}{32C_1}}\right);\frac{2}{\alpha-6}\right]. \tag{39}$$

The tachyon potential in this case is

$$V(t) = \frac{C^2(\alpha-4)}{3\left(C^2t^2(\alpha-4)^2-16C_1\right)^2}\times\sqrt{\left[C^2t^2(\alpha-4)^2(\alpha-2)-32C_1\right]\left[C^2t^2(\alpha-4)^2(\alpha-2)+32C_1(\alpha-5)\right]}. \tag{40}$$

Now, the kinetic and potential energies of tachyon model in terms of redshift parameter z, respectively, are

$$\phi(z)=\frac{4\sqrt{2C_1(\alpha-6)}}{\sqrt{C^2(\alpha-2)(\alpha-4)^2}}EllipticE\left[\sin^{-1}\left[\frac{\sqrt{C^2(\alpha-2)(\alpha-4)^2/C_1}\sqrt{4/(1+z)^6+48C_1/C(4-\alpha)}}{4\sqrt{6C(4-\alpha)}}\right],-\frac{2}{\alpha-6}\right], \quad (41)$$

$$V(z)=-\frac{1}{4}(1+z)^6\left(C(\alpha-2)-12(1+z)^6C_1\right)\sqrt{1-\frac{1}{2}(1+z)^6\left(-c(\alpha-2)+24(1+z)^6C_2\right)}. \quad (42)$$

To investigate the behavior of potential, we plot a graph of $V(z)$ versus z.

The graph plotted for the potential $V(z)$ of tachyon model against redshift parameter z shows the decreasing behavior irrespective of the values of parameters α, C, and C_1. This graph is plotted by taking particular values of these parameters to elaborate the result.

4.3. K-Essence. Armendariz et al. [47] introduced the dynamical concept of k-essence to explain the fact of accelerated expansion of universe. This model solves the fine-tuning problem of parameters. Actually, k-essence is developed on the principle of dynamical attractor solution; that is why it works as cosmological constant at the onset of matter domination. The energy and pressure densities of this model are given as

$$\rho=V(\phi)\left(-X+3X^2\right),$$
$$p=V(\phi)\left(-X+X^2\right). \quad (43)$$

$X=\dot{\phi}^2/2$; the EoS parameter for this model is

$$\omega=\frac{1-X}{1-3X} \quad (44)$$

Equating (25) and (44), we obtain

$$X(t)=\frac{16C_1(\alpha-4)}{C^2t^2(\alpha-4)^2(\alpha-2)+16C_1(3\alpha-14)}. \quad (45)$$

Since $X(t)=\dot{\phi}^2/2$, the integration of the above equation provides $\phi(t)$:

$$\phi(t)=\frac{4C_1}{\sqrt{(\alpha-4)(\alpha-2)}}\ln\left[C\left(Ct\left(\alpha^2-6\alpha+8\right)+\sqrt{\alpha-2}\sqrt{C^2t^2(\alpha-4)^2(\alpha-2)+16C_1(3\alpha-14)}\right)\right]. \quad (46)$$

The kinetic energy ϕ in terms of redshift parameter z is turned as

$$\phi(z)=\frac{4C_1}{c\sqrt{\alpha-2}\sqrt{(\alpha-4)C_1}}\times\log\left[\frac{2C}{\sqrt{3}}\sqrt{\frac{(\alpha-4)\left(-C(\alpha-2)+48(1+z)^6C_1\right)}{(1+z)^6}}+\frac{\sqrt{C}\left(8-6\alpha+\alpha^2\right)\sqrt{4/(1+z)^6+48C_1/C(4-\alpha)}}{2\sqrt{4-\alpha}}\right]. \quad (47)$$

The k-essence potential is calculated using (43), (44), and (25) as

$$V(t)=-\frac{C^2\left(C^2t^2(\alpha-4)^2(\alpha-2)+16C_1(3\alpha-14)\right)^2}{48C_1\left(C^2t^2(\alpha-4)^2-16C_1\right)^2}. \quad (48)$$

Now, we convert this function in terms of redshift parameter z and investigate its behavior.

$$V(z)=-\frac{\left(c(\alpha-2)-48(1+z)^6C_1\right)^2}{48C_2}. \quad (49)$$

The potential $V(z)$ is increasing function for all values of C and α at value of constant of integration $C_1=-1$. We plot a graph for particular values of these parameters just for example. After present epoch $z=0$, it increases exponentially.

4.4. Dilaton Model. The negative kinetic energy of the phantom field creates the problem of quantum instability. To resolve this puzzle of instability, dilaton model is proposed and further used to study the nature of DE. The dilaton model is defined as 4-dimensional effective low-energy model in the context of string theory. The pressure and energy densities are presented as

$$\rho=-X+c_1e^{\lambda\phi}X^2,$$
$$p=-X+3c_1e^{\lambda\phi}X^2. \quad (50)$$

$X=\dot{\phi}^2/2$, and c_1 and λ are positive constants. The EoS parameter $\omega=p/\rho$ for these densities is calculated as

$$\omega=\frac{1-c_1e^{\lambda\phi}}{1-3c_1e^{\lambda\phi}}. \quad (51)$$

The comparison of (25) with (51) yields

$$c_1e^{\lambda\phi}\dot{\phi}^2=\frac{16C_1(\alpha-4)}{C^2t^2(\alpha-4)^2(\alpha-2)+16C_1(3\alpha-14)}. \quad (52)$$

Solving for $\phi(t)$, we arrive at

$$\phi(t)=\frac{2}{\lambda}\ln\left[\frac{\lambda}{2}\cdot\sqrt{\frac{16C_1}{C^2(\alpha-4)}}\sinh^{-1}\left(t\sqrt{\frac{C^2(\alpha-4)^2(\alpha-2)}{16C_1(3\alpha-14)}}\right)\right]. \quad (53)$$

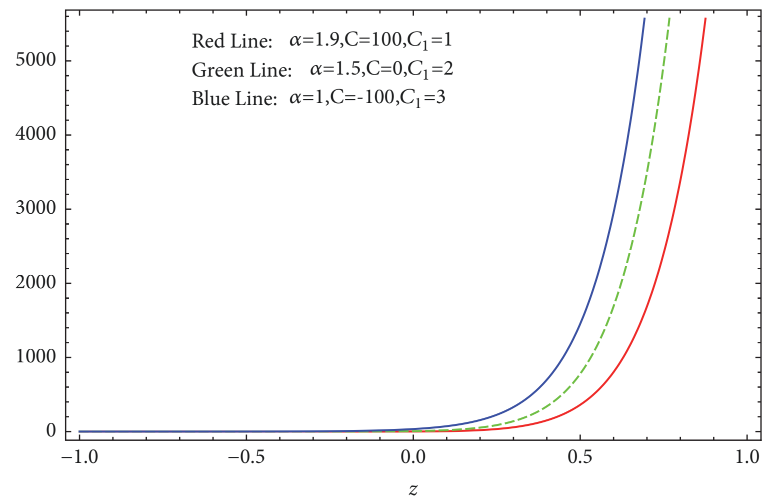

Figure 1: Density versus redshift parameter.

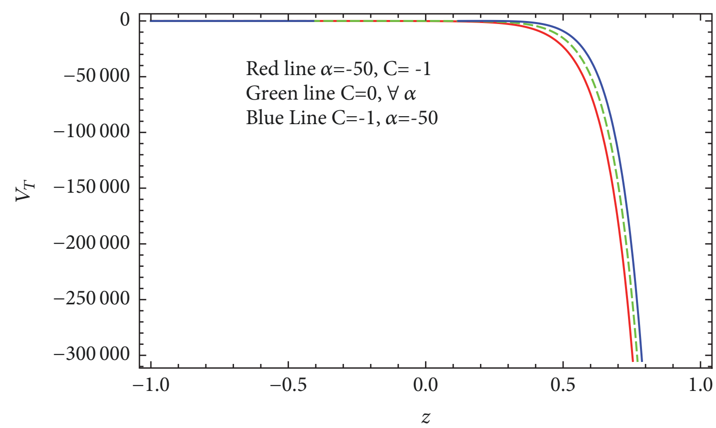

Figure 2: Potential versus redshift parameter.

In terms of redshift parameter z,

$$\phi(z) = \frac{2}{\lambda}\log\left[\sqrt{\frac{16C_1}{C^2(\alpha-2)}}\sinh^{-1}\frac{\left(\sqrt{C^2(\alpha-4)^2(\alpha-2)/C_1(3\alpha-14)}\sqrt{4/(1+z)^6+48C_1/C(4-\alpha)}\right)}{4\sqrt{3C(4-\alpha)}}\right]. \quad (54)$$

Analytically, it is found that there is no combination of parameters α, C, C_1 for which this function is defined.

5. Summary and Discussion

This work is devoted to study the cosmological analysis of MHRDE model in the context of CS modified gravity. The energy density for this model is calculated and observed in Figure 1. From the graph, it is obvious that density of the universe for MHRDE model is increasing for all values of CS modified gravity constants C, $\alpha < 2$, and $C_1 > 0$. The deceleration parameter $q < 0$ for the different combination of C, C_1, and α which advocates the accelerating expansion. The EoS parameter $\omega < -1$ is found, which favors the fact that DE is dominant at present epoch in case of MHRDE model in the context of CS modified gravity.

Furthermore, we reconstructed different scalar filed models using MHRDE in the context of dynamical CS modified gravity and found interesting results plotting them graphically. It is obvious that the potential of quintessence model depends only on the value of constant of integration C_1. It shows the increasing behavior for all $C_1 < 0$ and decreasing behavior for $C_1 > 0$. It is interesting that the potential in (36) is independent of CS parameter C and MHRDE parameter α identically, although we are working in the framework of CS modified gravity using MHRDE model. The graph plotted in Figure 2 for the potential of tachyon model shows the exponentially decreasing behavior irrespective of the values

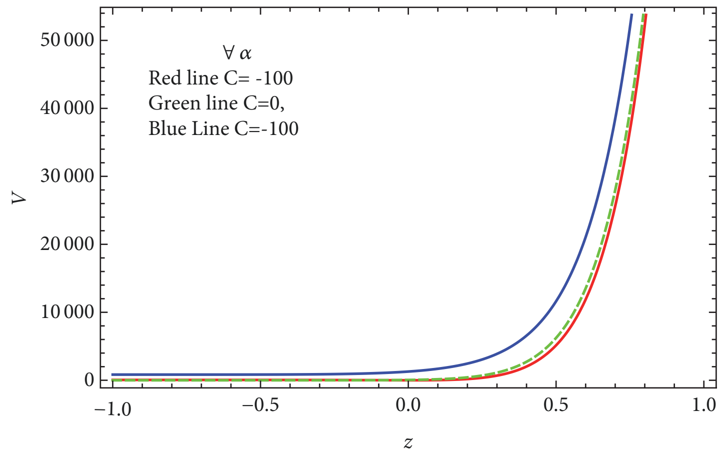

FIGURE 3: Potential versus redshift parameter.

of parameters α, C, and C_1. In case of k-essence, the potential is increasing function for all values of C and α at particular value of $C_1 = -1$. After present epoch $z = 0$, the graph increases exponentially as given by Figure 3. Analytically, it is found that there is no combination of parameters α, C, C_1 for which $\phi(z)$ is defined in case of dilaton model.

Data Availability

No data were used to support this study; these are the general results obtained mathematically using Mathematica software.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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