

Research Article

Out-Of-Equilibrium Transverse Momentum Spectra of Pions at LHC Energies

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In order to characterize the transverse momentum spectra (p_T) of positive pions measured in the ALICE experiment, two thermal approaches are utilized; one is based on degeneracy of nonperfect Bose-Einstein gas and the other imposes an *ad hoc* finite pion chemical potential. The inclusion of missing hadron states and the out-of-equilibrium contribute greatly to the excellent characterization of pion production. An excellent reproduction of these p_T -spectra is achieved at $\mu_\pi = 0.12$ GeV and this covers the entire range of p_T . The excellent agreement with the experimental results can be understood as a manifestation of not-yet-regarded anomalous pion production, which likely contributes to the long-standing debate on “anomalous” proton-to-pion ratios at top RHIC and LHC energies.

1. Introduction

The collective properties of strongly interacting matter (radial flow, for instance) and dynamics of colliding hadrons can be explored from the study of transverse momentum distributions (p_T) of produced particles. RHIC results on well-identified particles produced at low p_T , especially pions, have shown that the bulk matter created can be well described by hydrodynamics [1]. It should be emphasized that the high- p_T spectra, especially for the lowest-lying Nambu-Goldstone bosons, pions, likely manifest dynamics and interactions of partons and jets created in the earliest stage of nuclear collisions [2]. For instance, the collective expansion in form of radial flow might be caused by internal pressure gradients. Furthermore, the p_T -distributions are assumed to determine conditions, such as temperature and flow velocity, gaining dominance during the late eras of the evolution of the high-energy collision which is generically well-described as kinetic freeze-out, where the elastic interactions are ceased, conclusively.

From the theoretical point of view, the p_T -spectra are excellent measurements enabling us a better understanding of the QCD interactions. Soft nonperturbative QCD can

be well applied to low p_T -regime (below a few GeV/c) [3]. Fragmentation of QCD string [4], parton wave functions in flux tube [5], parton thermodynamics [6], and parton recombination [7] are examples on underlying physics. At high- p_T , hard-scattering cross-section from QCD perturbative calculations, parton distribution functions, and parton-to-hadron fragmentation functions have been successfully utilized in reproducing p_T -spectra of various produced particles [8].

It is worth mentioning that there is no well-defined line separating nonperturbative from perturbative p_T -regimes [3]. Even the various theoretical studies are not distinguishing sharply between both of them. For instance, when constructing partition functions, extensive and nonextensive statistical approaches are frequently misconducted [9–11]. For instance, the claim that high p_T -spectra of different produced particles are to be reproduced by Tsallis statistics seems being incomplete [11, 12]. This simply inspires a great contradiction between nonperturbative and perturbative QCD [12]. The statistical cluster decay could be scaled as power laws very similar to the ones of Tsallis statistics. The earlier is conjectured to cover a wide range of p_T , while the latter is limited to a certain p_T -regime. This would lead to an undesired mixing up that the observed power laws might be stemming from

the statistical cluster decay and interpreted as a Tsallis-type of nonextensivity.

In addition to the proposal of utilizing a generic (non)extensive statistical approach [9–11], we want here to recall another theoretical framework based on an *ad hoc* physically motivated assumption that the pion production might be interpreted due an out-of-equilibrium process [13]. Such an approach is stemming from the pioneering works of Bogolubov devoted to an explanation for the phenomenon of superfluidity on the basis of degeneracy of a non-perfect Bose-Einstein gas [14] and determining the general form of the energy spectrum, an ingenious application of the second quantization [15]. Finite pion chemical potential recalls Bogolubov dispersion relation for low-lying elementary excitations of pion fluid. In this case, degenerate state of statistical equilibrium is removed through inserting a noninvariant term to the Hamiltonian, e.g., pion chemical potential.

After a short review of the thermal approach, the Hadron Resonance Gas (HRG) model in equilibrium is introduced in Section 2. A discussion on how to drive it towards nonequilibrium through inclusion of repulsive interactions is added. In Section 3, we elaborate modifications carried out towards implementing nonperfect Bose-Einstein gas based on a proposal of pion superfluidity. Another out-of-equilibrium thermal approach is outlined in Section 4, where finite pion chemical potential is *ad hoc* imposed. The results shall be discussed in Section 5. The conclusions are given in Section 6.

2. A Short Review on Equilibrium Resonance Gas with Van der Waals

The hadron resonances treated as a noninteracting gas [16–22] are conjectured to determine the equilibrium thermodynamic pressure of QCD matter below chiral and deconfinement *critical* temperature, i.e., hadron phase. It has been shown that the thermodynamics of a strongly interacting system can also be approximated as an ideal gas composed of hadron resonances with masses ≤ 2 GeV [19, 23]. Interested readers are kindly advised to consult the most recent review article [24]. The resonances added in contribution with the degrees of freedom needed to characterize the hadron phase.

The grand canonical partition function can be constructed as

$$Z(T, \mu, V) = \text{Tr} \left[\exp^{(\mu N - H)/T} \right], \quad (1)$$

where H , T , and μ are the Hamiltonian, the temperature, and the chemical potential of the system, respectively. The Hamiltonian can be given by as summation of the kinetic energies of relativistic Fermi and Bose particles including the relevant degrees of freedom and the interactions resulting in formation of resonances and well describing the particle production in high-energy collisions. Under these assumptions, the sum over the *single-particle partition functions* Z_h^1

of existing hadrons and their resonances introduces dynamics to the partition function,

$$\ln Z(T, \mu_h, V) = V \sum_h \pm \frac{g_h}{2\pi^2} \int_0^\infty k^2 dk \ln \left\{ 1 \pm \exp \left[\frac{\mu_h - \varepsilon_h}{T} \right] \right\}, \quad (2)$$

where $\varepsilon_h = (k^2 + m_h^2)^{1/2}$ is the dispersion relation of h -th particle, g_h is spin-isospin degeneracy factor, and \pm stands for fermions and bosons, respectively.

In the present work, we include hadron resonances with masses ≤ 2 GeV compiling by the particle data group (PDG) 2018 [25]. This mass cut-off is assumed to define the validity of the HRG model in characterizing the hadron phase [26, 27]. The inclusion of hadron resonances with heavier masses leads to divergences in all thermodynamic quantities expected at temperatures larger than the Hagedorn temperature [16, 17]. In addition to these aspects, there are fundamental reasons (will be elaborated in forthcoming sections) favoring the utilization of even *ideal* HRG model in predicting the hadron abundances and their thermodynamics. For the sake of completeness, we highlight that the hadronic resonances which are not yet measured, including missing ones, can be parameterized as a spectral function [28].

As given earlier, we assume that the constituents of the HRG are free (collisionless) particles. Some authors prefer taking into account the repulsive (*electromagnetic*) van der Waals interactions in order to partly compensate strong interactions in the hadronic medium [29] and/or to drift the system towards even partial nonequilibrium. Accordingly, each constituent is allowed to have an *eigen*volume and the hadronic system of interest becomes thermodynamically partially out-of-equilibrium (how does a statistical thermal system, like HRG, become out-of-equilibrium? To answer this question, one might need to recall the main parameters describing particle production in equilibrium. These are T , μ_h , and V [30]. In the present work, we first focus on the third parameter and therefore describe this as a partial out-of-equilibrium process. The volume V , the normalization parameter typically constrained by pions, becomes a subject of modification through van der Waals repulsive interactions, for instance. Furthermore, it should be also noticed that the chemical potentials μ_h should be modified, as well, at least in connection with the modification in V . This would explain that taking into account van der Waals repulsive interactions, known as excluded volume corrections, contributes to deriving the system of interest towards nonequilibrium.). Thus, the total volume of HRG constituents should be subtracted from the fireball volume or that of heat bath. Considerable modifications in thermodynamics of HRG including energy, entropy, and number densities should be taken into consideration. It should be highlighted that the hard-core radius of hadron nuclei can be related to the multiplicity fluctuations.

How large can be the modification in V ? The answer to this question is conditioned, for instance, to the capability of the HRG model with finite-volumed constituents to reproduce first-principle lattice QCD simulations. At radius $r > 0.2$ fm, it was found that the disagreement with reliable

lattice QCD calculations becomes more and more larger [29]. It was concluded that such an excluded volume-correction becomes practically irrelevant, as it causes negligible effects at low temperatures [29]. But on the other hand, a remarkable deviation from the lattice QCD calculations is noticed at high T .

The repulsive interactions between hadrons are considered as a phenomenological extension of the HRG model. Exclusively, this is based on van der Waals excluded volume [31–34]. Intensive theoretical works have been devoted to the estimation of the excluded volume and its effects on the particle production and the fluctuations [35], for instance. It is conjectured that the hard-core radius of the hadrons can be related to the multiplicity fluctuations of the produced particles [36]. In the present work, we simply assume that the hadrons are spheres and all have the same radius. On the other hand, the assumption that the radii would be depending on the hadron masses and sizes could come up with a very small improvement. Various types of interactions have been assumed, as well [37, 38]. For the sake of possible comparison with existing literature, we focus on the van der Waals repulsive interaction. By replacing the system volume V by the actual one, V_{act} , the van der Waals excluded volume can be deduced [31]

$$V_{act} = V - \sum_h v_h N_h, \quad (3)$$

where $v_h = 4(4\pi r_h^3/3)$ is volume and N_h is the number of each constituent hadron. r_h is the corresponding hard sphere radius of h -th particle. The procedure encoded in (3) leads to modification in the chemical potentials $\tilde{\mu}_h = \mu_h - v_h p$, where the thermodynamic pressure p is self-consistently expressed as $\sum_h p_h^{id}(T, \tilde{\mu}_h)$ and

$$n = \frac{\sum_h n_h^{id}(T, \tilde{\mu}_h)}{1 + \sum_h v_h n_h^{id}(T, \tilde{\mu}_h)}, \quad (4)$$

$$\epsilon = \frac{\sum_h \epsilon_h^{id}(T, \tilde{\mu}_h)}{1 + \sum_h v_h n_h^{id}(T, \tilde{\mu}_h)}, \quad (5)$$

$$s = \frac{\sum_h s_h^{id}(T, \tilde{\mu}_h)}{1 + \sum_h v_h n_h^{id}(T, \tilde{\mu}_h)}, \quad (6)$$

where the superscript *id* refers to thermodynamic quantities calculated in HRG model with point-like constituents, i.e., ideal gas.

In the section that follows, we work out out-of-equilibrium p_T spectra of the positive pions, where finite pion chemical potential shall be *ad hoc* inserted in.

3. Out-of-Equilibrium p_T -Spectra of Pions

In U(1) global symmetry, where the scalar field $\phi(x)$ has a unitary transformation by the phase factor $\exp(-i\alpha)$,

the Bose-Einstein condensation of lowest-lying Nambu-Goldstone bosons could be studied from the partition function [39]

$$\ln z(T, \mu_\pi) = \frac{V}{T} (\mu_\pi^2 - m^2) \xi^2 - V \int \frac{d^3 p}{(2\pi)^3} \cdot \left[\frac{\epsilon}{T} + \ln(1 - e^{-(\epsilon - \mu_\pi)/T}) + \ln(1 - e^{-(\epsilon + \mu_\pi)/T}) \right], \quad (7)$$

where ξ is a parameter carrying full infrared characters of the scalar field. This can be treated as a variational parameter relating to the charge of condensed boson particle. At $|\mu_\pi| < m$, (2) can obviously be recovered. When the volume element $d^3 p$ is expressed in p_T , rapidity y and azimuthal angle ϕ as $d^3 p = p_T m_T \cosh(y) dp_T dy d\phi$ and the energy becomes $\epsilon = m_T \cosh(y)$, then at $\mu_\pi \rightarrow m$, where $m_T = (p_T^2 + m^2)^{1/2}$ is the pion transverse mass, the transverse momentum spectrum of pions is given as

$$\begin{aligned} \frac{1}{2\pi p_T} \frac{d^2 N_\pi}{dp_T dy} &= V \cdot \frac{m_T^{(\pi)} \cosh(y)}{(2\pi)^3} \left\{ \exp \left[\frac{m_T^{(\pi)} \cosh(y) - \mu_\pi}{T} \right] \right. \\ &\quad \left. - 1 \right\}^{-1} + \sum_{\text{reson.} \rightarrow \pi} V \\ &\quad \cdot \frac{m_T^{(\text{reson.})} \cosh(y)}{(2\pi)^3} \left\{ \exp \left[\frac{m_T^{(\text{reson.})} \cosh(y) - \mu_\pi}{T} \right] \right. \\ &\quad \left. - 1 \right\}^{-1} \times b_{\text{reson.} \rightarrow \pi}, \end{aligned} \quad (8)$$

where $b_{\text{reson.} \rightarrow \pi}$ is the branching ratio of resonances decaying into pions.

The pion p_T -spectrum is calculated from direct pions plus all contributions stemming from the heavier hadron resonances decaying into pions, in which the corresponding branching ratio should be taken into consideration (8)

$$\begin{aligned} \frac{1}{2\pi p_T} \frac{d^2 N_\pi}{dp_T dy} \Big|_\pi^{\text{total}} &= \frac{1}{2\pi p_T} \frac{d^2 N_\pi}{dp_T dy} \Big|_\pi \\ &\quad + \sum_{\text{reson.} \rightarrow \pi} \frac{1}{2\pi p_T} \frac{d^2 N_{\text{reson.}}}{dp_T dy} \Big|_{\text{reson.} \rightarrow \pi} \\ &\quad \times b_{\text{reson.} \rightarrow \pi}. \end{aligned} \quad (9)$$

In the results shown in Figure 1, we distinguish between the hadron resonances with the given mass cut-off and that without sigma states. In both cases, we also distinguish between results at chemical equilibrium of pion production, i.e., $\mu_\pi = 0$, and that at out-of-equilibrium, i.e., $\mu_\pi \neq 0$.

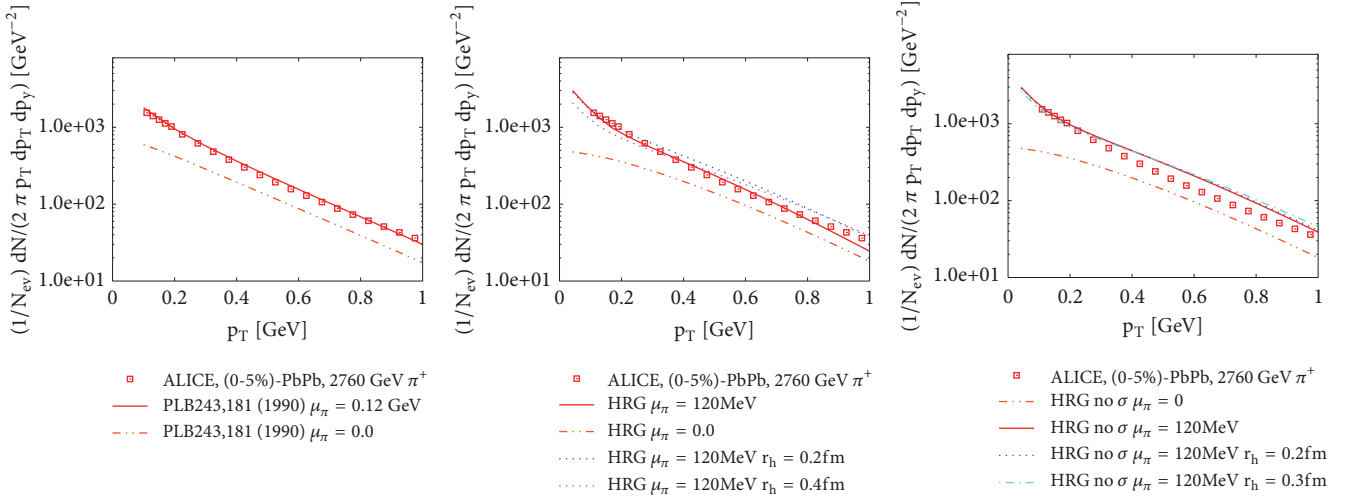


FIGURE 1: Number of positive pions per transverse momentum per rapidity is given dependent on the transverse momentum. The symbols refer to the ALICE measurements in the most central (0 – 5%) Pb+Pb collisions at 2.67 TeV. The left-hand panel shows a comparison with (10) at vanishing $\mu_\pi = 0$ and finite $\mu_\pi = 0.12$ GeV [41]. In middle and right-hand panels, HRG calculations (10), with and without scalar σ states and excluded volume corrections are confronted to the experimental measurements, as well.

4. Out-of-Equilibrium Thermal Distribution

In this section, we recall another thermal distribution at out-of-equilibrium [13], inspired by the proposal of Bogolubov to explaining the phenomenon of superfluidity as degeneracy of a nonperfect Bose-Einstein gas and determine a general form of energy spectra, a kind of ingenious application of second quantization.

In the approach introduced in [13], the p_T -spectra of *negatively* charged bosons measured from 200 A GeV O+Au and S+S collisions by NA35 experiment have been successfully reproduced. It was assumed that a cylindrical tube of matter with radius R expands, longitudinally, but without transverse flow $v_z = z/t$. When replacing ε in (2), for instance, by the azimuthal angle ϕ and the covariant form $p_\mu u^\mu$ (four-momentum and velocity) and then integrating over the freeze-out time $\tau_{fo} = \tau$, the p_T -distribution could be determined.

Then, at finite rapidity $y \neq 0$, the pion p_T -spectrum reads

$$\frac{1}{2\pi p_T} \frac{d^2 N_\pi}{dp_T dy} = (\pi R^2 \tau_{fo}) \frac{m_T \cosh(y)}{(2\pi)^3} \cdot \sum_{n=1}^{\infty} (\pm)^{n+1} \exp\left(n \frac{\mu_\pi}{T}\right) K_1\left[n \frac{m_T}{T} \cosh(y)\right]. \quad (10)$$

The contributions likely added by heavy resonances are contributing straightforwardly to (10). In Figure 1, we distinguish between results at chemical equilibrium of pion production, i.e., $\mu_\pi = 0$ and that at out-of-equilibrium, i.e., $\mu_\pi \neq 0$. Accordingly, we can examine the possibility of proposing a plausible explanation for the long-standing baryon-to-meson ratios, such as proton-to-pion ratios, especially at top RHIC and LHC energies [40]. We propose that this would be due to anomaly in the pion production.

5. Results

Figure 1 shows p_T -spectra of positive pions in dependence on the transverse momentum. The symbols refer to the ALICE measurements in the most central (0 – 1%) Pb+Pb collisions at 2.67 TeV [41]. The curves represent the present calculations.

The calculations from (10) at vanishing (dash-double-dotted curve) and finite $\mu_\pi = 0.12$ GeV (solid curve) are depicted in the left panel. It is obvious that finite μ_π leads to excellent agreement with the experimental results. The middle and right panels show the impacts for the excluded volume corrections and the scalar sigma states, respectively.

As done in the left panel, the contributions added by finite μ_π are examined in the middle panel. We find that the results from ideal HRG model at vanishing μ_π underestimate the experimental results. Finite μ_π and finite r_h improve the reproduction of the experimental results.

It intends to investigate the importance of these sigma states in reproducing p_T -spectra of pions at LHC energies, right panel. Various sigma states are to be in(ex)cluded: $I = 1/2$, $K_0^*(800)$ known as κ , which was excluded from 2014 particle data group and from our calculations as well, and $K_0^*(1430)$, $I = 1$, $a_0(980)$ and $a_0(1450)$ and $I = 0$, $f_0(500)$ widely known as σ , $f_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$.

The HRG calculations (8) with and without sigma states are presented in the right panels. Also, here we distinguish between vanishing (dash-double-dotted curve) and finite $\mu_\pi = 0.12$ GeV (solid curve). As the case with out-of-equilibrium thermal distributions, almost the same conclusion can be drawn here. Furthermore, the impacts of the inclusion of scalar sigma states are analyzed. It is obvious that these states seem to enhance the out-of-equilibrium, especially at large p_T . The excluded volume corrections are taken into consideration in the way that all stable hadrons and resonances with masses < 2 GeV are assumed to equally

have finite volume, r_h . As discussed earlier, at $r_h = 0.2$ fm, the corrections seem very small. There is no such a large variety to increase r_h and simultaneously conserve the thermodynamics consistency. A systematic analysis has been discussed in [29]; see Figure 1 and the related text.

The small difference when r_h is increased from 0.2 to 0.4 fm can basically be understood from the corresponding freeze-out temperature, with the inclusion of scalar sigma states $T_{ch} \simeq 151$ MeV, at $r_h = 0.2$ fm. But at $r_h = 0.4$ fm, T_{ch} becomes $\simeq 167$ MeV. These results are confirmed when scalar sigma states are removed, $T_{ch} \simeq 159$ MeV at $r_h = 0.2$ fm. Here, it was not possible to increase r_h to 0.4 fm due to divergence in the thermodynamic quantities, at the chemical freeze-out, for instance, T_{ch} should jump to ~ 1 GeV to fulfill the freeze-out conditions [20–22]! Therefore we have checked $r_h = 0.3$ fm. The corresponding $T_{ch} \simeq 164$ MeV. It seems in order to highlight that these results are limited to the validity of the approximation that all hadrons are conjectured to have the same radius, $r_h = 0.3$ fm.

We find that at vanishing μ_π , the results from (8) with and without scalar sigma states are almost identical. But at finite μ_π , the HRG calculations with and without scalar sigma states become distinguishable. This can be understood due to the remarkable characteristics and the great experimental abundances of the sigma states, which would appear as two-pion scalar-isoscalar resonances and are regarded as excitations of the scalar condensates, i.e., playing similar roles as the Higgs boson does for strong interactions [42]. Their $\bar{q}q$ -channel interaction is a maximally attractive one. This could be so strong that it breaks spontaneously the chiral symmetry and produces a quark condensate. Instanton-induced 't Hooft interaction is also conjectured as a mechanism for that attraction [42].

In order to elaborate more about the reasons why we have chosen another alternative and thought of confronting measured p_T -spectra of pions to Bogolubov superfluidity of Bose-Einstein gas and/or a thermal approach for out-of-equilibrium production of pions, i.e., finite μ_π , some remarks are now in order. Interpreting out-of-equilibrium in heavy-ion collisions at LHC in terms of the nonextensive Tsallis statistics [43] should be a subject of a fundamental revision. This has been discussed in great detail in [9–11]. On the one hand, the basic idea behind the thermal approach such as the HRG model is apparently additivity. Obviously, the Tsallis-type approach applies extensive statistics to the *additive* HRG, where both exponential and logarithm functions are merely replaced with the corresponding Tsallis counterpart function. On the other hand, the p_T -spectra of positive pions from the most central (0 – 5%) Pb+Pb collisions at 2.67 TeV are not reproducible by (20) in [43] (not drawn in Figure 1). But an excellent reproduction of these p_T -spectra is achieved at $\mu_\pi = 0.12$ GeV, Section 4. The excellent agreement apparently covers the entire range of p_T , left panel of Figure 1.

6. Conclusions and Outlook

The standard statistical thermal approach was reported as not being able to reproduce various baryon-to-boson ratios,

especially at top RHIC and LHC energies [40]. In reproducing the measured p_T -spectra of positive pions at LHC energies, we have decided in favor for another alternative thermal approach assuming an out-of-equilibrium production of pions, ($\mu_\pi \neq 0$). In addition to this, we have introduced out-of-equilibrium production of pions to the well-known HRG model. In ancillary to the baryon, the strangeness chemical potential and the electric charge potential, we have imposed $\mu_\pi \neq 0$, as well.

We have shortly highlighted the incompleteness of nonextensive Tsallis statistics, especially when confronted to bulk matter created at relativistic energies. The basic idea of implementing an *additive* resonance gas even with Tsallis algebra, where both exponential and logarithm functions are properly replaced by Tsallis algebra, seems not at all modifying the extensivity. The latter obviously contradicts the intention of taking into account out-of-equilibrium particle production.

We conclude that p_T -spectra of positive pions produced in most central (0 – 5%) Pb+Pb collisions at 2.67 TeV are well reproduced at $\mu_\pi = 0.12$ GeV. This is the case in two different thermal approaches. The first one is based on degeneracy of a nonperfect Bose-Einstein gas or an ingenious application of second quantization known as Bogolubov superfluidity. The second one is the well-known HRG model, in which $\mu_\pi \neq 0$ was *ad hoc* introduced. The baryon, strangeness, and electric charge chemical potentials, etc. can also be taken into consideration. This approach seems being successful in reproducing p_T -spectra of positive pions, at LHC energies, when PDG sigma states are taken into account.

Future works shall be devoted to a systematic comparison with other approaches and to investigating the impacts that the PDG sigma states in reproducing p_T -spectra of other hadrons. Furthermore, p_T -spectra of other bosons shall be extended to understand whether they require out-of-equilibrium processes, as the positive pions do. The behavior of the pion production at high energies, where the production of kaons, protons, and antiprotons seem requiring anomalously large contributions from the exponential term to describe the shape of their transverse momenta, could be a subject of a similar out-of-equilibrium analysis, as well.

Data Availability

The availability data are available.

Conflicts of Interest

The author declares that he has no conflicts of interest.

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