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Research Article

Constraining the Effective Mass of Majorana Neutrino with Sterile Neutrino Mass for Inverted Ordering Spectrum

Jaydip Singh

Department of Physics, Lucknow University, Lucknow 226007, India

Correspondence should be addressed to Jaydip Singh; jaydip.singh@gmail.com

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Inspired by the experimental anomalies in neutrino physics and recent oscillation data from short baseline and another neutrino experiment, the realization of one extra neutrino flavor seems to be favoring. This extra flavor may change the observable, $|m_{\beta\beta}|$, of current data taking and next-generation $(\beta\beta)_{0\nu}$-decay experiments aim to probe and possibly look at the Inverted Ordering region ($|m_{\beta\beta}| \simeq 10^{-2}$eV) of parameter space. This observation would allow establishing physics beyond the standard model and phenomena like lepton number violation and Majorana nature of neutrino. The range of this observable ($|m_{\beta\beta}|$) is not very well defined for both the ordering of mass spectrum (Normal Ordering and Inverted Ordering). Several attempts have been made for defining exactly the range for three active neutrino states. For contrasting this range, I have worked with an extra mass state, ν_4, and its effect on the observable with various combinations of CP violation Majorana phases by taking into account the updated data on the neutrino oscillation parameters for IO case. Based on the Monte Carlo technique, a parameter region is obtained using the fourth Majorana-Dirac phase of sterile parameters that lead to an effective mass below 0.01 eV or .05 eV for inverted mass ordering case which is planned to be observed in the near future experiment.

1. Introduction

In the near future, the positive observation of the neutrinoless double beta decay process would be clear evidence for the lepton number violation and the confirmation of the Majorana nature of neutrinos [1]. In addition to the phenomena discussed earlier, it can also address some of the yet unresolved issues in the neutrino physics and physics beyond the standard model, such as the origin of the tiny neutrino masses, the absolute neutrino mass scale, and the mass ordering and the dark matter physics; see [2–5] for more detail. In general, three active neutrinos are considered in the standard neutrino oscillation picture with mass-squared differences of orders 10^{-4} and 10^{-3} eV^2 [6]. From tritium beta decay experiments and cosmological observations, the absolute mass scale of neutrino is also constrained to be below 1 eV. Considering the three neutrino formalisms we have various successful achievements in atmospheric neutrino physics, solar neutrino physics, accelerator, and reactor neutrino experiments but there are experimental anomalies that cannot be explained within the standard three-neutrino framework. Mainly the issue of LSND [7], MiniBooNE[8], the Gallium neutrino anomaly [9, 10], and the reactor antineutrino anomaly [11] are yet not understood clearly. It is frequently interpreted as a hint towards the existence of one or two sterile neutrino states with masses at the eV scale. Such neutrinos are called "sterile" since they cannot participate in the weak interactions due to the collider constraints and point towards the nonstandard neutrino physics. The presence of sterile neutrinos would significantly change the observables in neutrino experiments, specifically the oscillation probabilities in short-baseline experiments and the effective mass in neutrino-less double beta decay [12–14]. Current data taking reactor and solar and gallium experiments support the sterile neutrino oscillations phenomena and recent analysis and discussion can be found in [15, 16]. Constantly growing current and future experiments for verifying the short-baseline neutrino oscillation anomalies and perhaps revealing sterile neutrinos are STEREO [17], DANSS [18], NEOS [19], PROSPECT [20], Neutrino-4 [21], BEST [22], and SOLID [23].

In the oscillation experiment, total lepton number is conserved while in the neutrinoless double beta decay experiment total lepton number changes by two units. So, some of the fundamental questions like lepton number violation and Majorana nature of neutrino cannot be answered through the neutrino oscillation experiments but can be answered through the neutrino-less double beta decay ($0\nu\beta\beta$) experiment. Therefore, the most promising process which can be unambiguous Majorana nature of neutrinos is neutrinoless double beta decay and the search for this phenomena has a long history [24]. Current running experiments like KamLAND-Zen, CUORE, CUORE-0, Cuoricino, and GERDA-II are trying to find out the lower bounds on the $T^{0\nu}_{1/2}$ of this decay for several nucleus samples. Recent lower bound obtained by the KamLAND-Zen collaboration for sample ^{136}Xe(xenon-136) is $T^{0\nu}_{1/2} > 1.07 \times 10^{26}$ yr [25], GERDA-II collaboration obtaining the lower bound for sample ^{76}Ge(germanium-76) is $T^{0\nu}_{1/2} > 8.0 \times 10^{25}$ yr [26], and CUORE, CUORE-0 and Cuoricino experiments collectively obtaining the lower bound for sample ^{130}Te(tellurium-130) is $T^{0\nu}_{1/2} > 1.5 \times 10^{25}$ yr[26].

The $(\beta\beta)_{0\nu}$-decay rate is proportional to the effective Majorana mass $|m_{\beta\beta}|$ in the three-Majorana neutrino picture. The range of this effective Majorana mass is not very well understood but, based on the present neutrino oscillation data, it is bounded from below $|m_{\beta\beta}|_{IO} > 1.4 \times 10^{-2}$ eV in the case of three neutrinos mass spectrum with Inverted Ordering [27]. While for the Normal Ordering configuration of the mass spectrum the lower bound is $|m_{\beta\beta}|_{NO} << 10^{-3}$ eV [28] and it can be extremely small depending on the values of the Dirac, Majorana phases, and the smallest neutrino mass. In the article [29] the possible range of $|m_{\beta\beta}|$ is determined by varying the Majorana and CPV phases to show the dependency of $|m_{\beta\beta}|$ on the phases. Then a condition is established using the full range of Majorana and Dirac CPV phases and also for some particular Majorana and Dirac CPV phases with NO (Normal Ordering) spectrum, under which $|m_{\beta\beta}|$ in the 3-neutrino mixing exceeds the milli-electron-volt value. In this article, a similar approach is followed to define the possible range of $|m_{\beta\beta}|$ by considering one extra neutrino states (3+1). Neutrino double beta decay experiments are trying to cover the range of $|m_{\beta\beta}|$ from the top and the current reachable upper limit reported by KamLAND-Zen collaboration is $|m_{\beta\beta}| < (0.061 - 0.165)eV$ [25]. This upper limit is obtained by the KamLAND-Zen collaboration using the lower limit on the half-life of the Xenon-136 sample and they have considered the uncertainties in the NMEs (Nuclear Matrix Elements) in their analysis of the relevant process.

New-generation experiments plan to look the Inverted Ordering region of parameter space and possibly work for the $|m_{\beta\beta}| \sim 10^{-2}$ eV energy range. Various running experiments are considered [30, 31] for upgradation and new experiments are also proposed to achieve this goal. Some of those experiments are MAJORANA, LEGEND(^{76}Ge), CANDLES(^{48}Ca), AMoRE, PandaX-III, SuperNEMO and DCBA(^{82}Se,^{150}Nd), ZICOS (^{96}Zr), MOON (^{100}Mo), COBRA (^{116}Cd,^{130}Te), SNO+(^{130}Te), NEXT, and nEXO(^{136}xE). If these planned experiments did not find positive response in this energy range ($10^{-2}eV$), then next generation experiment will be very interesting for sterile neutrino which will correspond to $|m_{\beta\beta}| \sim 10^{-3}$ eV or more below in the $(\beta\beta)_{o\nu}$-decay experiment.

The introduction of a sterile neutrino at the eV mass scale can change the prediction for the possible range of $|m_{\beta\beta}|$ values in the neutrinoless double-beta decay [12, 32–36]. In the present article, I have determined a sterile parameter region using Monte Carlo technique under which the effective Majorana mass is pushed below a certain value (.01 eV or .05 eV) in the case of 3+1 neutrino mixing with Inverted Ordering mass spectrum. For completeness Normal Ordering scenario of the spectrum with three active and one sterile neutrino is also discussed with the recent global data. Considering the latest global data and various possible CP violation Majorana phases possible ranges of observables in $(\beta\beta)_{0\nu}$-decay, tritium beta decay experiments and cosmological observations are also discussed.

2. The Effective Mass in 3 and 3+1 Neutrino States

In this section, I will discuss the general formalism of neutrino parameters and evaluate the contributions to the effective mass relevant for neutrino-less double beta decay. First I outline the formalism of three active neutrinos' mixing and then mixing in the presence of one extra sterile states (3+1). I have worked with the 3+1 scenario when the sterile neutrino is heavier than the active neutrino with Normal Ordering and Inverted Ordering scheme. Another alternative scheme is 1+3; here new nonstandard massive neutrinos ν_4 is lighter than the three standard massive neutrinos. The 1+3 schemes are disfavored by the cosmological upper bound on the neutrino masses that is smaller than 1 eV, and by the upper bound on the effective neutrino mass in neutrinoless double-β decay if neutrinos are Majorana particles, detail discussion can be found in [37, 38]. Hence I will not discuss this scheme further; other possible schemes (3+2/2+3 or 1+3+1) are also available in the literature but not discussed here; more detailed information can be found in [12].

2.1. Three Active Neutrinos' Mixing. For three active neutrino states configuration of the effective Majorana mass $|m_{\beta\beta}|$ is defined as

$$\left|m_{\beta\beta}\right| = \left|\sum_{k=1}^{3} U_{ek}^2 \mu_k\right| \quad (1)$$

with the partial contribution of the massive Majorana neutrino ν_k with mass μ_k and U being the leptonic mixing matrix and known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, which exactly defined the mixing of the electron neutrino with the three massive neutrinos. The first row of this mixing matrix is the one relevant for $(\beta\beta)_{0\nu}$-decay, and in standard parameterization it is defined [28] as

Table 1: Recent global data table for best-fit values of the oscillation parameters and their range at 1σ, 2σ, and 3σ in the case of three neutrinos with Normal Ordering spectrum [6].

parameter	$\Delta m^2_{21}/10^{-5}[eV]$	$\Delta m^2_{31}/10^{-3}[eV]$	$\sin^2\theta_{12}/10^{-1}$	$\sin^2\theta_{13}/10^{-2}$
Best fit	7.34	2.455	3.04	2.14
1σ	7.20 - 7.51	2.46 - 2.53	2.91 - 3.18	2.07 - 2.23
2σ	7.05 - 7.69	2.43 - 2.56	2.78 - 3.32	1.98 - 2.31
3σ	6.92 - 7.91	2.39 - 2.59	2.65 - 3.46	1.90 - 2.39

Table 2: Recent global data table for best-fit values of the oscillation parameters and their range at 1σ, 2σ, and 3σ in the case of three neutrinos with Inverted Ordering spectrum [6].

parameter	$\Delta m^2_{21}/10^{-5}[eV]$	$\Delta m^2_{23}/10^{-3}[eV]$	$\sin^2\theta_{12}/10^{-1}$	$\sin^2\theta_{13}/10^{-2}$
Best fit	7.34	2.441	3.03	2.18
1σ	7.20 - 7.51	2.41 - 2.47	2.90 - 3.17	2.11 - 2.26
2σ	7.05 - 7.69	2.39 - 2.52	2.77 - 3.31	2.02 - 2.35
3σ	6.92 - 7.91	2.38 - 2.58	2.64 - 3.45	1.95 - 2.43

$$U_{ek} = \left(c_{13}c_{12}, c_{13}s_{12}e^{i\phi_{21}/2}, s_{13}e^{-i\delta}e^{i\phi_{31}/2}\right) \tag{2}$$

where $c_{kl} \equiv \cos\theta_{kl}$ and $s_{kl} \equiv \sin\theta_{kl}$, where $\theta_{kl} \in [0, \pi/2]$ are the mixing angles, and δ and ϕ_{kl} are the Dirac and Majorana phases [39], respectively, and these are the complex phases and the possible range of these parameters are $[0, 2\pi]$. The values of these phases are not known and all possible values of $|m_{\beta\beta}|$ must take into account all the possible range of these phases. The results for the neutrino squared-mass differences are expressed in terms of the solar and atmospheric squared mass differences, which are defined as

$$\Delta m^2_{sol} = \Delta\mu^2_{21} \tag{3}$$

$$\Delta m^2_{atm} = \frac{1}{2}\left|\Delta\mu^2_{31} + \Delta\mu^2_{32}\right| \tag{4}$$

where $\Delta\mu^2_{jk} = \mu^2_j - \mu^2_k$. Given this assignment of the squared mass differences, it is currently unknown if the ordering of the neutrino masses is normal, i.e., $\mu_1 < \mu_2 < \mu_3$, or inverted, i.e., $\mu_3 < \mu_1 < \mu_2$. I also defined $m_{low} \equiv \mu_1(\mu_3)$ in the NO(IO). In terms of the lightest neutrino mass, CPV phases, neutrino mixing angles, and neutrino mass-squared differences, the effective Majorana mass reads

$$\begin{aligned}\left|m_{\beta\beta}\right|_{NO} = \Big| & m_{low}c^2_{12}c^2_{13} + \sqrt{\Delta m^2_{sol} + m^2_{low}}s^2_{12}c^2_{13}e^{i\phi_{21}} \\ & + \sqrt{\Delta m^2_{atm} + m^2_{low}}s^2_{13}e^{i\phi'_{31}}\Big|\end{aligned} \tag{5}$$

$$\begin{aligned}\left|m_{\beta\beta}\right|_{IO} = \Big| & \sqrt{\Delta m^2_{atm} - \Delta m^2_{sol} + m^2_{low}}c^2_{12}c^2_{13} \\ & + \sqrt{\Delta m^2_{atm} + m^2_{low}}s^2_{12}c^2_{13}e^{i\phi_{21}} + m_{low}s^2_{13}e^{i\phi'_{31}}\Big|,\end{aligned} \tag{6}$$

where we have defined $\phi'_{31} \equiv \phi_{31} - 2\delta$. Effective mass for both cases, $|m_{\beta\beta}|_{NO}$ and $|m_{\beta\beta}|_{IO}$, can be written in the form

$$\left|m_{\beta\beta}\right| = \left|\widetilde{\mu_1} + \widetilde{\mu_2}e^{i\phi_{21}} + \widetilde{\mu_3}e^{i\phi'_{31}}\right|, \tag{7}$$

It is then clear that the effective Majorana mass is the length of the vector sum of three vectors in the complex plane for three neutrino mixing case. And their relative orientations are determined by the CPV phase factors ϕ_{21} and ϕ'_{31} that can push the range of $|m_{\beta\beta}|$ significantly up and down with Normal or Inverted ordering of the neutrino mass spectrum. Tables 1 and 2 show the best fit, 1σ, 2σ, and 3σ ranges of the three neutrino oscillation parameters used for this analysis with NO and IO of the mass spectrum obtained from the recently updated global data [6].

Generally, the observation of the absolute values of neutrino masses is done through the measurements of the effective electron neutrino mass in the β-experiment and it is expressed as

$$m_\beta = \sqrt{\left|U_{e1}\right|^2\mu^2_1 + \left|U_{e2}\right|^2\mu^2_2 + \left|U_{e3}\right|^2\mu^2_3} \tag{8}$$

and the sum of the neutrino masses in cosmological experiments, which is defined as

$$\sum = \mu_1 + \mu_2 + \mu_3 \tag{9}$$

Therefore, it is helpful to estimate the allowed regions in the $m_\beta - |m_{\beta\beta}|$ and $\sum - |m_{\beta\beta}|$ planes, as shown in Figure 3.

2.2. Four-Neutrino Mixing Case. In this section, I consider the case of 3+1 mixing in which there is a new massive neutrino ν_4 at the eV scale which is mainly sterile. This 3+1 scheme mixing is motivated by the explanation of the reactor, Gallium and LSND anomalies, which requires the existence of a new squared-mass difference $\Delta m^2_{sbl} \sim 1eV^2$, recent analysis using global data can be found in [15]. By accommodating one sterile neutrino in three neutrino mixing mechanism, the effective Majorana mass in $0\nu\beta\beta$ is expressed as [12]

$$\begin{aligned}\left|m_{\beta\beta}\right|_{4\nu} = \Big| & c^2_{12}c^2_{13}c^2_{14}\mu_1 + s^2_{12}c^2_{13}c^2_{14}\mu_2e^{i\phi_{21}} + s^2_{13}c^2_{14}\mu_3e^{i\phi'_{31}} \\ & + s^2_{14}\mu_4e^{i\phi_{41}}\Big|\end{aligned} \tag{10}$$

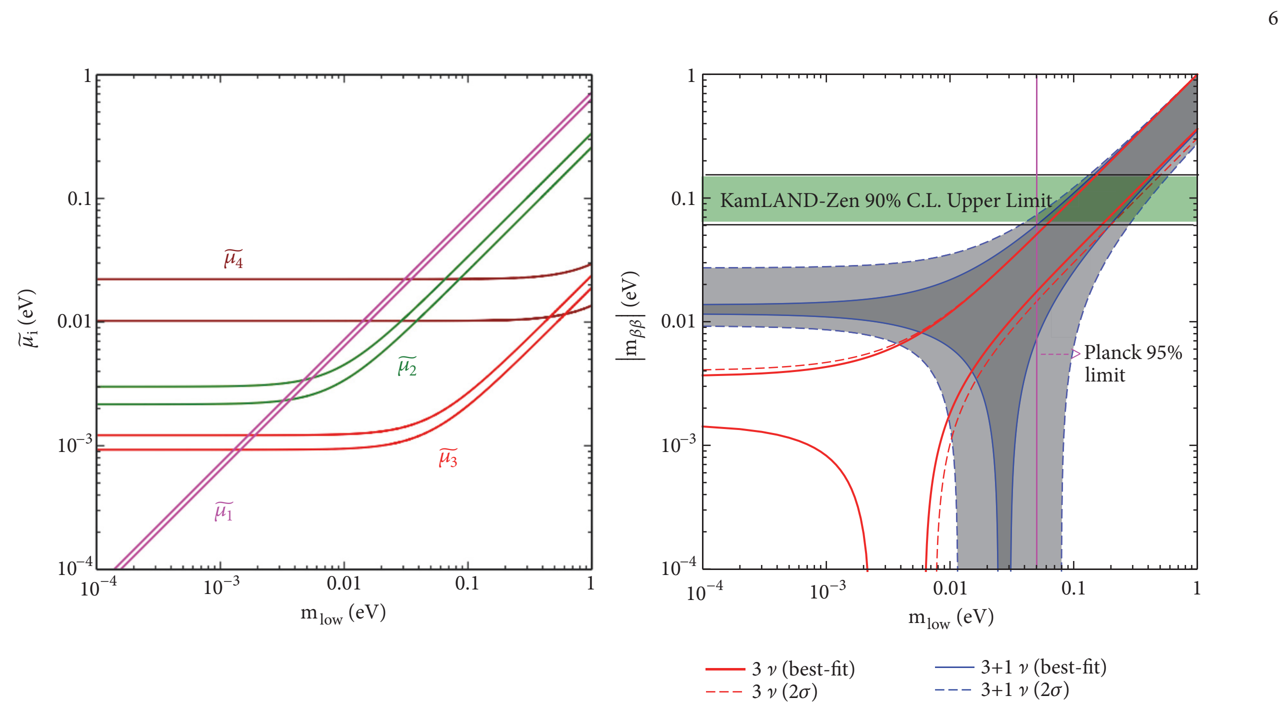

FIGURE 1: Left panel shows the four partial mass contribution to $|m_{\beta\beta}|$ in (12) as function of the lightest mass m_{low} for 3σ allowed intervals in the case of 3+1ν mixing with Normal Ordering. Right panel shows the allowed ranges in the $|m_{\beta\beta}|$ - m_{low} parameter space, both in the standard three-neutrino picture (unfilled band space) and with one sterile neutrino (filled band space) for the 3+1 case as defined by (14).

and this expression can be written as

$$\left|m_{\beta\beta}\right| = \left|\widetilde{\mu_1} + \widetilde{\mu_2}e^{i\phi_{21}} + \widetilde{\mu_3}e^{i\phi'_{31}} + \widetilde{\mu_4}e^{i\phi_{41}}\right|. \quad (11)$$

This extra flavor ν_4 contribution comes with an extra complex phase factor ϕ_{41} that must be varied between 0 and 2π for full physics analysis as ϕ_{21} and ϕ_{31} to estimate the all possible value of $|m_{\beta\beta}|$. However, for both orderings (NO and IO), $\widetilde{\mu_4}$ remains the same and defined as

$$\widetilde{\mu_4} \simeq \sqrt{m_{low}^2 + \Delta m_{sbl}^2} s_{14}^2, \quad (12)$$

where one can neglect the contributions of Δm_{sol}^2 and Δm_{atm}^2, which are much smaller than Δm_{sbl}^2. If three active neutrinos are lighter than the sterile neutrino ones, the expression for $|m_{\beta\beta}|$ can be expressed as

$$\left|m_{\beta\beta}\right|_{(3+1)\nu} \simeq \left|c_{14}^2 \left\langle m_{\beta\beta}\right\rangle_{3\nu} + s_{14}^2 \sqrt{\delta\mu_{41}^2} e^{i\phi_{41}}\right| \quad (13)$$

where $\langle m_{\beta\beta}\rangle_{3\nu}$ is the standard expression for three active neutrinos as defined by (5) and (6). Table 3 shows the best-fit, 2σ, and the 3σ range of the oscillation parameter for fourth sterile states used in this work for this analysis. The right panel of Figures 1 and 2 shows the allowed range of $|m_{\beta\beta}|_{3+1\nu}$ as a function of the lightest mass for NO and IO case, respectively, using data from [15] for 3+1 case and [6] for 3 active neutrinos' state.

TABLE 3: Best-fit, 2σ and 3σ values (from [15])of the oscillation parameters with three standard neutrinos' and one sterile neutrino's states case.

parameter	$\Delta m_{41}^2\,[eV]$	$\lvert U_{e4}\rvert^2$
best-fit	1.29	0.0089
2σ	1.21 - 1.50	0.0025 - 0.0190
3σ	1.18 - 1.60	0.009 - 0.0195

3. Normal Ordering Configuration

The aim of this work is to constrain the $|m_{\beta\beta}|$ range for IO spectrum only but for completeness; I will start my analysis with the normal ordering of the mass spectrum, discussed in this section; then in the following sections I will discuss the inverted ordering of mass spectrum in detail. The length of the vector, that determines $|m_{\beta\beta}|$ with the phase factor can be obtained from (11) and (12), and the vectors as a function of lightest neutrino mass with NO are $\widetilde{\mu_1} = m_{low}c_{12}^2c_{13}^2c_{14}^2$, $\widetilde{\mu_2} = \sqrt{\Delta m_{sol}^2 + m_{low}^2}s_{12}^2c_{13}^2c_{14}^2$, $\widetilde{\mu_3} = \sqrt{\Delta m_{atm}^2 + m_{low}^2}s_{13}^2c_{14}^2$, and $\widetilde{\mu_4} \simeq \sqrt{m_{low}^2 + \Delta m_{sbl}^2}s_{14}^2$. The lengths of the three partial mass contribution to $|m_{\beta\beta}|$ in (6) are shown in the left panel of Figure 1 as functions of lightest neutrino mass; m_{low} for 3σ variations of oscillation parameters is provided in the Table 1. One can see that contribution of μ_4 is dominant for $m_{low} \leq$ .01 eV while μ_2 is dominant for $m_{low} \leq$ .001 eV and cannot be cancelled by the smaller contribution of μ_1 and μ_3 for any values of the relative phase differences as shown in the right

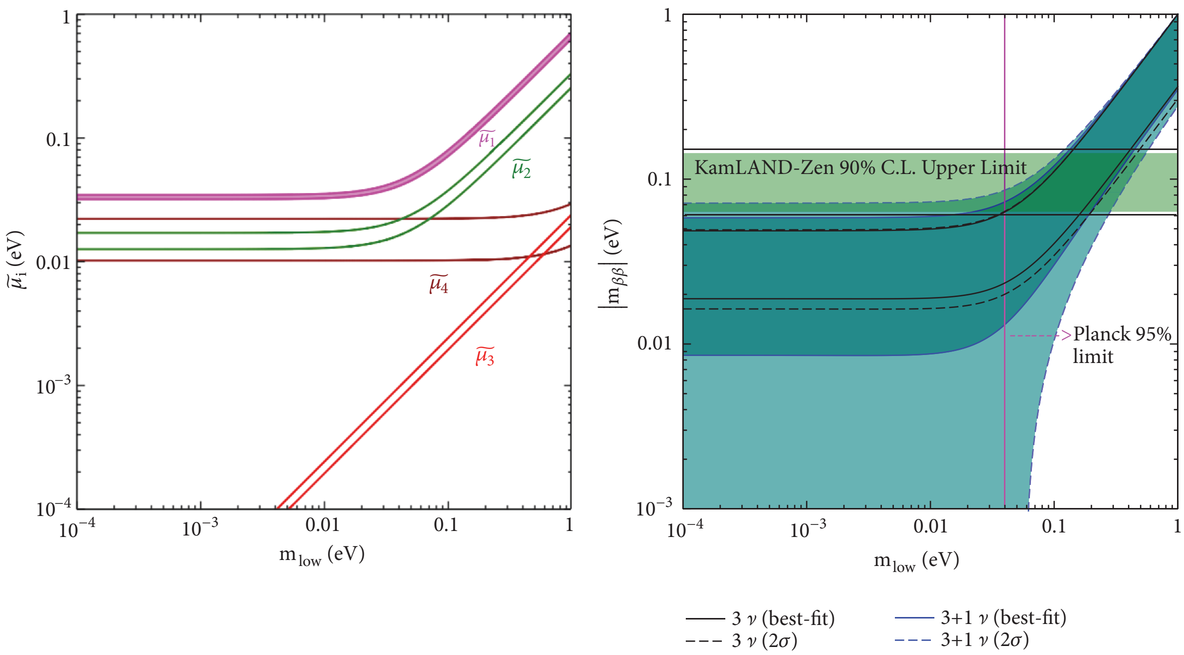

FIGURE 2: Left panel shows the four partial mass contribution to $|m_{\beta\beta}|$ in (12) as function of the lightest mass m_{low} for 3σ allowed intervals in the case of 3+1ν mixing with Inverted Ordering. Right pannel shows the allowed ranges in $|m_{\beta\beta}|$ - m_{low} parameter space, both in the standard three-neutrino picture (unfilled band space) and with one sterile neutrino (filled band space) for the 3+1 case as defined by (14).

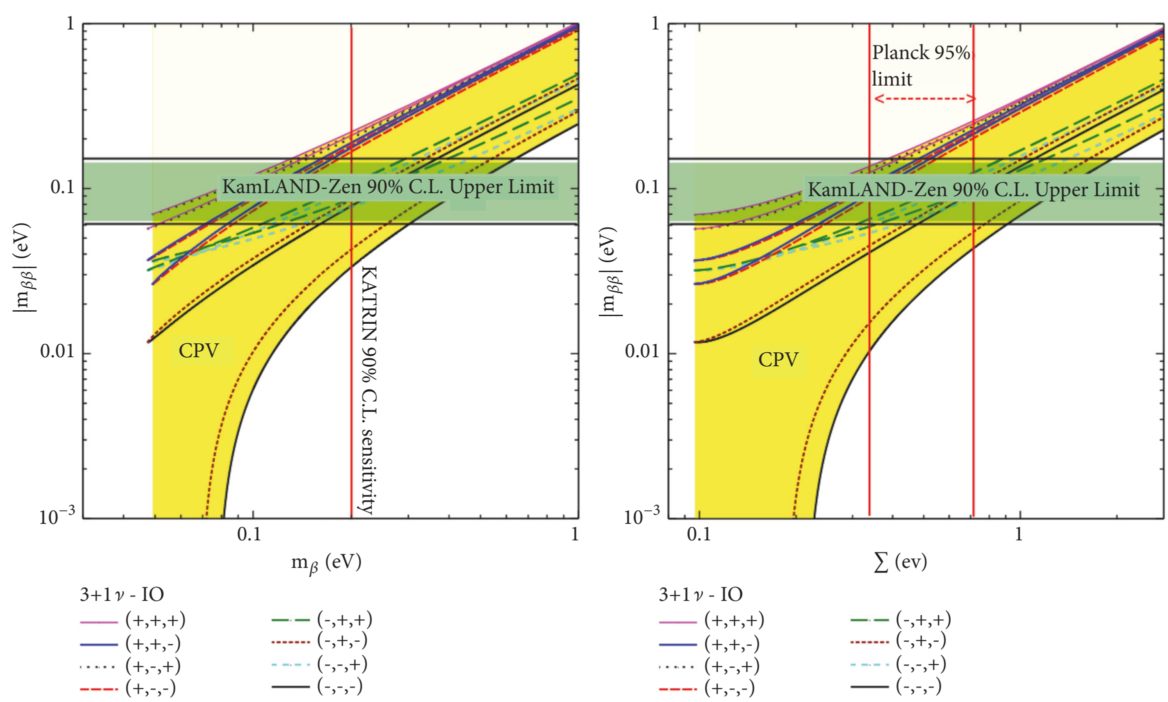

FIGURE 3: Left and right panel, respectively, show the value of effective Majorana mass $|m_{\beta\beta}|$ as a function of effective electron neutrino mass m_β and sum of the neutrino masses $\sum$ in the case of 3+1 neutrino mixing with Inverted Ordering. The signs combination as shown in the legend implies the signs of $e^{i\phi_{21}}, e^{i\phi_{31'}}, e^{i\phi_{41}} = \pm 1$ for the eight possible cases in which CP is conserved.

panel of Figure 1. In the other region cancellation is possible mainly between the values .001 and .1 eV. Result for effective Majorana mass, $|m_{\beta\beta}|$, as a function of m_{low} is shown in the right panel of Figure 1 with three active neutrinos' state and one extra flavour state. The minimum and maximum values of the allowed bands for $|m_{\beta\beta}|$ are obtained from the relative phase and the coefficients.

4. Inverted Ordering Configuration

For 3+1 neutrino states with Inverted Ordering, the length of the vectors that determines $|m_{\beta\beta}|$ with the phase factor can be obtained from (6), (11), and (12). These vectors as a function of the lightest neutrino mass m_{low} are $\widetilde{\mu_1} = \sqrt{|\Delta m^2_{atm}| - \Delta m^2_{sol} + m^2_{low}}c^2_{12}c^2_{13}c^2_{14}$, $\widetilde{\mu_2} = \sqrt{|\Delta m^2_{atm}| + m^2_{low}}s^2_{12}c^2_{13}c^2_{13}$, $\widetilde{\mu_3} = m_{low}s^2_{13}c^2_{14}$, and $\widetilde{\mu_4} \simeq \sqrt{m^2_{low} + \Delta m^2_{sbl}}s^2_{14}$. The lengths are shown in the left panel of Figure 2 as functions of m_{low} for 3σ variations of oscillation parameters as given in Tables 2 and 3. Right panel shows the value of effective Majorana mass $|m_{\beta\beta}|$ as a function of lightest neutrino mass in the 3 and 3+1 neutrino case as defined by (6) and (13), respectively. One can see that μ_1 is always dominant to, because θ_{13} is smaller than $\pi/4$ and $|U_{e1}| > |U_{e2}| > |U_{e3}|$. Therefore, there can not be a complete cancellation of three mass contribution to $|m_{\beta\beta}|$ for three active neutrinos with Inverted Ordering, as shown in the right panel of Figure 3. So one can obtain the lower and upper bounds of $|m_{\beta\beta}|$ for three active neutrino states with Inverted Ordering. Here, maximum and minimum values of the allowed bands for $|m_{\beta\beta}|$ are obtained from the relative phase factor and the coefficients of the oscillations parameters as given in Table 2. From the right panel of Figure 2, one can obtain the lower and upper bounds of the $|m_{\beta\beta}|$ in the case of three active neutrino with Inverted Ordering mass spectrum,

$$.02eV \le \left|m_{\beta\beta}\right| \le .05\,(best - fit)\,eV. \tag{14}$$

The next generation and current data-taking neutrinoless double-beta decay experiments will try to explore the range of $|m_{\beta\beta}|$ between the limits in (14) and testing the Majorana nature of neutrinos in the case of an Inverted Ordering. This range is estimated by measurements of the half-life of the nucleus samples, so the available major source of uncertainty in the measurements is uncertainty in NME. And second uncertainty is the accurate determination of the oscillation parameters which are directly dependent on $|m_{\beta\beta}|$. So these systematic uncertainties must be taken into account to accurately determine the range of $|m_{\beta\beta}|$ and this is the major aim of the next generation neutrino experiments.

While for the sterile neutrino case there can be a total cancellation between the partial contributions μ_4 and μ_2 for $m_{low} \le .06$ eV as shown in the right panel of Figure 2. Comparing 3ν and 3+1ν mixing as shown in the right panel of Figure 2, one can see that the predictions of effective Majorana mass $|m_{\beta\beta}|$ are completely different in the 3ν and 3+1ν cases if there is an Inverted Ordering [12, 32–36, 40, 41]. This difference in the effective Majorana mass $|m_{\beta\beta}|_{3+1\nu}$

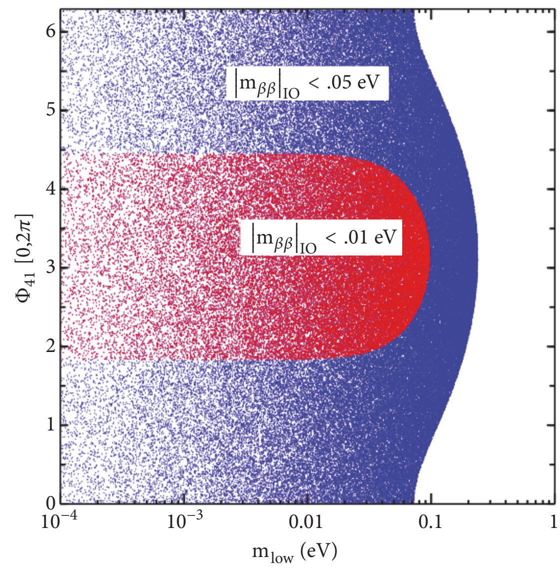

FIGURE 4: Regions in the (m_{low}, ϕ_{21}) plane where two different conditions on $|m_{\beta\beta}|_{IO}$ apply. In the blue region $|m_{\beta\beta}|_{IO}$ satisfies $|m_{\beta\beta}|_{IO} < .05eV$ and red region satisfies $|m_{\beta\beta}|_{IO} < .01eV$, for all values of θ_{kl}, $\delta\mu^2_{jk}$, and ϕ'_{31} from the corresponding 2σ intervals of oscillation parameter.

comes mainly from the phase factors, ϕ_{21}, ϕ'_{31}, and ϕ_{41}. The various possible ranges of $|m_{\beta\beta}|$ for the particular choice of the phase cases are shown in the Appendix.

Since $|m_{\beta\beta}|$ strongly depends on the phases ϕ_{21}, ϕ'_{31}, and ϕ_{41} and this phase combination determines the maximum and minimum of $|m_{\beta\beta}|$ with coefficients of oscillation parameters. These phase combinations can significantly change the range of the $|m_{\beta\beta}|$, region of $|m_{\beta\beta}| - m_\beta$ plane, and region of $|m_{\beta\beta}| - \sum$ plane as shown in Figure 2 (right panel and Figure 5) and Figure 3 (right and left panel), respectively. Here I consider the phase combination which gives the maximum and minimum possible range of $|m_{\beta\beta}|$ as shown in the right panel of Figure 2 and its dependency on the fourth sterile phase parameter (ϕ_{41}) only for estimating the lower bound. For understanding these phase dependencies on $|m_{\beta\beta}|$, a Monte Carlo code is developed, which is based on root package[42]. In the code, all parameters are varied in the full range to get the maxima and minima of $|m_{\beta\beta}|$. Then to find the particular parameter region, in my case (ϕ_{41} versus m_{low}), as shown in Figure 4, that range causes $|m_{\beta\beta}|$ below a certain values to be estimated. I have varied the full range of ϕ_{41}, i.e., $(0, 2\pi)$, and lightest neutrino mass in the range of 1.00-0.0001 eV and then find a region between ϕ_{41} and m_{low} that can push the effective mass below the certain values (.01 eV or .05 eV) shown in Figure 4. If this parameter lies within the red dotted region, the effective Majorana mass will be less than .01 eV, similarly, if this parameter lies within the blue dotted region, $|m_{\beta\beta}|$ will be $\le .05$ as shown in Figure 4. If the scenario 3+1ν mixing with Inverted Ordering of mass spectrum exists and $|m_{\beta\beta}|$ goes below 0.01 eV or 0.05 eV and observed in the near future; then sterile parameter must lie within the region shown in Figure 4.

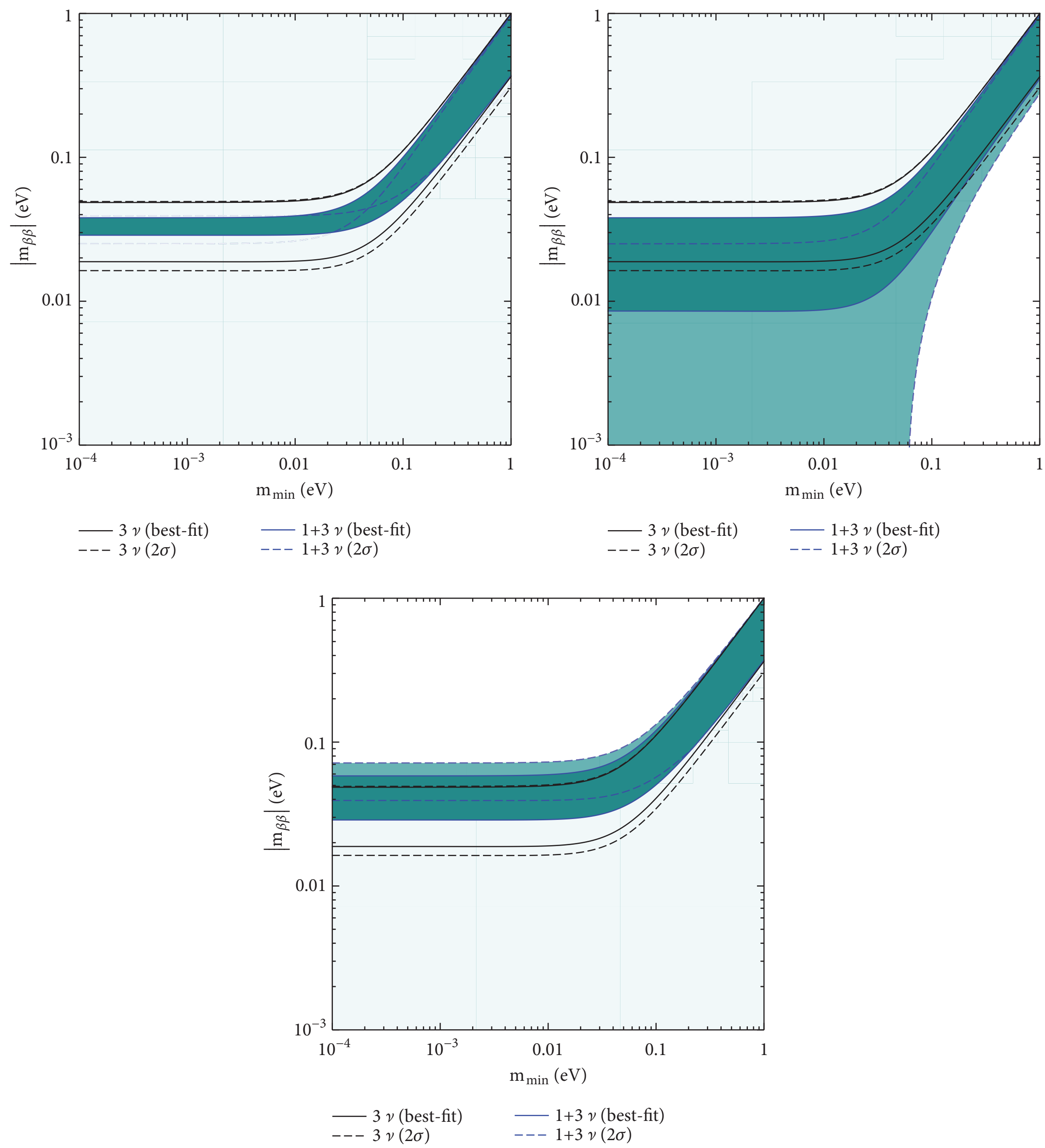

FIGURE 5: The allowed ranges in the $|m_{\beta\beta}|$ - m_{low} parameter space, both in the standard three-neutrino picture and with one sterile neutrino for the 3+1 IO case. Top left panel upper and lower band range are obtained in the mass term combination of (+,+,-) and (-,-,+), while right panel upper and lower band range are obtained in the mass term combination of (+,+,-) and (-,-,-). Bottom upper and lower band range are obtained in the mass term combination of (+,+,+) and (-,-,+). The signs indicate the signs of $e^{i\alpha_{21}}, e^{i\alpha_{31}}, e^{i\alpha_{41}} = \pm 1$; these are the extreme cases which determine the minimum and maximum value of $|m_{\beta\beta}|$ in which CP is conserved.

For completeness, left and right panel of Figure 3 show the correlation between $|m_{\beta\beta}|$ and the measurable quantities m_β and $\sum$ for all the possible combination of phases that will be observed in beta decay experiment [43–45] and cosmological and astrophysical data [46, 47], respectively. Figure 3 shows the 90% C.L. upper limit band for $|m_{\beta\beta}|$ from the results of the KamLAND-Zen experiment [25] taking into account the uncertainties of the nuclear matrix element calculations. In addition, in Figure 3, left panel also shows the 90% C.L. sensitivity on m_β of the KATRIN experiment [48] and right panel shows the 90% C.L. upper limit interval on $\sum$ of the Planck collaboration [46]. By considering the 3σ ranges for the mass-squared differences, this last bound gives $m_{low} <$ 0.05 eV in the NO case as shown in the right panel of Figure 1 and $m_{low} <$ 0.04 eV in the IO case [29], which is shown in the right panel of Figure 2.

5. Result and Discussion

Current data taking and next-generation $(\beta\beta)_{0\nu}$-decay experiments aiming to look and possibly cover the Inverted Ordering range $|m_{\beta\beta}| \simeq 10^{-2}$ eV is obtained for three-neutrino case. For contrasting this range in four-neutrino case by considering recent global-fit data on the neutrino mixing angles and the neutrino mass-squared differences, I have determined the parameter space under which the effective Majorana mass in the Inverted Ordering case $|m_{\beta\beta}|_{IO}$ goes down to .05 eV and .01 eV as shown in Figure 4. For obtaining this parameter space I have considered the full range $(0, 2\pi)$ of fourth Dirac-Majorana phase ϕ_{41} while keeping other phase fixed and varying the lightest neutrino mass in the range of 1.00 eV - 1×10^{-4} eV.

Finally one can conclude now by saying that if we obtained the $|m_{\beta\beta}|$ below the range discussed then it will be very interesting for sterile neutrino physics. And the parameter range obtained in this analysis will be useful for the development of models and experiments in 10^{-3} eV range to explore the sterile neutrino physics. Several oscillation types of experiment are planned mainly to investigate the LSND/MiniBooNE anomaly, particularly the Short-Baseline Neutrino program at Fermilab, which will use three detectors: Short-Baseline Near Detector [49], MicroBooNE [50], and ICARUS[51].

Appendix

IO Case with Other Phases

See Figure 5.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The author declares that they have no conflicts of interest.

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