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Research Article

Dark Energy in Spherically Symmetric Universe Coupled with Brans-Dicke Scalar Field

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The phenomenon of dark energy and its manifestations are studied in a spherically symmetric universe considering the Brans-Dicke scalar tensor theory. In the first model, the dark energy behaves like a phantom type and in such a universe the existence of negative time is validated with an indication that our universe started its evolution before $t = 0$. Dark energy prevalent in this universe is found to be more active at times when other types of energies remain passive. The second model of universe begins with big bang. On the other hand, the dark energy prevalent in the third model is found to be of the quintessence type. Here, it is seen that the dark energy triggers the big bang and after that much of the dark energy reduces to dark matter. One peculiarity in such a model is that the scalar field is prevalent eternally; it never tends to zero.

1. Introduction

The type Ia (SNIa) supernovae observations suggested that our universe is not only expanding, but also the rate of expansion is in accelerating way [1, 2] and this acceleration is caused by some mysterious object, so called dark energy. The matter species in the universe are broadly classified into relativistic particle, nonrelativistic particle, and dark energy. Another component, apparently a scalar field, dominated during the period of inflation in the early universe. In the present universe, the sum of the density parameters of baryons, radiation, and dark matter does not exceed 30% [3]; we still need to identify the remaining 70% of the cosmic matter. We call this 70% unknown component as dark energy, and it is supposed to be responsible for the present cosmic acceleration of the universe. According to the cosmological principle, our universe is homogeneous and isotropic in large scale. By assuming isotropicity and homogeneity, the acceleration equation of the universe in general theory of relativity can be written as $\ddot{a}/a = -(1/6)\kappa^2(\rho + 3p)$. The acceleration and deceleration of the universe depend on the sign of $\ddot{a}$; that is, the universe will accelerate if $\rho + 3p < 0$ or decelerate if $\rho + 3p > 0$. So, the condition $\rho + 3p < 0$ has to be satisfied in general relativity to explain accelerated expansion of the universe. This implies that the strong energy condition is violated; moreover, the strong energy condition is violated, meaning that the universe contains some abnormal (something not normal) matter. Hence, without violating strong energy condition, the accelerated expansion of the universe is not possible in general theory of relativity. Therefore, the modification of the general theory of relativity is necessary. Essentially, there are two approaches, out of which one is to modify the right-hand side of Einstein's field equations (i.e., matter part of the universe) by considering some specific forms of the energy momentum tensor $T_{\mu\nu}$ having a huge negative pressure and which is concluded in the form of some mysterious energy dubbed as dark energy. In this approach, the simplest candidate for dark energy is cosmological constant Λ, which is described by the equation of state $p = -\rho$ [4]. The second approach is by modifying Einstein Hilbert action, that is, the geometry of the space-time, which is named as modified gravity theory. So many modifications of general relativity theory have been done, namely, Brans-Dicke (BD) [5] and Saez-Ballester scalar-tensor theories [6], $f(R)$ gravity [7–12], $f(T)$ gravity [13–16], Gauss-Bonnet theory [17–20], Horava-Lifshitz gravity [21–23], and recently $f(R,T)$ gravity [24]. Subsequently, so many authors [25–54] have been studying modified gravity theory to understand the nature of the dark energy and accelerated expansion of the universe.

Apart from this, the Hubble parameter H may provide some important information about the evolution of our universe. It is dynamically determined by the Friedmann equations and then evolves with cosmological red-shift. The evolution of Hubble parameter is closely related with radiation, baryon, cold dark matter, and dark energy or even other exotic components available in the universe. Further, it may be impacted by some interactions between these cosmic inventories. Thus, one can look out upon the evolution of the universe by studying the Hubble parameter. Besides dark energy, there exists a dark matter component of the universe. One can verify whether these two components can interact with each other. Theoretically, there is no evidence against their interaction. Basically, they may exchange their energy which affects the cosmic evolution of the universe. Furthermore, it is not clear whether the nongravitational interactions between two energy sources produced by two different matters in our universe can produce acceleration. We can assume for a while that the origin of nongravitational interaction is related to emergence of the space-time dynamics. However, this is not of much help, since this hypothesis is not more fundamental compared with other phenomenological assumptions within modern cosmology [55–59]. However, the authors [60, 61] studied the finding that the interacting cosmological models make good agreement with observational data. The aim of this paper is to study a cosmological model, where a phenomenological form of nongravitational interactions is involved. In this article, we are interested in the problem of accelerated expansion of the large-scale universe; we follow the well-known approximation of the energy content of the recent universe. Namely, we consider the interaction between dark energy and other matters (including dark matter).

2. Space-Time and Field Equations

We consider the spherically symmetric space-time

$$ds^2 = dt^2 - e^{\lambda}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right), \tag{1}$$

where λ is a function of time. The energy momentum tensor for the fluid comprising our universe is taken as

$$T_{\mu\nu} = (p+\rho)\, u_\mu u_\nu - p g_{\mu\nu}, \tag{2}$$

where ρ and p are, respectively, the total energy density and total pressure which are taken as

$$\rho = \rho_m + \rho_d \tag{3}$$

and

$$p = p_m + p_d, \tag{4}$$

with ρ_d and ρ_m being, respectively, the densities of dark energy and other matters in this universe. p_d and p_m are, respectively, the pressures of dark energy and other matters (including dark matter) in this universe. And u_μ is the flow vector satisfying the relations

$$\begin{aligned} u_\mu u^\mu &= 1; \\ u_\mu u^\nu &= 0. \end{aligned} \tag{5}$$

The Brans-Dicke scalar tensor field equations are given by

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = {} & -8\pi\phi^{-1} T_{\mu\nu} \\ & - \omega\phi^{-2}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2} g_{\mu\nu}\phi_{,\gamma}\phi^{,\gamma}\right) \\ & - \phi^{-1}\left(\phi_{\mu;\nu} - g_{\mu\nu}\phi^{;\gamma}_{;\gamma}\right) \end{aligned} \tag{6}$$

with

$$\phi^{;\gamma}_{;\gamma} = 8\pi (3+2\omega)^{-1} T, \tag{7}$$

where ω is the coupling constant and ϕ is the scalar field. Energy conservation gives the equation

$$T^{\mu\nu}_{;\mu} = 0 \tag{8}$$

Here the field equations take the form

$$\frac{3}{4}\dot{\lambda}^2 - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{3}{2}\dot{\lambda}\frac{\dot{\phi}}{\phi} = 8\pi\phi^{-1}\left(\rho_m + \rho_d\right) \tag{9}$$

$$\ddot{\lambda} + \frac{3}{4}\dot{\lambda}^2 + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} + \dot{\lambda}\frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi} = -8\pi\phi^{-1}\left(p_m + p_d\right) \tag{10}$$

$$\frac{3}{2}\left(\ddot{\lambda} + \dot{\lambda}^2\right) + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{3}{2}\dot{\lambda}\frac{\dot{\phi}}{\phi} = -8\pi\phi^{-1}\left(p_m + p_d\right) \tag{11}$$

$$\ddot{\phi} + \frac{3}{2}\dot{\lambda}\dot{\phi} = 8\pi (3+2\omega)^{-1}\left(\rho_m + \rho_d - 3p_m - 3p_d\right) \tag{12}$$

Here, we take the equation of state parameter for dark energy as α so that

$$p_d = \alpha\rho_d. \tag{13}$$

And the conservation equation gives

$$\dot{\rho} + (p+\rho)\frac{3}{2}\dot{\lambda} = 0 \tag{14}$$

Since the dark energy and other matters are interacting in this universe, (14) can be written as

$$\dot{\rho_m} + \left(\rho_m + p_m\right)\frac{3}{2}\dot{\lambda} = -Q \tag{15}$$

and

$$\dot{\rho_d} + \left(\rho_d + p_d\right)\frac{3}{2}\dot{\lambda} = Q, \tag{16}$$

where Q is the interaction between dark energy and other matters (including dark matter) which this universe comprises. Here Q can take different forms like $3z^2\rho$, $3z^2\rho_m$, $3z^2\rho_d$, and so forth, where z^2 is a coupling constant. It can also take other forms which are functions of ρ and $\dot{\rho}$. Now from (10) and (11) we get the relation

$$\ddot{\lambda} + \frac{3}{4}\dot{\lambda}^2 + \dot{\lambda}\frac{\dot{\phi}}{\phi} = \frac{3}{2}\ddot{\lambda} + \frac{3}{2}\dot{\lambda}^2 + \frac{3}{2}\dot{\lambda}\frac{\dot{\phi}}{\phi} \quad (17)$$

which gives

$$e^{(3/2)\lambda}\dot{\lambda} = a_0\phi^{-1} \quad (18)$$

where a_0 is an arbitrary constant.

3. Analytical Solutions

In this section, we try to obtain the analytical solutions of the field equations in three different cases based on the different forms of the interaction parameter Q.

3.1. Case-I. From (17) and (18) we get

$$\lambda = \frac{2}{3}\log b_2 + \log (b_0 + b_1 t)^{a_1} \quad (19)$$

$$\phi = \frac{a_0}{a_1 b_1 b_2}(b_0 + b_1 t)^{1-(3/2)a_1} \quad (20)$$

where a_1, b_0, b_1, and b_2 are arbitrary constants. Here in this case we take

$$Q = 3z^2 H\rho, \quad (21)$$

where H is Hubble's parameter so that the conservation equation takes the form of the equations

$$\dot{\rho_m} + (\rho_m + p_m)\frac{3}{2}\dot{\lambda} = -\frac{3}{2}z^2\dot{\lambda}\rho \quad (22)$$

and

$$\dot{\rho_d} + (\rho_d + p_d)\frac{3}{2}\dot{\lambda} = \frac{3}{2}z^2\dot{\lambda}\rho. \quad (23)$$

Now from (23) we get

$$\rho_d = b_4 (b_0 + b_1 t)^{-(3/2)(\alpha+1)a_1} + b_3\left(1 - \frac{3}{2}a_1\alpha\right)^{-1}(b_0 + b_1 t)^{-1-(3/2)a_1} \quad (24)$$

where b_4 is an arbitrary constant and

$$b_3 = \frac{3z^2 a_0 b_1}{16 b_2 \pi}\left[\frac{4}{3}a_1^2 - \frac{\omega}{2}\left(1 - \frac{3}{2}a_1\right)^2 + \frac{3}{2}a_1\left(1 - \frac{3}{2}a_1\right)\right] \quad (25)$$

Thus using (24) in (9) we have

$$\rho_m = \frac{2b_3}{3a_1 z^2}(b_0 + b_1 t)^{-1-(3/2)a_1} + b_3\left(\frac{3}{2}a_1\alpha - 1\right)^{-1}(b_0 + b_1 t)^{-1-(3/2)a_1} - b_4 (b_0 + b_1 t)^{-(3/2)(1+\alpha)a_1} \quad (26)$$

Therefore (22) gives

$$p_m = \left[\frac{z^2 a_0 b_1}{8\pi a_1 b_2}\left\{\frac{\omega}{2}\left(1 - \frac{3}{2}a_1\right)^2 - \frac{4}{3}a_1^2 - \frac{3}{2}a_1\left(1 - \frac{3}{2}a_1\right)\right\} - \frac{2}{3a_1 b_1}\left\{\frac{2b_1 b_3}{3a_1 z^2} + b_1 b_3\left(\frac{3}{2}a_1\alpha - 1\right)^{-1}\right\} - \frac{2b_3}{3a_1 z^2} - b_3\left(\frac{3}{2}a_1\alpha - 1\right)^{-1}\right](b_0 + b_1 t)^{-1-(3/2)a_1} + \left(b_4 + \frac{2b_4}{3a_1}\right)(b_0 + b_1 t)^{-(3/2)(1+\alpha)a_1} \quad (27)$$

Now using (27) in (10) we have

$$p_d = \left[\frac{z^2 a_0 b_1}{8\pi a_1 b_2}\left\{\frac{4}{3}a_1^2 - \frac{\omega}{2}\left(1 - \frac{3}{2}a_1\right)^2 + \frac{3}{2}a_1\left(1 - \frac{3}{2}a_1\right)\right\} + \frac{2}{3a_1 b_1}\left\{\frac{2b_1 b_3}{3a_1 z^2} + b_1 b_3\left(\frac{3}{2}a_1\alpha - 1\right)^{-1}\right\} + b_3\left(\frac{3}{2}a_1\alpha - 1\right)^{-1} + \frac{2b_3}{3a_1 z^2} - \frac{a_0}{8\pi a_1 b_1 b_2}\left\{\frac{3}{4}a_1^2 b_1^2 - a_1 b_1^2 + \frac{\omega}{2}b_1^2\left(1 - \frac{3}{2}a_1\right)^2 + a_1 b_1^2\left(1 - \frac{3}{2}a_1\right) - b_1^2\left(1 - \frac{3}{2}a_1\right)\right\}\right](b_0 + b_1 t)^{-1(3/2)a_1} - b_4\left(1 + \frac{2}{3a_1}\right)(b_0 + b_1 t)^{-(3/2)(1+\alpha)a_1} \quad (28)$$

Again from (13) and (24) we get

$$p_d = \alpha b_4 (b_0 + b_1 t)^{-(3/2)(1+\alpha)a_1} + \alpha b_3\left(1 - \frac{3}{2}a_1\alpha\right)^{-1}(b_0 + b_1 t)^{-1-(3/2)a_1} \quad (29)$$

Thus comparing coefficients of $(b_0+b_1t)^{-(3/2)(1+\alpha)a_1}$ and $(b_0+b_1t)^{-(3/2)a_1-1}$ of the two expressions of p_d in (28) and (29), we obtain

$$\begin{aligned}
&\frac{z^2a_0a_1b_1}{6\pi b_2}-\frac{z^2\omega a_0b_1}{16\pi a_1b_2}\left(1-\frac{3}{2}a_1\right)^2 \\
&\quad+\frac{z^2a_0b_1}{16\pi b_2}\left(1-\frac{3}{2}a_1\right)+\frac{4b_3}{9a_1^2z^2} \\
&\quad+\frac{2b_3}{3a_1}\left(\frac{3}{2}a_1\alpha-1\right)^{-1}+\frac{2b_3}{3a_1z^2} \\
&\quad+b_3\left(\frac{3}{2}a_1\alpha-1\right)^{-1}-\frac{3a_0a_1b_1}{32\pi b_2}+\frac{a_0b_1}{8\pi b_2} \\
&\quad-\frac{\omega a_0b_1}{16\pi a_1b_2}\left(1-\frac{3}{2}a_1\right)^2-\frac{a_0b_1}{8\pi b_2}\left(1-\frac{3}{2}a_1\right) \\
&\quad+\frac{a_0b_1}{8\pi a_1b_2}\left(1-\frac{3}{2}a_1\right) \\
&=\frac{z^2a_0a_1b_1}{6\pi b_2}-\frac{z^2\omega a_0b_1}{16\pi a_1b_2}\left(1-\frac{3}{2}a_1\right)^2 \\
&\quad+\frac{z^2a_0b_1}{16\pi b_2}\left(1-\frac{3}{2}a_1\right)+\frac{4b_3}{9a_1^2z^2}+\frac{2b_3}{3a_1z^2} \\
&\quad+b_3\left(\frac{3}{2}a_1\alpha-1\right)^{-1}\left(\frac{2}{3a_1}+1\right)+\frac{3a_0a_1b_1}{32\pi b_2} \\
&\quad-\frac{\omega a_0b_1}{16\pi a_1b_2}\left(1-\frac{3}{2}a_1\right)^2+\frac{a_0b_1}{8\pi a_1b_2}\left(1-\frac{3}{2}a_1\right)
\end{aligned}\tag{30}$$

which is automatically satisfied; and

$$\alpha=-\left(1+\frac{2}{3a_1}\right)\tag{31}$$

In this case

$$\rho=\frac{2b_3}{3a_1z^2}\left(b_0+b_1t\right)^{-1-(3/2)a_1}\tag{32}$$

And the interaction Q is given by

$$\begin{aligned}
Q=\frac{3z^2a_0b_1^2}{16b_2\pi}&\left[\frac{4}{3}a_1^2-\frac{\omega}{2}\left(1-\frac{3}{2}a_1\right)^2\right. \\
&\left.+\frac{3}{2}a_1\left(1-\frac{3}{2}a_1\right)\right]\left(b_0+b_1t\right)^{-2-(3/2)a_1}
\end{aligned}\tag{33}$$

The physical and kinematical properties of the model are obtained as follows:

Volume is

$$V=b_2\left(b_0b_1t\right)^{(3/2)a_1}\tag{34}$$

Hubble's parameter is

$$H=\frac{1}{2}a_1b_1\left(b_0+b_1t\right)^{-1}\tag{35}$$

Expansion factor is

$$\theta=\frac{3}{2}a_1b_1\left(b_0+b_1t\right)^{-1}\tag{36}$$

Deceleration parameter is

$$q=\frac{2}{a_1}-1\tag{37}$$

Jerk parameter is

$$j=\frac{2b_1}{a_1}\left(\frac{a_1}{2}-1\right)\left(\frac{a_1}{2}-2\right)\left(b_0+b_1t\right)^{-1}\tag{38}$$

And state-finder parameters $\{r,s\}$ are obtained as

$$r=4a_1^2\left(\frac{a_1}{2}-1\right)\left(\frac{a_1}{2}-2\right)\tag{39}$$

$$s=\frac{2}{3}a_1^{-1}\left(a_1-2\right)\left(a_1-4\right)\left(4-3a_1\right)^{-1}\tag{40}$$

Dark energy parameter is

$$\begin{aligned}
\Omega_d&=\frac{\rho_d}{3H^2} \\
&=a_1^{-2}b_1^{-2}b_3\left(4+3a_1\right)\left(b_0+b_1t\right)^{-1-(3/2)a_1} \\
&\quad+\frac{4}{3}a_1^{-2}b_1^{-2}b_4\left(b_0+b_1t\right)^3
\end{aligned}\tag{41}$$

3.2. Case-II. As another solution we get from (17) and (18),

$$\lambda=\left(c_1t+c_0\right)^{c_2}\tag{42}$$

$$\phi=\frac{a_0}{c_1c_2}\left(c_1t+c_0\right)^{1-c_2}e^{(-3/2)(c_1t+c_0)^{c_2}}\tag{43}$$

where c_0, c_1, and c_2 are arbitrary constants. Here in the case we take the interaction Q as

$$Q=3z^2H\rho_d\tag{44}$$

where H is the mean Hubble's parameter. Then (15) and (16), respectively, take the forms

$$\dot{\rho_m}+\left(\rho_m+p_m\right)\frac{3}{2}\dot{\lambda}=-3z^2\rho_d\frac{\dot{\lambda}}{2}\tag{45}$$

and

$$\dot{\rho_d}+\left(\rho_d+p_d\right)\frac{3}{2}\dot{\lambda}=3z^2\rho_d\frac{\dot{\lambda}}{2}\tag{46}$$

From (46) we get

$$\rho_d=c_3e^{(3/2)(z^2-\alpha-1)(c_1t+c_0)^{c_2}}\tag{47}$$

where c_3 is an arbitrary constant. Now using (42), (43), and (47) in (9) we get

$$\begin{aligned}\rho_m &= \frac{a_0c_1c_2}{8\pi}\left(\frac{3}{4}-\frac{9\omega}{8}\right)(c_1t+c_0)^{c_2-1}e^{-(3/2)(c_1t+c_0)^{c_2}} \\ &- \frac{\omega a_0c_1}{16\pi c_2}(1-c_2)^2(c_1t+c_0)^{-1-c_2}e^{-(3/2)(c_1t+c_0)^{c_2}} \\ &+ \frac{3a_0c_1}{16\pi}(1-c_2)(1+\omega)(c_1t+c_0)^{-1}e^{-(3/2)(c_1t+c_0)^{c_2}} \\ &- \frac{9a_0c_1c_2}{32\pi}(c_1t+c_0)^{c_2-1}e^{-(3/2)(c_1t+c_0)^{c_2}} \\ &- c_3e^{(3/2)(z^2-\alpha-1)(c_1t+c_0)^{c_2}}\end{aligned} \tag{48}$$

Here, using (47), (48) and (13), (12) gives

$$\begin{aligned}p_m &= \left(\frac{21a_0c_1c_2}{96\pi}+\frac{27\omega a_0c_1c_2}{48\pi}-\frac{9}{32\pi}-\frac{9\omega}{48\pi}\right) \\ &\times(c_1t+c_0)^{c_2-1}e^{-(3/2)(c_1t+c_0)^{c_2}}+\frac{a_0c_1}{16\pi}(1-c_2) \\ &\cdot(1+\omega)(c_1t+c_0)^{-1}e^{-(3/2)(c_1t+c_0)^{c_2}} \\ &+\left[\frac{a_0c_1^2(1-c_2)}{24\pi}(3+2\omega)-\frac{\omega a_0c_1(1-c_2)^2}{48\pi c_2}\right] \\ &\cdot(c_1t+c_0)^{-c_2-1}e^{-(3/2)(c_1t+c_0)^{c_2}} \\ &-\alpha c_3e^{(3/2)(z^2-\alpha-1)(c_1t+c_0)^{c_2}}\end{aligned} \tag{49}$$

In this case

$$\begin{aligned}\rho &= \frac{a_0c_1c_2}{8\pi}\left(\frac{3}{4}-\frac{9\omega}{8}\right)(c_1t+c_0)^{c_2-1}e^{-(3/2)(c_1t+c_0)^{c_2}} \\ &- \frac{\omega a_0c_1}{16\pi c_2}(1-c_2)^2(c_1t+c_0)^{-1-c_2}e^{-(3/2)(c_1t+c_0)^{c_2}} \\ &+ \frac{3a_0c_1}{16\pi}(1-c_2)(1+\omega)(c_1t+c_0)^{-1}e^{-(3/2)(c_1t+c_0)^{c_2}} \\ &- \frac{9a_0c_1c_2}{32\pi}(c_1t+c_0)^{c_2-1}e^{-(3/2)(c_1t+c_0)^{c_2}}\end{aligned} \tag{50}$$

And the interaction Q is obtained as

$$Q = \frac{3}{2}z^2c_1c_2c_3(c_1t+c_0)^{c_2-1}e^{(3/2)(z^2-\alpha-1)(c_1t+c_0)^{c_2}} \tag{51}$$

3.3. Case-II(a). In this case, we take the interaction Q as $3z^2\rho_m(\dot{\lambda}/2)$, so that the conservation equations take the forms

$$\dot{\rho_m}+(\rho_m+p_m)\frac{3}{2}\dot{\lambda} = -3z^2\rho_m\frac{\dot{\lambda}}{2} \tag{52}$$

$$\dot{\rho_d}+(\rho_d+p_d)\frac{3}{2}\dot{\lambda} = 3z^2\rho_m\frac{\dot{\lambda}}{2} \tag{53}$$

Now, from (9), using (42) and (43), we get

$$\begin{aligned}\rho_m &= \frac{a_0}{8\pi c_1c_2}e^{-(3/2)(c_1t+c_0)^{c_2}}\left[\left(\frac{3}{2}\omega c_1^2c_2(1-c_2)\right.\right. \\ &\left.+\frac{3}{2}c_1^1c_2(1-c_2)\right)\times(c_1t+c_0)^{-1}-\frac{\omega}{2}c_1^2(1-c_2)^2 \\ &\left.\cdot(c_1t+c_0)^{-1-c_2}-\left(\frac{3}{2}+\frac{9\omega}{8}\right)c_1^2c_2^2(c_1t+c_0)^{c_2-1}\right] \\ &-\rho_d\end{aligned} \tag{54}$$

From (53), using (54), (13) and (42), we have

$$\begin{aligned}\dot{\rho_d} &= -\frac{3}{2}c_1c_2(c_1t+c_0)^{c_2-1}\rho_d+\frac{3a_0z^2}{16\pi} \\ &\cdot e^{-(3/2)(c_1t+c_0)^{c_2}}\left[-\frac{\omega}{2}c_1^2(1-c_2)^2(c_1t+c_0)^{-2}\right. \\ &+\frac{3}{2}c_1^2c_2(1+\omega)(1-c_2)(c_1t+c_0)^{c_2-2} \\ &\left.-\left(\frac{3}{2}+\frac{9\omega}{8}\right)c_1^2c_2^2(c_1t+c_0)^{2c_2-1}\right],\end{aligned} \tag{55}$$

where

$$\alpha = -z^2 \tag{56}$$

And this is possible without loss of generality as α can take values such that $-1 \le \alpha < 0$ as well as $\alpha < -1$ which is the characteristic of different forms of dark energy which can be attained according to different values of z^2. Now (55) gives

$$\begin{aligned}\rho_d &= \frac{3a_0z^2}{16\pi}e^{-(3/2)(c_1t+c_0)^{c_2}}\left[\frac{\omega}{2}c_1(1-c_2)^2(c_1t+c_0)^{-1}\right. \\ &-\frac{1}{2}\left(\frac{3}{2}+\frac{9\omega}{8}\right)c_1c_2(c_1t+c_0)^{2c_2} \\ &\left.-\frac{3}{2}(1+\omega)c_1c_2(c_1t+c_0)^{c_2-1}\right]\end{aligned} \tag{57}$$

From (54) and (57), we have

$$\begin{aligned}\rho_m &= e^{-(3/2)(c_1t+c_0)^{c_2}}\left[\left(\frac{3a_0c_1}{16\pi}(1+\omega)(1-c_2)\right.\right. \\ &\left.-\frac{3a_0\omega c_1z^2}{32\pi}(1-c_2)^2\right)\times(c_1t+c_0)^{-1} \\ &+\left(\frac{9a_0c_1c_2}{32\pi}(1+\omega)z^2-\frac{a_0}{8\pi}c_1c_2\left(\frac{3}{2}+\frac{9\omega}{8}\right)\right)(c_1t \\ &+c_0)^{c_2-1}+\frac{3a_0c_1c_2}{32\pi}z^2\left(\frac{3}{2}+\frac{9\omega}{8}\right)(c_1t+c_0)^{2c_2} \\ &\left.-\frac{a_0\omega c_1}{16\pi c_2}(1-c_2)^2(c_1t+c_0)^{-c_2-1}\right]\end{aligned} \tag{58}$$

Now from (57) and (13) we take

$$p_d = \frac{3a_0 z^2 \alpha}{16\pi} e^{-(3/2)(c_1 t + c_0)^{c_2}} \left[\frac{\omega}{2} c_1 (1 - c_2)^2 (c_1 t + c_0)^{-1} - \frac{1}{2}\left(\frac{3}{2} + \frac{9\omega}{8}\right) c_1 c_2 (c_1 t + c_0)^{2c_2} - \frac{3}{2}(1+\omega) c_1 c_2 (c_1 t + c_0)^{c_2 - 1} \right] \tag{59}$$

Thus, now using (59) in (10), we get

$$\begin{aligned} p_m = {} & \left[\left\{ \frac{3a_0 \omega c_1}{32\pi} z^4 (1 - c_2)^2 + \frac{3\omega a_0 c_1}{16\pi}(1 - c_2) + \frac{3a_0 c_1}{16\pi}(1 - c_2) \right\} (c_1 t + c_0)^{-1} - \frac{3a_0 z^4 c_1 c_2}{32\pi} \left(\frac{3}{2} + \frac{9\omega}{8} \right) (c_1 t + c_0)^{2c_2} \right. \\ & + \left(\frac{9a_0 z^4}{32\pi} (1+\omega) c_1 c_2 + \frac{3a_0 c_1 c_2}{32\pi} - \frac{9\omega a_0 c_1 c_2}{32\pi} - \frac{9a_0 c_1}{32\pi} \right) (c_1 t + c_0)^{c_2 - 1} \\ & \left. + \left(\frac{a_0 c_1 (1 - c_2)}{8\pi} - \frac{\omega a_0 c_1}{16\pi c_2} (1 - c_2)^2 \right) (c_1 t + c_0)^{-1-c_2} \right] e^{-(3/2)(c_1 t + c_0)^{c_2}} \end{aligned} \tag{60}$$

And in this case

$$\begin{aligned} \rho = {} & \frac{a_0}{8\pi c_1 c_2} e^{-(3/2)(c_1 t + c_0)^{c_2}} \left[\left(\frac{3\omega}{2} c_1^2 c_2 (1 - c_2) + \frac{3}{2} c_1^2 c_2 (1 - c_2) \right) \times (c_1 t + c_0)^{-1} - \frac{\omega}{2} c_1^2 (1 - c_2)^2 \right. \\ & \left. \cdot (c_1 t + c_0)^{-1-c_2} - \left(\frac{3}{2} + \frac{9\omega}{8} \right) c_1^1 c_2^2 (c_1 t + c_0)^{c_2 - 1} \right] \end{aligned} \tag{61}$$

Here the interaction Q is obtained as

$$\begin{aligned} Q = {} & \frac{3}{2} c_1 c_2 z^2 (c_1 t + c_0)^{c_2 - 1} e^{-(3/2)(c_1 t + c_0)^{c_2}} \times \left[\left(\frac{3a_0 c_1}{16\pi} (1+\omega)(1 - c_2) - \frac{3a_0 \omega c_1 z^2 (1 - c_2)^2}{32\pi} \right) \right. \\ & \cdot (c_1 t + c_0)^{-1} + \left(\frac{9a_0 c_1 c_2}{32\pi} (1+\omega) z^2 - \frac{a_0}{8\pi} c_1 c_2 \left(\frac{3}{2} + \frac{9\omega}{8} \right) \right) \\ & \cdot (c_1 t + c_0)^{c_2 - 1} + \frac{3a_0 c_1 c_2}{32\pi} z^2 \left(\frac{3}{2} + \frac{9\omega}{8} \right) (c_1 t + c_0)^{2c_2} \\ & \left. - \frac{a_0 \omega c_1}{16\pi c_2} (1 - c_2)^2 (c_1 t + c_0)^{-1-c_2} \right] \end{aligned} \tag{62}$$

The physical and kinematical properties of the models are as follows:

$$V = e^{(3/2)(c_1 t + c_0)^{c_2}} \tag{63}$$

$$H = \frac{1}{2} c_1 c_2 (c_1 t + c_0)^{c_2 - 1} \tag{64}$$

$$\theta = \frac{3}{2} c_1 c_2 (c_1 t + c_0)^{c_2 - 1} \tag{65}$$

$$q = -\frac{2}{c_1 c_2} (c_2 - 1)(c_1 t + c_0)^{-c_2} - 1 \tag{66}$$

$$j = \frac{1}{2} c_1 c_2 (c_1 t + c_0)^{c_2 - 1} + 3c_1 (c_2 - 1)(c_1 t + c_0)^{-1} + 2c_1 c_2^{-1} (c_1 t + c_0)^{-c_2 - 1} \tag{67}$$

$$r = 1 + 4c_2^{-2} (c_2 - 1)(c_2 - 2)(c_1 t + c_0)^{-2c_2} + 6c_2^{-1} (c_2 - 1)(c_1 t + c_0)^{-c_2} \tag{68}$$

$$\begin{aligned} s = {} & -\frac{1}{3} \left[\frac{2}{c_1 c_2} (c_2 - 1)(c_1 t + c_0)^{-c_2} + \frac{3}{2} \right]^{-1} \times \left[6c_2^{-1} (c_2 - 1)(c_1 t + c_0)^{-c_2} \right. \\ & \left. + 4c_2^{-2} (c_2 - 1)(c_2 - 2)(c_1 t + c_0)^{-2c_2} \right] \end{aligned} \tag{69}$$

3.4. Case-III. Equations (17) and (18) give

$$\lambda = \beta (\log t)^n, \quad \beta > 0, \ n > 1 \tag{70}$$

$$\phi = a_0 (n\beta)^{-1} t (\log t)^{1-n} e^{-(3\beta/2)(\log t)^n} \tag{71}$$

Here in this case we assume the interaction between dark energy and other matters of the universe in the form

$$Q = 3z^2 H \rho_m \tag{72}$$

so that (15) and (16), respectively, take the forms

$$\dot{\rho_m} + (\rho_m + p_m) 3\frac{\dot{\lambda}}{2} = -3z^2 \rho_m \frac{\dot{\lambda}}{2} \tag{73}$$

$$\dot{\rho_d} + (\rho_d + p_d) 3\frac{\dot{\lambda}}{2} = 3z^2 \rho_m \frac{\dot{\lambda}}{2} \tag{74}$$

Now from (9) we get

$$\begin{aligned}
\rho_m = {} & \frac{a_0 n\beta}{8\pi t}(\log t)^{n-1} e^{-(3/2)\beta(\log t)^n} \\
& + \frac{3a_0}{16\pi t}(1-\omega)(1-n)(\log t)^{-1} e^{-(3/2)\beta(\log t)^n} \\
& + \frac{a_0}{8\pi t}\left(1-\frac{\omega}{2}\right) e^{-(3/2)\beta(\log t)^n} \\
& - \frac{a_0\omega}{16\pi n\beta t}(\log t)^{1-n} e^{-(3/2)\beta(\log t)^n} \\
& - \frac{(1-n)^2\omega a_0}{16\pi n\beta t}(\log t)^{-n-1} e^{-(3/2)\beta(\log t)^n} \\
& - \frac{(1-n)\omega a_0}{8\pi n\beta t}(\log t)^{-n} e^{-(3/2)\beta(\log t)^n} - \rho_d
\end{aligned} \tag{75}$$

Thus, from (74) and (75), using relation (13), we have

$$\rho_d = e^{-(3/2)\beta(\alpha+1+z^2)(\log t)^n} \times \psi(t) \tag{76}$$

where

$$\begin{aligned}
\psi(t) = {} & \int \frac{3z^2 n\beta}{2t^2}(\log t)^{n-1} \\
& \cdot e^{(3/2)\beta(\alpha+z^2)(\log t)^n}\left[\frac{na_0\beta}{8\pi}(\log t)^{n-1}\right. \\
& + \frac{3a_0(1-n)}{16\pi}(1-\omega)(\log t)^{-1} + \left(1-\frac{\omega}{2}\right)\frac{a_0}{8\pi} \\
& - \frac{(1-n)^2\omega a_0}{16\pi n\beta}(\log t)^{-n-1} - \frac{\omega a_0}{16\pi n\beta}(\log t)^{1-n} \\
& \left. - \frac{(1-n)\omega a_0}{8\pi n\beta}(\log t)^{-n}\right] dt
\end{aligned} \tag{77}$$

Again using relations (70), (71), (75), (76), and (13) in (12), we have

$$\begin{aligned}
p_m = {} & \frac{1}{24\pi t}(3+2\omega) \\
& \cdot e^{-(3/2)\beta(\log t)^n}\left[\frac{a_0}{\beta}(1-n)(\log t)^{-1-n} + \frac{3}{2}a_0\right. \\
& + \frac{3}{2}a_0(1-n)(\log t)^{-1} - \frac{a_0}{n\beta}(1-n)(\log t)^{-n} \\
& \left. - \frac{9na_0\beta}{4}(\log t)^{n-1}\right] - \frac{n\beta(3+2\omega)}{16\pi t}(\log t)^{n-1} \\
& \cdot e^{-(3/2)\beta(\log t)^n} \times \left[\frac{a_0}{n\beta}(\log t)^{1-n} - \frac{3a_0}{2}\right. \\
& \left. + \frac{a_0}{n\beta}(1-n)(\log t)^{-n}\right] + \frac{1}{3}\left[\frac{na_0\beta}{8\pi t}(\log t)^{n-1}\right. \\
& + \frac{3(1-\omega)a_0(1-n)}{16\pi t}(\log t)^{-1} + \frac{a_0}{8\pi t}\left(1-\frac{\omega}{2}\right) \\
& - \frac{\omega a_0}{16\pi n\beta t}(\log t)^{1-n} - \frac{(1-n)^2\omega a_0}{16\pi n\beta t}(\log t)^{-1-n} \\
& \left. - \frac{(1-n)\omega a_0}{8\pi n\beta t}(\log t)^{-n}\right] e^{-(3/2)\beta(\log t)^n} - \alpha\psi(t) \\
& \cdot e^{-(3/2)\beta(\alpha+1+z^2)(\log t)^n}
\end{aligned} \tag{78}$$

Here

$$\begin{aligned}
\rho_m = {} & \frac{na_0\beta}{8\pi t}(\log t)^{n-1} e^{-(3/2)\beta(\log t)^n} \\
& + \frac{3a_0}{16\pi t}(1-\omega)(1-n)(\log t)^{-1} e^{-(3/2)\beta(\log t)^n} \\
& + \frac{a_0}{8\pi t}\left(1-\frac{\omega}{2}\right) e^{-(3/2)\beta(\log t)^n} \\
& - \frac{\omega a_0}{16\pi n\beta t}(\log t)^{1-n} e^{-(3/2)\beta(\log t)^n} \\
& - \frac{\omega a_0}{16\pi n\beta t}(1-n)^2(\log t)^{-n-1} e^{-(3/2)\beta(\log t)^n} \\
& - \frac{\omega a_0(1-n)}{8\pi n\beta t}(\log t)^{-n} e^{-(3/2)\beta(\log t)^n} \\
& - e^{-(3/2)\beta(\alpha+1+z^2)(\log t)^n}\psi(t)
\end{aligned} \tag{79}$$

And

$$\begin{aligned}
\rho = {} & \frac{a_0 n\beta}{8\pi t}(\log t)^{n-1} e^{-(3/2)\beta(\log t)^n} \\
& + \frac{3a_0}{16\pi t}(1-\omega)(1-n)(\log t)^{-1} e^{-(3/2)\beta(\log t)^n} \\
& + \frac{a_0}{8\pi t}\left(1-\frac{\omega}{2}\right) e^{-(3/2)\beta(\log t)^n} \\
& - \frac{\omega a_0}{16\pi n\beta t}(\log t)^{1-n} e^{-(3/2)\beta(\log t)^n} \\
& - \frac{(1-n)^2\omega a_0}{16\pi n\beta t}(\log t)^{-n-1} e^{-(3/2)\beta(\log t)^n} \\
& - \frac{(1-n)\omega a_0}{8\pi n\beta t}(\log t)^{-n} e^{-(3/2)\beta(\log t)^n}
\end{aligned} \tag{80}$$

In this case, the interaction Q is given by

$$\begin{aligned}
Q = {} & \frac{3z^2 n\beta}{2t}(\log t)^{n-1}\left[\frac{na_0\beta}{8\pi t}(\log t)^{n-1} e^{-(3/2)\beta(\log t)^n}\right. \\
& + \frac{3a_0}{16\pi t}(1-\omega)(1-n)(\log t)^{-1} e^{-(3/2)\beta(\log t)^n}
\end{aligned}$$

$$+\frac{a_0}{8\pi t}\left(1-\frac{\omega}{2}\right)e^{-(3/2)\beta(\log t)^n}$$
$$-\frac{\omega a_0}{16\pi n\beta t}(\log t)^{1-n}e^{-(3/2)\beta(\log t)^n}$$
$$-\frac{\omega a_0}{16\pi n\beta t}(1-n)^2(\log t)^{-n-1}e^{-(3/2)\beta(\log t)^n}$$
$$-\frac{\omega a_0(1-n)}{8\pi n\beta t}(\log t)^{-n}e^{-(3/2)\beta(\log t)^n}$$
$$\left.-e^{-(3/2)\beta(\alpha+1+z^2)(\log t)^n}\psi(t)\right] \tag{81}$$

The physical and kinematical properties of the model are obtained as follows:

$$V=e^{(3/2)\beta(\log t)^n} \tag{82}$$

$$H=\frac{n\beta}{2t}(\log t)^{n-1} \tag{83}$$

$$\theta=\frac{3n\beta}{2t}(\log t)^{n-1} \tag{84}$$

$$q=\frac{2}{n\beta}(\log t)^{n-1}+\frac{2(n-1)}{n\beta}(\log t)^{n-2}-1 \tag{85}$$

$$\begin{aligned}j={}&\frac{n\beta}{2}t^{-1}(\log t)^{n-1}+3(n-1)t^{-3}(\log t)^{-1}\\&+\frac{4}{n\beta}t^{-1}(\log t)^{-n+1}-3t^{-3}\\&-\frac{6}{n\beta}(n-1)t^{-1}(\log t)^{-n}\\&+\frac{2}{n\beta}(n-1)(n-2)t^{-1}(\log t)^{n-3}\end{aligned} \tag{86}$$

$$\begin{aligned}r={}&1+\frac{6}{n\beta}(n-1)t^{-2}(\log t)^{-n}-\frac{6}{n\beta}t^{-2}(\log t)^{-n+1}\\&+\frac{8}{n^2\beta^2}(\log t)^{-2n+2}\\&-\frac{12}{n^2\beta^2}(n-1)(\log t)^{-2n+1}\\&+\frac{4}{n^2\beta^2}(n-1)(n-2)(\log t)^{-2}\end{aligned} \tag{87}$$

$$\begin{aligned}s={}&\frac{1}{3}\left[\frac{2}{n\beta}(\log t)^{n-1}+\frac{2(n-1)}{n\beta}(\log t)^{n-2}-\frac{3}{2}\right]^{-1}\\&\times\left[\frac{6}{n\beta}(n-1)t^{-2}(\log t)^{-n}\right.\\&-\frac{6}{n\beta}t^{-2}(\log t)^{-n+1}+\frac{8}{n^2\beta^2}(\log t)^{-2n+2}\\&-\frac{12}{n^2\beta^2}(n-1)(\log t)^{-2n+1}\\&\left.+\frac{4}{n^2\beta^2}(n-1)(n-2)(\log t)^{-2}\right]\end{aligned} \tag{88}$$

4. Study of the Solutions and Conclusions

For the model universe in Case-I, we see that, at $t = 0$, the energy density has finite value dependent on the coupling constant of the interaction between dark energy and other matters in this universe, and the total energy density is found to decrease with time until it tends to zero at infinite time and the interaction term is also found to follow the same behavior with respect to time. But during this time the rate of decrease of the dark energy density is slower in comparison to the rate of decrease of the energy density of other matters in the universe. Therefore, as time passes by, dark energy seems to dominate over other matters. Thus, the scenario in such type of universe is that dark energy plays a vital role and it seems that as the energy density of dark energy increases the expansion of the universe increases.

In this model, it is seen that, at $t = 0, V = b_2(b_0+b_1t)^{(3/2)a_1}$, which shows that if we have to accept the big bang theory, the universe begins its evolution at time $t \longrightarrow -b_0/b_1$, thereby implying the existence of negative time which is almost possible from the presence of dark energy in this universe. Again the value of the deceleration parameter obtained here implies that the value of a_1 is limited by the condition ($a_1 > 2$). And in this model the expansion of the universe is accelerating though the rate decreases with time. Therefore, this universe may be taken as a reasonable model; otherwise, if the rate of expansion increases with time, there will be a singularity where the universe ends transforming itself into a cloud of dust.

In this universe, we see that the interaction between dark energy and other matters decreases with time, and perhaps there is a tendency where the dark energy decays into cold dark matter. Again it is seen that dark energy density is zero only at time $t = -b_0/b_1$ showing that dark energy exists before $t = 0$ also. Thus perhaps there exists an epoch before our cosmic time begins in the history of evolution of the universe.

Here, in this case, if either $a_1 = 2$ or $a_1 = 4$, then we get the state-finder parameter $\{r, s\}$ as $r = 0$ and $s = 0$ for which the dark energy model reduces to a flat ΛCDM model which predicts a highly accelerated expansion before these events of time. In this model, the interaction between dark energy and other matters is found to exist at these events of time not interrupted by the high speed of expansion.

For this universe, the equation of state parameter for dark energy is found to be less than −1 which indicates that the dark energy contained is of the phantom type. From the study of the interaction, we also see that the action of such type of dark energy is more when the energies from other types of sources remain idle or not so active. It is also opposite to or against the light energy. It acts also against the living energy or the energy possessed by the human beings.

In Case-II, though the universe is expanding and the rate of expansion is accelerating, it depends much on the value of c_1, which indicates that the expansion is related to the dark energy density. There it may be taken that dark energy enhances the accelerated expansion. And this enhancement is also dependent on the value of z^2 which is the coupling constant of the interaction between dark energy and other matters in this universe. This implies that the expansion of the universe is very much interrelated with the interaction between dark energy and other components of the universe. Thus, all the members of this universe may be taken to expand due to also the presence of dark energy. Hence, considering a small-scale structure of the universe, the earth may be taken to expand due to also the presence of dark energy.

In this universe, we see that when $c_2 \longrightarrow 0$, it goes to the asymptotic static era with $r \longrightarrow \infty$ and $s \longrightarrow \infty$. And when $c_2 = 1$, the universe goes to ΛCDM model for which $r = 1$ and $s = 0$. Thus, the state-finder parameters $\{r, s\}$ show the picture of the evolution of our universe, starting from the asymptotic static era and then coming to the ΛCDM model era. Here we see that the interaction $Q \longrightarrow 0$ as $c_2 \longrightarrow 0$ which means that the interaction almost stops at the cosmic time when the scale factor of the universe becomes or takes the value $e^{1/2}$.

Again it is seen that the energy density of this model universe tends to infinity at $c_2 = 0$ and decreases gradually as $c_2 \longrightarrow 1$; thus it seems that our universe started with a big bang. From the above behaviors of this model, it is also implied that at the beginning of the evolution of this universe there was no interaction between dark energy and other components of the universe, and after that the interaction between them becomes active and increases with time, and at present they are highly interacting. But this interaction also depends much on the values of α and z^2 which are, respectively, equation of state parameter of the dark energy contained and the coupling constant of interaction.

Case-III represents the logamediate scenario of the universe where the cosmological solutions have indefinite expansion [62]. In this case, the dark energy has a quintessence-like behavior. Here the matter content of the universe is seen to increase slowly due to the interaction and the cosmic effect. In this universe, there is an interesting event of time, that is, the cosmic time, when $t = 1$. At this point, the universe has the tendency of accelerating expansion, but the expansion suddenly stops at this moment and the volume of the universe takes the value of 1 at this juncture. So it seems that there is a bounce and a new epoch begins from this juncture. Also we see that, at $t = 1$, the state-finder parameters $\{r, s\}$ take the values $r = 1$ and $s = 0$. Thus at this instant our universe will go to that of a ΛCDM model which implies that at this event of time most of the dark energy contained will reduce to cold dark matter.

The energy density of this model universe tends to infinity at $t = 0$ which indicates that this universe begins with a big bang, and it (energy density) decreases gradually until it tends to a finite quantity at infinite time, of course with a bounce at $t = 1$. And at $t = 0$, the dark energy density is found to be exceptionally high which indicates that it helps much in triggering the big bang. It is also seen that the dark energy is highly interacting with other components of the universe at $t = 0$, with the interaction decreasing slowly with the passing away of the cosmic time. In this universe, we see that both the scalar field ϕ and the interaction Q tend to vanish as $a_0 \longrightarrow 0$. Thus, the scalar field is very much interconnected with the dark energy content of this universe and plays a vital role in the production and existence of it. One peculiarity in this model is that the scalar field does not vanish at $t \longrightarrow \infty$; thus, the dark energy seems to be prevalent eternally in this universe due to this scalar field.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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