

Research Article

Stability of the Universe Model Coupled with Phantom and Tachyon Fields

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Received 14 May 2019; Revised 8 July 2019; Accepted 15 July 2019; Published 31 July 2019

Academic Editor: Sally Seidel

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In this paper, we study the phase space analysis of locally rotationally symmetric Bianchi type I universe model by taking interactions between dark matter and scalar field models. We define normalized dimensionless variables to develop an autonomous system of equations. We also find the corresponding critical points in order to study the dynamics of the system. The dynamical analysis indicates that all the critical points correspond to accelerated cosmic expansion for tachyon coupled field. We observe that positive values of m provide more stable future attractors as compared to its negative values. We also analyze the behavior of power-law scale factor which shows different cosmological phases. It is found that the region for decelerated expansion gets larger for the phantom coupled matter by increasing m while this region decreases for tachyon coupled field.

1. Introduction

Recent observations suggest that our universe is expanding at an accelerating rate [1–3]. These observational probes indicate two cosmic phases, i.e., the cosmos phase before radiation and ultimately the current cosmic era. Many substantial attempts have been taken to explore the facts behind the current cosmic acceleration. An unusual form of energy having large negative pressure was proposed by the physicists, known as dark energy (DE). This energy dominates over the matter content undergoing current cosmic expansion.

There have been a massive literature devoted to study the ambiguous nature of DE. The cosmological constant (Λ) is taken as the elementary candidate having two conventional problems, i.e., fine-tuning and cosmic coincidence. Several alternative dynamical models can be considered as a substitute of Λ like quintessence [4, 5], phantom model [6], tachyon field [7], and k-essence [8] that also show the expanding nature of cosmos. The extension of general barotropic equation of state (EoS) to some exotic candidates like Chaplygin gas [9] and its generalization [10] also represents the DE candidates.

The astonishing idea of scalar field, with an EoS parameter other than -1 , has been proved to be useful to interpret

the universe evolution due to its applications in different cosmological problems like cosmic acceleration as well as coincidence problems. The scalar field candidates also yield the inflationary era of cosmic evolution. We can adopt scalar field models (in particular, phantom and tachyon) as an alternative of DE candidates which interact with DM to resolve the coincidence problem. Researchers have paid an extensive attention to the tachyon cosmology in which the tachyon is fundamentally attributed by string theory [11, 12]. Gibbons studied the cosmological impact of tachyon field turning down to its ground phase [13]. The tachyon matter may execute inflationary era and eventually a new form of DM at late times [14]. A phantom field was also presented as an alternative of DE that constitutes large negative pressure with EoS parameter $w < -1$ and plays an important role for accelerated expansion of the universe [15–18]. One of the significant features of the phantom model reveals that the universe will end with a big-rip (future singularity). The interaction of DE (phantom or tachyon) and DM may alleviate the coincidence problem [19, 20].

A phase space represents various states including position as well as momentum corresponding to each point of a dynamical system. This analysis yields dynamical characterization of a cosmological model by reducing complexity

of the system of equations. It has always been helpful to study different stages of evolution by converting the system of equations to an autonomous one. This investigates the influence of initial data on the stability of a system by checking whether the respective system will remain stable or not [21].

Copeland et al. [22] discussed phase space analysis of the inflationary model that was unable to solve density problem. Guo et al. [23] studied stability of FRW universe model filled with barotropic fluid and phantom scalar field. They found that phantom dominated solution is a stable late-time attractor. Guo et al. [24] used this analysis to study the dynamical behavior of cosmos by interacting phantom energy with DM. Chen et al. [25] discussed a detailed phase space analysis of various phantom cosmological models for which the DE sector interacts with the DM. Acquaviva and Beesham [26] explored this analysis for FRW model and found that the nonlinear extension of viscous cosmic model provides accelerated expansion of the universe. We studied the influence of nonlinear electrodynamics on stability of accelerated expansion in the background of FRW cosmology [27]. Shahalam et al. [28] presented phase space analysis for various combinations of the coupling between phantom and tachyon fields taking FRW universe. We also discussed phase space analysis of FRW universe model by considering a power-law model for bulk viscosity coefficient [29].

Bianchi universe models have also been discussed in literature to explore primordial anisotropy and some large angle anomalies identified by CMBR which give rise to the negligence of statistical cosmic isotropy [30]. Belinskii and Khalatnikov [31] applied phase plane approach to Bianchi type I (BI) model and studied the effects of shear and bulk viscosity. Coley and Dunn [32] used this technique to investigate the dynamics of Bianchi type V model by taking a viscous fluid. Burd and Coley [33] discussed the influence of shear as well as bulk viscosity on stability of Bianchi universe models. Goliath and Ellis [34] studied dynamical evolution of Bianchi universe model by including the cosmological constant. Sharif and his collaborators discussed phase space analysis of Bianchi I universe in Brans-Dicke gravity with chameleon scalar field [35] as well as $f(T)$ gravity taking nonphantom, vacuum, and phantom phases [36]. Chaubey and Raushan [37] studied phase space analysis of LRS BI model in the presence of scalar field.

This paper aims to investigate the stability of LRS BI universe model by taking linear interactions of phantom and tachyon fields coupled with DM through phase space analysis. The plan of the paper is as follows. In Section 2, we provide some basic formalism for the evolution equations. An autonomous system of equations is developed by defining normalized dimensionless variables. Section 3 deals with the dynamical analysis of the universe model by taking three different forms of interactions between phantom energy and DM. We also study the phase space analysis of tachyon field coupled with DM in Section 4. Section 5 deals with the formulation of power-law scale factor. Finally, we summarize our findings in the last section.

2. General Equations

Bianchi universe models are the simplest extensions of FRW universe with the addition of anisotropic effects. The LRS BI model is described by the line element [38, 39]:

$$ds^2 = -dt^2 + a(t) dx^2 + b(t) (dy^2 + dz^2), \quad (1)$$

where $a(t)$ and $b(t)$ are the cosmic scale factors. The mean Hubble parameter, in terms of the directional Hubble parameters ($H_1 = \dot{a}/a$ and $H_2 = \dot{b}/b$), can be defined as

$$H = \frac{1}{3} [H_1 + H_2] = \frac{1}{3} \left[\frac{\dot{a}}{a} + \frac{2\dot{b}}{b} \right] = \frac{1}{3} \left(\frac{\dot{v}}{v} \right). \quad (2)$$

We consider a power-law relation $a = b^m$, $m \neq 0, 1$, where m represents a constant anisotropic parameter that gives the deviation of anisotropy from the isotropic model. This leads to the following relation:

$$H_1 = mH_2 = \left(\frac{3m}{m+2} \right) H. \quad (3)$$

This assumption can be rationalized by the velocity redshift expression for extragalactic sources through which the Hubble cosmic expansion may achieve isotropy when shear to expansion scalar ratio is constant [40].

We assume a cosmic fluid by interacting phantom field with matter such that the conservation of energy yields

$$\dot{\sigma}_m + 3(\sigma_m + p_m)H = Q, \quad (4)$$

$$\dot{\sigma}_\phi + 3(\sigma_\phi + p_\phi)H = -Q, \quad (5)$$

$$\dot{\sigma} + 3(\sigma + p)H = 0, \quad (6)$$

where Q is the interaction term, dot corresponds to the derivative with respect to time, $\sigma = \sigma_m + \sigma_\phi$, $p = p_m + p_\phi$, σ_m , σ_ϕ , p_m , and p_ϕ represent energy densities and pressures of the matter and phantom energy, respectively. It should be noted that the transfer of energy between two components depends on the sign of interaction term. The energy passes from phantom to matter for $Q > 0$ while $Q < 0$ yields the opposite. The conservation equations clearly show that $Q = Q(H, \sigma_m, \sigma_\phi)$. The constraint and Raychaudhuri equations are formulated from the field equations as

$$H^2 = \frac{(m+2)^2}{9(2m+1)} (\sigma_m + \sigma_\phi), \quad (7)$$

$$0 = \left(\frac{6}{m+2} \right) \dot{H} + \frac{27}{(m+2)^2} H^2 + p_\phi, \quad (8)$$

where $\sigma_\phi = -(1/2)\dot{\phi}^2 + V(\phi)$ and $p_\phi = -(1/2)\dot{\phi}^2 - V(\phi)$. Due to many arbitrary parameters, it seems difficult to find analytical solution of the evolution equation. For this purpose, we define the following normalized dimensionless quantities:

$$\begin{aligned}\mu &= \frac{(m+2)\dot{\phi}}{\sqrt{6}(2m+1)H}, \\ \nu &= \frac{(m+2)\sqrt{V}}{\sqrt{3}(2m+1)H}, \\ \lambda &= -\frac{V'}{V},\end{aligned}\quad (9)$$

that transform the dynamical equations into an autonomous system. Differentiating μ and ν with respect to $N = ((m+2)/3m)\ln a$, we have

$$\mu' = \frac{3m}{m+2}\mu \left[\frac{\ddot{\phi}}{H\dot{\phi}} - \frac{\dot{H}}{H^2} \right], \quad (10)$$

$$\nu' = -\frac{3m}{m+2}\nu \left[\sqrt{\frac{3}{2}} \frac{2m+1}{m+2} \mu\lambda + \frac{\dot{H}}{H^2} \right]. \quad (11)$$

The Raychaudhuri and conservation equations through these dimensionless parameters become

$$\frac{\dot{H}}{H^2} = \frac{1}{2(m+2)} [-9 + (2m+1)^2 \{\mu^2 + \nu^2\}], \quad (12)$$

$$\frac{\ddot{\phi}}{H\dot{\phi}} = -3 - \sqrt{\frac{3}{2}} \frac{2m+1}{m+2} \frac{\lambda\nu^2}{\mu} + \frac{Q}{H\dot{\phi}^2}, \quad (13)$$

where λ is a constant. The constraint equation can be written in the form

$$\Omega_\phi = \frac{(m+2)^2}{9(2m+1)} \frac{\sigma_\phi}{H^2} = \frac{2m+1}{3} [-\mu^2 + \nu^2]. \quad (14)$$

The effective EoS for the cosmic fluid and phantom field are given by

$$\begin{aligned}w_{eff} &= -1 - \frac{2\dot{H}}{3H^2}, \\ w_\phi &= \frac{w_e f f'}{\Omega_\phi}.\end{aligned}\quad (15)$$

3. Dynamics of Interacting Phantom Energy

In this section, we discuss dynamics of LRS BI universe by the phase space analysis taking an interaction between phantom energy and matter. In this context, we impose the condition $\mu' = \nu' = 0$ to solve the system of (10) and (11) and also determine the corresponding critical points $\{\mu, \nu\}$. The stability of universe model depends on the behavior of critical points and their eigenvalues. We take three different types of coupling between phantom field and matter as follows.

3.1. Coupling $Q = \alpha\dot{\sigma}_m$. Firstly, we consider $Q = \alpha\dot{\sigma}_m$ as an interaction model for the universe where both phantom field and DM are coupled. Equation (13) for this coupling takes the form

$$\frac{\ddot{\phi}}{H\dot{\phi}} = -3 - \sqrt{\frac{3}{2}} \frac{2m+1}{m+2} \frac{\lambda\nu^2}{\mu} - \frac{9\alpha\Omega_m}{2(2m+1)(1-\alpha)\mu^2}, \quad (16)$$

where $\Omega_m = 1 - \Omega_\phi$. The autonomous system of equations becomes

$$\begin{aligned}\mu' &= \frac{3m}{m+2}\mu \left[-3 - \sqrt{\frac{3}{2}} \frac{2m+1}{m+2} \frac{\lambda\nu^2}{\mu} \right. \\ &\quad \left. - \frac{9\alpha\Omega_m}{2(2m+1)(1-\alpha)\mu^2} - \frac{1}{2(m+2)} \right. \\ &\quad \left. \times \{-9 + (2m+1)^2(\mu^2 + \nu^2)\} \right],\end{aligned}\quad (17)$$

$$\begin{aligned}\nu' &= -\frac{3m}{m+2}\nu \left[\sqrt{\frac{3}{2}} \left(\frac{2m+1}{m+2} \right) \mu\lambda \right. \\ &\quad \left. + \frac{1}{2(m+2)} \{-9 + (2m+1)^2(\mu^2 + \nu^2)\} \right].\end{aligned}\quad (18)$$

We can find the eigenvalues by the Jacobian matrix

$$A = \begin{pmatrix} \frac{\partial f}{\partial \mu} & \frac{\partial f}{\partial \nu} \\ \frac{\partial g}{\partial \mu} & \frac{\partial g}{\partial \nu} \end{pmatrix}_0, \quad (19)$$

where suffix 0 defines the values at critical points (μ_c, ν_c) . The fixed point is called a source or unstable point (resp., a sink or stable point) if both eigenvalues consist of positive (resp., negative) real parts. The real parts of the eigenvalues having opposite signs correspond to a saddle point of the system. We evaluate the following critical points in this case. For $P_1 = (\mu_c, \nu_c) = ((-\sqrt{6}\lambda + \sqrt{6(\lambda^2 + 6)})/2(2m+1), 0)$, the eigenvalues of Jacobian matrix are obtained as

$$\begin{aligned}\eta_1 &= \frac{9m}{m+2} \left[-1 \right. \\ &\quad \left. - \frac{3\alpha}{2(1-\alpha)(2m+1)} \left\{ \frac{-4(2m+1)^2}{\left[-\sqrt{6}\lambda + \sqrt{6(\lambda^2 + 6)} \right]^2} \right. \right. \\ &\quad \left. \left. + \frac{2m+1}{3} \right\} + \frac{3}{2(m+2)} \right. \\ &\quad \left. - \frac{\left[-\sqrt{6}\lambda + \sqrt{6(\lambda^2 + 6)} \right]^2}{8(m+2)} \right],\end{aligned}\quad (20)$$

$$\eta_2 = -\frac{3m}{m+2} \left[\sqrt{\frac{3}{2}} \lambda \frac{\sqrt{6(\lambda^2+6)} - \sqrt{6}\lambda}{m+2} + \frac{1}{2(m+2)} \left\{ -9 + \frac{[\sqrt{6(\lambda^2+6)} - \sqrt{6}\lambda]^2}{4} \right\} \right]. \quad (21)$$

We are interested to explore the influence of physical parameters (α and m) on the stability of cosmos coupled with scalar field. We plot the dynamical characteristics of critical points for $Q = \alpha \dot{\sigma}_m$ in the vicinity of different values of α and m as shown in Figure 1. We find negative eigenvalues which show the point P_1 as a stable future attractor for $m > 0$ in the physical phase space except for $\alpha = 1$ at which the system becomes undetermined. For $\alpha, m < 0$, this point behaves as an unstable node.

The effective potential for the cosmic fluid is defined as

$$w_{eff} = -1 + \frac{1}{m+2} [-9 + (2m+1)^2 (\mu^2 + \nu^2)]. \quad (22)$$

We also evaluate the effective EoS parameter and deceleration parameter as

$$w_\phi = \frac{1}{\mu^2 + \nu^2} \left[-1 + \frac{1}{m+2} \left\{ -9 + (2m+1)^2 (\mu^2 + \nu^2) \right\} \right], \quad (23)$$

$$q = -1 + \frac{1}{m+2} [-9 + (2m+1)^2 (\mu^2 + \nu^2)]. \quad (24)$$

The dynamical analysis shows accelerated expansion of the universe model for P_1 as $q < 0$.

For $P_2 = ((-\sqrt{6}\lambda - \sqrt{6(\lambda^2+6)})/2(2m+1), 0)$, the corresponding eigenvalues are

$$\eta_1 = \frac{9m}{m+2} \left[-1 - \frac{3\alpha}{2(1-\alpha)(2m+1)} \left\{ \frac{-4(2m+1)^2}{[\sqrt{6}\lambda + \sqrt{6(\lambda^2+6)}]^2} + \frac{2m+1}{3} \right\} + \frac{3}{2(m+2)} - \frac{[\sqrt{6}\lambda + \sqrt{6(\lambda^2+6)}]^2}{8(m+2)} \right], \quad (25)$$

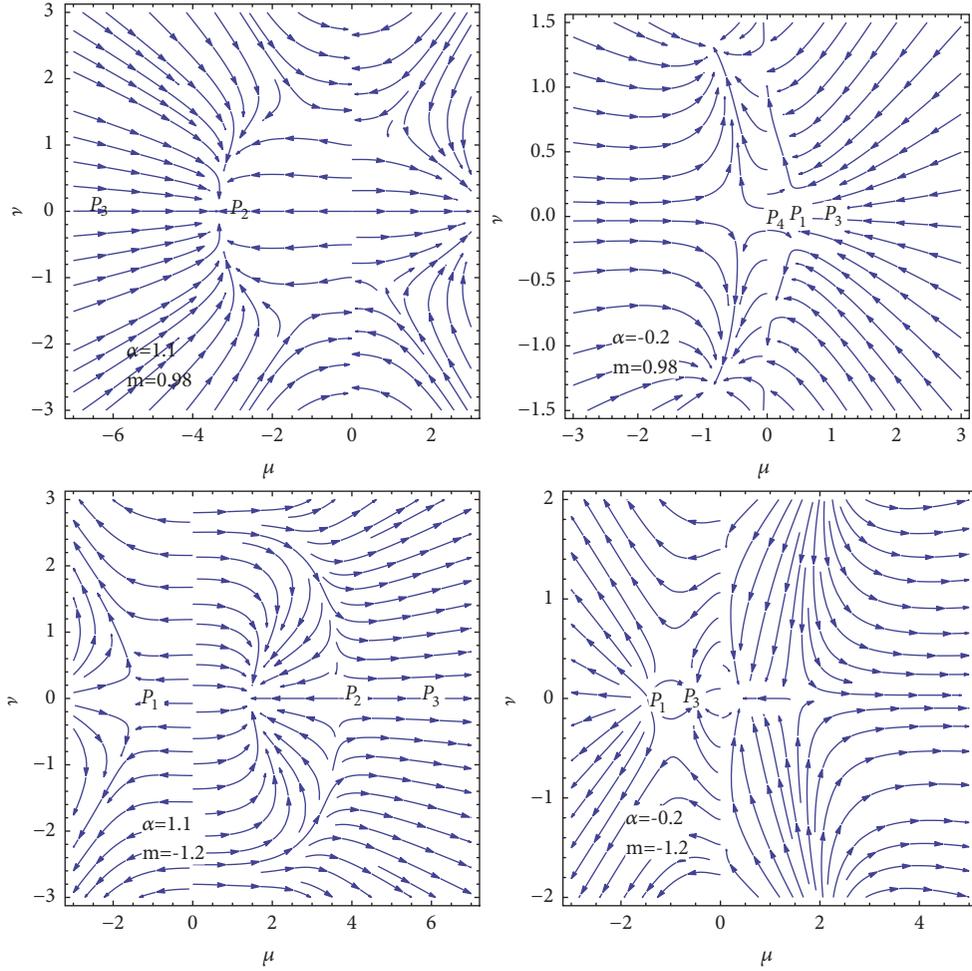
$$\eta_2 = -\frac{3m}{m+2} \left[\sqrt{\frac{3}{2}} \lambda \frac{\sqrt{6(\lambda^2+6)}\lambda + \sqrt{6}\lambda}{m+2} + \frac{1}{2(m+2)} \left\{ -9 + \frac{[\sqrt{6(\lambda^2+6)} + \sqrt{6}\lambda]^2}{4} \right\} \right]. \quad (26)$$

This point shows a stable attractor with decelerated expansion for $m > 0$. This point becomes unstable past attractor for negative values of m . We find that $\alpha < 0$ gives accelerated expansion while the universe undergoes decelerated expansion for $\alpha = 1.1$ as $q > 0$.

For $P_3 = ((3(m+2)(2-\alpha) + \sqrt{[3(m+2)(2-\alpha)]^2 + 36\alpha(1-\alpha)(m+2)})^{1/2}/2(1-\alpha)(2m+1), 0)$, we have

$$\eta_1 = \frac{9m}{m+2} \left[\frac{\alpha}{(m+2)(2-\alpha) + \sqrt{[(m+2)(2-\alpha)]^2 + 4\alpha(1-\alpha)(m+2)}} + \frac{\alpha-2}{2(1-\alpha)} + \frac{3}{2(m+2)} + \frac{2m+1}{2(1-\alpha)} \times \left\{ 3(m+2)(2-\alpha) + \sqrt{[3(m+2)(2-\alpha)]^2 + 36\alpha(1-\alpha)(m+2)} \right\} \right], \quad (27)$$

$$\eta_2 = -\frac{3m}{m+2} \left[-\frac{9}{2(m+2)} + \sqrt{\frac{3}{2}} \lambda \left(\frac{2m+1}{m+2} \right) \times \frac{\sqrt{3(m+2)(2-\alpha) + \sqrt{[3(m+2)(2-\alpha)]^2 + 36\alpha(1-\alpha)(m+2)}}}{2(1-\alpha)(2m+1)} + \frac{(2m+1)^2}{4(m+2)(2m+1)(1-\alpha)} \left\{ 3(m+2)(2-\alpha) + \sqrt{[3(m+2)(2-\alpha)]^2 + 36\alpha(1-\alpha)(m+2)} \right\} \right]. \quad (28)$$


 FIGURE 1: Plots for phantom coupled cosmic model with $Q = \alpha \dot{\sigma}_m$ and $\lambda = 2$.

For positive values of m , this behaves as a saddle point showing decelerated expansion ultimately followed by accelerated expansion. For $m < 0$, this point is saddle or stable showing accelerated expanding universe model corresponding to $\alpha < 0$ or $\alpha > 0$, respectively.

For $P_4 = \left(\frac{((3(m+2)(2-\alpha) - \sqrt{[3(m+2)(2-\alpha)]^2 + 36\alpha(1-\alpha)(m+2)})^{1/2}}{2(1-\alpha)})(2m+1), 0 \right)$, we have

$$\eta_1 = \frac{9m}{m+2} \left[\frac{\alpha}{(m+2)(2-\alpha) - \sqrt{[(m+2)(2-\alpha)]^2 + 4\alpha(1-\alpha)(m+2)}} + \frac{\alpha-2}{2(1-\alpha)} + \frac{3}{2(m+2)} + \frac{2m+1}{2(1-\alpha)} \right. \\ \left. \times \left\{ 3(m+2)(2-\alpha) - \sqrt{[3(m+2)(2-\alpha)]^2 + 36\alpha(1-\alpha)(m+2)} \right\} \right], \quad (29)$$

$$\eta_2 = -\frac{3m}{m+2} \left[-\frac{9}{2(m+2)} + \sqrt{\frac{3}{2}} \lambda \left(\frac{2m+1}{m+2} \right) \times \frac{\sqrt{3(m+2)(2-\alpha) - \sqrt{[3(m+2)(2-\alpha)]^2 + 36\alpha(1-\alpha)(m+2)}}}{2(1-\alpha)(2m+1)} \right]$$

$$+ \frac{(2m+1)^2}{4(m+2)(2m+1)(1-\alpha)} \left\{ 3(m+2)(2-\alpha) - \sqrt{[3(m+2)(2-\alpha)]^2 + 36\alpha(1-\alpha)(m+2)} \right\}. \quad (30)$$

This point behaves as a saddle point for all choices except for $\alpha > 0$ and $m > 0$ that show stable future attractor. It is mentioned here that all the points lie in accelerated expanding phase of the universe for negative values of α as $q < 0$ and $\omega_\phi < -1$. We summarize the results for stability of LRS BI universe coupled with phantom energy and matter in Table 1.

3.2. *Coupling* $Q = \beta\dot{\sigma}_\phi$. For this coupling, (13) leads to

$$\frac{\ddot{\phi}}{H\dot{\phi}} = -3 - \sqrt{\frac{3}{2}} \frac{2m+1}{m+2} \frac{\lambda\nu^2}{\mu} + \frac{3\beta}{1+\beta}. \quad (31)$$

Similarly, the autonomous system of equations takes the form

$$\mu' = \frac{3m}{m+2} \mu \left[-3 - \sqrt{\frac{3}{2}} \frac{2m+1}{m+2} \frac{\lambda\nu^2}{\mu} + \frac{3\beta}{1+\beta} - \frac{1}{2(m+2)} \times \{-9 + (2m+1)^2(\mu^2 + \nu^2)\} \right], \quad (32)$$

$$\nu' = -\frac{3m}{m+2} \nu \left[\sqrt{\frac{3}{2}} \frac{2m+1}{m+2} \mu\lambda + \frac{1}{2(m+2)} \{-9 + (2m+1)^2(\mu^2 + \nu^2)\} \right]. \quad (33)$$

We follow the same procedure to find the critical points. For $P_1 = (0, 0)$, we have

$$\eta_1 = -\frac{9m[2m+3\beta+7]}{2(m+2)^2(1+\beta)}, \quad (34)$$

$$\eta_2 = \frac{27m}{2(m+2)^2}.$$

This point gives a changing behavior of critical points corresponding to different values of β and m . For $\beta = -1.2, 0.8$, the point P_1 is an unstable node by taking $\lambda = 2$ (Figure 2). For $\beta = -1$, the eigenvalues become undetermined; hence, we neglect it. The negative value of m gives stable future attractor. It is found that point P_1 indicates accelerated expansion of cosmos since $q < 0$ for different choices of parameters.

For $P_2 = ((1/(2m+1))\sqrt{3[3\beta-(2m+1)]}/(1+\beta), 0)$, the eigenvalues are given by

$$\eta_1 = \frac{9m}{(m+2)^2(1+\beta)} [3\beta - 4m - 5], \quad (35)$$

$$\eta_2 = -\frac{9m}{(m+2)^2} \left[\lambda \sqrt{\frac{3\beta-(2m+1)}{2(1+\beta)}} - \frac{m+2}{1+\beta} \right]. \quad (36)$$

Here, all values of m and β correspond to saddle or unstable points except $m > 0$ and $\beta < 0$ that yield stable future attractor. The universe undergoes decelerated expansion for negative values of β while $\beta > 0$ corresponds to the accelerated expansion. For $P_3 = (-1/(2m+1))\sqrt{3[3\beta-(2m+1)]}/(1+\beta), 0)$, the eigenvalues are obtained as

$$\eta_1 = \frac{9m}{(m+2)^2(1+\beta)} [3\beta - 4m - 5], \quad (37)$$

$$\eta_2 = \frac{9m}{(m+2)^2} \left[\lambda \sqrt{\frac{3\beta-(2m+1)}{2(1+\beta)}} + \frac{m+2}{1+\beta} \right]. \quad (38)$$

We find that point P_3 is a saddle node for all choices of β and m except for $\beta < 0$ and $m > 0$ at which it behaves as a stable point. This point yields decelerated expansion for negative values of β . For $P_4 = (0, \pm 3/(2m+1))$, the eigenvalues are

$$\eta_1 = -\frac{9m}{(m+2)(1+\beta)}, \quad (39)$$

$$\eta_2 = -\frac{27m}{(m+2)^2}.$$

This point is a saddle node for negative values of β while $\beta > 0$ corresponds to stable/unstable attractors depending on parameter m . It is found that P_4 lies in a region of accelerated expansion for all the choices of parameters. Table 2 shows the dynamical analysis for LRS BI model.

3.3. *Coupling* $Q = \gamma(\dot{\sigma}_m + \dot{\sigma}_\phi)$. In this section, we assume a combination of the form $\dot{\sigma}_m$ and $\dot{\sigma}_\phi$ for the coupling through which (13) becomes

$$\frac{\ddot{\phi}}{H\dot{\phi}} = -3 - \sqrt{\frac{3}{2}} \frac{2m+1}{m+2} \frac{\lambda\nu^2}{\mu} - \frac{3\gamma\Omega_m}{2(1-\gamma)\mu^2} + \frac{3\gamma}{1+\gamma}. \quad (40)$$

TABLE 1: Phase Space Analysis for the Phantom Coupled System with $Q = \alpha \dot{\sigma}_m$.

Ranges of m and α for Critical Points	Stability	ω_ϕ	q	Acceleration
P_1				
$m = 0.98, \alpha = 1.1 (\alpha \neq 1)$	Stable	-6.41	-1.87	Yes
$m = 0.98, \alpha = -0.2$	Stable	-3.38	-0.73	Yes
$m = -1.2, \alpha = -0.2 (m \neq -2)$	Unstable	-1.18	-0.003	Yes
$m = -1.2, \alpha = 1.1$	Saddle	-3.92	-4.27	Yes
P_2				
$m = 0.98, \alpha = 1.1 (\alpha \neq 1)$	Stable	1.301	1.096	No
$m = 0.98, \alpha = -0.2$	Stable	-1.67	-1.23	Yes
$m = -1.2, \alpha = -0.2 (m \neq -0.5, -2)$	Unstable	-1.69	-1.84	Yes
$m = -1.2, \alpha = 1.1$	Unstable	1.3	6.8	No
P_3				
$m = 0.98, \alpha = 1.1 (\alpha \neq 1)$	Saddle	2.85	61.76	No
$m = 0.98, \alpha = -0.2$	Saddle	-2.306	-1.383	Yes
$m = -1.2, \alpha = -0.2 (m \neq -0.5, -2)$	Saddle	-11.57	-5.56	Yes
$m = -1.2, \alpha = 1.1$	Stable	2.16	46.25	No
P_4				
$m = 0.98, \alpha = 1.1 (\alpha \neq 1)$	Stable	1.11	0.72	No
$m = 0.98, \alpha = -0.2$	Saddle	-301.77	-2.49	Yes
$m = -1.2, \alpha = -0.2 (m \neq -0.5, -2)$	Saddle	-194.81	-6.54	Yes
$m = -1.2, \alpha = 1.1$	Saddle	1.42	8.003	No

TABLE 2: Phase Space Analysis for the Phantom Coupled System with $Q = \beta \dot{\sigma}_\phi$.

Ranges of m and β for Critical Points	Stability	ω_ϕ	q	Acceleration
P_1				
$m = 0.98, \beta = 1.2$	Saddle	Undetermined	-2.51	Yes
$m = 0.98, \beta = -1.2, (\beta \neq -1)$	Unstable	Undetermined	-2.51	Yes
$m = -1.2, \beta = -1.2$	Stable	Undetermined	-6.63	Yes
$m = -1.2, \beta = 1.2$	Stable	Undetermined	-6.63	Yes
P_2				
$m = 0.98, \beta = 1.2$	Saddle	-37.41	-2.36	Yes
$m = 0.98, \beta = -1.2, (\beta \neq -1)$	Stable	2.58	14	No
$m = -1.2, \beta = -1.2, (m \neq -0.5)$	Saddle	1.72	14	No
$m = -1.2, \beta = 1.2$	Unstable	-1.07	-2.36	Yes
P_3				
$m = 0.98, \beta = 1.2$	Saddle	-37.42	-2.36	Yes
$m = 0.98, \beta = -1.2, (\beta \neq -1)$	Stable	2.58	14	No
$m = -1.2, \beta = -1.2, (m \neq -0.5)$	Saddle	1.72	14	No
$m = -1.2, \beta = 1.2$	Saddle	-1.07	-2.36	Yes
P_4				
$m = 0.98, \beta = 1.2$	Stable	-0.97	-1	Yes
$m = 0.98, \beta = -1.2, (\beta \neq -1)$	Saddle	-0.97	-1	Yes
$m = -1.2, \beta = -1.2$	Saddle	-0.217	-1	Yes
$m = -1.2, \beta = 1.2$	Unstable	-0.217	-1	Yes

The evolution and conservation equations are given by

$$\begin{aligned} \mu' = & \frac{3m}{m+2} \mu \left[-3 - \sqrt{\frac{3}{2}} \frac{2m+1}{m+2} \frac{\lambda \nu^2}{\mu} - \frac{3\gamma\Omega_m}{2(1-\gamma)\mu^2} \right. \\ & + \frac{3\gamma}{1+\gamma} - \frac{1}{2(m+2)} \\ & \left. \times \{-9 + (2m+1)^2 (\mu^2 + \nu^2)\} \right], \end{aligned} \quad (41)$$

$$\begin{aligned} \nu' = & -\frac{3m}{m+2} \nu \left[\sqrt{\frac{3}{2}} \mu \lambda \left(\frac{2m+1}{m+2} \right) \right. \\ & \left. + \frac{1}{2(m+2)} \{-9 + (2m+1)^2 (\mu^2 + \nu^2)\} \right]. \end{aligned} \quad (42)$$

For $P_1 = ((-(2m+1)/(m+2))\sqrt{3/2}\lambda + \sqrt{(3/2)\lambda^2 + (3(2m+1)/(m+2))^2})/((2m+1)^2/(m+2)), 0)$, the corresponding eigenvalues are

$$\begin{aligned} \eta_1 = & \frac{9m}{m+2} \left[\frac{1-2m}{2(m+2)} \right. \\ & + \frac{\gamma}{2(1-\gamma)} \left\{ \frac{(2m+1)^4}{(m+2)^2 \left\{ -((2m+1)/(m+2))\sqrt{3/2}\lambda + \sqrt{(3/2)\lambda^2 + (3(2m+1)/(m+2))^2} \right\}^2} - \frac{2m+1}{3} \right\} + \frac{\gamma}{1+\gamma} \\ & \left. - \frac{1}{2(m+2)} \left\{ \frac{-((2m+1)/(m+2))\sqrt{3/2}\lambda + \sqrt{(3/2)\lambda^2 + (3(2m+1)/(m+2))^2}}{(2m+1)^2/(m+2)} \right\}^2 \right], \\ \eta_2 = & -\frac{3m}{m+2} \left[\sqrt{\frac{3}{2}} \lambda \left\{ \frac{-((2m+1)/(m+2))\sqrt{3/2}\lambda + \sqrt{(3/2)\lambda^2 + (3(2m+1)/(m+2))^2}}{(2m+1)} \right\} + \frac{1}{2(m+2)} \right. \\ & \left. \times \left\{ -9 + \left(\frac{m+2}{2m+1} \right)^2 \left[-\left(\frac{2m+1}{m+2} \right) \sqrt{\frac{3}{2}} \lambda + \sqrt{\frac{3}{2} \lambda^2 + \left(\frac{3(2m+1)}{m+2} \right)^2} \right]^2 \right\} \right]. \end{aligned} \quad (43)$$

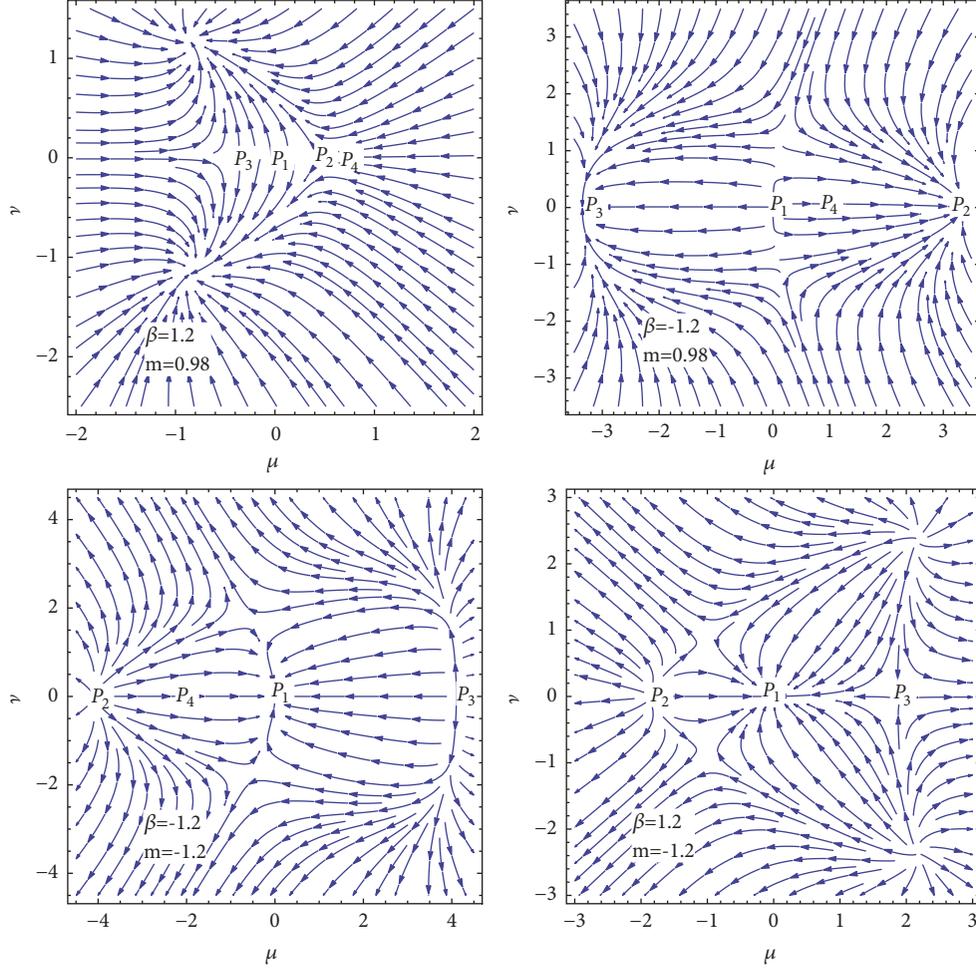
In this case, we examine unstable nodes for $m < 0$ by varying γ that shows decelerated expanding universe except for $m =$

$-0.5, -2$ (Figure 3). On the other hand, the positive values of m give accelerated expansion of the universe.

For $P_2 = ((-(2m+1)/(m+2))\sqrt{3/2}\lambda - \sqrt{(3/2)\lambda^2 + (3(2m+1)/(m+2))^2})/((2m+1)^2/(m+2)), 0)$, the corresponding eigenvalues are given as

$$\begin{aligned} \eta_1 = & \frac{9m}{m+2} \left[\frac{1-2m}{2(m+2)} \right. \\ & + \frac{\gamma}{2(1-\gamma)} \left\{ \frac{(2m+1)^4}{(m+2)^2 \left\{ -((2m+1)/(m+2))\sqrt{3/2}\lambda - \sqrt{(3/2)\lambda^2 + (3(2m+1)/(m+2))^2} \right\}^2} - \frac{2m+1}{3} \right\} + \frac{\gamma}{1+\gamma} \\ & \left. - \frac{1}{2(m+2)} \left\{ \frac{-((2m+1)/(m+2))\sqrt{3/2}\lambda - \sqrt{(3/2)\lambda^2 + (3(2m+1)/(m+2))^2}}{(2m+1)^2/(m+2)} \right\}^2 \right], \end{aligned} \quad (44)$$

$$\eta_2 = -\frac{3m}{m+2} \left[\sqrt{\frac{3}{2}} \lambda \left\{ \frac{-((2m+1)/(m+2))\sqrt{3/2}\lambda - \sqrt{(3/2)\lambda^2 + (3(2m+1)/(m+2))^2}}{(2m+1)} \right\} + \frac{1}{2(m+2)} \right. \\ \left. \times \left\{ -9 + \left(\frac{m+2}{2m+1} \right)^2 \left[-\left(\frac{2m+1}{m+2} \right) \sqrt{\frac{3}{2}} \lambda - \sqrt{\frac{3}{2} \lambda^2 + \left(\frac{3(2m+1)}{m+2} \right)^2} \right]^2 \right\} \right]. \quad (45)$$


 FIGURE 2: Plots for phantom coupled cosmic model with $Q = \beta\dot{\sigma}_\phi$ and $\lambda = 1.5$.

$$\eta_2 = -\frac{3m}{m+2} \left[\sqrt{\frac{3}{2}}\lambda \left\{ \frac{-((2m+1)/(m+2))\sqrt{3/2}\lambda - \sqrt{(3/2)\lambda^2 + (3(2m+1)/(m+2))^2}}{(2m+1)} \right\} \right. \\ \left. + \frac{1}{m+2} \left\{ -9 + \frac{(m+2)^2}{m+2} \left[-\left(\frac{2m+1}{m+2}\right)\frac{3}{2}\lambda - \sqrt{\frac{3}{2}\lambda^2 + \left(\frac{3(2m+1)}{m+2}\right)^2} \right]^2 \right\} \right]. \quad (46)$$

This point corresponds to either stable or unstable nodes for $m > 0$ or $m < 0$, respectively. It is found that P_2 lies in the matter dominated phase for all choices of the parameters. For

$P_3 = \left(\frac{((\sqrt{m+2})/(2m+1))\sqrt{\gamma(2m+1)/2(1-\gamma) - 3/(1+\gamma) + 9/2(m+2) + \xi}, 0 \right)$, the eigenvalues are

$$\eta_1 = \frac{9m}{m+2} \left[\frac{2(2m+1)^2\gamma(1+\gamma)}{2\gamma(m+2) + 4(m+1)(1-\gamma^2)(\xi-3) + 9} + \frac{1}{2(1+\gamma)} + \frac{5\gamma(2m+1)}{12(1-\gamma)} - \frac{3}{4(m+2)} - \frac{\xi}{2} \right], \quad (47)$$

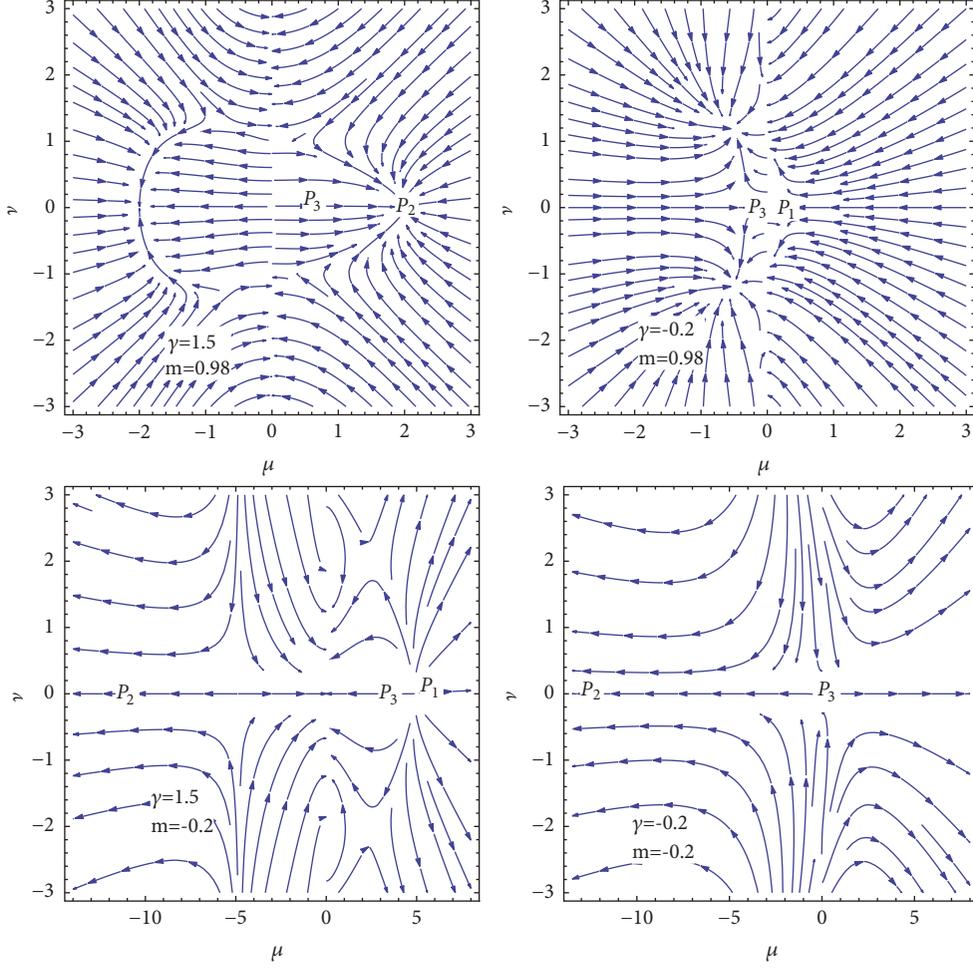


FIGURE 3: Plots for phantom coupled cosmic model with $Q = \gamma(\dot{\sigma}_m + \dot{\sigma}_\phi)$ and $\lambda = 1.5$.

$$\eta_2 = -\frac{3m}{m+2} \left[\sqrt{\frac{3}{2(m+2)} \left(\frac{\gamma(2m+1)}{2(1-\gamma)} - \frac{3}{1+\gamma} + \frac{9}{2(m+2)} + \xi \right)} \right] \quad (48)$$

$$+ \frac{1}{2(m+2)} \left\{ -9 + (m+2) \left(\frac{\gamma(2m+1)}{2(1-\gamma)} - \frac{3}{1+\gamma} + \frac{9}{2(m+2)} + \xi \right) \right\} \right], \quad (49)$$

where

$$\xi = \sqrt{\frac{3}{1+\gamma} - \frac{\gamma(2m+1)}{2(1-\gamma)} - \frac{9}{2(m+2)} - \frac{3(2m+1)^2 \gamma}{(m+2)(1-\gamma)}} \quad (50)$$

The behavior of eigenvalues as well as trajectories indicate that point P_3 is a saddle node for all choices of the parameters. The summary of the above results is provided in Table 3.

4. Coupled Tachyon Dynamics

Now we study the evolution of LRS BI universe by considering tachyon coupled cosmic fluid. The conservation equations are

$$\dot{\sigma}_m + 3(\sigma_m + p_m)H = Q, \quad (51)$$

$$\dot{\sigma}_\phi + 3(\sigma_\phi + p_\phi)H = -Q, \quad (52)$$

where $\sigma_\phi = V(\phi)/\sqrt{1-\dot{\phi}^2}$ and $p_\phi = -V(\phi)\sqrt{1-\dot{\phi}^2}$. In this case, the evolution equations take the form

TABLE 3: Phase Space Analysis for $Q = \gamma(\dot{\sigma}_m + \dot{\sigma}_\phi)$.

Ranges of m and γ for Critical Points	Stability	ω_ϕ	q	Acceleration
P_1				
$m = 0.98, \gamma = 1.5, (\gamma \neq 1)$	Stable	-14.18	-2.16	Yes
$m = 0.98, \gamma = -0.2, (\gamma \neq -1)$	Saddle	-14.18	-2.16	Yes
$m = -0.2, \gamma = -0.2, (m \neq -0.5, -2)$	Unstable	0.128	4.86	No
$m = -0.2, \gamma = 1.5$	Unstable	0.128	4.86	No
P_2				
$m = 0.98, \gamma = 1.5, (\gamma \neq 1)$	Stable	2.06	4.22	No
$m = 0.98, \gamma = -0.2, (\gamma \neq -1)$	Stable	2.06	4.22	No
$m = -0.2, \gamma = -0.2, (m \neq -0.5, -2)$	Unstable	0.179	26.46	No
$m = -0.2, \gamma = 1.5$	Unstable	0.179	26.46	No
P_3				
$m = 0.98, \gamma = 1.5, (\gamma \neq 1)$	Saddle	57.94	-2.62	Yes
$m = 0.98, \gamma = -0.2, (\gamma \neq -1)$	Saddle	57.94	-2.62	Yes
$m = -0.2, \gamma = -0.2, (m \neq -2)$	Saddle	-1.504	-3.503	Yes
$m = -0.2, \gamma = 1.5$	Saddle	-1.504	-3.503	Yes

$$H^2 = \frac{(m+2)^2}{9(2m+1)} \left[\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} + \sigma_m \right], \quad (53)$$

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} = - \left[3H\dot{\phi} + \frac{V'(\phi)}{V(\phi)} + \frac{Q\sqrt{1-\dot{\phi}^2}}{V(\phi)\dot{\phi}} \right]. \quad (54)$$

Here, we define the following dimensionless quantities:

$$\begin{aligned} \mu &= \frac{(m+2)\dot{\phi}}{2m+1}, \\ \nu &= \frac{(m+2)\sqrt{V}}{\sqrt{3}(2m+1)H}, \\ \lambda &= -\frac{V'}{V\sqrt{V}}, \end{aligned} \quad (55)$$

such that the dynamical system yields

$$\mu' = \frac{3m}{2m+1} \frac{\ddot{\phi}\mu}{H\dot{\phi}}, \quad (56)$$

$$\nu' = -\frac{3\sqrt{3}m(2m+1)^2\lambda\mu\nu^2}{2(m+2)^3} - \frac{3m\nu}{m+2} \frac{\dot{H}}{H^2}. \quad (57)$$

We consider inverse square potential with constant parameter λ . In this case, we take the only coupling $Q = \beta\dot{\sigma}_\phi$ through which (53) and (54) lead to

$$\frac{\dot{H}}{H^2} = \frac{(2m+1)^2}{2(m+2)} \nu^2 \sqrt{1 - \left(\frac{2m+1}{m+2}\right)^2 \mu^2} - \frac{9}{2(m+2)}, \quad (58)$$

$$\frac{\ddot{\phi}}{\dot{\phi}H} = \left[1 - \left(\frac{2m+1}{m+2}\right)^2 \mu^2 \right] \left[\frac{\sqrt{3}\nu\lambda}{\mu} + \frac{3\beta}{1+\beta} - 3 \right]. \quad (59)$$

The effective EoS and deceleration parameters can be written as

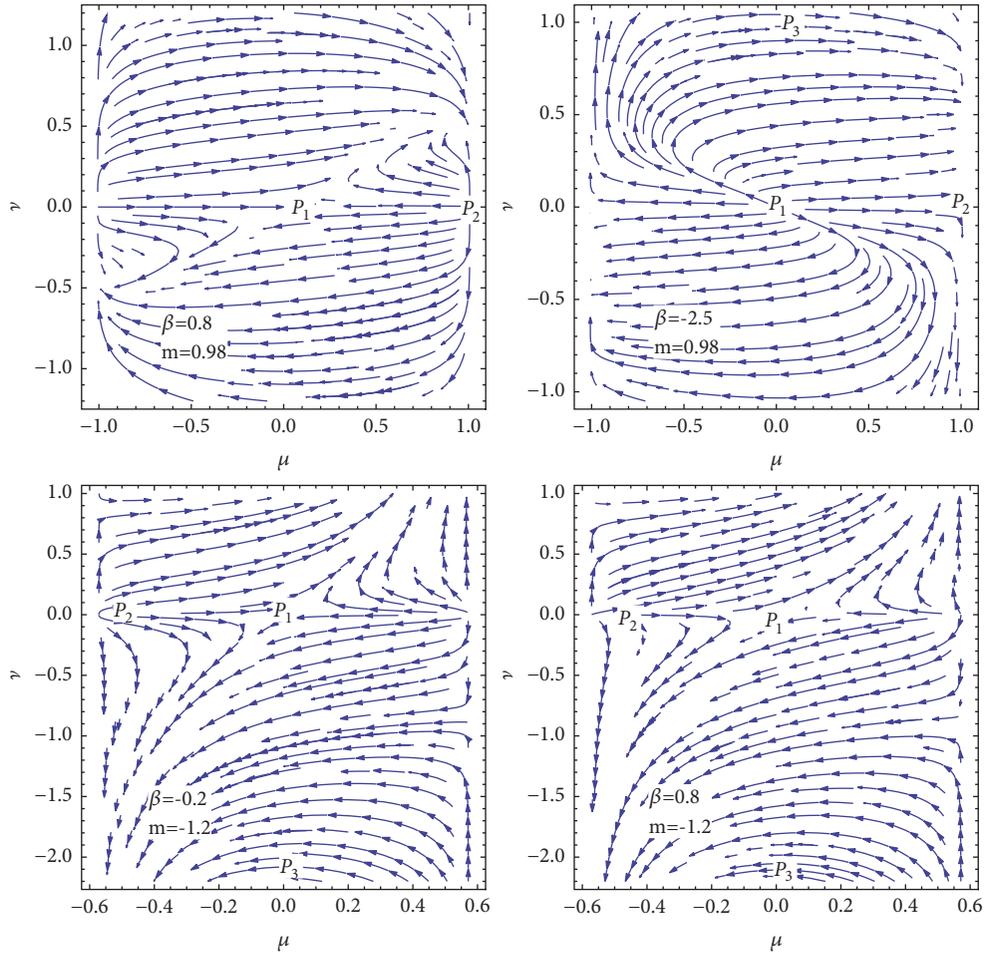
$$w_{eff} = -1 - \frac{1}{3(m+2)} \left[(2m+1)^2 \nu^2 \sqrt{1 - \left(\frac{2m+1}{m+2}\right)^2 \mu^2} - 9 \right], \quad (60)$$

$$q = -1 - \frac{1}{2(m+2)} \left[(2m+1)^2 \nu^2 \sqrt{1 - \left(\frac{2m+1}{m+2}\right)^2 \mu^2} - 9 \right]. \quad (61)$$

For $P_1 = (0, 0)$, we find the eigenvalues as

$$\begin{aligned} \eta_1 &= -\frac{9m}{(2m+1)(1+\beta)}, \\ \eta_2 &= \frac{27m}{2(m+2)^2}. \end{aligned} \quad (62)$$

We find that point P_1 is saddle/unstable for $m >$ which becomes stable for negative values of m ($m \neq -0.5, \beta \neq -1$) undergoing accelerated expansion as $q < 0$ (Figure 4). We provide the obtained results in Table 4.

FIGURE 4: Plots for tachyon coupled cosmic model with $Q = \beta\dot{\sigma}_\phi$ and $\lambda = 2$.TABLE 4: Phase Space Analysis of Tachyon Coupled System with $Q = \beta\dot{\sigma}_\phi$.

Ranges of m and β for Critical Points	Stability	ω_ϕ	q	Acceleration
P_1				
$m = 0.98, \beta = 0.8$	Saddle	-2.006	-2.51	Yes
$m = 0.98, \beta = -0.2, (\beta \neq -1)$	Unstable	-2.006	-2.51	Yes
$m = -1.2, \beta = -0.2, (m \neq -0.5)$	Stable	-4.75	-6.625	Yes
$m = -1.2, \beta = 0.8$	Stable	-4.75	-6.625	Yes
P_2				
$m = 0.98, \beta = 0.8$	Unstable	-2.006	-2.51	Yes
$m = 0.98, \beta = -0.2, (\beta \neq -1)$	Saddle	-2.006	-2.51	Yes
$m = -1.2, \beta = -0.2, (m \neq -0.5)$	Saddle	-4.75	-6.625	Yes
$m = -1.2, \beta = 0.8$	Saddle	-4.75	-6.625	Yes
P_3				
$m = 0.98, \beta = 0.8$	Stable	-1	-1	Yes
$m = 0.98, \beta = -0.2, (\beta \neq -1)$	Saddle	-1	-1	Yes
$m = -1.2, \beta = -0.2, (m \neq -0.5)$	Saddle	-1	-1	Yes
$m = -1.2, \beta = 0.8$	Saddle	-1	-1	Yes

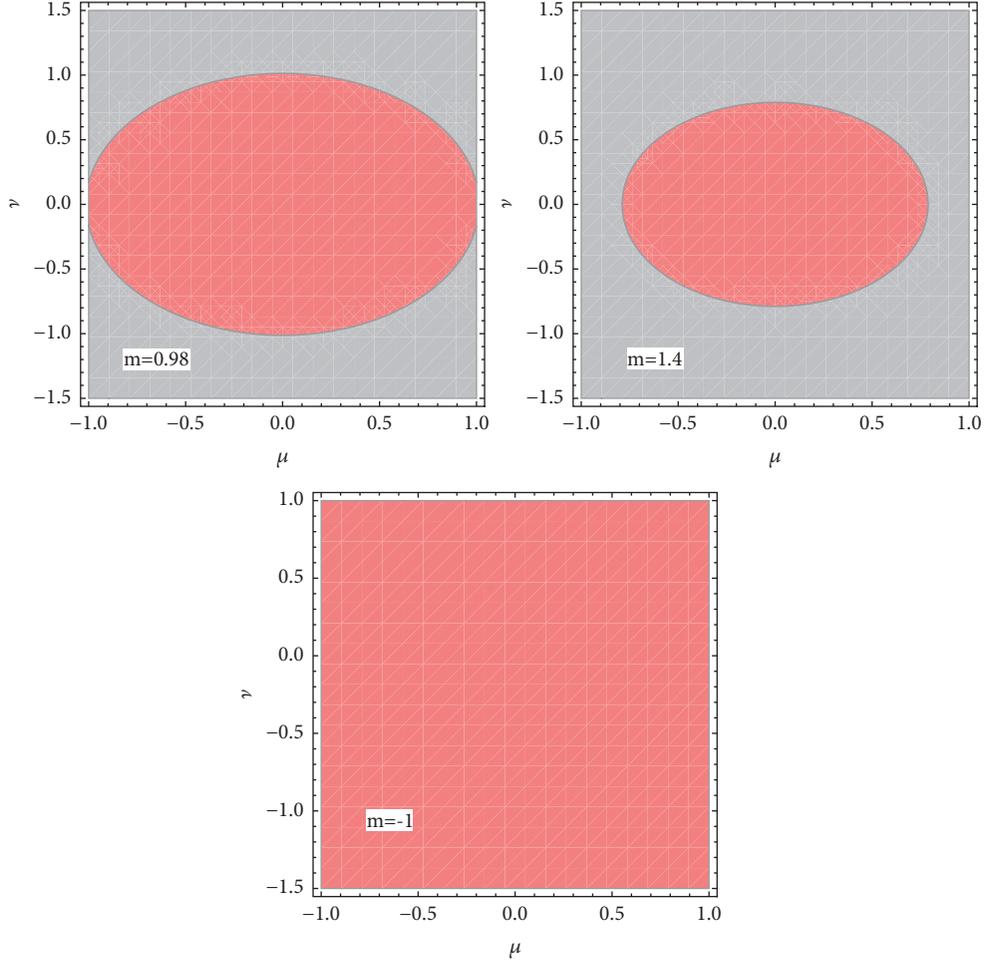


FIGURE 5: Plots for power-law scale factor coupled with phantom fluid. Blue and gray zones represent contraction and accelerated expansion of cosmos, respectively.

For $P_2 = (\pm(m+2)/(2m+1), 0)$, the eigenvalues become

$$\begin{aligned} \eta_1 &= \frac{18m}{(2m+1)(1+\beta)}, \\ \eta_2 &= \frac{27m}{2(m+2)^2}, \end{aligned} \quad (63)$$

which correspond to unstable nodes for $m, \beta > 0$ lying in accelerated expanding phase of the universe model. We have saddle nodes for the remaining choices of parameters m and β . The cosmic portrait indicates accelerated expansion phase of the universe for all choices of the parameters. When $P_3 = (0, \pm 3/(2m+1))$, the eigenvalues yield

$$\begin{aligned} \eta_1 &= -\frac{9m}{(2m+1)(1+\beta)}, \\ \eta_2 &= -\frac{27m}{(m+2)^2}. \end{aligned} \quad (64)$$

This point is a stable future attractor for $\beta > 0$ and $m > 0$ undergoing accelerated expansion of the cosmic model. We

find that the respective point corresponds to saddle nodes for all other choices of parameters except at $m = -0.5, \beta = 1$. This point indicates de Sitter ($q = -1, w_{eff} = -1$) phase of cosmos.

5. Power-Law Scale Factor

Here we explore the power-law behavior of the scale factor by applying some assumptions to both the phantom and tachyon coupled fields. In this context, (12) becomes

$$\dot{\Theta} = \frac{1}{6(m+2)} [-9 + (2m+1)^2(\mu^2 + \nu^2)] \Theta^2, \quad (65)$$

where $\Theta = 3H$ is the expansion scalar. For $\Theta \neq 0$, we formulate power-law scale factor only if $-9 + (2m+1)^2(\mu^2 + \nu^2) \neq 0$. We obtain the respective generic critical point by solving $\Theta = \dot{a}/a + 2\dot{b}/b$ for $a(t)$ and $b(t)$ as

$$b^{(m+2)} = b_0^{(m+2)} (t - t_0)^{-6(m+2)/(-9+(2m+1)^2(\mu^2+\nu^2))}. \quad (66)$$

It is noticed that behavior of the term “ $-9 + (2m+1)^2(\mu^2 + \nu^2)$ ” is quite important to assess different cosmological phases.

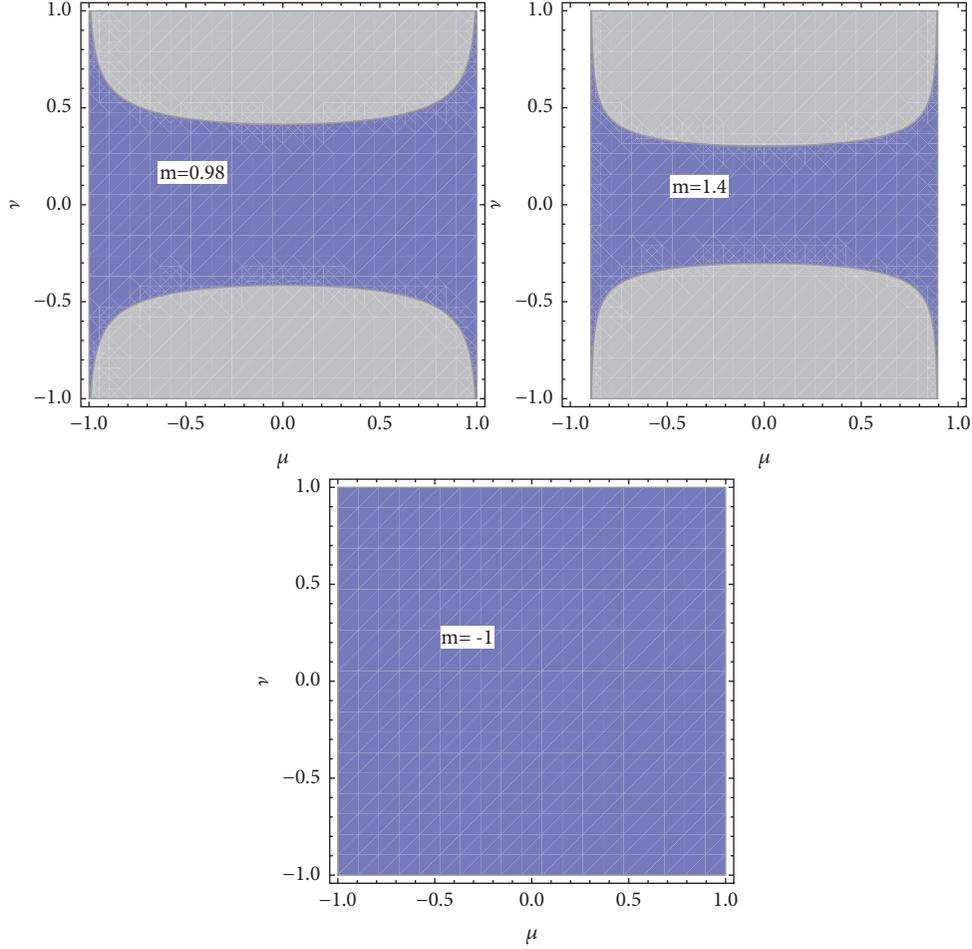


FIGURE 6: Plots of qualitative phase space analysis for power-law scale factor with tachyon coupled matter.

It shows exponential cosmic expansion for $-9 + (2m + 1)^2(\mu^2 + \nu^2) = 0$ while $-9 + (2m + 1)^2(\mu^2 + \nu^2) \geq 0$ gives accelerated cosmic expansion or contraction, respectively. Figure 5 depicts different states for power-law scale factor, where red and gray regions show contraction and accelerated expansion of cosmic model, respectively. It is found that decelerated expansion tends to increase by increasing m whereas $m < 0$ undergoes decelerated expansion.

For tachyon coupled fluid, (58) becomes

$$\dot{\Theta} = \frac{1}{6(m+2)} \left[(2m+1)^2 \nu^2 \sqrt{1 - \left(\frac{2m+1}{m+2}\right)^2 \mu^2} - \frac{9}{2(m+2)} \right] \Theta^2. \quad (67)$$

For $\Theta \neq 0$, we again find power-law scale factor if $(2m + 1)^2 \nu^2 \sqrt{1 - ((2m + 1)/(m + 2))^2 \mu^2} - 9/2(m + 2) \neq 0$. The

generic critical point is obtained through $\Theta = \dot{a}/a + 2\dot{b}/b$ as

$$b^{(m+2)} = b_0^{(m+2)} (t - t_0)^{-6(m+2)/((2m+1)^2 \nu^2 \sqrt{1 - ((2m+1)/(m+2))^2 \mu^2} - 9/2(m+2))}. \quad (68)$$

We examine various cosmological states according to $(2m + 1)^2 \nu^2 \sqrt{1 - ((2m + 1)/(m + 2))^2 \mu^2} - 9/2(m + 2) \geq 0$. Figure 6 corresponds to the different cosmological phases, where blue and gray zones indicate contraction as well as accelerated expansion of cosmos, respectively. It is found that the region for decelerated expansion decreases by increasing m whereas $m < 0$ corresponds to contraction of the cosmic model (Figure 6).

6. Summary

This work is devoted to discuss stability of LRS BI universe model by taking scalar field coupled fluid. An autonomous system of equations has been developed by introducing various dimensionless parameters. We have obtained the corresponding critical points with different choices of the

parameters. The eigenvalues have been formulated which characterize the obtained critical points. We summarize the results of the dynamical analysis as follows.

Firstly, we have studied stability of the universe model dominated by the coupling of phantom energy and DM for different values of m (Figures 1–3). We have considered three different linear combinations for the coupling parameter. For $Q = \alpha\dot{\sigma}_m$ and (P_1) , we have observed stable DE dominated state with $m > 0$ and different choices of α (Figure 1). This analysis indicates a deceleration phase eventually followed by accelerated expansion of the cosmos for point P_3 . It is observed that point P_2 lies in nonaccelerating phase of the universe as $q > 0$ for $\alpha > 0$ which may alleviate coincidence problem as compared to [25, 28]. For $m > 0$, all the points show stable future attractors except P_3 and P_4 which shows saddle nodes depending on the choices of α .

For $Q = \beta\dot{\sigma}_\phi$, all the eigenvalues and trajectories show saddle nodes for positive values of m, β (except P_4) which become stable/unstable for $m < 0$ corresponding to different choices of β (Figure 2). It is noted that P_1 and P_4 undergo accelerated expansion for all choices of the parameters. For the coupling $Q = \gamma(\dot{\sigma}_m + \dot{\sigma}_\phi)$, we have found P_1 as stable attractor for positive values of m showing accelerated cosmic expansion. When $m < 0$, unstable node is observed for both P_1 and P_2 undergoing decelerated expansion (Figure 3). In this case, point P_3 shows saddle node in accelerating phases. For the coupling $Q = \beta\dot{\sigma}_\phi$, P_3 is the only point that indicates accelerated expanding universe for all the chosen values of parameters.

Secondly, we have discussed dynamics of the universe model by considering tachyon coupled field. In this case, we consider $Q = \beta\dot{\sigma}_\phi$ only. The cosmic portrait indicates accelerated expansion of the universe model (Figure 4). For $m > 0$, points P_1 and P_2 behave as unstable/saddle nodes whereas P_3 provides stable future attractor in accelerated expansion scenario. For $m < 0$, the point P_1 is stable showing accelerated expansion of cosmos while the remaining points act as saddle nodes that correspond to de Sitter cosmic phase. We conclude that tachyon coupled field yields an era of accelerated expansion for all choices of parameters.

Finally, we have investigated the characteristics of power-law scale factor by varying m . It indicates various phases of evolution (accelerated or exponential expansion) for the respective cosmic model as shown (Figures 5 and 6). In case of phantom coupled matter, the region of decelerated expansion increases by increasing m while $m < 0$ corresponds to deceleration phase only. For tachyon coupled field, the expansion rate increases by increasing m which directly affects the contraction rate such that blue region gets smaller. Also, the negative values of parameter m shows only decelerated expansion of the cosmic model.

Data Availability

No data were used to support this study.

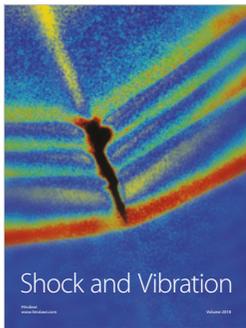
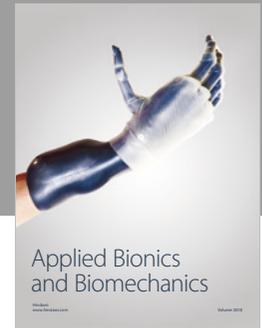
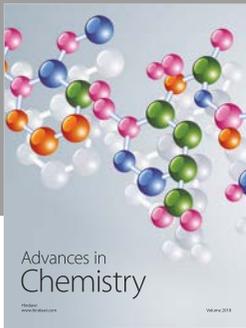
Conflicts of Interest

The authors declare that they have no conflicts of interest.

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