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Research Article

$f(R)$ Gravity Effects on Charged Accelerating AdS Black Holes Using Holographic Tools

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We investigate numerically $f(R)$ gravity effects on certain AdS/CFT tools including holographic entanglement entropy and two-point correlation functions for a charged single accelerated Anti-de Sitter black hole in four dimensions. We find that both holographic entanglement entropy and two-point correlation functions decrease by increasing the acceleration parameter A, matching perfectly with literature. Taking into account the $f(R)$ gravity parameter η, the decreasing scheme of the holographic quantities persist. However, we observe a transition-like point where the behavior of the holographic tools changes. Two regions meeting at such a transit-like point are shown up. In such a nomination, the first one is associated with slow accelerating black holes while the second one corresponds to a fast accelerating solution. In the first region, the holographic entanglement entropy and two-point correlation functions decrease by increasing the η parameter. However, the behavioral situation is reversed in the second one. Moreover, a cross-comparison between the entropy and the holographic entanglement entropy is presented, providing another counterexample showing that such two quantities do not exhibit similar behaviors.

1. Introduction

Anti-de Sitter/Conformal field theory (AdS/CFT) conjecture has brought new roads to understand the quantum gravity [1–5]. Motivated by the holographic entanglement entropy, an intrinsic potential interplay between quantum information theory and gravity physics has been established [6, 7].

The entanglement entropy, which may be viewed as the von-Neumann entropy reflecting the entanglement between two different subsystems A and B, can be applied to the black hole. It has been considered also as a candidate not only for the statistical origin of the black hole entropy [8] but also for solving the information puzzle in black hole physics [9]. It is worth noting that the entanglement entropy acquires a similar geometric description to the bulk thermal entropy using holography [10–12]. Unfortunately, many attempts that have been made to explain the BH entropy in terms of quantum entanglement suffer from conceptual and technical difficulties. Indeed, the first difficulty is associated with the usual statistical interpretation of the black hole entropy in terms of microstates. This is conceptually very different from the entanglement entropy measuring the observer's lack of information by regarding the system as quantum states in an inaccessible region of the spacetime [13, 14]. The second one comes from the fact that the entanglement entropy depends on the number of species of matter fields. The associated entanglement should reproduce the BH entropy. It depends also on the value of the ultraviolet (UV) cut-off, which may be needed to regularize the divergences corresponding to sharp boundaries between the accessible and inaccessible regions of spacetime. However, the Hawking-Bekenstein entropy should be universal. In the context of the extended phase space, it has been shown that the entanglement entropy undergoes qualitatively a similar behavior as entropy [15]. In paralleled ways, many investigations have been devoted to such a similitude [16–19]. However, it has been revealed that behavioral coincidence is associated with the first law of black hole thermodynamics. This can also be supported by the fact the entanglement entropy depends on the black hole mass [20, 21]. It turns out that the holographic entanglement entropy is not dual to the black hole one. It should not be expected that they present the same behavior as observed in [20, 21].

On the other hand, the inflation theory and dark energy can be exploited to build Einstein modified gravity. In this way, $f(R)$ gravity can be considered a good example to approach such modified theories. Adding $f(R)$ as higher powers of R, one can implement the Ricci and Riemann tensors and their derivatives in Lagrangians, which could describe possible physical models [22–31]. In this context, considerable efforts have been made to investigate the AdS black hole thermodynamics in the background of $f(R)$ gravity with constant curvature [32–34] as in the context of the AdS/CFT conjecture [34–36]. Recently, several results have been elaborated and refined to support such investigation lines, including accelerating black holes. The latters, being known to be described by the so-called C-metric [37–40], have been used not only to study the pair creation of black holes [41], the splitting of cosmic strings [42, 43], but also to construct the black rings in five-dimensional gravity, motivated by string theory and related inspired models [44]. The thermodynamics of accelerating black holes [45], generalized results to the case of varying conical deficits for C-metric, and their applications to holographic heat engines, have been studied in many places, see for instance [45–50].

The aim of this paper is to contribute to these activities by investigating numerically the effect of the $f(R)$ gravity on certain AdS/CFT tools including holographic entanglement entropy and two-point correlation functions for a charged single accelerated Anti-de Sitter black hole in four dimensions. It has been shown that both holographic entanglement entropy and two-point correlation functions decrease by increasing the acceleration parameter A, matching perfectly with the literature [51, 52]. Taking into account the $f(R)$ gravity parameter η, the decreasing scheme of the holographic quantities persist. However, it has been found a critical-like point where the behavior of the holographic tools changes. Two possible regions intersecting at such a transition-like point are observed. The first one corresponds to slow accelerating black holes while the second one is associated with a fast accelerating solution. In the first region, the holographic entanglement entropy and two-point correlation functions decrease by increasing the η parameter. However, the second one corresponds to a reversed behavioral situation.

The organization of the paper is as follows. We first review, in Section 2, the essential of the solution describing charged single accelerated AdS Black holes in $f(R)$ gravity background and the associated relevant thermodynamics. In Section 3 and Section 4, we, numerically, study the entanglement entropy and two-point correlation functions and we prob the effect of the $f(R)$ gravity parameter η on such holographic quantities. The last section is devoted to the conclusion and some open questions.

2. Charged Accelerating $f(R)$ Black Hole Background

To start, one reconsiders the study of the physical content of $f(R)$ gravity. According to [53], its action containing a Maxwell gauge term takes the following general form

$$\mathcal{S} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\mathcal{L}. \tag{1}$$

In this action, $\mathcal{L}$ can be expressed as

$$\mathcal{L} = R + f(R) - F_{ab}F^{ab}, \tag{2}$$

where R is the Ricci scalar. It is noted that $f(R)$ is an auxiliary function of R, which determines the gravity model [54, 55]. The field strength $F_{ab} = \nabla_a B_b - \nabla_b B_a$ is derived from the gauge potential one form B_a. Using field theory technics associated with the variation with respect to the metric, and the gauge field, one can get field equations of motion. Roughly, they are listed as follows

$$\begin{aligned} R_{ab} - \frac{1}{2}Rg_{ab} - \frac{1}{2}f(R)g_{ab} + R_{ab}f'(R) - \nabla_b\nabla_a Rf''(R) \\ + g_{ab}\nabla_c\nabla^c Rf''(R) - \nabla_a R\nabla_b Rf^{(3)}(R) + g_{ab}\nabla_c R\nabla^c Rf^{(3)}(R) = 2T_{ab}, \end{aligned} \tag{3}$$

and

$$\nabla_b\nabla^b B^a - \nabla_b\nabla^a B^b = 0, \tag{4}$$

where one has used

$$f'(R) = \frac{df(R)}{dR}, \tag{5}$$

$$T_{ab} = F_a^c F_{bc} - \frac{1}{4}F_{cd}F^{cd}g_{ab}. \tag{6}$$

One can check that the electromagnetic field T_{ab} is traceless

$$T_a^a = 0. \tag{7}$$

It turns out that a maximally symmetry solution can be obtained by considering a constant Ricci scalar [56]. In the situation given by $R = R_0 (R_0 \neq 0)$, the metric field equation reduces to [32, 57].

$$R_0 - R_0 f'(R_0) + 2f(R_0) = 0. \tag{8}$$

In this way, Eq. (3) can be rewritten as follows

$$\eta R_{ab} - \frac{\eta}{4}R_0 g_{ab} = 2T_{ab}. \tag{9}$$

Here, η is defined as

$$\eta = 1 + f'(R_0). \tag{10}$$

Exploiting the results reported in [53] and using the equation of motion given by Eq. (9), we can derive a charged accelerating black hole solution based on the following line element

$$ds^2 = \frac{1}{\Omega^2}\left[-\frac{N(r)dt^2}{\alpha^2} + \frac{dr^2}{N(r)} + r^2\left(\frac{d\theta^2}{g(\theta)} + g(\theta)\sin^2\theta\frac{d\phi^2}{K^2}\right)\right]. \tag{11}$$

The terms involving such a line element are given by

$$\Omega = 1 + Ar\cos\theta, \tag{12}$$

$$N(r) = \left(1 - A^2r^2\right)\left(1 - \frac{2m}{r} + \frac{q^2}{\eta r^2}\right) - \frac{R_0 r^2}{12}, \tag{13}$$

$$g(\theta) = 1 + 2mA\cos\theta + \frac{q^2}{\eta}A^2\cos^2\theta, \tag{14}$$

$$\alpha = \sqrt{\Xi\left(1 + \frac{12A^2\Xi}{R_0}\right)}, \tag{15}$$

$$\Xi = 1 + \frac{q^2A^2}{\eta}. \tag{16}$$

In such a black hole solution, the conformal factor Ω denotes the conformal boundary r_b of the black hole given by $r_b = -1/(A\cos\theta)$. A, m, and q indicate individually the acceleration, the mass parameter, and the electric charge parameter of the black hole, respectively. K is the conical deficits of the black hole on the north and south poles. The present space-time involves conical singularities localized at $(\theta = 0, \pi)$. In this way, the metric regularity condition at the poles, $\theta_+ = 0$ and $\theta_- = \pi$ imposes that

$$K_\pm = g(\theta_\pm) = 1 \pm 2mA + \frac{q^2}{\eta}A^2. \tag{17}$$

Generally, K is explored to regularize one pole producing a conical deficit or a conical excess along the other pole. Due to a negative energy corresponding to the source of a conical excess, it has been supposed, throughout this study, that the black hole is regular on the north pole where $\theta = 0$ and $K = K_+$. It turns out that a conical deficit can exist on the other pole for $\theta = \pi$.

In order to get a normalized Killing vector at the conformal infinity [58–60], the parameter α can be exploited to rescale the time coordinate. When A goes to zero, it is observed that one can recover the usual charged AdS black hole in the $f(R)$ gravity [61].

It is noted that $R_0 < 0, R_0 = 0$ and $R_0 > 0$ are associated with asymptotically AdS, flat, and dS accelerating black holes, respectively. In the present investigation, we will be only interested in the case $R_0 < 0$. For $A = 0, K = 1$ and $R_0 < 0$, the solution reduces to the $f(R)$ black hole [32]. In this way, the condition $\eta > 0$ is needed to insure the existence of inner and outer horizons [57]. For certain physical reasons, we will only deal with the case $\eta > 0$ for the accelerating $f(R)$ AdS black holes. We hope other nontrivial cases could be dealt with in future works. Solving the equation of motion (4) for the gauge field B_a, one can obtain the electromagnetic tensor

$$F_{ab} = (dB)_{ab}. \tag{18}$$

The corresponding calculation gives

$$B_a = \frac{1}{\alpha}\left(\frac{q}{r_+} - \frac{q}{r}\right)(dt)_a, \tag{19}$$

where r_+ (the complete form of the r_+ is given in the appendix) is the outer horizon. It is worth noting that for

$$f'(R_0) = 0, R_0 = -\frac{12}{\ell^2}, \tag{20}$$

the solution coincides with the charged accelerating AdS black hole in the Einstein gravity framework. For $R_0 = -12/\ell^2$, the blackening factor of the black hole can be written as

$$N(r) = \left(1 - \frac{2m}{r} + \frac{q^2}{\eta r^2}\right)\left(1 - A^2r^2\right) + \frac{r^2}{\ell^2}, \tag{21}$$

where η and ℓ are considered as independent physical parameters.

3. Holographic Entanglement Entropy of Charged Accelerating Black Holes in $f(R)$ Gravity Background

In the present investigation, we ignore the acceleration horizons of C-metrics and consider only the black hole horizon in order to get a well-defined temperature. This kind of simplification is known as the "slowly accelerating C-metric", proposed in [62]. It has been remarked that various solutions can appear depending on the parameter A. For $A < 1/\ell$, it has been suggested that a single black hole can be built in the AdS geometry space with the only horizon being that of such a black hole. For $A > 1/\ell$, however, it involved two black holes separated by the acceleration horizon [62–64]. According to [65], a C-metric with a cosmological constant and charge in the Hong-Teo coordinates can be represented by the metric given in Eq. (11). The first law of thermodynamics for accelerating black holes with a varying conical deficit and a critical behavior has been investigated in many places including [45]. It has been extended to $f(R)$ gravity backgrounds in [53]. The mass M of the slowly accelerating $f(R)$ AdS black hole can be calculated as

$$M = \frac{\eta m\left(1 - A^2l^2\Xi\right)}{K\alpha}. \tag{22}$$

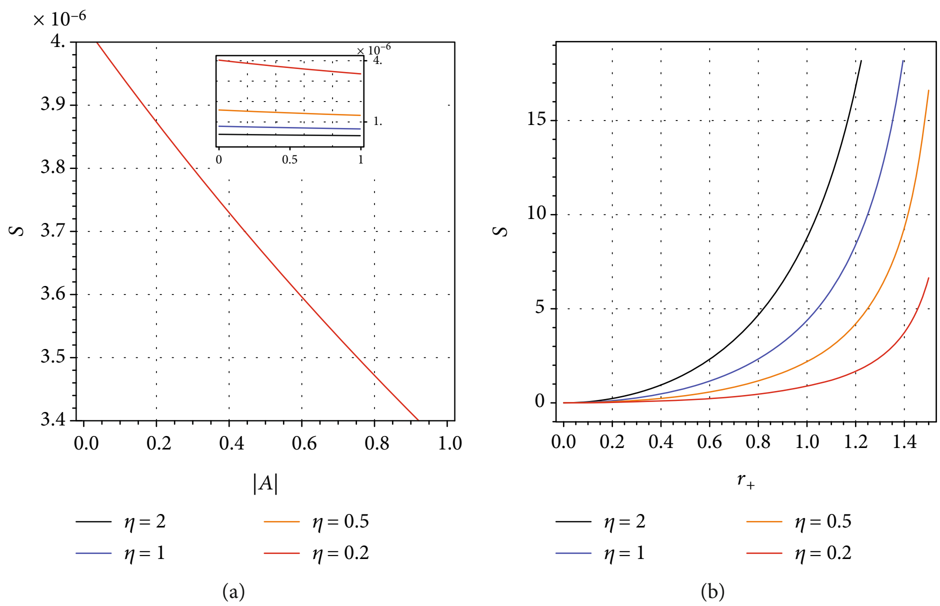

Figure 1: (a) The entropy of the charged accelerating AdS Black hole with respect to the acceleration parameter A. Here, we take $q = 0.01$, $m = 0.1$, $\ell = 1$, $\eta = 0.2$, and $r_+ = 0.190382$ given by Eq. (A.1). (b) The entropy of the charged accelerating AdS Black hole in terms of the horizon radius r_+ within different values of accelerating parameter A. Here, we take $q = 0.01$, $m = 0.1$, and $\ell = 1$.

While its charge is given by

$$Q = \frac{1}{4\pi} \lim_{\Omega \to 0} \int * F = \frac{1}{4\pi} \int \frac{q}{K} \sin\theta d\theta d\phi = \frac{q}{K}. \tag{23}$$

Roughly, the entropy of the black hole reads as

$$S = -2\pi \oint d^2 x \sqrt{\hat{h}} \frac{\partial \mathscr{L}}{\partial R_{abcd}} \varepsilon_{ab} \varepsilon_{cd} = \frac{\eta \pi r_+^2}{K\left(1 - A^2 r_+^2\right)}, \tag{24}$$

where $\hat{h}$ indicates the determinant of the induced metric on the $t =$ const. and $r = r_+$ hypersurface. The quantity ε_{ab} is a normal tensor verifying $\varepsilon_{ab}\varepsilon^{ab} = -2$. In Figure 1, we illustrate the variation of such an entropy quantity in terms of the accelerating parameter A and the horizon radius r_+.

The left side of Figure 1, associated with the fixed horizon radius, shows that the entropy presents a decreasing variation in terms of the accelerating parameter A and the $f(R)$ gravity parameter η. For a fixed acceleration parameter A (right side of Figure 1), it follows that the entropy grows up within the horizon radius r_+. Moreover, we observe that the entropy increases by increasing r_+ and blows up at $(Ar_+)^2 = 1$. Following the η parameter, the entropy grows as η increases.

Roughly, the temperature of the black hole is given by

$$\begin{aligned} T = \frac{N'(r_+)}{4\pi\alpha} &= \frac{\ell^2 \left(A^2 r_+^2 - 1\right)^2 \left(q^2 - \eta r_+^2\right) - \eta r_+^4 \left(3 - A^2 r_+^2\right)}{4\ell^2 \pi \alpha \eta r_+^3 \left(A^2 r_+^2 - 1\right)} \\ &= \frac{r_+}{2\pi\alpha\ell^2 \left(1 - A^2 r_+^2\right)} + \frac{\left(A^2 r_+^2 - 1\right)\left(q^2 - \eta m r_+\right)}{2\pi\alpha\eta r_+^3}. \end{aligned} \tag{25}$$

It has been quite well-known that the entanglement entropy (EE) is considered as a good approach to measure the amount of quantum information of a bipartite system. One way to quantify this information is to compute the von-Neumann entropy of a bipartite system where the system is divided into two parts. For this proposal, we first consider a time slice located at the AdS spacetime. Then, we calculate the area of the minimal surface γ_A. This can be parameterized by $r = r(\theta)$ given by a time slice $t = 0$ in the line-element appearing in Eq. (11). In this case, the entanglement entropy can be expressed as

$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G} = \frac{\mathscr{A}}{4G}, \tag{26}$$

where G is the Newton's constant [6, 7]. Using Eq. (26) and the metric presented in Eq. (11), the holographic entanglement entropy (HEE) for charged accelerated black holes in $f(R)$ gravity can be examined. The area, associated with the minimal hyper-surface in the presence of a charged accelerating AdS black hole in the bulk, is computed using the following relation

$$A = 2\pi \int_0^{\theta_0} \frac{\mathrm{r}(\theta) \sin\theta \sqrt{g(\theta)}}{\Omega^2 K} \sqrt{\frac{r'(\theta)^2}{N(r(\theta))} + \frac{r(\theta)^2}{g(\theta)}} d\theta. \tag{27}$$

In fact, this surface area should be minimized. A Lagrangian, associated with such an area, can be worked out to give the equation of motion. Indeed, the latter reads as

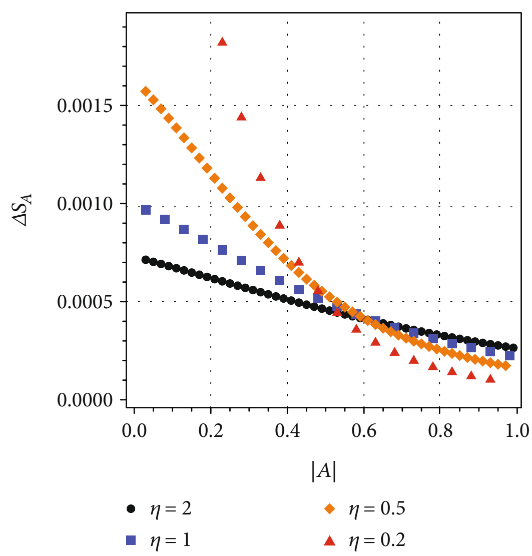

FIGURE 2: The holographic entanglement entropy for the charged accelerating AdS Black hole with respect to the acceleration parameter A. Here, we take $q = 0.01$, $m = 0.1$, $\ell = 1$, and $\theta_0 = 0.16$. The holographic entanglement entropy decreases with increasing acceleration exhibiting behaviour similar to the one observed in relativistic quantum information studies [51, 52].

$$\begin{aligned} & g(\theta)^2\Omega \sin\theta N'(r(\theta))r'(\theta)^3 + 2N(r(\theta))^2 r(\theta) \\ & \cdot \left[r'(\theta)\left(2g(\theta)\sin\theta r(\theta)\Omega' - \Omega\left[\sin\theta r(\theta)g'(\theta)\right.\right.\right. \\ & \left.\left. + g(\theta)\left(\cos\theta r(\theta) - 3\sin\theta r'(\theta)\right)\right]\right) - g(\theta)\Omega\sin\theta r \\ & \cdot \left.(\theta)r''(\theta)\right] + 4N(r(\theta))^3\Omega\sin\theta r(\theta)^3 N(r(\theta))g(\theta)r'(\theta) \\ & \cdot \left(2\Omega\sin\theta r(\theta)^2 N'(r(\theta)) - r'(\theta)^2\left[\Omega\sin\theta g'(\theta) - 4g\right.\right. \\ & \left.\left.\cdot(\theta)\sin\theta\Omega' + 2g(\theta)\Omega\cos\theta\right]\right) = 0. \end{aligned} \tag{28}$$

A priori, they are many ways to solve this equation. However, the analytic one seems to be hard. For such a reason, we could use numerical procedures to get $r(\theta)$. The Wolfram Mathematica program used to support the findings of this study is available from the corresponding author upon request.

In this way, the existence of the conformal factor Ω changes significantly the location of the AdS boundary from infinity $r = \infty$ to finite values $r = -1/A\cos\theta$ associated with vanishing Ω. Then, now all the boundary conditions are $r'(\theta) = 0$ and $r = r_0$ at $\theta = 0$ and $r = -1/(A\cos\theta_0)$ at $\theta = \theta_0$. To regularize the entanglement entropy, we follow the result of [66, 67]. Indeed, we subtract the area of the minimal surface in Rindler-AdS whose boundary is also $\theta = \theta_0$ with

$$\begin{aligned} r_{\mathrm{RAdS}}(\theta) = & \frac{\ell}{\sqrt{\cos^2\theta/\cos^2\theta_0 - 1}} + \frac{\ell^2\cos\theta}{\cos^2\theta - \cos^2\theta_0}A \\ & + \frac{\ell\cos\theta_0}{4(\cos^2\theta - \cos^2\theta_0)^{3/2}} \times \left[\left(4\ell^2 + 1\right)\left(2\cos^2\theta - 1\right)\right. \\ & \left. - \left(2\cos^2\theta_0 - 1\right)\left(2\cos^2\theta + 2\ell^2\right) + 6\ell^2 + 1\right]A^2 + O(A^3). \end{aligned} \tag{29}$$

Substituting $r(\theta)$ from Eq. (28) into Eq. (27) and with the help of Eq. (29), we can illustrate the holographic entanglement entropy with respect to the acceleration parameter A as shown in Figure 2, where $e = 0.01$, $m = 0.1$, $\ell = 1$, and $\theta_0 = 0.16$. The ultraviolet cutoff is chosen to be $\theta_c = 0.1599$.

It follows that the values of ΔS_A decrease as a function of the acceleration parameter when the charged accelerating AdS black hole is located in the bulk. At this level, one may give some discussing points. The first one concerns the similarity with the behaviors reported in the study of the entanglement entropy within the framework of the relativistic quantum information theory [51, 52]. The second point is that the variation of such a quantity is affected by the values of η. Two regions associated with such behaviors can appear. For the small values of the accelerating parameter A, one can note that the entanglement entropy is large as the value of η is small. While in the large values of A, the behavior changes contrarily with respect to the first region. These regions meet at a transition-like point, where all the graphs intersect.

Before going ahead, a brief comment on the behavior of the entropy given by Eq. (24), and the holographic entanglement entropy given by Eq.(26) is needed. Indeed, both quantities share the same behavior apparently. Indeed, they both decrease when the accelerating parameter A increase. For the η parameter, however, the transition point observed in the holographic picture is absent in the entropy one. Since the holographic entanglement entropy is not dual to the black hole entropy, they should not necessarily present the same behavior. This difference observed here commensurate with [20, 21], where the authors demonstrate that the Maxwell law observed in (T, S) plane is not verified in $(T, \Delta S_A)$ one. The apparently similitude, observed in this case, is due to the fact that HEE depends on mass directly.

4. Two-Point Correlation Function of Charged Accelerating Black Holes in $f(R)$ Gravity Background

Having investigated the behavior of the holographic entanglement entropy as a function of the accelerating parameter A and the parameter η, we move to discuss two-point correlation functions. In particular, we will show that such a local observable exhibits the same scheme found in the previous section. To show that, we first give a concise review of the Maldacena correspondence in order to introduce the time two-point correlation functions under the saddle-point approximation and in the large limit of Δ as [47].

$$\left\langle \mathcal{O}(t_0, x_i)\mathcal{O}(t_0, x_j)\right\rangle \approx e^{-\Delta L}. \tag{30}$$

It is noted that Δ stands for the conformal dimension of the scalar operator $\mathcal{O}$ in the dual field theory. L is the length of the bulk geodesic between the two points (t_0, x_i) and (t_0, x_j) on the AdS boundary. Using spacetime symmetry arguments of the associated black hole, we can redefine x_i as θ with the boundary θ_0 to parameterize the trajectory. In this way, the proper length takes the following form

$$L = \int_0^{\theta_0} \mathscr{L}(r(\theta), \theta) d\theta, \mathscr{L} = \frac{1}{\Omega^2}\sqrt{\frac{r'(\theta)^2}{N(r(\theta))} + \frac{r(\theta)^2}{g(\theta)}}, \quad (31)$$

where one has used $r' = dr/d\theta$. Treating $\mathscr{L}$ as Lagrangian and θ as time, the equation of motion associated with $r(\theta)$ reads as

$$\begin{aligned}&4r(\theta)^3 N(r(\theta))^2 \Omega'(r(\theta)) - 4g(\theta)N(r(\theta))\Omega(r(\theta))r'(\theta)^2 \\ &\quad - 2r(\theta)^2 N(r(\theta))^2 \Omega(r(\theta)) + r(\theta)\Big[r'(\theta)^2\Big(4g(\theta)N(r(\theta))\Omega' \\ &\quad \cdot (r(\theta)) - g(\theta)\Omega(r(\theta))N'(r(\theta))\Big) + N(r(\theta))g'(\theta)\Omega \\ &\quad \cdot (r(\theta))r'(\theta) + 2g(\theta)N(r(\theta))\Omega(r(\theta))r''(\theta)\Big] = 0.\end{aligned}$$

Using the same boundary conditions used in the previous section $r'(\theta) = 0$ and $r = r_0$ at $\theta = 0$ and $r = -1/(A\cos\theta_0)$ at $\theta = \theta_0$, we can solve such an equation by choosing the same values of θ_0 with the same UV cutoff in the dual field theory. Labeling the regularized two-point correlation functions as $\Delta L_A = L - L_0$, where L_0 is the geodesic length in Rindler-AdS under the same boundary region obtained with the help of Eq. (29), we present the variation of the function of ΔL_A in terms of the acceleration parameter A. The associated computation is illustrated in Figure 3. It is observed, from this figure, that the same behavior is held as the HEE picture.

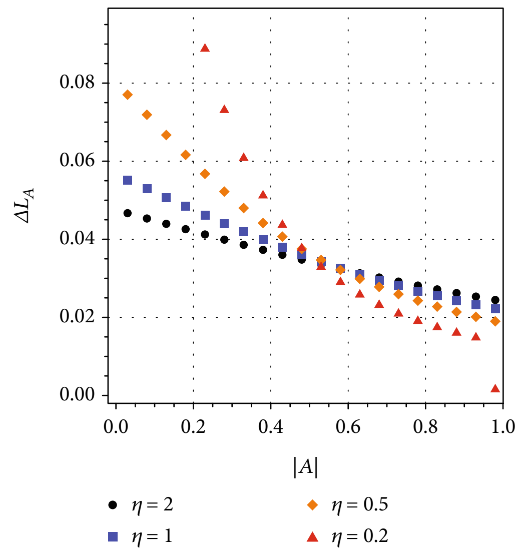

FIGURE 3: The two-point correlation functions for a charged accelerating AdS Black hole with respect to the acceleration parameter A. Here, we take $q = 0.01$, $m = 0.1$, $\ell = 1$, $\theta_0 = 0.16$ and $\theta_c = 0.1599$.

5. Discussions and Concluding Remarks

In this paper, the holographic tools including entanglement entropy and two-point correlation functions have been investigated for accelerating observers in four-dimensional $f(R)$ gravity backgrounds. In particular, we have considered a C-metric with a cosmological constant in the Hong-Teo coordinates to evaluate the minimal surface area associated with HEE and two-point correlation functions of a charged single accelerated AdS black hole in four dimensions.

Concretely, we have dealt with the acceleration parameter less than the inverse of the cosmological length ℓ, where a single black hole appears in the AdS geometry with the only horizon being that of a black hole. Due to the appearance of a communicating horizon, there is no possibility for a uniformly accelerated observer to get information about space-time behaviours. A loss of information takes place generating an entanglement degradation which implies that, in a space-time with horizons, one might expect some kinds of information loss puzzle [2, 54]. For such a reason, various activities have been made to figure out how the holographic entanglement entropy and two-point correlation functions behave as a function of the accelerating parameter A and the parameter η controlling $f(R)$ gravity effects. It has been observed that the holographic entanglement entropy and two-point correlation functions decrease by increasing the acceleration parameters of such black hole solutions. This indicates that the holographic entanglement entropy follows the universal behavior of entanglement entropy for accelerating observers [51, 52].

Moreover, we have investigated the effects of the $f(R)$ gravity parameter η on such nonlocal observers. Rigorously, the holographic quantities including HEE and two-point correlation functions exhibit nontrivial behaviors. It has been observed a transition point where the behavior of the holographic tools changes. We have found two regions intersecting at such a transition point. These two regions correspond to slow and fast accelerating black holes in our nomination, retrospectively. In the first region, HEE and two-point correlation functions decrease by increasing the η parameter. However, the second one is associated with a reversed behavioral situation. A comparison between the nonlocal observables and the entropy has been investigated. The absence of the transition point in the entropy picture provides a counterexample showing that both entropy and nonlocal observable exhibit a similar behavior.

This work comes up with some open questions. It will be interesting to extend the calculation for $A > 1/\ell$ with two black holes separated by an acceleration horizon. Motivated by string theory and related topics, higher dimensional solutions could be developed and investigated in the presence of nontrivial contributions including dark energy effects. Moreover, the thermodynamical behavior within the nonlocal observable quantities could be considered as interesting investigated approaches. We leave these questions for future works.

Appendix

The complete form of the event horizon radius is

$$\begin{aligned} r_+ &= \frac{1}{2}\sqrt{\frac{3a_2^2 - 8a_1a_3}{12a_1^2} + X + Y} \\ &\quad + \frac{1}{2}\sqrt{\frac{a_2^2}{2a_1^2} - \frac{8a_1^2a_4 - 4a_1a_2a_3 + a_2^3}{4a_1^3\sqrt{(3a_2^2 - 8a_1a_3/12a_1^2) + X + Y}} - \frac{4a_3}{3a_1} - X - Y} - \frac{a_2}{4a_1}, \end{aligned} \quad (A.1)$$

where

$$X = \frac{\sqrt[3]{2}\left(12a_1a_5 - 3a_2a_4 + a_3^2\right)}{3a_1Z},$$
$$Y = \frac{Z}{3\sqrt[3]{2}a_1},$$
$$Z = \left(\sqrt{\left(-72a_1a_3a_5 + 27a_1a_4^2 + 27a_2^2a_5 - 9a_2a_3a_4 + 2a_3^3\right)^2 - 4\left(12a_1a_5 - 3a_2a_4 + a_3^2\right)^3} - 72a_1a_3a_5 + 27a_1a_4^2 + 27a_2^2a_5 - 9a_2a_3a_4 + 2a_3^3\right)^{\frac{1}{3}}, \tag{A.2}$$

$$a_1 = \eta - A^2\eta\ell^2,$$
$$a_2 = 2A^2\eta m\ell^2,$$
$$a_3 = \eta\ell^2 - A^2q^2\ell^2, \tag{A.3}$$
$$a_4 = -2\eta m\ell^2,$$
$$a_5 = q^2\ell^2.$$

Data Availability

Since this work is theoretical, there is no data used to support the findings of this study. The Mathematica notebooks associated with this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that they have no conflicts of interest.

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References

[1] J. Maldacena, "The large N limit of superconformal field theories and supergravity," *International Journal of Theoretical Physics*, vol. 38, no. 4, pp. 1113–1133, 1999.

[2] H. Boyuan, X. Lianchao, and Z. Shanyou, "On the q -Deformation of Heisenberg Algebra," *Advances in Theoretical and Mathematical Physics*, vol. 29, no. 2, p. 231, 1998.

[3] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," *Physics Letters B*, vol. 428, no. 1-2, pp. 105–114, 1998.

[4] E. Witten, "Anti de Sitter space and holography," *Advances in Theoretical and Mathematical Physics*, vol. 2, no. 2, pp. 253–291, 1998.

[5] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, "Large *N* field theories, string theory and gravity," *Physics Reports*, vol. 323, no. 3-4, pp. 183–386, 2000.

[6] S. Ryu and T. Takayanagi, "Holographic derivation of entanglement entropy from the anti–de Sitter Space/Conformal Field Theory Correspondence," *Physical Review Letters*, vol. 96, no. 18, article 181602, 2006.

[7] S. Ryu and T. Takayanagi, "Aspects of holographic entanglement entropy," *Journal of High Energy Physics*, vol. 2006, no. 8, p. 45, 2006.

[8] A. Strominger and C. Vafa, "Microscopic origin of the Bekenstein-Hawking entropy," *Physics Letters B*, vol. 379, no. 1-4, pp. 99–104, 1996.

[9] A. Strominger, "The Black hole information puzzle: a progress report," in *PASCOS '94: Proceedings, 4th International Symposium on Particles, Strings and Cosmology*, pp. 428–432, Syracuse, New York, 1994.

[10] G. 't. Hooft, "On the quantum structure of a black hole," *Nuclear Physics B*, vol. 256, pp. 727–745, 1985.

[11] V. Frolov and I. Novikov, "Dynamical origin of the entropy of a black hole," *Physical Review D*, vol. 48, pp. 4545–4551, 1993.

[12] R. Emparan, "Black hole entropy as entanglement entropy: a holographic derivation," *Journal of High Energy Physics*, vol. 2006, no. 6, p. 12, 2006.

[13] M. Cadoni and M. Melis, "Entanglement entropy of AdS black holes," *Entropy*, vol. 12, no. 11, pp. 2244–2267, 2010.

[14] D. V. Fursaev, "Entanglement entropy in quantum gravity and the Plateau problem," *Physical Review D*, vol. 77, no. 12, article 124002, 2008.

[15] C. V. Johnson, "Large n phase transitions, finite volume, and entanglement entropy," *Journal of High Energy Physics*, vol. 2014, no. 3, p. 47, 2014.

[16] P. H. Nguyen, "An equal area law for holographic entanglement entropy of the AdSRN black hole," *Journal of High Energy Physics*, vol. 2015, no. 12, pp. 1–17, 2015.

[17] J.-X. Mo, "An alternative perspective to observe the critical phenomena of dilaton black holes," *The European Physical Journal C*, vol. 77, no. 8, p. 529, 2017.

[18] X.-X. Zeng and L.-F. Li, "Van der Waals phase transition in the framework of holography," *Physics Letters B*, vol. 764, pp. 100–108, 2017.

[19] H. El Moumni, "Revisiting the phase transition of AdS-Maxwell-power-Yang-Mills black holes via AdS/CFT tools," *Physics Letters B*, vol. 776, pp. 124–132, 2018.

[20] Y. Sun, H. Xu, and L. Zhao, "Thermodynamics and holographic entanglement entropy for spherical black holes in 5D Gauss-Bonnet gravity," *Journal of High Energy Physics*, vol. 2016, no. 9, p. 60, 2016.

[21] F. McCarthy, D. Kubizňák, and R. B. Mann, "Breakdown of the equal area law for holographic entanglement entropy," *Journal of High Energy Physics*, vol. 2017, no. 11, p. 165, 2017.

[22] S. Upadhyay, B. Pourhassan, and S. Capozziello, "Thermodynamics and phase transitions of galactic clustering in higher-order Modified Gravity," *International Journal of Modern Physics D*, vol. 28, no. 1, article 1950027, 2019.

[23] M. Khurshudyan, B. Pourhassan, and A. Pasqua, "Higher derivative corrections off(R) gravity with varying equation of state in the case of variable G and Λ," *Canadian Journal of Physics*, vol. 93, no. 4, pp. 449–455, 2015.

[24] S. Capozziello, M. Faizal, M. Hameeda, B. Pourhassan, V. Salzano, and S. Upadhyay, "Clustering of galaxies with f(R) gravity," *Monthly Notices of the Royal Astronomical Society*, vol. 474, no. 2, pp. 2430–2443, 2018.

[25] H. A. Buchdahl, "Non-linear Lagrangians and cosmological theory," *Monthly Notices of the Royal Astronomical Society*, vol. 150, no. 1, pp. 1–8, 1970.

[26] A. A. Starobinsky, "A new type of isotropic cosmological models without singularity," *Physics Letters B*, vol. 91, no. 1, pp. 99–102, 1980.

[27] A. De Felice and S. Tsujikawa, "f(R) theories," *Living Reviews in Relativity*, vol. 13, no. 1, 2010.

[28] S. Capozziello and M. De Laurentis, "Extended theories of gravity," *Physics Reports*, vol. 509, no. 4-5, pp. 167–321, 2011.

[29] S.'i. Nojiri and S. D. Odintsov, "Unified cosmic history in modified gravity: From F(R) theory to Lorentz non- invariant models," *Physics Reports*, vol. 505, no. 2-4, pp. 59–144, 2011.

[30] J. Sadeghi, B. Pourhassan, A. S. Kubeka, and M. Rostami, "Logarithmic corrected polynomial $f(R)$ inflation mimicking a cosmological constant," *International Journal of Modern Physics D*, vol. 25, no. 7, article 1650077, 2016.

[31] P. Bueno, P. A. Cano, V. S. Min, and M. R. Visser, "Aspects of general higher-order gravities," *Physical Review D*, vol. 95, no. 4, article 044010, 2017.

[32] T. Moon, Y. S. Myung, and E. J. Son, "f(R) black holes," *General Relativity and Gravitation*, vol. 43, no. 11, pp. 3079–3098, 2011.

[33] M. Chabab, H. El Moumni, S. Iraoui, K. Masmar, and S. Zhizeh, "Joule-Thomson expansion of RN-AdS black holes in $f(R)$ gravity," *LHEP*, vol. 2, p. 5, 2018.

[34] A. Belhaj and H. El Moumni, "Entanglement entropy and phase portrait of f(R)-AdS black holes in the grand canonical ensemble," *Nuclear Physics B*, vol. 938, pp. 200–211, 2019.

[35] D. Bazeia, L. Losano, G. J. Olmo, and D. Rubiera-Garcia, "Black holes in five-dimensional Palatinif(R)gravity and implications for the AdS/CFT correspondence," *Physical Review D*, vol. 90, no. 4, article 044011, 2014.

[36] R. Pourhasan, "Spacetime entanglement with $f(R)$ gravity," *Journal of High Energy Physics*, vol. 2014, no. 6, 2014.

[37] W. Kinnersley and M. Walker, "Uniformly accelerating charged mass in general relativity," *Physics Review D*, vol. 2, no. 8, pp. 1359–1370, 1970.

[38] J. F. Plebanski and M. Demianski, "Rotating, charged, and uniformly accelerating mass in general relativity," *Annals of Physics*, vol. 98, no. 1, pp. 98–127, 1976.

[39] J. Oscar, C. Dias, and J. P. S. Lemos, "Pair of accelerated black holes in anti-de Sitter background: AdS C metric," *Physical Review D*, vol. 67, article 064001, 2003.

[40] J. B. Griffiths and J. Podolsky, "A new look at the Plebanski-Demianski family of solutions," *International Journal of Modern Physics D*, vol. 15, pp. 335–370, 2012.

[41] F. Dowker, J. P. Gauntlett, D. A. Kastor, and J. H. Traschen, "Pair creation of dilaton black holes," *Physical Review D*, vol. 49, no. 6, pp. 2909–2917, 1994.

[42] R. Gregory and M. Hindmarsh, "Smooth metrics for snapping strings," *Physical Review D*, vol. 52, no. 10, pp. 5598–5604, 1995.

[43] D. M. Eardley, G. T. Horowitz, D. A. Kastor, and J. H. Traschen, "Breaking cosmic strings without monopoles," *Physical Review Letters*, vol. 75, no. 19, pp. 3390–3393, 1995.

[44] R. Emparan and H. S. Reall, "A rotating black ring solution in Five Dimensions," *Physical Review Letters*, vol. 88, no. 10, article 101101, 2002.

[45] M. Appels, R. Gregory, and D. Kubiznak, "Thermodynamics of accelerating black holes," *Physical Review Letters*, vol. 117, no. 13, article 131303, 2016.

[46] M. Appels, R. Gregory, and D. Kubiznak, "Black hole thermodynamics with conical defects," *Journal of High Energy Physics*, vol. 2017, no. 5, p. 116, 2017.

[47] V. Balasubramanian and S. F. Ross, "Holographic particle detection," *Physical Review D*, vol. 61, no. 4, article 044007, 2000.

[48] J. Zhang, Y. Li, and H. Yu, "Accelerating AdS black holes as the holographic heat engines in a benchmarking scheme," *The European Physical Journal C*, vol. 78, no. 8, p. 645, 2018.

[49] J. Zhang, Y. Li, and Y. Hongwei, "Thermodynamics of charged accelerating AdS black holes and holographic heat engines," *Journal of High Energy Physics*, vol. 2019, no. 2, p. 144, 2019.

[50] M. Rostami, J. Sadeghi, S. Miraboutalebi, A. A. Masoudi, and B. Pourhassan, "Charged accelerating AdS black hole of f(R) gravity and the Joule-Thomson expansion," *International Journal of Geometric Methods in Modern Physics*, 2020.

[51] I. Fuentes-Schuller and R. B. Mann, "Alice falls into a black hole: Entanglement in non-inertial frames," *Physical Review Letters*, vol. 95, no. 12, article 120404, 2005.

[52] C. Y. Huang, W. Ma, D. Wang, and L. Ye, "How the Relativistic Motion Affect Quantum Fisher Information and Bell Non-locality for Multipartite state," *Scientific Reports*, vol. 7, no. 1, p. 38456, 2017.

[53] M. Zhang and R. B. Mann, "Charged accelerating black hole inf(R)gravity," *Physical Review D*, vol. 100, no. 8, article 084061, 2019.

[54] S. Hod, "Discrete black-hole radiation and the information loss paradox," *Physics Letters A*, vol. 299, no. 2-3, pp. 144–148, 2002.

[55] S.'i. Nojiri and S. D. Odintsov, "Introduction to modified gravity and gravitational alternative for dark energy," *International*

Journal of Geometric Methods in Modern Physics, vol. 4, pp. 115–145, 2011.

[56] T. P. Sotiriou and V. Faraoni, "*f(R)* theories of gravity," *Reviews of Modern Physics*, vol. 82, no. 1, pp. 451–497, 2010.

[57] A. Sheykhi, "Higher-dimensional charged $f(R)$ black holes," *Physics Letters D*, vol. 86, article 024013, 2012.

[58] J. Podolsky, "Accelerating black holes in anti-de Sitter universe," *Czechoslovak Journal of Physics*, vol. 52, no. 1, pp. 1–10, 2002.

[59] A. Anabalón, M. Appels, R. Gregory, D. Kubizňák, R. B. Mann, and A. Övgün, "Holographic thermodynamics of accelerating black holes," *Physical Review D*, vol. 98, no. 10, article 104038, 2018.

[60] A. Anabalón, F. Gray, R. Gregory, D. Kubizňák, and R. B. Mann, "Thermodynamics of charged, rotating, and accelerating black holes," *Journal of High Energy Physics*, vol. 2019, no. 4, p. 96, 2019.

[61] S.'i. Nojiri and S. D. Odintsov, "Instabilities and anti-evaporation of Reissner– Nordstro¨m black holes in modified $F(R)$ gravity," *Physical Review B*, vol. 735, pp. 376–382, 2014.

[62] J. Podolsky, M. Ortaggio, and P. Krtous, "Radiation from accelerated black holes in an anti-de Sitter universe," *Physical Review D*, vol. 68, no. 12, article 124004, 2003.

[63] O. J. C. Dias and J. P. S. Lemos, "Pair of accelerated black holes in an anti-de Sitter background: The AdSCmetric," *Physical Review D*, vol. 67, no. 6, article 064001, 2003.

[64] P. Krtous, "Accelerated black holes in an anti-de Sitter universe," *Physical Review D*, vol. 72, no. 12, article 124019, 2005.

[65] K. Hong and E. Teo, "A new form of the C metric," *Classical and Quantum Gravity*, vol. 20, no. 14, pp. 3269–3277, 2003.

[66] M. Tavakoli, B. Mirza, and Z. Sherkatghanad, "Holographic entanglement entropy for charged accelerating AdS black holes," *Nuclear Physics B*, vol. 943, article 114620, 2019.

[67] X. Hao, "Minimal surfaces in AdS C-metric," *Physics Letters B*, vol. 773, pp. 639–643, 2017.