Hindawi
Advances in High Energy Physics
Volume 2021, Article ID 4613870, 5 pages
https://doi.org/10.1155/2021/4613870

Research Article

Nonequilibrium Black Hole Thermodynamics in Anti-de Sitter Spacetime

Kai Lin

Hubei Subsurface Multi-Scale Imaging Key Laboratory, Institute of Geophysics and Geomatics, China University of Geosciences, Wuhan, 430074 Hubei, China

Correspondence should be addressed to Kai Lin; lk314159@hotmail.com

Received 2 October 2021; Revised 1 November 2021; Accepted 26 November 2021; Published 28 December 2021

Academic Editor: Luciano Petruzziello

This work discusses the black hole thermodynamics in a weak dynamical Anti-de Sitter spacetime, which should be described by the nonequilibrium thermodynamics, because the metric depends on the time coordinate. Taking the Vaidya-Anti-de Sitter black hole spacetime as an example, the local entropy balance equations and principle of minimum entropy generation are derived, and finally, some irreversible effects in nonequilibrium thermodynamics are studied by using the Onsager reciprocal relation.

1. Introduction

In 1975, by using the quantum field theory in curved spacetime, Hawking proposed that a black hole could produce radiation at horizon [1, 2], and the effect is named as Hawking radiation, which implies a profound relation among gravitational theory, quantum theory, and thermodynamics. The Hawking radiation theory has attracted plenty of attention, because it is believed that studies on Hawking radiation can help to understand and construct the quantum gravity theory. Another interesting theory is AdS/CFT correspondence: due to the similar symmetry, researchers guess that there is a correspondence between the conformal field theory (CFT) and the physics in Anti-de Sitter (AdS) spacetime [3]. According to the hypothesis, the holographic superconductor theory is proposed, and the phase transition curve of the superconductor is obtained in AdS spacetime [4, 5]. Recently, Kubiznak et al. developed a new subdiscipline called the *Black Hole Chemistry* [6, 7] by considering the cosmological constant $\Lambda = -8\pi P$ as the thermodynamic pressure P. Here, the pressure is positive while the cosmological constant is negative at the AdS case. According to the definition, the thermodynamic volume is given by $V = (\partial M/\partial P)_{S,Q,J} = (4/3)\pi r_h^3$, where the geometric units can be expressed as $G_N = \hbar = c = k = 1$, and M, r_h, $S = A/4$, Q, J represent the black hole's mass, horizon, entropy, electric charge and angular momentum, respectively. (In black hole chemistry and extended black hole thermodynamics, the V plays the role of the volume in the thermodynamical equation, so it is named as the thermodynamic volume. However, in [8], the physical definition of the black hole volume is considered as $3\sqrt{3}\pi m^2 v$, where m is the Schwarzschild black hole mass, and $v = t + r + 2m \ln |r - 2m|$.) Therefore, the first law of black hole thermodynamics requires [7]

$$\delta M = T\delta S + V\delta P + \phi\delta Q + \Omega\delta J, \tag{1}$$

where the black hole temperature $T = \kappa/2\pi$, the surface gravity is κ, the electric potential is ϕ, the rotational potential is Ω, and the black hole's area is A. After giving the above physical quantity meaning, the van der Waals gas-like phase transition curve is plotted by adjusting a few parameters.

The above studies are based on the static and stationary black holes, which perfectly correspond to equilibrium thermodynamics. However, in our universe, the real black hole is time-dependent, because black holes always swallow matter and radiation due to the extreme gravity, or evaporate by the Hawking radiation effect. As a result, real black holes are characterized by varying their mass, electric charge, and angular momentum. The fact implies that a dynamical black hole should satisfy nonequilibrium thermodynamics, which includes many dynamical irreversible processes, such as the electrodynamical effect, thermodynamical effect, and transfer phenomenon. These effects can give a good explanation for the actual phenomena of life. As we all know, near the equilibrium state, there is a nonequilibrium state region which is called a linear region. The physical phenomena in the linear region can be well described by modern nonequilibrium thermodynamics, and we believe that the linear region corresponds to the black hole thermodynamics of a weak dynamical black hole, which means the time-dependent part is far smaller than the stationary part of this black hole metric, so that several physical effects could be investigated by expanding the perturbation near the equilibrium state and the Onsager reciprocal relation.

In this paper, we study nonequilibrium thermodynamics of weak dynamical black holes in AdS spacetime. In the next section, the local entropy balance equation is derived by the first law of weak dynamical black hole thermodynamics, and then, we take the Vaidya-AdS black hole as an example to show the principle of minimum entropy, which means the entropy production rate goes to a stable final value (or extreme value) in weak dynamical black hole spacetime. Because of the Onsager reciprocal relation, the nonequilibrium thermodynamics suppose the existence of some irreversible effect in the linear region, and we show the derivation in Section 3. Section 4 includes some conclusions, and we also give some discussion about the possible physical phenomena of strong dynamical black hole thermodynamics.

2. The Balance Relation of Local Entropy in Weak Dynamical Black Hole Spacetime

Let us start with the dynamical Eddington coordinates metric that is formulated as

$$ds^2 = -f(r, v)dv^2 + 2drdv + r^2 d\Omega^2, \tag{2}$$

where $d\Omega^2$ is the sphere metric, r is the radial space coordinate, and v is the advanced Eddington coordinate that plays the role of time coordinate in the spacetime. $f = f(r, v)$ is the function of the radial space coordinate and advanced time, which means that it is a dynamical metric. The surface of the dynamical black hole can be defined by an apparent horizon r_A. According to [9, 10], for a dynamical metric $ds^2 = h_{ab}dx^a dx^b + r^2 d\Omega^2$ with $a, b = t, r$, the apparent horizon r_A satisfies the equation $h^{ab}\partial_a r \partial_b r = 0$, so it requires $f(r_A, v) = 0$. The black hole temperature is $T = (\partial_r f/4\pi)|_{r=r_A}$. In this work, we only study weak dynamical black hole cases, so we require

$$\partial_v f(r, v) \ll f(r, v). \tag{3}$$

The construction allows us to linearly expand the perturbation near the equilibrium thermodynamics state (where $\partial_v f = 0$), so the nonequilibrium thermodynamics is in the linear region.

A famous dynamical black hole solution is the Vaidya metric [11], which can change the black hole mass due to the contribution of the pure radiation field. Therefore, the mass of the Vaidya black hole can be written as the function of the time coordinate, and the Vaidya black hole reduces to the Schwarzschild black hole when the mass is a constant.

In Anti-de Sitter spacetime, the Vaidya-AdS black hole metric requires [12]

$$f(r, v) = 1 - \frac{2M(v)}{r} - \frac{\Lambda}{3} r^2, \tag{4}$$

where $M = M(v)$ is the mass of black holes, and the first law of black hole thermodynamics is simplified as

$$\delta M = T\delta S + V\delta P. \tag{5}$$

For the reason of constant Λ, the thermodynamic pressure P does not depend on time, so we have

$$\frac{\partial M}{\partial v} = T\frac{\partial S}{\partial v}. \tag{6}$$

The above relation shows that the rate of change of entropy with time only is determined by the rate of change of mass in the dynamical black hole spacetime without charge and spin. The mass increase of the Vaidya black hole comes from ingoing pure thermal radition flux J_K, and the continuity equation for energy (or mass) is given by

$$\frac{\partial M}{\partial v} = -\nabla \cdot J_K, \tag{7}$$

and thus, the local entropy balance equation in nonequilibrium thermodynamics is given as

$$\frac{\partial S}{\partial v} = -\frac{1}{T}\nabla\cdot J_K = -\nabla \cdot \left(\frac{J_K}{T}\right) + J_K \cdot \nabla\left(\frac{1}{T}\right) \equiv -\nabla \cdot J_S + \sigma, \tag{8}$$

where $J_S \equiv J_K/T$ is the entropy flow density and $\sigma \equiv J_K \cdot X_K = J_K \cdot \nabla(1/T)$ is the entropy production rate. $X_K = \nabla(1/T)$ is the driving force for flux J_K, and it is obvious that the force originates from the gradient of the temperature of the system. The fact requires that the black hole temperature should be different from the external temperature, so the flux J_K can flow into the black hole.

In the Vaidya-AdS spacetime without charge and angular momentum, we can directly consider that the strength of flux J_K is in direct proportion to the driving force X_K in the linear region, so we set

$$J_K = L_{KK} X_K = L_{KK}\nabla\left(\frac{1}{T}\right), \tag{9}$$

where L_{KK} is a proportional factor, and the relation reflects the properties of a weak dynamical black hole, which satisfies condition (3). Therefore, the entropy production rate is given by

$$\sigma = L_{KK}\left[\nabla\left(\frac{1}{T}\right)\right]^2, \tag{10}$$

and the total entropy production rate is defined as

$$P = \int \sigma dV = \int dV\left\{L_{KK}\left[\nabla\left(\frac{1}{T}\right)\right]^2\right\}, \tag{11}$$

so that the rate of change of P with time is given by

$$\begin{aligned}\frac{dP}{dv} &= 2\int\left\{J_K\cdot\nabla\left[\frac{\partial}{\partial v}\left(\frac{1}{T}\right)\right]\right\}dV \\ &= 2\int dV\left\{\nabla\cdot\left[J_K\frac{\partial}{\partial v}\left(\frac{1}{T}\right)\right]-\frac{\partial}{\partial v}\left(\frac{1}{T}\right)\nabla\cdot J_K\right\}.\end{aligned} \tag{12}$$

We considered the combination of the black hole and the pure thermal radiation beyond the apparent horizon as a system, so the first term before the second equal sign could be written as a surface integral by using the Gauss theorem, and this term vanishes because the temperature does not change at the surface of the system. In order to simplify the above equation, we apply the weak dynamical black hole condition (3) and concretely set

$$M = M_0 + \delta M(v), \tag{13}$$

where M_0 represents the constant part of the mass, and the time-dependent part $\delta M(v) \ll M_0$. According to the expression of temperature [9, 10]

$$T = \left(\frac{\partial M}{\partial S}\right)_P = \left.\frac{\partial_r f}{4\pi}\right|_{r=r_A} = \frac{3M - r_A}{2\pi r_A^2}, \tag{14}$$

we have $\partial_v T \approx \eta M_0^{-2}\partial_v M = \eta M_0^{-2}\partial_v \delta M$ and $\partial_v \ln T \approx \varepsilon M_0^{-1}\partial_v M$ where $\eta = \eta(\lambda)$ and $\varepsilon = \varepsilon(\lambda)$ with $\lambda \equiv \Lambda M_0^2$ are two time-independent parameters. The relation between the above parameters and λ is shown in Figure 1.

Figure 1 shows the existence of critical value $\lambda_c = -(4/9)$, which lead to $\eta < 0$ as $\lambda > \lambda_c$, while $\eta > 0$ as $\lambda < \lambda_c$. Substituting the above relations into Equation (12), we finally obtain

$$\frac{dP}{dv} = -\frac{2M_0^2}{\eta}\int[\partial_v(\ln T)]^2 dV = -\frac{2\varepsilon^2}{\eta}\int(\partial_v M)^2 dV, \tag{15}$$

or

$$d\sigma = -\frac{2}{\eta}[\partial_v(\ln T)]d(\ln T) = -\frac{2\varepsilon^2}{\eta}(\partial_v M)dM. \tag{16}$$

Therefore, when $\lambda < \lambda_c$, we can obtain the principle of minimum entropy generation, just like the case in normal nonequilibrium thermodynamics: $P = P(v)$ is a monotone decreasing function, so P and σ go to a positive minimal value, according to the entropy increase theory. However, $P = P(v)$ becomes a monotone increasing function at the case $\lambda > \lambda_c$, so we must discuss the two cases, respectively.

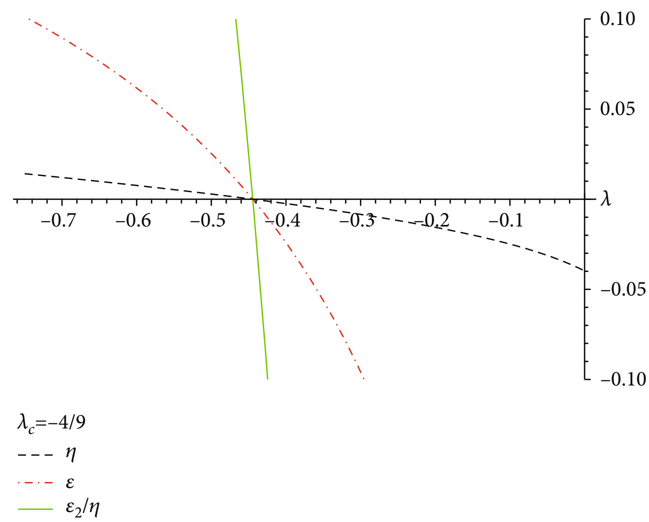

Figure 1: $\eta = \eta(\lambda)$ and $\varepsilon = \varepsilon(\lambda)$.

Let us take $M = 1 + \pi/40 + \arctan(3v - 15)/20$ as an example to show the relation $\sigma = \sigma(v)$ in Figure 2.

In fact, what we study is a weak dynamical black hole which satisfies constraint (3), while the mass of the Vaidya black hole should be monotone increasing with time, so that we should set that the black hole's mass will reach a final value as $v \longrightarrow \infty$. According to the condition, the entropy production rate σ goes to a final value. When $\lambda < \lambda_c$, it monotonically decreases to this value, but σ monotonically increases to the value at $\lambda > \lambda_c$ case. Therefore, it may be that the principle of minimum entropy generation should be renamed as the principle of extremum entropy generation. It should be noted that, with different initial value $\sigma_0 = \sigma(v = 0)$, we can plot two groups of translational curves for σ at the two cases, respectively, so it shows that the form of function σ are the same but the position of the curves is determined by initial value σ_0.

3. Irreversible Dynamical Effect in Linear Region

The no-hair theorem reveals that the black hole only has 3 properties: mass, electric charge, and angular momentum. By considering the black hole with an angular momentum and electric charge in Anti-de Sitter spacetime, the first law of black hole thermodynamics is given by Equation (1) (of course, the area, entropy, surface gravity, and temperature of the black hole should be defined by apparent horizon r_A in dynamical black hole spacetime). Finally, Equation (6) is generalized as

$$\frac{\partial S}{\partial v} = \frac{1}{T}\frac{\partial M}{\partial v} - \frac{\phi}{T}\frac{\partial Q}{\partial v} - \frac{\Omega}{T}\frac{\partial J}{\partial v}, \tag{17}$$

where J_K is from Equation (7), and electrical current J_Q and angular momentum flux J_J can be defined by the equations of continuity:

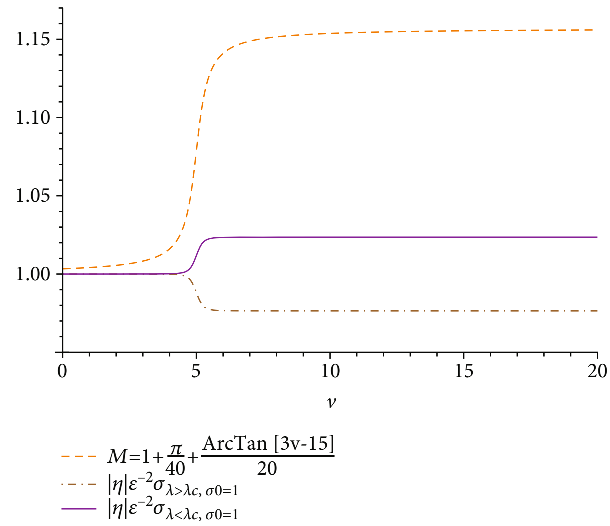

FIGURE 2: $M = M(v)$ and $\sigma = \sigma(v)$.

$$\begin{aligned} \frac{\partial Q}{\partial v} &= -\nabla \cdot J_Q, \\ \frac{\partial J}{\partial v} &= -\nabla \cdot J_J, \end{aligned} \tag{18}$$

and, respectively, the corresponding driving forces are given by

$$\begin{aligned} X_Q &= \nabla\left(\frac{\phi}{T}\right), \\ X_J &= \nabla\left(\frac{\Omega}{T}\right), \end{aligned} \tag{19}$$

so the entropy flow density and the entropy production rate are rewritten as

$$\begin{aligned} J_S &\equiv \frac{J_K - \phi J_Q - \Omega J_J}{T}, \\ \sigma &\equiv J_K \cdot X_K - J_Q \cdot X_Q - J_J \cdot X_J. \end{aligned} \tag{20}$$

Without loss of generality, we can assume the above 3 fluxes all originate from 3 driving forces in the linear region (or weak dynamical black hole spacetime), so that they satisfy the relations:

$$J_i = \sum_{n=1}^{3} L_{\text{in}} X_n, \tag{21}$$

where $(J_1, J_2, J_3) \equiv (J_K, J_Q, J_J)$ and $(X_1, X_2, X_3) \equiv (X_K, X_Q, X_J)$. According to the Onsager reciprocal relation,

$$L_{\text{in}} = L_{ni}. \tag{22}$$

There are 6 independent coefficients ofL_{ni}, and Equation (21) can be rewritten as the matrix form

$$\begin{pmatrix} J_K \\ J_Q \\ J_J \end{pmatrix} = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{bmatrix} \begin{pmatrix} X_K \\ X_Q \\ X_J \end{pmatrix}. \tag{23}$$

Equation (23) can describe several irreversible dynamical processes. For the sake of simplicity, let us just consider a simple system: charged irrational dynamical black hole with charged ingoing particle without angular momentum, so that the above equation can be simplified as

$$\begin{aligned} J_K &= L_{11}\nabla\left(\frac{1}{T}\right) + L_{12}\nabla\left(\frac{\phi}{T}\right), \\ J_Q &= L_{12}\nabla\left(\frac{1}{T}\right) + L_{22}\nabla\left(\frac{\phi}{T}\right). \end{aligned} \tag{24}$$

According to the above equations, if $\nabla(\phi/T) = 0$ but $\nabla(1/T)$=0, $(J_Q/J_K)_{\nabla(\phi/T)=0} = L_{12}/L_{11}$ represents the electric current caused by the mass flux; if $\nabla(1/T) = 0$ but $\nabla\phi$=0, $(J_K/J_Q)_{\nabla(1/T)=0} = L_{12}/L_{22}$ is the mass flux caused by the electric current. The irreversible effect of nonequilibrium thermodynamics could be observed in physical experiments or astronomical observations in the future.

4. Conclusion

We constructed a nonequilibrium thermodynamics in weak dynamical Anti-de Sitter black hole spacetime. We obtain the local entropy balance equation in nonequilibrium thermodynamics, and then by using the Onsager reciprocal relation, several irreversible processes are revealed. Speaking of the principle of extremum entropy generation, we find there is a critical value λ_c, and the principle of minimum entropy generation is satisfied as $\lambda < \lambda_c$, but the entropy production rate becomes a monotone increasing function when $\lambda > \lambda_c$. Thanks to the weak dynamical black hole condition (3), the area of the black hole does not increase immoderately, so the entropy production rate is also not divergent. Therefore, we can get a weak dynamical black hole thermodynamics which is similar to the quasiequilibrium thermodynamics.

Nevertheless, quasiequilibrium thermodynamics only studies the physical properties near the equilibrium state region, so the conclusion in this paper cannot describe the nature of the region far away from the equilibrium state, namely, the strong dynamical black hole spacetime case. According to the dissipative structure theory proposed by Prigogine, nonlinear property will dominate the evolution of the system at this region, so that the solution from the linear region becomes the unstable mode, but another stable solution exhibits some self-organizing phenomena which show some ordered property due to the breaking of the symmetry. It is reasonable to believe that the self-organizing-like phenomena or the fractal phenomena also will appear in strong dynamical black hole thermodynamics, so it could show some very meaningful new phenomena, such as the following: a pure spherically symmetric black hole is impossible and some pictures could present on the surface of the black hole. We also guess that the information from self-organizing and fractal phenomena could be recorded in the gravitational wave, so we hope to try to read them from gravitational wave signals in the future. We will continue to resolve these problems.

Data Availability

No data were used to support the findings of this study.

Conflicts of Interest

The author declares that they have no conflicts of interest.

Acknowledgments

The author gratefully acknowledges the financial support from the National Natural Science Foundation of China (NNSFC) under contract No. 11805166.

References

[1] S. W. Hawking, "Black hole explosions?," *Nature (London)*, vol. 248, no. 5443, pp. 30-31, 1974.

[2] S. W. Hawking, "Particle creation by black holes," *Communications in Mathematical Physics*, vol. 43, no. 3, pp. 199–220, 1975.

[3] P. K. Kovtun and A. O. Starinets, "Quasinormal modes and holography," *Physical Review D*, vol. 72, no. 8, article 086009, 2005.

[4] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, "Building a holographic superconductor," *Physical Review Letters*, vol. 101, no. 3, article 031601, 2008.

[5] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, "Holographic superconductors," *Journal of High Energy Physics*, vol. 2008, no. 12, p. 015, 2008.

[6] S. Gunasekaran, D. Kubizňák, and R. B. Mann, "Extended phase space thermodynamics for charged and rotating black holes and Born-Infeld vacuum polarization," *Journal of High Energy Physics*, vol. 2012, no. 11, 2012.

[7] W. Cong, D. Kubiznak, and R. B. Mann, "Thermodynamics of AdS black holes: critical behavior of the central charge," *Physical Review Letters*, vol. 127, no. 9, article 091301, 2021.

[8] M. Christodoulou and C. Rovelli, "How big is a black hole?," *Physical Review D*, vol. 91, no. 6, article 064046, 2015.

[9] R. G. Cai and S. P. Kim, "First law of thermodynamics and Friedmann equations of Friedmann–Robertson–Walker universe," *Journal of High Energy Physics*, vol. 2005, no. 2, p. 50, 2005.

[10] Y.-P. Hu, X.-X. Zeng, and H.-Q. Zhang, "Holographic thermalization and generalized Vaidya-AdS solutions in massive gravity," *Physics Letters B*, vol. 765, pp. 120–126, 2017.

[11] P. C. Vaidya, "'Newtonian' time in general relativity," *Nature*, vol. 171, no. 4345, pp. 260-261, 1953.

[12] K. Lin, Y.-Y. Sun, and H. Zhang, "Quasinormal modes for dynamical black holes," *Physical Review D*, vol. 103, no. 8, article 084015, 2021.