Research Article

Significance of Broken $\mu - \tau$ Symmetry in Correlating $\delta_{CP}$, $\theta_{13}$, Lightest Neutrino Mass, and Neutrinoless Double Beta Decay $0\nu\beta\beta$

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1. Introduction

In 1950, Bruno Pontecorvo for the first time emphasized the idea of neutrino oscillations which resembled $K^0 - \bar{K}^0$ oscillations. In neutrino oscillations, a neutrino originated with a definite flavor, $(\nu_e, \nu_\mu, \nu_\tau)$ oscillates to a distinct contrasting lepton flavor. Neutrino oscillation reveals that each of the three states of neutrino $\nu_\alpha$ in flavor basis is a superposition of three mass eigen states $(m_1, m_2, m_3)$ [1]. Neutrinos are massive, and they mix with each other. The massive neutrinos are formed in their gauge eigen states $(\nu_\alpha)$ which are linked to their mass eigen states $\nu_i$. Gauge eigen states participate in gauge interactions as

$$|\nu_\alpha\rangle = \sum U_{\alpha i} |\nu_i\rangle,$$

where $\alpha = e, \mu, \tau$, $\nu_i$ is the neutrino of distinct mass $m_i$. $U$ is parameterized as

$$U = \begin{pmatrix}
    c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{i \delta} \\
    -s_{12} c_{23} - s_{13} s_{23} c_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} c_{13} & s_{23} c_{13} \\
    s_{12} s_{23} - c_{12} s_{23} s_{13} e^{i \delta} & -c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & c_{23} c_{13}
\end{pmatrix},$$

where $\delta$ is the atmospheric mixing angle ($\delta = \pi/4$). Precise breaking of the $\mu - \tau$ symmetry is achieved by adding a 120-plet Higgs to the SO(10) representation of Higgs. The estimated three-dimensional density parameter space of the lightest neutrino mass $m_1$, $\delta_{CP}$, and reactor mixing angle $\theta_{13}$ is constrained here for the requirement of producing the observed value of baryon asymmetry of the Universe through the mechanism of leptogenesis. Carrying out numerical analysis, the allowed parameter space of $m_1$, $\delta_{CP}$, and $\theta_{13}$ is found out which can produce the observed baryon to photon density ratio of the Universe.
where $\theta_{12} = 33^\circ, \theta_{23} = 38^\circ - 53^\circ$, and $\theta_{13} = 8^\circ$ [2] are the solar, atmospheric, and reactor angles according to the global fits, respectively. The Majorana phases $\alpha$ and $\beta$ dwell in $P$, where

$$ P = \text{diag}(1, e^{i\alpha}, e^{i(\beta-\delta)}). \quad (3) $$

$U \ast P$ is known as the Pontecorvo-Maki-Nakagawa-Sakata $U_{\text{PMNS}}$ matrix [3]. Since a $n$ of a given flavor $\alpha$ is a mixed state of at least three $\nu$ with distinct masses, this three-generation mixing could result into the flavor mixing mass matrix or PMNS matrix possessing an irreducible imaginary component. This irreducible imaginary component is responsible for CP asymmetry. CP violation interchanges every particle into its antiparticle. $\delta_{\text{CP}}$ in PMNS matrix can induce CP violation. CP asymmetry can be observed in neutrino oscillations. $\delta_{\text{CP}}$ phase measures the amount of asymmetries between lepton oscillations and antilepton oscillations. Neutrinos are massive, and they mix with each other. This may be a source of CP violation if $\sin \delta_{\text{CP}} \neq 0$. The amount of $\delta_{\text{CP}}$ violation phase in this case is estimated by the Jarksgog invariant [4].

$$ J_{\text{CP}} = \frac{1}{8} \cos \theta_{13} \sin 2 \theta_{13} \sin 2 \theta_{23} \sin 2 \theta_{13} \sin \delta_{\text{CP}}, \quad (4) $$

when $\sin \delta_{\text{CP}} \neq 0$. In leptogenesis, lepton-antilepton asymmetry is explained if there are complex imaginary irreducible terms in the Yukawa couplings of lepton mass matrices. The lepton number generation of the early Universe can be estimated by the complex CPV phase term in the fermion mass matrices. The paper gives the impression that the neutrino Dirac CP phase is automatically connected to the baryon matrices. The paper gives the impression that the neutrino mated by the complex CPV phase term in the fermion mass generation of the early Universe can be estimated if there are complex imaginary irreducible terms in the Yukawa couplings of lepton mass matrices. The $\delta_{\text{CP}}$ phase is maximized, i.e., $\theta_{13} = 0$. The deviation of $\delta_{\text{CP}}$ from the maximal angle $\theta_{13}$, the explanation of reactor angle $\theta_{13}$, and the existence of CP violating phase necessitate the spontaneous breaking of the $\mu - \tau$ exchange symmetry in the neutrino sector. The measurement of the neutrino mixing angle $\theta_{13}$ in concurrence with a measurement of the departure from maximality of the atmospheric mixing angle can be a very strong way to probe any possible $\nu \leftrightarrow \tau$ symmetry present in the neutrino mass matrix. Different types of plausible mechanism of breaking of $\mu - \tau$ symmetry and the possible types of resultant gauge symmetry for generation of nonzero $\theta_{13}$ are introduced in [7].

$\mu - \tau$ symmetry is an important idea in neutrino physics in view of the near-maximal atmospheric mixing angle. The original papers which introduced the concept of $\mu - \tau$ symmetry are cited in [8-10].

Here, in this work, an explicit form of the Dirac neutrino mass matrix in broken $\mu - \tau$ [11] symmetry framework in type I seesaw mechanism is used in our calculation for generating baryon asymmetry of the Universe via leptogenesis. This scenario is characterized by small divergence of $\theta_{13}$ from the maximal angle $\theta_{23}$, which is consistent with a liberal size of $\theta_{13} \sim 8^\circ - 9^\circ$ and a large $\delta_{\text{CP}}$ phase in the neutrino sector. The renormalizable Dirac neutrino Yukawa couplings of the Dirac mass matrices are determined from the fermion Yukawa couplings to the 10, 126, and 120 dimensional fields of Higgs multiplet in the SO(10) group. Higgs field under the 10 and 126 representations is symmetrical under the general $\mu - \tau$ symmetry, while the 120-dimensional representation changes sign. This spontaneously breaks the $\mu - \tau$ invariant symmetry, which in turn allows a generalized $\delta_{\text{CP}}$ phase in the PMNS matrix.

Here, we made an effort for correlating or constraining the values of $\delta_{\text{CP}}$ phase, non-zero reactor angle $\theta_{13}$, and the lightest neutrino mass space for both the hierarchies in the context of leptogenesis and current ratio of baryon to photon density of the Universe. Both CPV phase $\delta_{\text{CP}}$ and reactor angle $\theta_{13}$ have good vibes between each other. A precise value of $\theta_{13}$ plays an imperative role in its CP violation phase measurements. On the basis of this fact, nonzero values of $\theta_{13}$ are predicted here in consistency with the $\delta_{\text{CP}}$ phase. Taking into account constraints from the global fit values of $\nu$ oscillation parameters and cosmology, a density plot of the favourable values of the $\delta_{\text{CP}}$ phase, lightest neutrino mass, and $\theta_{13}$ is being initiated, which is compatible with the constraints on the sum of the absolute neutrino masses, $\sum_i m_i < 0.23$ eV from CMB, Planck 2015 data (CMB15 + LRG + lensing + $H_0$) [12]. Constraints from the leptonic asymmetry of the Universe are also considered for further restricting the $\delta_{\text{CP}}$ phase space and lightest neutrino mass. The leptonic CP asymmetry is being deliberated via leptogenesis in terms of baryon density to photon density ratio of the Universe $\eta_b$ accessible as $5.8 \times 10^{-10} < \eta_b \leq 6.6 \times 10^{-10}$ [13]. We also calculate the effective mass spectrum for neutrinoless double beta decay, $\nu\beta\beta$ decay given by

$$ m_{\nu e} = |m_1 c_{12}^2 s_{13}^2 + m_2 c_{12}^2 s_{13}^2 e^{i\delta_{12}} + m_3 s_{13}^2 e^{i(\delta_{12} - 2\delta_{\text{CP}})}|, \quad (5) $$

for favourable values of the $\delta_{\text{CP}}$ phase and lightest $\nu$ mass explored here in this work. In this paper, we apply the broken $\mu - \tau$ symmetry to the Dirac neutrino Yukawa couplings in type I seesaw mechanism in the SO(10) model in predicting favourable values of the $\delta_{\text{CP}}$ phase, lightest neutrino mass, and $\theta_{13}$. We then scan free parameters in these models and search for the allowed region in which the neutrino oscillation data can be fitted. For the allowed parameter sets, we show the predictions of observables like the $\delta_{\text{CP}}$ phase, lightest neutrino mass, and $\theta_{13}$ in the neutrino sector. Finally, we show our predictions for the effective mass spectrum for
neutrinoless double beta, $0\nu\beta\beta$, decay for favourable values of the $\delta_{CP}$ phase.

This paper deals with an important aspect of neutrino physics, i.e., its CP violating Dirac phase and its possible connection to the matter-antimatter asymmetry of the universe. In this work, we have used user-defined Dirac Neutrino Yukawa couplings [11] for the Yukawa interactions associated with the broken $\mu - \tau$ symmetry model for the generation of nonzero reactor mixing angle $\theta_{13}$ and leptonic CP phase $\delta_{CP}$ in type I seesaw mechanism in the light of leptogenesis; there can be a transformation of the lepton asymmetry into a baryon asymmetry by nonperturbative $B + L$ violating ( sphaleron, Sakharov conditions) processes as discussed in [6]. A small explicit breaking of $\mu - \tau$ symmetry [11] by hand with the specified numerical value in Equation (30) inherits the property of generating nonzero CP violation in $\Delta m_{MN}$ matrices and results in $\theta_{13}$ being nonzero. Here, we consider the type I seesaw as the main donor to neutrino mass. We also take into account both inverted and normal ordering of neutrino mass spectrum as well as two different types of the lightest neutrino mass $m_1$ ($m_1 = 0.07118 eV(0.0657 eV)$) to visualise the results of hierarchical v mass spectrum. In the case of normal ordering of v masses, the dependance of leptonic CPV phase, $\delta_{CP}$, on the lightest v mass is predicted in Figures 1–3 (in the light of recent ratio of the baryon to photon density bounds, $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$). The favoured values of $\delta_{CP}$ phase is found to lie between $\delta_{CP} \in [30^\circ, 30^\circ]$ for best fit values of $\theta_{13} = 8.41$ corresponding to $\Delta \chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) as a result of contribution of type I seesaw mechanism to neutrino mass matrix. The favoured values of the lightest v mass, $m_1$, in this case come out to be $v < 0.085, 0.1 eV$. In case of inverted hierarchy, the variation of $\delta_{CP}$ phase is found to be very intense with the best fit values of $\theta_{13} = 8.49$ corresponding to $\Delta \chi^2 = 9.5$ w/o SK-ATM [14]. Values of the $\delta_{CP}$ phase favoured are $\delta_{CP} = 220^\circ, 223^\circ, 252^\circ, 268^\circ, 293^\circ, 309^\circ, 345^\circ$ (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) as is evident from Figure 4. The allowed spectrum of the lightest v mass is $m_1 \in [0.02, 0.055]$ eV.

The paper is organized as follows. In Section 2, we introduce our broken $\mu - \tau$ symmetry models. In Section 3, we perform parameter scan to fit neutrino oscillation data and provide some information in predicting observables like the $\delta_{CP}$ phase in the neutrino sector. Also, we show our predictions for the effective mass spectrum for neutrinoless double beta, $0\nu\beta\beta$, decay for favourable values of the $\delta_{CP}$ phase. Section 4 is our results and conclusions for the Yukawa interactions associated with broken $\mu - \tau$ symmetry model discussed here.

2. Broken $\mu - \tau$ Symmetry with Type I Seesaw Mechanism

When $\mu - \tau$ symmetry is unified with grand unification subsequently, a more general symmetry results that interchanges the second and third generations of fermions. This generalized symmetry is aimed at explaining why the Cabibbo angle is greater than the other two angles, and a soft explicit breaking of this $\mu - \tau$ symmetry leads to correct explanation of the quark mixing angles and masses. In this work, we consider a model based on the $SO(10) \otimes Z^2_{\mu} \otimes Z^2_{\tau}$. First, $Z_2$ symmetry represents the generalized $\mu - \tau$ symmetry. Second, $Z_2$ symmetry indicates parity [15] transformation between two components of the 16-dimensional representation of the field acting as $(4, 2, 1)$ and $(1, 2, 4)$ under the Pati-Salam group decomposition of $SO(10)$. Sixteen-dimensional representation of fermions gets their masses from coupling to three Higgs multiplets which transform as 10, 126 and 120 representations under $SO(10)$. The $SO(10)$ breaking is fulfilled with 210 plets. In a supersymmetric realm, an additional 126 plets of Higgs conserve the supersymmetry at the GUT breaking scale. These Higgs multiplets have in them altogether six doublets which match with the quantum numbers of the minimal supersymmetric standard model (MSSM) field $H_u$ and six other quantum numbers which complement the quantum field numbers of $H_d$. Two linear superpositions of these Higgs doublets stay light and play the role of the $H_d$ and $H_u$ fields. This is controlled by the fine-tuning conditions [16, 17]. After this fine-tuning results, the fermion masses can be written as

$$-L_{mass} = \bar{f}_L M_f f_R + \bar{v}_L M_D v_R + \frac{1}{2} \bar{v}_L M_M v_L + \frac{1}{2} \bar{v}_R M_R v_R + h.c.,$$  

where $f = u, d, l$ represents the up and down quarks and the charged leptons, respectively. The mass matrices representing different scalar representations to obtain the flavor texture of interest in the paper appearing in the above Lagrangian can be [18, 19] written as

$$M_d = H + F + iG,$$

$$M_u = rH + sF + itG,$$

$$M_l = H - 3F + ipG,$$

$$M_D = rH - 3F + iqG,$$

$$M_{1l} = r_{1l}F,$$

$$M_{1r} = r_{1r}F.$$ 

Here, $M_D$ is the Dirac v mass matrix. In the basis where $M_{1l}(M_{1r})$, the Majorana mass matrix for the left(right)-handed neutrinos, gets benefaction only from the vacuum expectation value (vev) of the 126 Higgs field, gauge coupling unification in the minimal model needs that the vacuum expectation value (vev) contributing to $M_{1l}$ is adjacent to the GUT scale. $r, s, t, p, q, r_{1l},$ and $r_{1r}$ are the dimensionless parameters which are determined by the Clebsch-Gordan coefficients, ratios of vevs, and mixing among the Higgs fields [18]. The matrices $H, F,$ and $G$ rise from the fermion couplings to the 10, 126, and 120 Higgs fields, respectively. Normally, $(G)H, F$ are complex (anti) symmetric matrices. Nonetheless, generalized parity symmetry keeps $(G)H, F$ real. Moreover, when all vevs and thus $r, s, t, p, q, r_{1l},$ and $r_{1r}$ are real, then the Dirac masses defined in Equation (10) are Hermitian and $M_{1l}$ and $M_{1r}$ are...
real. The 10- and 126-dimensional Higgs field representations are invariant under the generalized \( \mu - \tau \) symmetry while the 120-dimensional representation changes sign. This allows spontaneous explicit breaking of the \( \mu - \tau \) symmetry.

Let \( H, F, G \) be any complex symmetric (anti) matrices in general, which are a measure of the fermion Yukawa couplings to the 10, 126, and 120 Higgs field, respectively. Here,

\[
H = \begin{pmatrix} h_{11} & h_{12} & h_{12} \\ h_{12} & h_{22} & h_{23} \\ h_{12} & h_{23} & h_{22} \end{pmatrix},
\]

Similarly,

\[
F = \begin{pmatrix} f_{11} & f_{12} & f_{12} \\ f_{12} & f_{22} & f_{23} \\ f_{12} & f_{23} & f_{22} \end{pmatrix},
\]

Similarly,

\[
G = \begin{pmatrix} 0 & g_{12} & -g_{12} \\ -g_{12} & 0 & g_{23} \\ g_{12} & -g_{23} & 0 \end{pmatrix}.
\]

The matrices \( H, F, G \) originate from the fermion couplings to the 10, 126, and 120 fields, respectively. \( G \) is complex (anti) symmetric matrices in general. However, generalized parity makes them real. In addition, if all vevs and hence \( r, s, t, p, q, r_1, \) and \( r_8 \) are real, then all the Dirac masses in Equations (7), (8), (9), (10), (11), and (12) are Hermitian and \( M_1 \) and \( M_8 \) are real. Here, the Higgs field in the 10 and 126 representations are symmetric and invariant under the generalized \( \mu - \tau \) symmetry while the 120-dimensional representation changes sign. This assumption in turn allows spontaneous breaking of the \( \mu - \tau \) symmetry. In the \( \mu - \tau \) symmetric mass matrices in Equations (13), (14), and (15), Higgs field 120 induces antisymmetry of the Yukawa matrix. If that matrices as well as the 10 and 126 couplings are complex, it is well known that if there are no extra symmetries, they will induce a Dirac phase. Explicit breaking of \( \mu - \tau \) symmetry leads to nonzero \( \theta_{13} \). Small tiny breaking of the \( \mu - \tau \) symmetry allows a large Dirac CP violating phase in neutrino oscillation. Fermion mass spectrum can be explained by Hermitian mass matrices derived from the renormalizable Yukawa couplings of the 16 plts of fermions with the Higgs fields transforming as 10, 126, and 120 representations of the SO(10) group. The \( \mu - \tau \) symmetry upon spontaneously broken down through the 120 plts leads to nonzero reactor angle \( \theta_{13} \), which in turn induces leptonic dirac CPV phase in the \( U_{\text{PMNS}} \) matrix. Tiny explicit breaking of \( \mu - \tau \) symmetry leads to nonzero \( \theta_{13} \). This scenario implies a generalized CP invariance of the fermion mass matrices and vanishing CP violating phases if the Yukawa couplings are symmetric under the \( \mu - \tau \) symmetry. Small tiny breaking of the \( \mu - \tau \) symmetry allows a large Dirac CP violating phase in neutrino oscillation. Explicit breaking of the \( \mu - \tau \) symmetry by hand as evident from Equation (30) provides a nice spectrum of all the fermion masses and mixing and leads to nonzero \( \theta_{13} \), which in turn induces the \( \delta_{\text{CP}} \) phase and allows a large required Dirac CP violating phase in neutrino oscillation. Detailed fits to the fermion spectrum are presented in several scenarios in [11].
$h, f,$ and $g$ are all real. $\mu - \tau$ symmetry is invariant under the exchange of second- and third-generation fermions. When $\mu - \tau$ symmetry is added with SO(10) grand unified theory, then a general symmetry results which satisfies

$$S^T (H, F, G) S = (H, F, -G),$$

where

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (17)$$

The implicit neutrino mass matrix $M_{\nu}$ can be written as

$$M_{\nu} = r_L M_D F^{-1} M_D^T, \quad (18)$$

$r_L$ are inversely proportional to the vev of the RH triplet component in 126 dimensional Higgs field.

The Lagrangian of the type I seesaw model is [20, 21]

$$L_N^M = \frac{i}{2} N_i^R \delta N_i^R - y_{1a} N_i^R H \bar{L}_a - \frac{1}{2} N_i^R M_{1j} (N_j^R)^C + h.c. \quad (19)$$

Here, $y_{1a}$ is the complex Yukawa coupling matrix; $L_a = (\nu_{aL}^d, L_a)^T$ is the standard model left-handed lepton doublet of flavor $\alpha$, when $\alpha = e, \mu, \tau$; and $\bar{\phi}$ is the hypercharge-conjugated Higgs doublet, $(\phi^*_0 - \phi)^T$.

The Lagrangian describes the scenario of generation of $\nu$ masses via Higgs mechanism. Electroweak symmetry breaking process allows neutral part of the Higgs field to acquire a VEV, $v = 246$ GeV, and $\sqrt{2} \langle \phi_0 \rangle = v$ so that the left-handed and right-handed neutrinos form massive Dirac fermions.

**Figure 2:** Predictions in the broken $\mu - \tau$ symmetry model for normal ordering: (a) the three-dimensional plot of preferred values of the $(m_1, \delta_{CP})$ plane for the best fit values of $\theta_{13} = 8.41$ of $\Delta \chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Similarly, (b) shows the three-dimensional plot of favourable values of the $(m_1, \theta_{13})$ plane, for the best fit value of $\delta_{CP} = 222^\circ$ of $\Delta \chi^2 = 6.2$ [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$).
In Equation (6), \( M_{IJ} \) is a symmetric matrix of right-handed violating Majorana masses, where \( M_{IJ} = M_J \delta_{IJ} \) is real and diagonal. Here, right-handed neutrino masses are larger than the electroweak scale. The \( \nu \) masses are then suppressed by right-handed neutrino Yukawa couplings and also by

\[
\frac{\nu}{M_I} < 1. \tag{20}
\]

In type I seesaw, the baryon asymmetry of the Universe (BAU) occurs via leptogenesis mechanism via out of equilibrium decay of heavy RH Majorana neutrinos in the early Universe via electroweak sphaleron processes [22]. The resulting Majorana mass matrix of light SM neutrinos is

\[
M_{\alpha\beta} = -(m_D^T)_{\alpha i} (M_{ij}^{-1}) (m_D)_{j\beta}, \tag{21}
\]

where \( (m_D)_{\alpha i} \) is the Dirac mass matrix. Equation (8) shows that in the type I seesaw mechanism, SM \( \nu \) masses are suppressed by the combination of small Yukawa couplings and large RH \( \nu \) masses. Neutrino mass matrix on diagonalisation gives two eigenvalues—light neutrino \( \sim m_D^2/M_R \) and a heavy neutrino state \( \sim M_R \). This is known as type I seesaw mechanism.

In SO(10), heavy right-handed Majorana neutrino couples to the left-handed \( \nu \) via Dirac mass matrix \( m_D \). Out of
the decay of the lightest of the RH Majorana neutrinos, $M_1$, i.e., $M_3 \gg M_1$, will contribute to CP asymmetry [6, 23] (for leptogenesis), i.e., $\epsilon_{\nu}^{CP}$ and leptogenesis [24–30].

At the end of inflation [31], a certain number density of right-handed neutrinos, $n_{\nu_R}$, was created, which is linked to the present cosmological scenario. Right-handed neutrinos decayed, with a decay rate that reads, at tree level,

$$\Gamma_{\nu_R} = \frac{1}{8} (Y_\nu Y_\nu^\dagger)_{ii} M_i. \quad (22)$$

It is convenient to work in a basis of right-handed neutrinos, where the RH $\nu$ mass matrix is diagonal, the type I seesaw mechanism contribution to $\epsilon_{\nu}^{CP}$ is given by decay of $M_1$, or the CP violating parameter is given as

$$\epsilon_{\nu}^{CP} = \frac{\Gamma_{D_i} - \Gamma_{D_j}}{\Gamma_{D_j} + \Gamma_{D_i}}, \quad (23)$$

where

$$\Gamma_{D_j} = \Gamma(v_R \longrightarrow l_i H_u) + \Gamma(v_R \longrightarrow \tilde{l}_i h_u) = \frac{1}{8} (Y_\nu Y_\nu^\dagger)_{ij} M_j. \quad (24)$$

where $\Gamma(v_R \longrightarrow l_i H_u)$ means decay rate of heavy Majorana RH $\nu$ of mass $M_1$ to a lepton and Higgs. In the electroweak sphaleron process, asymmetries produced by the out of equilibrium decay of $M_2$ and $M_3$ get washed out by lepton number violating interactions after $v_2$ or $M_1$ decay. In lepton number violating interactions, decays, inverse decays, and scatterings must be out of equilibrium when the right-handed neutrinos decay. In the basis where the RH $\nu$ mass matrix is diagonal, the type I seesaw mechanism contribution to $\epsilon_{\nu}^{CP}$ is given by [32]

$$\epsilon_{\nu}^{CP} = \frac{\Delta m_{\nu}^{2} Q_{12} + \Delta m_{\nu}^{2} Q_{13}}{\nu^2 \Sigma Q_{1j}^2 m_j}, \quad (25)$$

where $\nu$ is the Higg’s vev. $Q$ is a complex unitary orthogonal matrix where $Q$ is parameterized as [33] $Q = D_{\sqrt{M_{\nu}}} Y_u U_{\nu} D_{\sqrt{M_{\nu}}}^{-1}$, where $Y_u$ is the Dirac neutrino Yukawa couplings. To reproduce the physical, low-energy, parameters, i.e., the light neutrino masses (contained in $D_{\nu}$) and mixing angles and CP phases (contained in $U_{\nu}$), we have taken the most general Dirac neutrino mass matrix in broken $\mu - \tau$ symmetry framework as [11]

$$Y_\nu = \frac{M_{\nu}}{246 \, \text{GeV}} = \frac{1}{246 \, \text{GeV}} \left( \begin{array}{ccc} 11353.7 & -11193.7 & -11193.7 \\ -11193.7 & 12692.2 & 62440.4 \\ 11193.7 & 12692.2 & 62279.4 \end{array} \right) \left( \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right). \quad (26)$$

The numerical values of the above Dirac Neutrino Yukawa coupling matrix come from the best fit values of the parameters of $H, F, G$, and Equation (10) by performing a fit using the best fit solutions for fermion masses and mixing obtained assuming the type I seesaw dominance in [11]. The Dirac $\nu$ mass matrix is expressed in meaelectron volt units.

In the flavor basis, where the charged-lepton Yukawa matrix $Y_\ell$ and gauge interactions are flavor-diagonal,

$$D_K = U^T Y_\nu D_{\nu}^T Y_\nu U. \quad (27)$$

In terms of user-defined Dirac neutrino mass matrices, [11]

$$K = Y_\nu^T M_{\nu}^{-1} Y_\nu. \quad (28)$$

$U$ is the PMNS matrix and $M_R$ is the RH neutrino Majorana scale. We can always choose to work in a basis of right neutrinos where $M$ is diagonal $D_M = \text{Diag}(M_1, M_2, M_3)$ where $M_3 \gg M_1$. Equation (12) expresses $\epsilon_{\nu}^{CP}$ in terms of both the solar ($\Delta m_{\nu}^{2}$) and atmospheric ($\Delta m_{\nu}^{2}$) mass squared differences. Equation (12) also reveals that CP asymmetry is linked to the Dirac CPV phase. Here, we utilise this fact to generate the allowed region of the $\delta_{\nu}^{CP}$ phase in the context of leptogenesis. As has been discussed in [32], the lepton-antilepton asymmetry gets connected to both the solar and the atmospheric mass squared differences. The transformation of the leptonic asymmetry into a baryon asymmetry by nonperturbative $B + L$ violating (sphaleron) processes is discussed in [6].

Neutrino masses and mixings are connected with the atmospheric and solar neutrino fluxes; this is suitable to explain flavor changing neutral current processes and FCNC processes, like $\mu \rightarrow e + \gamma$ processes. In supersymmetric theories like cMSSM, NUHM, NUGM, and NUSM where the origin of the $\nu$ masses is via the seesaw mechanism, it can be shown that the prediction for $BR(\mu \rightarrow e, \gamma)$, $BR(\tau \rightarrow \mu, \gamma)$, and $BR(\tau \rightarrow e, \gamma)$ is in general larger than the experimental upper MEG bound [34, 35]. Also, some studies ondecays of $b$-flavored hadrons in the context of cMSSM/mSUGRA models is being done in [36].

A small explicit breaking of $\mu - \tau$ symmetry is put by hand, by inheriting the property in Equation (13).

$$h_{22} \neq h_{33}. \quad (29)$$

This introduces CP violation in PMNS matrices and results in $\theta_{13}$ being nonzero. Although an explicit breaking of the $\mu - \tau$ symmetry is used in [11], the magnitude of the breaking needed in order to get a large CP violating phase is very minute. This tiny amount of breaking which is used here for generating nonzero CP asymmetry producing a measurable CP violating phase via Dirac neutrino Yukawa couplings used from [11] is fixed to a well specific numerical value, which in turn allows one to replicate mixing angles precisely.

$$\frac{h_{22} - h_{33}}{h_{22} + h_{33}} = 0.0045. \quad (30)$$
The mass matrices defined in Equations (7), (8), (9), (10), (11), and (12) in the model are symmetric under CP invariance if Yukawa couplings are taken to be $\mu - \tau$ symmetric. Small explicit breaking of this symmetry defined in Equation (30) is enough to produce the required CP violating phase.

The Higgs field in the 10 and 126 representations are symmetric and invariant under the generalized $\mu - \tau$ symmetry while the 120-dimensional representation changes sign. This assumption in turn allows spontaneous breaking of the $\mu - \tau$ symmetry. 120 Higgs vev only contributes to off-diagonal elements. Also, one can break the exact $\mu - \tau$ symmetry explicitly through the use of Equation (30) which also involves both diagonal and off-diagonal elements. H, F (G) are any complex symmetric (anti) matrices in general, which are a measure of the fermion Yukawa couplings to the 10, 126, and 120 Higgs field, respectively. So the 22 entries and 33 entries in symmetric H matrix can be broken down by explicitly breaking the $\mu - \tau$ symmetry by hand. A required amount of the CP violating phase $\delta_{CP}$ is generated by explicitly breaking the $\mu - \tau$ symmetry. This assumption by using Equation (30) leads to $\sin^2 \theta_{23} \sim 0.42 - 0.63$ and $\sin^2 \theta_{13} > 0.006$. All these remarks enable one to use Dirac neutrino Yukawa coupling mass matrices reproduced by the explicit use of broken $\mu - \tau$ symmetry embedded in Equation (30).

The $\mu - \tau$ exchange symmetry in the neutrino mass matrix restricts the 2-3 and 1-3 neutrino mixing angles as $\theta_{23} = \pi/4$ and $\theta_{13} = 0$. We find that the $\mu - \tau$ symmetry breaking prefers a large CP violation to realize the symmetric and invariant under the generalized $\mu - \tau$ symmetry model for the generation of non-zero reactor mixing angle $\theta_{13}$ and lepton CP phase $\delta_{CP}$ in type I seesaw mechanism; in the light of leptogenesis, there can be a transformation of the lepton asymmetry into a baryon asymmetry by nonperturbative $B + L$ violating (sphaleron, Sakharov conditions) processes as discussed in [6]. A small explicit breaking of the $\mu - \tau$ symmetry model for the generation of non-zero lepton CP phase $\delta_{CP}$ in type I seesaw mechanism; in the light of leptogenesis, there can be a transformation of the lepton asymmetry into a baryon asymmetry by nonperturbative $B + L$ violating (sphaleron, Sakharov conditions) processes as discussed in [6].

The deviation of $\delta_{23}$ from maximal $\theta_{23} = \pi/4$ and the explanation of nonzero $\theta_{13} = 0$ are major predictions, goals, and motivation for breaking the $\mu - \tau$ symmetry.

The input parameters defined here are $r, s, t, p, \text{and } q$; Equations (7), (8), (9), (10), (11), and (12); the real elements of the matrices G, H, and F; Equations (13), (14), and (15); and the overall scales $r_{RL}$. There can be an overall rotation $R$ on $G, H, F : (G, H, F)^{RT}(G, H, F)R$. This equals to a choice of initial basis for the 16 plets of fermions. We can then set $h_{12} = 0$. This is done with a specific choice $R = R_{23}^T(\pi/4)R_{12}(\theta_{12})R_{23}(\pi/4)$. Here, $R_{ij}(\theta)$ symbolises rotation in the $i j$ plane by an angle $\theta$ and

$$\tan 2\theta_{12} = \frac{2\sqrt{2}h_{12}}{h_{11} - h_{22} - h_{23}}. \quad (31)$$

This rotation equals to reformulation of elements of $F$ and $G$ which still retain the same form as in Equations (13), (14), and (15). With the option $h_{12} = 0$, there are 15 input parameters in the case of type I seesaw mechanism. These input parameters when well organized set up 12 fermion masses and six mixing angles. The exact $\mu - \tau$ symmetric $H, F$ and $G$ are not able to provoke CP violation. CP violation is introduced by adding a small $\mu - \tau$ breaking difference between the 22 and 33 elements in H as seen in Equation (30).

We concentrated here in building comprehensive fits to fermion masses and mixings rather than taking into account the whole parameter space of the theory provided by the Yukawa couplings and basic parameters in the superpotential Lagrangian. Parameters in fermion mass matrices are a measure of the strengths of the light Higgs components in different $SO(10)$ Higgs representations.

Many models make use of complex vev to obtain $\tau^\mu - \tau$ breaking. In our approach, the breaking is by hand; i.e., we introduce small explicit breaking of $\mu - \tau$ symmetry in H. The models which use complex vev have 20 free parameters compared to 15 used here.

The explicit breaking of the $\mu - \tau$ symmetry is methodologically natural in the supersymmetric circumstances. However, one can achieve such breaking by introducing an additional 10 plets of the Higgs field which changes sign under the $\mu - \tau$ symmetry. Integrated benefaction of these two 10 plets would then provide an explicitly $\mu - \tau$ noninvariant H.

Also, if we abide by the best fit values of leptonic CP phase $\delta_{CP} = 222^\circ$ discussed in the literature [14, 37], then our scenario of explicit breaking of the $\mu - \tau$ symmetry by hand, evident from Equation (30), leads to $\delta_{CP} \in 222^\circ$ for inverted ordering of $\nu$ masses corresponding to $\langle m_{\nu} \rangle = 0.01$ eV which exactly matches with the best fit values of $\delta_{CP} = 222^\circ$ with $\Delta \chi^2 = 6.2$ w/o SK-ATM [14].

The novelty of this work lies in the successful explicit breaking of the $\mu - \tau$ symmetry within the $SO(10)$ framework in order to obtain a constrained picture of fermion masses. This framework provides a user-defined neutrino Yukawa coupling [11] for the Yukawa interactions associated with the broken $\mu - \tau$ symmetry model for the generation of non-zero reactor mixing angle $\theta_{13}$ and leptonic CP phase $\delta_{CP}$ in type I seesaw mechanism; in the light of leptogenesis, there can be a transformation of the lepton asymmetry into a baryon asymmetry by nonperturbative $B + L$ violating (sphaleron, Sakharov conditions) processes as discussed in [6]. A small explicit breaking of the $\mu - \tau$ symmetry model for the generation of non-zero reactor mixing angle $\theta_{13}$ and leptonic CP phase $\delta_{CP}$ in type I seesaw mechanism; in the light of leptogenesis, there can be a transformation of the lepton asymmetry into a baryon asymmetry by nonperturbative $B + L$ violating (sphaleron, Sakharov conditions) processes as discussed in [6].
for best fit values of $\theta_{13} = 8.41$ corresponding to $\Delta \chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. The favoured values of the lightest $v$ mass, $m_1$, in this case come out to be $[0.09, 0.1]$ eV. In case of inverted hierarchy, the variation of the $\delta_{\text{CP}}$ phase is found to be very intense with best fit values of $\theta_{13} = 8.49$ corresponding to $\Delta \chi^2 = 9.5$ w/o SK-ATM [14]. Values of $\delta_{\text{CP}}$ phase favoured are $\delta_{\text{CP}} = 220^\circ, 223^\circ, 252^\circ, 268^\circ, 293^\circ, 309^\circ, 345^\circ$ (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$). The allowed spectrum of the lightest $v$ mass is $m_1, e[0.02, 0.055]$ eV.

We also plot the allowed values of $|m_{ee}|$ eV for neutrinoless double beta decay and the Jarkslog invariant, $I_{\text{CP}}$, for normal ordering of $v$ masses. Prediction of future leptonic CP violation experiments should be able to rule out or take into account some of the results discussed in this work. If we abide by the best fit values of leptonic CP phase $\delta_{\text{CP}} = 222^\circ$ discussed in the literature [14, 37], then our scenario, $\delta_{\text{CP}} \in 222^\circ$, for inverted ordering of $v$ masses corresponding to $|m_{ee}| = 0.01$ eV exactly matches with the best fit values of $\delta_{\text{CP}} = 222^\circ$ with $\Delta \chi^2 = 6.2$ w/o SK-ATM [14].

The model here is motivated in the sense that it provides a specific form of Dirac neutrino Yukawa coupling matrix in the constraint of explicit breaking of the $\mu - \tau$ symmetry as is evident from the relation $(h_{32} - h_{31})/(h_{32} + h_{33}) = 0.0045$ between the $h_{32}$ and the $h_{33}$ elements of the H matrix. If we use this specific form of Dirac neutrino Yukawa coupling matrix to calculate baryon asymmetry of the Universe, then in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$ as a result of contribution of the type I seesaw mechanism to neutrino mass matrix, we get a range of allowed values of nonzero $\theta_{13}$ and large $\delta_{\text{CPV}}$ phase.

The motivation of this work is to relate baryogenesis through leptogenesis and the hint of CP violation in the neutrino oscillation data to a breaking of the $\mu - \tau$ symmetry. Small explicit tiny breaking of the $\mu - \tau$ symmetry allows a large Dirac CP violating phase in neutrino oscillation which in turn is characterized by awareness of measured value of nonzero $\theta_{13}$ and to provide a hint towards a better understanding of the experimentally observed near-maximal value of $\nu_\mu - \nu_\tau$ mixing angle $\theta_{23} = \pi/4$. Precise breaking of the $\mu - \tau$ symmetry can be achieved by adding a 120-plet Higgs to the 10 + 126-dimensional representation of Higgs.

Here in this work, the estimated three-dimensional density parameter space of the lightest neutrino mass $m_1$, $\delta_{\text{CP}}$, and reactor mixing angle $\theta_{13}$ is constrained here for the requirement of producing the observed value of baryon asymmetry of the Universe through the mechanism of leptogenesis. Carrying out numerical analysis, the allowed parameter space of $m_1$, $\delta_{\text{CP}}$, and $\theta_{13}$ is found out which can produce the observed baryon to photon density ratio of the Universe, the details of which are discussed below.

### 3. Numerical Analysis

In this section, numerical analysis has been carried out. Firstly, the free parameter called the lightest $v$ mass, the $v$ oscillation parameters like reactor angle $\theta_{13}$, Dirac CPV phase, $\delta_{\text{CP}}$, Majorana phases $\alpha_{21}$ and $\alpha_{31}$ in broken $\mu - \tau$ symmetry, and type I seesaw model are scanned to be very intense with best fit values of leptonic CP phase $\delta_{\text{CP}} = 222^\circ$ discussed in the literature [14, 37], then our scenario, $\delta_{\text{CP}} \in 222^\circ$, for inverted ordering of $v$ masses corresponding to $|m_{ee}| = 0.01$ eV exactly matches with the best fit values of $\delta_{\text{CP}} = 222^\circ$ with $\Delta \chi^2 = 6.2$ w/o SK-ATM [14].

We use the best fit values of $v$ oscillation parameters. The two mass square differences $\Delta m_{12}^2$ and $\Delta m_{13}^2$ are embedded in neutrino mixing matrix so we are left out with lightest $v$ mass as the only free parameter in this model. In the charged lepton basis, we parameterize the PMNS matrix $U_{\text{PMNS}}$ by diagonalizing the neutrino mass matrix $m_\nu$, in terms of three mixing angles $\theta_{ij}, (i, j = 1, 2, 3; i < j)$, one CP violating Dirac CPV phase $\delta_{\text{CP}}$, and two Majorana phases ($\alpha_{21}$ and $\alpha_{31}$) as follows:

$$U^* P^* m_\nu P U^T = m_\nu'^T.$$

The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix is $U_{\text{PMNS}}$, where $U$ is

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{i \delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13} \end{pmatrix},$$

where $\theta_{12} = 33.82^\circ, \theta_{23} = 48.3^\circ(48.6^\circ), \text{ and } \theta_{13} = 8.61^\circ(8.65^\circ)$ [14] in case of normal hierarchy (inverted hierarchy) are the solar, atmospheric, and reactor angles, respectively. The Majorana phases reside in $P$, where

$$P = \text{diag} \left( 1, e^{i \alpha_{21}}, e^{i (\alpha_{31} + \delta)} \right).$$

We have taken complex and orthogonal matrix $R = U_{\text{PMNS}}^*$ in terms of user-defined Dirac neutrino Yukawa couplings defined in Equation (13) in order to produce correct baryon asymmetry of the Universe.

For the normally ordered light $v$ masses, we have

$$m_{\nu}^\text{PMNS} = \text{diag} \left( M_1, M_2, M_3 \right) = M_1 \text{ diag} \left( 1, \frac{M_2}{M_1}, \frac{M_3}{M_1} \right) = M_1 \text{ diag} \left( 1, \frac{m_1}{m_2}, \frac{m_1}{m_3} \right),$$

with $m_1 \in [10^{-6} \text{eV}, 10^{-1} \text{eV}], m_2^2 - m_1^2 = 7.39 \times 10^{-5} \text{eV}^2$, and $m_3^2 - m_1^2 = 2.48 \times 10^{-3} \text{eV}^2$ as is evident from the $v$ oscillation
data [14], \( m_1 \) being the lightest of three \( \nu \) masses. For the inverted ordered light \( \nu \) masses, we have

\[
M_{\alpha \nu}^{\text{inv}} = \text{diag}(M_1, M_2, M_3) = M_1 \text{ diag} \left( 1, \frac{M_2}{M_1}, \frac{M_3}{M_1} \right) = M_1 \text{ diag} \left( 1, \frac{m_1}{m_3}, \frac{m_3}{m_3} \right),
\]

with \( m_3 \) being the lightest of three \( \nu \) masses. Here, we take \( M_1 \sim 10^{12} \text{ GeV} \). For normal ordering, the choices of the lightest neutrino mass is \( m_1 = 0.07118 \text{ eV} \) whereas for inverted ordering, the choice of the lightest neutrino mass is \( m_3 = 0.0657 \text{ eV} \). This sustainable allowance of \( m_1(m_3) = 0.07(0.065) \text{ eV} \) signifies a neutrino mass spectrum where the sum of absolute neutrino masses lies below the cosmological upper bound, \( \sum_{i=1}^{3} m_i(v_i) < 0.23 \text{ eV} \) [12]. Next, random scan of the \( \nu \) mixing matrix parameter space for NH and IH in order to produce correct the baryon asymmetry of the Universe \( 5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10} \) is performed in the following 3 \( \sigma \) range of \( \delta_{\text{CP}} \) with respect to the tabulated \( \chi^2 \) map of the Super-Kamiokande analysis of the data within \( \Delta \chi^2 = 6.2 \) [14]:

(i) \( m_1(m_3) \in [10^6 \text{ eV}, 0.1 \text{ eV}] \) \( \left| 10^6 \text{ eV}, 0.1 \text{ eV} \right| \)

(ii) \( \delta_{\text{CP}} \in [141, 370] \) for normal ordering

(iii) \( \delta_{\text{CP}} \in [205, 354] \) for inverted ordering

(iv) \( \theta_{13} \in [7.9, 8.9] \) for normal ordering for tabulated \( \Delta \chi^2 = 9.5 \text{ w/o SK-ATM} \) [14]

(v) \( \theta_{13} \in [8.0, 9.0] \) for normal ordering for tabulated \( \Delta \chi^2 = 9.5 \text{ w/o SK-ATM} \) [14]

(vi) \( \theta_{23} \in [40.8, 51.3] \) for normal ordering for tabulated \( \Delta \chi^2 = 6.2 \text{ w/o SK-ATM} \) [14]

While doing parameter scan, we find favoured values of the lightest \( \nu \) mass and dirac CPV phase \( \delta_{\text{CP}} \), for producing correct baryon asymmetry of the Universe, \( 5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10} \).

The lepton flavor effects are significant if the lightest right-handed Majorana neutrino mass \( M_{\nu_R} \) is below \( 10^{12} \text{ GeV} \). Here, \( M_1 = 10^{12} \text{ GeV} \). In the type I seesaw mechanism, one can always find the right-handed neutrino mass matrix as

\[
M_{\alpha \beta} = -\left( m_{13}^T \right)_{\alpha \rho} \left( M_{11}^{-1} \right)_{\rho \beta},
\]

where \( (m_{13})_{\alpha \rho} \) is the Dirac mass matrix. We consider a Dirac neutrino mass matrix defined in Equation (13). Here, when we fix \( m_1(m_3), Y_{\nu}, M_{R}, \) the remaining free parameter in the neutrino sector within our broken \( \mu - \tau \) framework is the leptonic CPV phase \( \delta_{\text{CP}} \). When we vary the CPV phase \( \delta_{\text{CP}} \), we compute the favoured regions of \( m_1(m_3) \). The variations of leptonic CPV phase \( \delta_{\text{CP}} \) with \( m_1(m_3), \theta_{13}, I_{\text{CP}}, \) and \( m_{ee} \) for \( 0\nu\beta\beta \) as shown in the figures discussed here.

For global fit values of \( \nu \) oscillation parameters, we compute the Jarlskog invariant, \( \delta_{\text{CP}} \), given by PMNS matrix elements \( U_{ai} \). We also compute the Jarlskog invariant for allowed values of the \( \delta_{\text{CP}} \) phase, \( \theta_{13} \), and lightest \( \nu \) mass explored here in this work for both normal ordering and inverted ordering.

\[
I_{\text{CP}} = \text{Im} \left( U_{ei} U_{\mu i} U_{\tau i}^* U_{\mu i}^* \right) = \delta_{13} s_{12} \delta_{12} c_{12} \delta_{13} c_{13} \sin \delta_{\text{CP}}.
\]
Figure 6: Predictions in the broken $\mu - \tau$ symmetry model for normal ordering: (b) the preferred three-dimensional regions of the ($\delta_{CP}, \theta_{23}, J_{CP}$) plane for the best fit values of $\theta_{13} = 8.41$ of $\Delta \chi^2 = 9.5$ w/o SK-ATM [14]; (a) allowed two-dimensional space of the ($\delta_{CP}, J_{CP}$) plane for the best fit values of $\theta_{13} = 8.41$ of $\Delta \chi^2 = 9.5$ w/o SK-ATM [14].

Figure 7: Predictions in the broken $\mu - \tau$ symmetry model for normal ordering: (b) shows the allowed two-dimensional space of the ($\delta_{CP}, J_{CP}$) plane for an absolute range of $\delta_{CP} \in [-180,180]$. 
Figure 8: Predictions in the broken $\mu - \tau$ symmetry model for normal ordering: (b) preferred three-dimensional regions of the $(\delta_{CP}, \theta_{23}, J_{CP})$ plane for favoured values of $\delta_{CP} \in [303,308]$ (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) for the best fit values of $\theta_{13} = 8.41$ of $\Delta \chi^2 = 9.5$ w/o SK-ATM [14]; (a) allowed two-dimensional space of the $(\delta_{CP}, J_{CP})$ plane for favoured values of $\delta_{CP} \in [303,308]$ (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) for the best fit values of $\theta_{13} = 8.41$ of $\Delta \chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$).

Figure 9: Predictions in the broken $\mu - \tau$ symmetry model for normal ordering: (b) preferred three-dimensional regions of the $(\theta_{13}, \theta_{23}, J_{CP})$ plane for the best fit values of $\delta_{CP} = 222^\circ$ of $\Delta \chi^2 = 6.2$ w/o SK-ATM [14]; (a) allowed two-dimensional space of the $(\theta_{13}, J_{CP})$ plane for the best fit values of $\delta_{CP} = 222^\circ$ of $\Delta \chi^2 = 6.2$ w/o SK-ATM [14].

Figure 10: Predictions in the broken $\mu - \tau$ symmetry model for normal ordering: variation of $J_{CP}$ as a function of $\theta_{23}$, $\theta_{23} \in [40,85,1.3]$, for normal ordering for tabulated $\Delta \chi^2 = 6.2$. [14].
Figure 11: Predictions in the broken $\mu - \tau$ symmetry model for normal ordering: (a) depicts predicted three-dimensional space of $(m_1, \delta_{\text{CP}}, \theta_{13})$ for $m_{\nu}$ [eV], $0\nu \beta \beta$ decay for favoured values of $m_1$, $\delta_{\text{CP}}, \theta_{13}$ (in the light of recent ratio of the baryon to photon density bounds, $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$). (b) Depicts the predicted three-dimensional space of $(m_1, \delta_{\text{CP}}, \theta_{13})$ for $m_{\nu}$ [eV], $0\nu \beta \beta$ decay for values of $m_1$, $\delta_{\text{CP}}$, and $\theta_{13}$ in the given three $\sigma$ range, corresponding to $\Delta \chi^2 = 6.2$ and $\Delta \chi^2 = 9.5$ w/o SK-ATM [14].
We also calculate the favourable space of the effective mass for $0\nu\beta\beta$ decay for favourable values of the $\delta_{CP}$ phase, $\theta_{13}$, and lightest $\nu$ mass given by

$$m_{ee} = \left| m_1 c_{12} c_{13}^2 + m_2 s_{12} c_{13} e^{i\alpha_2} + m_3 s_{13} e^{i(\alpha_3 - 2\delta_{CP})} \right|. \tag{39}$$

The colour coding in the different figures imply the values of baryon asymmetry of the Universe in the allowed range, $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$.

In Figure 1, we have presented the predictions in the broken $\mu-\tau$ symmetry model for normal ordering. Panel (a) conveys the predicted three-dimensional space of $(m_{ee}, \delta_{CP}, m_1)$ for $m_{ee}$ $[eV]$, $0\nu\beta\beta$ decay for favoured values of $m_1$, $\delta_{CP}$, and $\theta_{13}$ (in the light of recent ratio of the baryon to photon density bounds, $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$). Depicts predicted three-dimensional space of $(m_{ee}, \delta_{CP}, m_1)$ for $m_{ee}$ $[eV]$, $0\nu\beta\beta$ decay for favoured values of $m_1$, $\delta_{CP}$, $m_{ee}$ for values of $\delta_{CP}$ in the given three $\sigma$ range, corresponding to $\Delta\chi^2 = 9.5$ w/o SK-ATM [14].
\[ -8.35^\circ \] in light of the correct baryon to photon density bounds \[ 5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10} \]. Panel (b) manifests itself in the predicted favoured value \((m_1, \delta_{\text{CP}}, \theta_{13})\) plane, for the best fit value of \(\theta_{13} = 8.41^\circ\) with \(\Delta \chi^2 = 9.5\) \cite{14}. The favoured value of \(\delta_{\text{CP}}\) is around 307° in the light of the correct baryon asymmetry of the Universe which can be manifested from the colour coding in the figure.

We have shown the contour plot for predicted favoured values of \((m_1, \theta_{13})\) plane for the best fit values of \(\delta_{\text{CP}} = 222^\circ\) of \(\Delta \chi^2 = 6.2\) w/o SK-ATM \cite{14} (as allowed by updated values of correct baryon asymmetry of the Universe) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix in Figure 5. The colour coding in the contour plot also shows that for \(\eta\) to lie in the interval, \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\), the allowed lightest neutrino mass \(m_1\) must lie around 0.095 eV.

In Figure 2, we have speculated the predictions in the broken \(\mu - \tau\) symmetry model for normal ordering. The (a) presents the three-dimensional plot of preferred values of \((\eta, m_1, \delta_{\text{CP}})\) plane for the best fit values of \(\theta_{23} = 48.6^\circ\) of \(\Delta \chi^2 = 6.2\) \cite{14} as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. The preferred value...
of the leptonic CPV phase came out to be around $304^\circ$-$307^\circ$. Similarly, the panel (b) communicates the three-dimensional plot of favourable values of the $(\eta, m_1, \theta_{13})$ plane, for the best fit value of $\delta_{\text{CP}} = 222^\circ$ of $\Delta \chi^2 = 0.2$ [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$).

In Figure 3, we have depicted the predictions in the broken $\mu - \tau$ symmetry model for normal ordering: panel (a) favours preferred values of $\delta_{\text{CP}}$ for the best fit values of $\theta_{13} = 8.41$ of $\Delta \chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. The preferred value of the leptonic CPV phase $\delta_{\text{CP}}$ came out to be around $304^\circ$-$307^\circ$ as is obvious from the figure. Similarly, panel (b) shows the favoured values of the lightest neutrino mass, $m_1$, for the best fit value of $\delta_{\text{CP}} = 222^\circ$ of $\Delta \chi^2 = 0.2$ [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$). The favoured values of $m_1$ is around 0.09 eV–0.1 eV.

Figure 6 reveals the predictions in the broken $\mu - \tau$ symmetry model for normal ordering: panel (b) shows the preferred three-dimensional regions of the $(\delta_{\text{CP}}, \theta_{13}, J_{\text{CP}})$ plane for the best fit values of $\theta_{13} = 8.41$ of $\Delta \chi^2 = 9.5$ w/o SK-ATM [14]. Panel (a) presents allowed two-dimensional space of the $(\delta_{\text{CP}}, J_{\text{CP}})$ plane for the best fit values of $\theta_{13} = 8.41$ of $\Delta \chi^2 = 9.5$ w/o SK-ATM [14].

In Figure 7, we present the predictions in the broken $\mu - \tau$ symmetry model for normal ordering. Panel (b) favours allowed two-dimensional space of the $(\delta_{\text{CP}}, J_{\text{CP}})$ plane for an absolute range of $\theta_{\text{CP}} \in [-180^\circ, +180^\circ]$.

In Figure 8, we have speculated the predictions in the broken $\mu - \tau$ symmetry model for normal ordering. Panel (b) conveys preferred three-dimensional regions of the $(\delta_{\text{CP}}, \theta_{23}, J_{\text{CP}})$ plane for favoured values of $\delta_{\text{CP}} \in [303,308]$ (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) for best fit values of $\theta_{13} = 8.41$ of $\Delta \chi^2 = 9.5$ w/o SK-ATM [14]. Panel (a) presents allowed two-dimensional space of the $(\delta_{\text{CP}}, J_{\text{CP}})$ plane for favoured values of $\delta_{\text{CP}} \in [303,308]$ (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$) for the best fit values of $\theta_{13} = 8.41$ of $\Delta \chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$).

In Figure 9, we have shown predictions in the broken $\mu - \tau$ symmetry model for normal ordering. Panel (b) presents preferred three-dimensional regions of the $(\theta_{23}, J_{\text{CP}})$ plane for the best fit values of $\delta_{\text{CP}} = 222^\circ$ of $\Delta \chi^2 = 6.2$ w/o SK-ATM [14]. The left panel communicates the allowed two-dimensional space of the $(\delta_{\text{CP}}, J_{\text{CP}})$ plane for the best fit values of $\delta_{\text{CP}} = 222^\circ$ of $\Delta \chi^2 = 6.2$ w/o SK-ATM [14].

Figure 10 depicts the predictions in the broken $\mu - \tau$ symmetry model for normal ordering. The variation of $J_{\text{CP}}$ as a function of $\theta_{23}$, $J_{\text{CP}} \in [40,85,153]$ for normal ordering for tabulated $\Delta \chi^2 = 6.2$. [14] is speculated.

In Figure 11, we have presented predictions in the broken $\mu - \tau$ symmetry model for normal ordering. Panel (a) depicts three-dimensional space of $(m_1, \delta_{\text{CP}}, \theta_{13})$ for $m_\alpha$ [eV], $0\nu\beta\beta$ decay for favoured values of $m_1$, $\delta_{\text{CP}}$, and $\theta_{13}$ (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$). Panel (b) demonstrated three-dimensional space of $(m_1, \delta_{\text{CP}}, \theta_{13})$ for $m_\alpha$ [eV], $0\nu\beta\beta$ decay for values of $m_1$, $\delta_{\text{CP}}$, and $\theta_{13}$ in the given three $\sigma$ range, corresponding to $\Delta \chi^2 = 6.2$ and $\Delta \chi^2 = 9.5$ w/o SK-ATM [14].

In Figure 12, we have introduced predictions in the broken $\mu - \tau$ symmetry model for normal ordering. The figure depicts the density plot of $(m_1, \delta_{\text{CP}})$ for $m_\alpha$ [eV], $0\nu\beta\beta$ decay.
Figure 17: Predictions in the broken $\mu - \tau$ symmetry model for inverted ordering. (a) The density plot of predicted favoured values of the $(m_3, \theta_{13})$ plane for the best fit values of $\delta_{\text{CP}} = 285^\circ$ of $\Delta \chi^2 = 6.2$ w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Similarly, (b) shows the predicted two-dimensional space of $(m_3, \theta_{13})$ for $m_{ee}$ [eV], $0\nu\beta\beta$ decay, for the best fit values of $\delta_{\text{CP}} = 285^\circ$ of $\Delta \chi^2 = 6.2$ w/o SK-ATM [14].
for favoured values of \(m_1\), \(\delta_{\text{CP}}\), and \(\theta_{13}\) (in the light of recent ratio of the baryon to photon density bounds, \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\))

In Figure 13, we have presented predictions in the broken \(\mu - \tau\) symmetry model for normal ordering. Panel (a) depicts predicted three-dimensional space of \((m_{ee}, \delta_{\text{CP}}, m_1)\) for \(m_{ee}\) [eV], \(0\nu\beta\beta\) decay for favoured values of \(m_1\), \(\delta_{\text{CP}}, \theta_{13}\) (in the light of recent ratio of the baryon to photon density bounds, \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)). Panel (b) depicts predicted three-dimensional space of \((m_{ee}, \delta_{\text{CP}}, m_1)\) for \(m_{ee}\) [eV], \(0\nu\beta\beta\) decay for favoured values of \(m_1\), \(\delta_{\text{CP}}, m_{ee}\) for values of \(\delta_{\text{CP}}\) in the given three \(\sigma\) range, corresponding to \(\Delta \chi^2 = 9.5\) w/o SK-ATM [14].

In Figure 14, we have conveyed the three-dimensional space of \((m_{ee}, \delta_{\text{CP}}, \theta_{13})\) for \(m_{ee}\) [eV], \(0\nu\beta\beta\) decay for favoured values of \(m_1\), \(\delta_{\text{CP}}, \theta_{13}\) (in the light of recent ratio of the baryon to photon density bounds, \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)) for the lightest \(\nu\) mass \(m_1 = 0.07118\) eV.

In Figure 15, we have shown the predictions in the broken \(\mu - \tau\) symmetry model for inverted ordering. Panel (a) predicts favoured values of \(\delta_{\text{CP}}\) (in the light of recent ratio of the baryon to photon density bounds, \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)) for the lightest \(\nu\) mass \(m_3 = 0.0657\) eV, as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Similarly, panel (b) shows predicted allowed three-dimensional space of the \((\delta_{\text{CP}}, \theta_{13}, J_{\text{CP}})\) plane for allowed regions of Jarkslog invariant, \(J_{\text{CP}}\), values for the best fit value of \(\theta_{13} = 48.6^\circ\) of \(\Delta \chi^2 = 6.2\) [14] as a result of contribution of the type I seesaw mechanism to neutrino mass matrix.

In Figure 4, we have shown predictions in the broken \(\mu - \tau\) symmetry model for inverted ordering. The three-dimensional plot for predicted favoured values of \((m_{13}, \delta_{\text{CP}}, \eta)\) plane for the best fit values of \(\theta_{13} = 8.49^\circ\) of \(\Delta \chi^2 = 9.5\) w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds, \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix is presented. The favoured values of the \(\delta_{\text{CP}}\) phase predicted here are 295° and 303°–306° in the light of recent ratio of baryon to photon density bounds, \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\). The value of the lightest \(\nu\) mass \(m_1\) depicted here from the figure is around 0.005 eV.

In Figure 16, panel (a) presents the density plot of predicted favoured values of the \((m_3, \theta_{13})\) plane for the best fit values of \(\delta_{\text{CP}} = 285^\circ\) of \(\Delta \chi^2 = 6.2\) w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. The favoured values of the lightest \(\nu\) mass depicted here are \(m_3 \sim 0.03\) eV–0.047 eV for producing correct baryon asymmetry of the Universe. Also, the favoured value reactor angle, \(\theta_{13}\), lies in the range 8°–9°. Similarly, in panel (b), we have shown the three-dimensional plot for predicted favoured values of the \((\theta_{13}, \delta_{\text{CP}}, \eta)\) plane for the lightest \(\nu\) mass, \(m_3 = 0.0657\) (in the light of recent ratio of the baryon to photon density bounds, \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix.

We also plot the allowed values of \(|m_{ee}|\) eV for neutrino-less double beta decay and the Jarkslog invariant, \(J_{\text{CP}}\), in Figures 15 and 17–20 for inverted ordering of \(\nu\) masses.

In Figure 17, we have shown the predictions in the broken \(\mu - \tau\) symmetry model for inverted ordering. Panel (a) shows the density plot of predicted favoured values of the \((m_{13}, \theta_{13})\) plane for the best fit values of \(\delta_{\text{CP}} = 285^\circ\) of \(\Delta \chi^2 = 6.2\) w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Similarly, panel (b) depicts the predicted two-dimensional space of \((m_{13}, \theta_{13})\) for \(m_{ee}\) [eV], \(0\nu\beta\beta\) decay, for the best fit values of \(\delta_{\text{CP}} = 285^\circ\) of \(\Delta \chi^2 = 6.2\) w/o SK-ATM [14]. The favoured values of the lightest \(\nu\) mass depicted here are \(m_3\).
...0.03 eV–0.047 eV for producing correct baryon asymmetry of the Universe. Also, the favoured value reactor angle, $\theta_{13}$, lies in the range 8°–9°. In Figure 18, we show the predicted allowed three-dimensional space of the $(\delta_{CP}, \theta_{23}, J_{CP})$ plane for allowed regions of Jarkslog invariant, $J_{CP}$, values for the best fit value of $\theta_{13} = 8.49^\circ$ corresponding to $\Delta \chi^2 = 9.5$ w/o SK-ATM [14].

In Figure 19, we depict in panel (a) the predicted three-dimensional space of $(m_{ee}, \delta_{CP}, m_3)$ for values of $m_3 \in [10^{-6}, 0.1]$ eV and $\delta_{CP}$ in the given 3 $\sigma$ range, corresponding to $\Delta \chi^2 = 6.2$, and the best fit values of $\theta_{13} = 8.49^\circ$ corresponding to $\Delta \chi^2 = 9.5$ w/o SK-ATM [14]. Panel (b) shows the predicted three-dimensional space of $(m_{ee}, \delta_{CP}, \theta_{13})$ for values of the lightest $\nu$ mass, $m_3 = 0.0657$ eV, $\delta_{CP}$, and $\theta_{13}$ in the given 3 $\sigma$ range, corresponding to $\Delta \chi^2 = 6.2$ and $\Delta \chi^2 = 9.5$ w/o SK-ATM [14].

The favoured values of the leptonic CPV phase, $\delta_{CP}$, predicted here are $235^\circ–237^\circ$ consistent with $m_{ee} = 0.01$ eV. Panel (b) shows the predicted three-dimensional space of $(m_{ee}, \delta_{CP}, \theta_{13})$ for values of the lightest $\nu$ mass, $m_3 = 0.0657$ eV, $\delta_{CP}$, and $\theta_{13}$ in the given 3 $\sigma$ range, corresponding to $\Delta \chi^2 = 6.2$ and $\Delta \chi^2 = 9.5$ w/o SK-ATM [14] for inverted ordering. The favoured values of the reactor angle predicted here are $\theta_{13} \sim 8.6^\circ–8.9^\circ$ corresponding to a value of $m_{ee} = 0.01$ eV [38]. The favoured values of the leptonic CPV phase, $\delta_{CP}$, predicted here are $222^\circ$ consistent with, limits on, $(m_{ee})$, $m_{ee} = 0.01$ eV [38]. In Figure 20, we have depicted the predicted density plot of $(\delta_{CP}, m_3)$ for values of $m_3 \in [10^{-6}, 0.1]$ eV and $\delta_{CP}$ in the given 3 $\sigma$ range, corresponding to $\Delta \chi^2 = 6.2$, and the best fit values of $\theta_{13} = 8.49^\circ$ corresponding to $\Delta \chi^2 = 9.5$. In Figure 19, we show the predicted allowed three-dimensional space of the $(\delta_{CP}, \theta_{23}, J_{CP})$ plane for allowed regions of Jarkslog invariant, $J_{CP}$, values for the best fit value of $\theta_{13} = 8.49^\circ$ corresponding to $\Delta \chi^2 = 9.5$ w/o SK-ATM [14].
by hand as is evident from Equation (30) with the specific numerical value 0.0045 is sufficient to generate the large required CP violating phase discussed here. We have considered the type I seesaw model with very tiny explicit $\mu - \tau$ symmetry breaking. This model with very tiny explicit $\mu - \tau$ symmetry breaking is motivated in the sense as it is described by the predictions $\sin^2\theta_{13} \sim 0.42 - 0.63$ and nonzero reactor angle $\sin^2\theta_{13} > 0.005$ and large required CP violation in neutrino oscillations.

In Figure 1, for the best fit values of $\delta_{\text{CP}} = 222^\circ$ with $\Delta \chi^2 = 6.2$ w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe), the allowed spectrum of the lightest $\nu$ mass here lies in the range $0.09 \text{ eV} - 0.095 \text{ eV}$ corresponding to favoured values of reactor angle, $\theta_{13}$, in the interval, $8.1^\circ - 8.35^\circ$ in light of the correct baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$. Panel (a) manifests itself in the predicted favoured value $(m_1, \delta_{\text{CP}})$ plane, for the best fit value of $\theta_{13} = 8.41$ with $\Delta \chi^2 = 9.5$ [14]. The favoured value of $\delta_{\text{CP}}$ is around $307^\circ$ in the light of the correct baryon asymmetry of the Universe which can be manifested from the colour coding in the figure. We have shown for the best fit values of $\delta_{\text{CP}} = 222^\circ$ of $\Delta \chi^2 = 6.2$ w/o SK-ATM [14] (as allowed by updated values of correct baryon asymmetry of the Universe) in Figure 5, (for $\eta$ to lie in the interval, $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$), $m_1$, must lie around $0.095 \text{ eV}$. In Figure 2, for the best fit values of $\theta_{13} = 8.41$ of $\Delta \chi^2 = 9.5$ w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$), the preferred value of the leptonic CPV
phase came out to be around 304°-307°. Similarly, panel (b) communicates the three-dimensional plot of favourable values of the \((\eta, m_1, \theta_{13})\) plane, for the best fit value of \(\delta_{\text{CP}} = 222°\) of \(\Delta \chi^2 = 6.2\) [14] (in the light of recent ratio of the baryon to photon density bounds \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)). In Figure 3, for the best fit values of \(\theta_{13} = 8.41\) of \(\Delta \chi^2 = 9.5\) w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)), the preferred value of the leptonic CPV phase \(\delta_{\text{CP}}\) came out to be around 304°-307° as is obvious from the figure. Similarly, panel (b) shows the favoured values of the lightest neutrino mass, \(m_1\), for the best fit value of \(\delta_{\text{CP}} = 222°\) of \(\Delta \chi^2 = 6.2\) [14] (in the light of recent ratio of the baryon to photon density bounds \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)). The favoured values of \(m_1\) is around 0.09 eV–0.1 eV.

In Figure 8, panel (a) presents the allowed two-dimensional space of the \((\delta_{\text{CP}}, I_{\text{CP}})\) plane for favoured values of \(\delta_{\text{CP}} \in [303, 308]\) (in the light of recent ratio of the baryon to photon density bounds \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)) for the best fit values of \(\theta_{13} = 8.41\) of \(\Delta \chi^2 = 9.5\) w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)). In Figure 4, for the best fit values of \(\theta_{13} = 8.49°\) of \(\Delta \chi^2 = 9.5\) w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds, \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)), the value of the lightest \(\nu\) mass \(m_1\) depicted here from the figure is around 0.005 eV. In Figure 16, for the best fit values of \(\delta_{\text{CP}} = 285°\) of \(\Delta \chi^2 = 6.2\) w/o SK-ATM [14] (allowed by updated values of correct baryon asymmetry of the Universe), the favoured values of the lightest \(\nu\) mass depicted here are \(m_1 \sim 0.03\) eV–0.047 eV for producing correct baryon asymmetry of the Universe. Also, the favoured values reactor angle, \(\theta_{13}\), lies in the range 8°–9°. Similarly, in panel (b), we have shown the three-dimensional plot for the predicted favoured values of the \((\theta_{13}, \delta_{\text{CP}}, \eta)\) plane for the lightest \(\nu\) mass, \(m_1 = 0.0657\) (in the light of recent ratio of the baryon to photon density bounds, \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. Figure 19 depicts in panel (a) the predicted three-dimensional space of \((m_{\text{ee}}, \delta_{\text{CP}}, m_3)\) for values of \(m_{\text{ee}} \in [10^{-6}, 0.1]\) eV and \(\delta_{\text{CP}}\) in the given 3 \(\sigma\) range, corresponding to \(\Delta \chi^2 = 6.2\), and the best fit values of \(\theta_{13} = 8.49°\) corresponding to \(\Delta \chi^2 = 9.5\) w/o SK-ATM [14]. The favoured values of the lightest \(\nu\) mass depicted here are \(m_3 \sim 0.045\) eV–0.06 eV corresponding to a value of \(m_{\text{ee}} = 0.01\) eV. The favoured values of the leptonic CPV phase, \(\delta_{\text{CP}}\), predicted here are 235°–237° consistent with \(m_{\text{ee}} = 0.01\) eV. Panel (b) shows the predicted three-dimensional space of \((m_{\text{ee}}, \delta_{\text{CP}}, \theta_{13})\) for values of the lightest \(\nu\) mass, \(m_1 = 0.0657\) eV, \(\delta_{\text{CP}}\), and \(\theta_{13}\) in the given 3 \(\sigma\) range, corresponding to \(\Delta \chi^2 = 6.2\) and \(\Delta \chi^2 = 9.5\) w/o SK-ATM [14] for inverted ordering. The favoured values of the reactor angle predicted here are \(\theta_{13} \sim 8.6° – 8.9°\) corresponding to a value of \(m_{\text{ee}} = 0.01\) eV. The favoured values of the leptonic CPV phase, \(\delta_{\text{CP}}\), predicted here are 222° consistent with, limits on, \(\langle m_{\text{ee}}\rangle\), \(m_{\text{ee}} = 0.01\) eV.

In this work, we learn that by using user-defined Dirac Neutrino Yukawa couplings [11] for the Yukawa interactions associated with the broken \(\mu - \tau\) symmetry model for the generation of the nonzero reactor mixing angle \(\theta_{13}\) and leptonic CP phase \(\delta_{\text{CP}}\) in the type I seesaw mechanism in the light of leptogenesis, there can be a transformation of the lepton asymmetry into a baryon asymmetry by nonperturbative \(B + L\) violating (sphaleron, Sakharov conditions) processes as discussed in [6]. A small explicit breaking of \(\mu - \tau\) symmetry [11] inherits the property of generating nonzero CP violation in \(U_{\text{PMNS}}\) matrices and \(\delta_{\text{CP}}\) phase and results in \(\theta_{13}\), being nonzero. Here, we consider the type I seesaw as the main donor to neutrino mass. We also take into account both inverted and normal ordering of neutrino mass spectrum as well as two different types of the lightest neutrino mass \(m_1\) (\(m_1 = 0.07118\) eV(0.0657 eV)) to visualise the results of hierarchical \(\nu\) mass spectrum. In the case of normal ordering of \(\nu\) masses, the dependence of the \(\delta_{\text{CP}}\) phase on the lightest \(\nu\) mass is predicted in Figures 1–3 (in the light of recent ratio of the baryon to photon density bounds, \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)). The favoured values of the \(\delta_{\text{CP}}\) phase is found to lie between \(\delta_{\text{CP}} \in [304°, 307°]\) for the best fit values of \(\theta_{13} = 8.41\) corresponding to \(\Delta \chi^2 = 9.5\) w/o SK-ATM [14] (in the light of recent ratio of the baryon to photon density bounds \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)) as a result of contribution of the type I seesaw mechanism to neutrino mass matrix. The favoured values of the lightest \(\nu\) mass, \(m_1\), in this case come out to be \(\in [0.09, 0.1]\) eV. In the case of inverted hierarchy, the variation of the \(\delta_{\text{CP}}\) phase is found to be very intense with the best fit values of \(\theta_{13} = 8.49\) corresponding to \(\Delta \chi^2 = 9.5\) w/o SK-ATM [14]. Values of the \(\delta_{\text{CP}}\) phase favoured are \(\delta_{\text{CP}} = 220°\), 223°, 252°, 268°, 293°, 309°, 345° (in the light of recent ratio of the baryon to photon density bounds \(5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}\)) as is evident from Figure 4. The allowed spectrum of the lightest \(\nu\) mass is \(m_3\), \(\in [0.02, 0.055]\) eV. We also plot the allowed values of \(m_{\text{ee}}\) eV for neutrinoless double beta decay and the Jarlskog invariant, \(J_{\text{CP}}\), in Figures 6–14 for normal ordering of \(\nu\) masses. Prediction of future leptonic CP violation experiments should be able to rule out or take into account some of the results discussed.

### Table 1: Values of the \(\delta_{\text{CP}}\) phase giving correct updated values of baryon asymmetry.

<table>
<thead>
<tr>
<th>(m_1)</th>
<th>(\delta_{\text{CP}})</th>
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<tr>
<td>0.085±0.1 eV (NH)</td>
<td>304°–307°</td>
</tr>
<tr>
<td>0.03±0.047, 0.045±0.06 eV (IH)</td>
<td>220°, 222°, 252°, 268°, 293°, 309°, 345°</td>
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in this work. If we abide by the best fit values of leptonic CP phase \( \delta_{\text{CP}} = 222^\circ \) discussed in the literature [14, 37], then our scenario, \( \delta_{\text{CP}} \in 222^\circ \), for inverted ordering of \( \nu \) masses corresponding to \( (m_{\nu_e}) = 0.01 \text{eV} \) exactly matches with the best fit values of \( \delta_{\text{CP}} = 222^\circ \) with \( \Delta \chi^2 = 6.2 \) w/o SK-ATM [14]. We show the variation of baryon asymmetry with the leptonic \( \delta_{\text{CP}} \) phase in Table 1.

Future LBL experiments will hunt for the leptonic CP phase and potentially will measure it with precision. Neutrinoless double beta decay will indicate towards the Majorana CPV phase. The fundamental mysteries in the Universe are no less double beta decay will indicate towards the Majorana phase and potentially will measure it with precision. Neutrino oscillations with external constraints in Super-Kamiokande I-IV, determination of the status of leptonic CP asymmetry (T2K, NOvA, T2HK, DUNE), determination of the type of neutrino mass ordering (T2K + NOvA, JUNO, PINGU, ORCA, T2HK, DUNE), and determination of the order of absolute neutrino mass scale (KATRIN, cosmology) are few of the most challenging tasks today. The ideas presented in this work may definitely will rule in or rule out some of the favoured space in few of the above experiments.

Conflicts of Interest
The author declares that she has no conflicts of interest.

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References


