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Research Article

Hunting for Direct CP Violation in $\bar{B}_s^0 \longrightarrow \pi^+\pi^-K^{*0}$

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In perturbative QCD approach, based on the first order of isospin symmetry breaking, we study the direct CP violation in the decay of $\bar{B}_s^0 \longrightarrow \rho(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$. An interesting mechanism is applied to enlarge the *CP* violating asymmetry involving the charge symmetry breaking between ρ and ω. We find that the CP violation is large by the $\rho-\omega$ mixing mechanism when the invariant masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. For the decay process of $\bar{B}_s^0 \longrightarrow \rho(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$, the maximum CP violation can reach −59.12%. Furthermore, taking $\rho-\omega$ mixing into account, we calculate the branching ratio for $\bar{B}_s^0 \longrightarrow \rho(\omega)K^{*0}$. We also discuss the possibility of observing the predicted *CP* violation asymmetry at the LHC.

1. Introduction

Charge-parity (CP) violation is an open problem, even though it has been known in the neutral kaon systems for more than five decades [1]. The study of CP violation in the heavy quark systems is important to our understanding of both particle physics and the evolution of the early universe. Within the standard model (SM), CP violation is related to the nonzero weak complex phase angle from the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes the mixing of the three generations of quarks [2, 3]. Theoretical studies predicted large CP violation in the B meson system [4–6]. In recent years, the LHCb collaboration has measured sizable direct CP asymmetries in the phase space of the three-body decay channels of $B^\pm \longrightarrow \pi^\pm\pi^+\pi^-$ and $B^\pm \longrightarrow K^\pm\pi^+\pi^-$ [7–9]. These processes are also valuable for studying the mechanism of multibody heavy meson decays. Hence, more attention has been focused on the nonleptonic B meson three-body decay channels in searching for CP violation, both theoretically and experimentally.

The direct CP violation in the b hadron decays occurs through the interference of at least two amplitudes with different weak phase ϕ and strong phase δ. The weak phase difference ϕ is determined by the CKM matrix elements, while the strong phase can be produced by the hadronic matrix elements and interference between the intermediate states. The hadronic matrix elements are not still well determined by the theoretical approach. The mechanism of two-body B decays is still not quite clear, although many physicists are devoted to this field. Many factorization approaches have been developed to calculate the two-body hadronic decays, such as the naive factorization approach [10–13], the QCD factorization (QCDF) [14–18], perturbative QCD (pQCD) [19–21], and soft-collinear effective theory (SCET) [22–24]. Most factorization approaches are based on heavy quark expansion and light-cone expansion in which only the leading power or part of the next to leading power contributions are calculated to compare with the experiments. However, the different methods may present different strong phases so as to affect the value of the CP violation. Meanwhile, in order to have a large signal of CP violation, we need an appeal to some phenomenological mechanism to obtain a large strong phase δ. In Refs. [25–30], the authors studied the direct CP violation in hadronic B (including B_s and Λ_b) decays through the interference of tree

and penguin diagrams, where ρ-ω mixing was used for this purpose in the past few years and focused on the naive factorization and QCD factorization approaches. This mechanism was also applied to generalize the pQCD approach to the three-body nonleptonic decays in $B^{0,\pm} \longrightarrow \pi^{0,\pm}\pi^+\pi^-$ and $B_c \longrightarrow D^+_{(s)}\pi^+\pi^-$, where even larger CP violation may be possible [31, 32]. In this paper, we will investigate direct CP violation of the decay process $\bar{B}^0_s \longrightarrow \rho(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ involving the same mechanism in the pQCD approach.

The three-body decays of heavy B mesons are more complicated than the two-body decays as they receive both resonant and nonresonant contributions. Unlike the two-body case, to date, we still do not have effective theories for hadronic three-body decays, though attempts along the framework of pQCD and QCDF have been used in the past [33–36]. As a working starting point, we intend to study ρ-ω mixing effect in three-body decays of the B meson. The $\rho-\omega$ mixing mechanism is caused by the isospin symmetry breaking from the mixing between the u and d flavors [37, 38]. In Ref. [39], the authors studied the $\rho-\omega$ mixing and the pion form factor in the time-like region, where $\rho-\omega$ mixing comes from three part contributions: two from the direct coupling of the quasi-two-body decay of $\bar{B}^0_s \longrightarrow \rho K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ and $\bar{B}^0_s \longrightarrow \omega K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ and the other from the interference of $\bar{B}^0_s \longrightarrow \omega K^{*0} \longrightarrow \rho K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ mixing. Generally speaking, the amplitudes of their contributions are as follows: $\bar{B}^0_s \longrightarrow \rho K^{*0} \longrightarrow \pi^+\pi^-K^{*0} > \bar{B}^0_s \longrightarrow \omega K^{*0} \longrightarrow \rho K^{*0} \longrightarrow \pi^+\pi^-K^{*0} > \bar{B}^0_s \longrightarrow \omega K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$. $\omega \longrightarrow \pi^+\pi^-$ and $\omega \longrightarrow \rho \longrightarrow \pi^+\pi^-$ were used to obtain the (effective) mixing matrix element $\tilde{\Pi}_{\rho\omega}(s)$ [40–42]. The magnitude has been determined by the pion form factor through the data from the cross section of $e^+e^- \longrightarrow \pi^+\pi^-$ in the ρ and ω resonance region [39, 42–45]. Recently, isospin symmetry breaking was discussed by incorporating the vector meson dominance (VMD) model in the weak decay process of the meson [27, 32, 46–49]. However, one can find that $\rho-\omega$ mixing produces the large CP violation from the effect of isospin symmetry breaking in the three and four bodies decay process. Hence, in this paper, we shall follow the method of Refs. [27, 32, 46–49] to investigate the decay process of $\bar{B}^0_s \longrightarrow \rho(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ by the isospin symmetry breaking.

The remainder of this paper is organized as follows. In Sec. 2, we will present the form of the effective Hamiltonian and briefly introduce the pQCD framework and wave functions. In Sec. 3, we give the calculating formalism and details of the CP violation from $\rho-\omega$ mixing in the decay process $\bar{B}^0_s \longrightarrow \rho(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$. In Sec. 4, we calculate the branching ratio for the decay process of $\bar{B}^0_s \longrightarrow \rho^0(\omega)K^{*0}$. In Sec. 5, we show the input parameters. We present the numerical results in Sec. 6. Summary and discussion are included in Sec. 7. The related functions defined in the text are given in the Appendix.

2. The Framework

Based on the operator product expansion, the effective weak Hamiltonian for the decay processes $\bar{B}^0_s \longrightarrow \rho^0(\omega)K^{*0}$ can be expressed as [50]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}}\left\{V_{ub}V^*_{ud}[C_1(\mu)Q^u_1(\mu) + C_2(\mu)Q^u_2(\mu)] - V_{tb}V^*_{td}\left[\sum_{i=3}^{10} C_i(\mu)Q_i(\mu)\right]\right\} + H.c., \tag{1}$$

where G_F represents the Fermi constant, $C_i(\mu)$ $(i = 1, \cdots, 10)$ are the Wilson coefficients, and V_{ub}, V_{ud}, V_{tb}, and V_{td} are the CKM matrix elements. The operators O_i have the following forms:

$$O^u_1 = \bar{d}_\alpha\gamma_\mu(1-\gamma_5)u_\beta\bar{u}_\beta\gamma^\mu(1-\gamma_5)b_\alpha, \tag{2}$$

$$O^u_2 = \bar{d}_\alpha\gamma_\mu(1-\gamma_5)u_\alpha\bar{u}_\beta\gamma^\mu(1-\gamma_5)b_\beta, \tag{3}$$

$$O_3 = \bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\alpha\sum_{q'}\overline{q'}_\beta\gamma^\mu(1-\gamma_5)q'_\beta, \tag{4}$$

$$O_4 = \bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\overline{q'}_\beta\gamma^\mu(1-\gamma_5)q'_\alpha, \tag{5}$$

$$O_5 = \bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\alpha\sum_{q'}\overline{q'}_\beta\gamma^\mu(1+\gamma_5)q'_\beta, \tag{6}$$

$$O_6 = \bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\overline{q'}_\beta\gamma^\mu(1+\gamma_5)q'_\alpha, \tag{7}$$

$$O_7 = \frac{3}{2}\bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\alpha\sum_{q'}e_{q'}\overline{q'}_\beta\gamma^\mu(1+\gamma_5)q'_\beta, \tag{8}$$

$$O_8 = \frac{3}{2}\bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\overline{q'}_\beta\gamma^\mu(1+\gamma_5)q'_\alpha, \tag{9}$$

$$O_9 = \frac{3}{2}\bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\alpha\sum_{q'}e_{q'}\overline{q'}_\beta\gamma^\mu(1-\gamma_5)q'_\beta, \tag{10}$$

$$O_{10} = \frac{3}{2}\bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\overline{q'}_\beta\gamma^\mu(1-\gamma_5)q'_\alpha, \tag{11}$$

where α and β are the SU(3) color indices, $e_{q'}$ is the electric charge of quark q' in the unit of $|e|$, and the sum extend over $q' = u, d, s, c$ or b quarks. In Eq. (11) O^u_1 and O^u_2 are the tree operators, O_3–O_6 are the QCD penguin operators, and O_7–O_{10} are the operators associated with electroweak penguin diagrams.

The Wilson coefficient $C_i(\mu)$ in Eq. (1) describes the coupling strength for a given operator and summarizes the physical contributions from scales higher than μ [51]. They are calculable perturbatively with the renormalization group improved perturbation theory. Usually, the scale μ is chosen to be of order $O(m_b)$ for B meson decays. Since we work in the leading order of perturbative QCD ($O(\alpha_s)$), it is consistent to use the leading order Wilson coefficients. So, we

use the numerical values of $C_i(m_b)$ as follow [19, 21]:

$$\begin{aligned} C_1 &= -0.2703, \\ C_2 &= 1.1188, \\ C_3 &= 0.0126, \\ C_4 &= -0.0270, \\ C_5 &= 0.0085, \\ C_6 &= -0.0326, \\ C_7 &= 0.0011, \\ C_8 &= 0.0004, \\ C_9 &= -0.0090, \\ C_{10} &= 0.0022, \end{aligned} \tag{12}$$

The combinations a_1–a_{10} of the Wilson coefficients are defined as usual [52–55]:

$$\begin{aligned} a_1 &= C_2 + \frac{C_1}{3}, \\ a_2 &= C_1 + \frac{C_2}{3}, \\ a_i &= C_i + \frac{C_{i\pm 1}}{3} \quad (i = 3 - 10), \end{aligned} \tag{13}$$

where the upper (lower) sign applies, when i is odd (even).

For the two-body decay processes of $\bar{B}_s^0 \longrightarrow M_2 M_3$, we denote the emitted or annihilated meson as M_2, while the recoiling meson is M_3. The meson M_2 (ρ or ω) and the final state meson M_3 (K^{*0}) move along the direction of $n = (1, 0, 0_T)$ and $v = (0, 1, 0_T)$ in the light-cone coordinates, respectively. The decay amplitude can be expressed as the convolution of the wave functions ϕ_{B_s}, ϕ_{M_2}, and ϕ_{M_3} and the hard scattering kernel T_H in the pQCD. The pQCD factorization theorem has been developed for the two-body nonleptonic heavy meson decays, based on the formalism of Botts, Lepage, Brodsky, and Sterman [56–59]. The basic idea of the pQCD approach is that it takes into account the transverse momentum of the valence quarks in the hadrons which results in the Sudakov factor in the decay amplitude. Then, the decay channels of $\bar{B}_s^0 \longrightarrow \rho^0(\omega) K^{*0}$ are conceptually written as the following:

$$\begin{aligned} A\left(\bar{B}_s^0 \longrightarrow \rho^0(\omega) K^{*0}\right) &= \int d^4k_1 d^4k_2 d^4k_3 \,\mathrm{Tr} \\ &\cdot \left[C(t)\phi_{B_s}(k_1)\phi_{M2}(k_2)\phi_{M3}(k_3) T_H(k_1, k_2, k_3, t)\right], \end{aligned} \tag{14}$$

where $k_i (i = 1, 2, 3)$ is the momentum of light quark in each meson. Tr denotes the trace over Dirac structure and color indices. $C(t)$ is the short distance Wilson coefficients at the hard scale t. The meson wave functions ϕ_{B_s} and $\phi_M (m = 2, 3)$, including all nonperturbative contribution during the hadronization of mesons, can be extracted from the experimental data or other nonperturbative methods. The hard kernel $T_H(k_1, k_2, k_3, \text{and } t)$ describes the four quark operator and the spectator quark connected by a hard gluon, which can be perturbatively calculated including all possible Feynman diagrams of the factorizable and nonfactorizable contributions without endpoint singularity.

The $\rho^0(\omega)$ and K^{*0} mesons are treated as a light-light system. At the B_s meson rest frame, they are moving very fast. We define the ratios $r_{K^{*0}} = M_{K^{*0}}/M_{B_s}$, $r_\rho = M_\rho/M_{B_s}$, and $r_\omega = M_\omega/M_{B_s}$. In the limits $M_{K^{*0}}$, M_ρ, and $M_\omega \longrightarrow 0$, one can drop the terms of proportional to $r_{K^{*0}}^2$, r_ρ^2, and r_ω^2 safely. The symbols P_B, P_2, and P_3 refer to the $\bar{B}_s^0$ meson momentum, the $\rho^0(\omega)$ meson momentum, and the final state K^{*0} meson momentum, respectively. The momenta of the participating mesons in the rest frame of the B_s meson can be written as:

$$P_B = \frac{M_{B_s}}{\sqrt{2}}(1, 1, 0_T), P_2 = \frac{M_{B_s}}{\sqrt{2}}(1, 0, 0_T), P_3 = \frac{M_{B_s}}{\sqrt{2}}(0, 1, 0_T). \tag{15}$$

One can denote the light (anti-)quark momenta k_1, k_2, and k_3 for the initial meson $\bar{B}_s^0$ and the final mesons $\rho^0(\omega)$ and K^{*0}, respectively. We can choose

$$\begin{aligned} k_1 &= \left(x_1 \frac{M_{B_s}}{\sqrt{2}}, 0, k_{1\perp}\right), k_2 = \left(x_2 \frac{M_{B_s}}{\sqrt{2}}, 0, k_{2\perp}\right), k_3 \\ &= \left(0, x_3 \frac{M_{B_s}}{\sqrt{2}}, k_{3\perp}\right), \end{aligned} \tag{16}$$

where x_1, x_2, and x_3 are the momentum fractions. $k_{1\perp}$, $k_{2\perp}$, and $k_{3\perp}$ refer to the transverse momentum of the quark, respectively. To extract the helicity amplitudes, we parameterize the following longitudinal polarization vectors of the $\rho^0(\omega)$ and K^{*0} as the following:

$$\begin{aligned} \epsilon_2(L) &= \frac{P_2}{M_{\rho(\omega)}} - \frac{M_{\rho(\omega)}}{P_2 \cdot v} v, \\ \epsilon_3(L) &= \frac{P_3}{M_{K^{*0}}} - \frac{M_{K^{*0}}}{P_3 \cdot n} n, \end{aligned} \tag{17}$$

which satisfy the orthogonality relationship of $\epsilon_2(L) \cdot P_2 = \epsilon_3(L) \cdot P_3 = 0$, and the normalization of $\epsilon_2^2(L) = \epsilon_3^2(L) = -1$. The transverse polarization vectors can be adopted directly as

$$\begin{aligned} \epsilon_2(T) &= (0, 0, 1_T), \\ \epsilon_3(T) &= (0, 0, 1_T). \end{aligned} \tag{18}$$

Within the pQCD framework, both the initial and the final state meson wave functions and distribution amplitudes are important as nonperturbative input parameters. For the B_s meson, the wave function of the meson can be

expressed as

$$\Phi_{B_s} = \frac{i}{\sqrt{6}}\left(P_{B_s} + M_{B_s}\right)\gamma_5\phi_{B_s}(k), \tag{19}$$

where the distribution amplitude ϕ_{B_s} is shown in Refs. [60–62]:

$$\phi_{B_s}(x,b) = N_{B_s}x^2(1-x)^2 \exp\left[-\frac{M_{B_s}^{2\ 2}}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right]. \tag{20}$$

The shape parameter ω_b is a free parameter and N_{B_s} is a normalization factor. Based on the studies of the light-cone sum rule, lattice QCD or be fitted to the measurements with good precision [63], we take $\omega_b = 0.50\,\text{GeV}$ for the B_s meson. The normalization factor N_{B_s} depends on the values of the shape parameter ω_b and decay constant f_{B_s}, which is defined through the normalization relation $\int_0^1 dx\phi_{B_s}(x,0) = f_{B_s}/(2\sqrt{6})$.

The distribution amplitudes of vector meson (V=ρ, ω, or K^*), ϕ_V, ϕ_V^T, ϕ_V^t, ϕ_V^s, ϕ_V^v, and ϕ_V^a, can be written in the following forms [64, 65]:

$$\begin{aligned}
\phi_\rho(x) &= \frac{3f_\rho}{\sqrt{6}}x(1-x)\left[1+0.15C_2^{3/2}(t)\right],\\
\phi_\omega(x) &= \frac{3f_\omega}{\sqrt{6}}x(1-x)\left[1+0.15C_2^{3/2}(t)\right],\\
\phi_{K^*}(x) &= \frac{3f_{K^*}}{\sqrt{6}}x(1-x)\left[1+0.03C_1^{3/2}(t)+0.11C_2^{3/2}(t)\right],\\
\phi_\rho^T(x) &= \frac{3f_\rho^T}{\sqrt{6}}x(1-x)\left[1+0.14C_1^{3/2}(t)\right],\\
\phi_\omega^T(x) &= \frac{3f_\omega^T}{\sqrt{6}}x(1-x)\left[1+0.14C_1^{3/2}(t)\right],\\
\phi_{K^*}^T(x) &= \frac{3f_{K^*}^T}{\sqrt{6}}x(1-x)\left[1+0.04C_1^{3/2}(t)+0.10C_2^{3/2}(t)\right],\\
\phi_V^t(x) &= \frac{3f_V^T}{2\sqrt{6}}t^2,\\
\phi_V^s(x) &= \frac{3f_V^T}{2\sqrt{6}}(-t),\\
\phi_V^v(x) &= \frac{3f_V}{8\sqrt{6}}\left(1+t^2\right),\\
\phi_V^a(x) &= \frac{3f_V}{4\sqrt{6}}(-t),
\end{aligned} \tag{21}$$

where $t = 2x - 1$. Here, $f_V^{(T)}$ is the decay constant of the vector meson with longitudinal (transverse) polarization. The Gegenbauer polynomials $C_n^\nu(t)$ can be defined as [66, 67]

$$\begin{aligned}
C_1^{3/2}(t) &= 3t,\\
C_2^{3/2}(t) &= \frac{3}{2}\left(5t^2-1\right).
\end{aligned} \tag{22}$$

3. CP Violation in $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ Decay Process

3.1. Formalism. The decay width Γ for the processes of $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$ is given by

$$\Gamma = \frac{P_c}{8\pi M_{B_s}^2}\sum_{\sigma=L,T} A^{(\sigma)\dagger}A^{(\sigma)}, \tag{23}$$

where P_c is the absolute value of the three-momentum of the final state mesons. The decay amplitude $A^{(\sigma)}$ which is decided by QCD dynamics will be calculated later in the pQCD factorization approach. The superscript σ denotes the helicity states of the two vector mesons with the longitudinal (transverse) components L(T). The amplitude $A^{(\sigma)}$ for the decays $B_s(P_{B_s}) \longrightarrow V_{\rho(\omega)}(P_2, \varepsilon_{2\mu}^*) + V_{K^{*0}}(P_3, \varepsilon_{3\mu}^*)$ can be decomposed as follows [67–70]:

$$\begin{aligned}
A^{(\sigma)} = {} & M_{B_s}^2 A_L + M_{B_s}^2 A_N \epsilon_2^*(\sigma=T)\cdot\epsilon_3^*(\sigma=T)\\
& + iA_T\varepsilon^{\alpha\beta\gamma\rho}\varepsilon_{2\alpha}^*(\sigma)\varepsilon_{3\beta}^*(\sigma)P_{2\gamma}P_{3\rho},
\end{aligned} \tag{24}$$

where ε^* is the polarization vector of the vector meson. The amplitude A_i (i refers to the three kinds of polarizations, longitudinal (L), normal (N), and transverse (T)) can be written as

$$\begin{aligned}
M_{B_s}^2 A_L &= a\,\varepsilon_2^*(L)\cdot\varepsilon_3^*(L) + \frac{b}{M_2M_3}\varepsilon_2^*(L)\cdot P_3\,\varepsilon_3^*(L)\cdot P_2,\\
M_{B_s}^2 A_N &= a,\\
A_T &= \frac{c}{M_2M_3},
\end{aligned} \tag{25}$$

where a, b, and c are the Lorentz-invariant amplitudes. M_2 and M_3 are the masses of the vector mesons $\rho^0(\omega)$ and K^{*0}, respectively.

The longitudinal H_0 and transverse $H_\pm$ of helicity amplitudes can be expressed

$$H_0 = M_{B_s}^2 A_L, \tag{26}$$

$$H_\pm = M_{B_s}^2 A_N \mp M_2M_3\sqrt{\kappa^2-1}A_T, \tag{27}$$

where H_0 and $H_\pm$ are the penguin level and tree level helicity amplitudes of the decay process $\bar{B}_s^0 \longrightarrow \rho(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ from the three kinds of polarizations, respectively. The

helicity summation satisfies the relation

$$\sum_{\sigma=L,R} A^{(\sigma)\dagger} A^{(\sigma)} = |H_0|^2 + |H_+|^2 + |H_-|^2. \tag{28}$$

In the vector meson dominance model [71, 72], the photon propagator is dressed by coupling to vector mesons. Based on the same mechanism, $\rho-\omega$ mixing was proposed and later gradually applied to B meson physics [29, 39, 47, 73]. According to the effective Hamiltonian, the amplitude A ($\bar{A}$) for the three-body decay process $\bar{B}_s^0 \longrightarrow \pi^+\pi^- K^{*0}$ ($B_s^0 \longrightarrow \pi^+\pi^- \bar{K}^{*0}$) can be written as follows [47]:

$$A = <\pi^+\pi^- K^{*0}|H^T|\bar{B}_s^0> + <\pi^+\pi^- K^{*0}|H^P|\bar{B}_s^0>, \tag{29}$$

$$\bar{A} = <\pi^+\pi^- \bar{K}^{*0}|H^T|B_s^0> + <\pi^+\pi^- \bar{K}^{*0}|H^P|B_s^0>, \tag{30}$$

where H^T and H^P are the Hamiltonian for the tree and penguin operators, respectively.

The relative magnitude and phases between the tree and penguin operator contribution are defined as follows:

$$A = <\pi^+\pi^- K^{*0}|H^T|\bar{B}_s^0> \left[1 + re^{i(\delta+\phi)}\right], \tag{31}$$

$$\bar{A} = <\pi^+\pi^- \bar{K}^{*0}|H^T|B_s^0> \left[1 + re^{i(\delta-\phi)}\right], \tag{32}$$

where δ and ϕ are the strong and weak phase differences, respectively. The weak phase difference ϕ can be expressed as a combination of the CKM matrix elements, and it is $\phi = \arg\left[(V_{tb}V_{td}^*)/(V_{ub}V_{ud}^*)\right]$ for the $b \longrightarrow d$ transition. The parameter r is the absolute value of the ratio of the tree and penguin amplitudes:

$$r \equiv \left|\frac{<\pi^+\pi^- K^{*0}|H^P|\bar{B}_s^0>}{<\pi^+\pi^- K^{*0}|H^T|\bar{B}_s^0>}\right|. \tag{33}$$

The parameter of CP violating asymmetry, A_{CP}, can be written as

$$\begin{aligned} A_{CP} &= \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \\ &= \frac{-2\left(T_0^2 r_0 \sin\delta_0 + T_+^2 r_+ \sin\delta_+ + T_-^2 r_- \sin\delta_-\right)\sin\phi}{\sum_{i=0+-} T_i^2\left(1 + r_i^2 + 2r_i\cos\delta_i\cos\phi\right)}, \end{aligned} \tag{34}$$

where $T_i\,(i = 0,+,-)$ represent the tree level helicity amplitudes of the decay process $\bar{B}_s^0 \longrightarrow \pi^+\pi^- K^{*0}$ from H_0, H_+, and H_- of Eq. (27), respectively. $r_j(j = 0,+,-)$ refer to the absolute value of the ratio of the tree and penguin amplitude for the three kinds of polarizations, respectively. $\delta_k(k = 0,+,-)$ are the relative strong phases between the tree and penguin operator contributions from the three kinds of the helicity amplitudes, respectively. We can see explicitly from Eq. (34) that both weak and strong phase differences are needed to produce CP violation. In order to obtain a large signal for the direct CP violation, we intend to apply the $\rho-\omega$ mixing mechanism, which leads to large strong phase differences in hadron decays.

With the $\rho-\omega$ mixing mechanism, the process of the $\bar{B}_s^0 \longrightarrow \rho(\omega)K^{*0} \longrightarrow \pi^+\pi^- K^{*0}$ decay is shown in Figure 1. In the isospin representation, the decay amplitude $M_{\bar{B}_s^0 \longrightarrow \rho(\omega) \longrightarrow \pi^+\pi^-}$ in Figure 1 can be written as [31, 39, 48, 74]

$$M_{\bar{B}_s^0 \longrightarrow \pi^+\pi^-} = M_{\rho_I \longrightarrow \pi\pi}\frac{1}{s_\rho}M_{\bar{B}_s^0 \longrightarrow \rho_I} + M_{\rho_I \longrightarrow \pi\pi}\frac{1}{s_\rho}\Pi_{\rho\omega}\frac{1}{s_\omega}M_{\bar{B}_s^0 \longrightarrow \omega_I}. \tag{35}$$

Introducing the $\varepsilon = \Pi_{\rho\omega}/s_\rho - s_\omega$ [31, 39, 48, 74], we have identified the physical amplitudes as

$$\begin{aligned} M_{\rho \longrightarrow \pi\pi} &= M_{\rho_I \longrightarrow \pi\pi}, \\ M_{\omega \longrightarrow \pi\pi} &= \varepsilon M_{\rho_I \longrightarrow \pi\pi}, \\ M_{\bar{B}_s^0 \longrightarrow \rho} &= M_{\bar{B}_s^0 \longrightarrow \rho_I} - \varepsilon M_{\bar{B}_s^0 \longrightarrow \omega_I}, \\ M_{\bar{B}_s^0 \longrightarrow \omega} &= M_{\bar{B}_s^0 \longrightarrow \omega_I} + \varepsilon M_{\bar{B}_s^0 \longrightarrow \rho_I}. \end{aligned} \tag{36}$$

From the physical representation, we can obtain the decay amplitude:

$$M_{\bar{B}_s^0 \longrightarrow \pi^+\pi^-} = M_{\rho \longrightarrow \pi\pi}\frac{1}{s_\rho}M_{\bar{B}_s^0 \longrightarrow \rho} + M_{\rho \longrightarrow \pi\pi}\frac{\Pi_{\rho\omega}s_\rho}{s_\rho - s_\omega}\frac{1}{s_\rho s_\omega}M_{\bar{B}_s^0 \longrightarrow \omega}, \tag{37}$$

where $\mathcal{O}(\varepsilon^2)$ corrections are neglected and $M_{\rho \longrightarrow \pi\pi} = g_\rho$, $M_{\bar{B}_s^0 \longrightarrow \rho} = t_\rho^i$ or p_ρ^i, and $M_{\bar{B}_s^0 \longrightarrow \omega} = t_\omega^i$ or p_ω^i are used. So, we can get $\tilde{\Pi}_{\rho\omega} = \Pi_{\rho\omega}s_\rho/s_\rho - s_\omega$. $\tilde{\Pi}_{\rho\omega}$ is the effective $\rho-\omega$ mixing amplitude which also effectively includes the direct coupling $\omega \longrightarrow \pi^+\pi^-$. At the first order of the isospin violation, we have the following tree and penguin amplitudes when the invariant mass of $\pi^+\pi^-$ pair is near the ω resonance mass [26, 47]:

$$<\pi^+\pi^- K^{*0}|H^T|\bar{B}_s^0> = \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}t_\omega^i + \frac{g_\rho}{s_\rho}t_\rho^i, \tag{38}$$

$$<\pi^+\pi^- K^{*0}|H^P|\bar{B}_s^0> = \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}p_\omega^i + \frac{g_\rho}{s_\rho}p_\rho^i, \tag{39}$$

where $t_\rho^i(p_\rho^i)$ and $t_\omega^i(p_\omega^i)$ are the tree (penguin) level helicity amplitudes for $\bar{B}_s^0 \longrightarrow \rho^0 K^{*0}$ and $\bar{B}_s^0 \longrightarrow \omega K^{*0}$, respectively. The amplitudes t_ρ^i, t_ω^i, p_ρ^i, and p_ω^i can be found in Sec. 3.2. g_ρ is the coupling constant for the decay process $\rho^0 \longrightarrow \pi^+\pi^-$. s_V, m_V, and $\Gamma_V(V=\rho$ or $\omega)$ are the inverse propagator, mass, and decay width of the vector meson V, respectively. s_V can be expressed as

$$s_V = s - m_V^2 + \mathrm{i}m_V\Gamma_V, \tag{40}$$

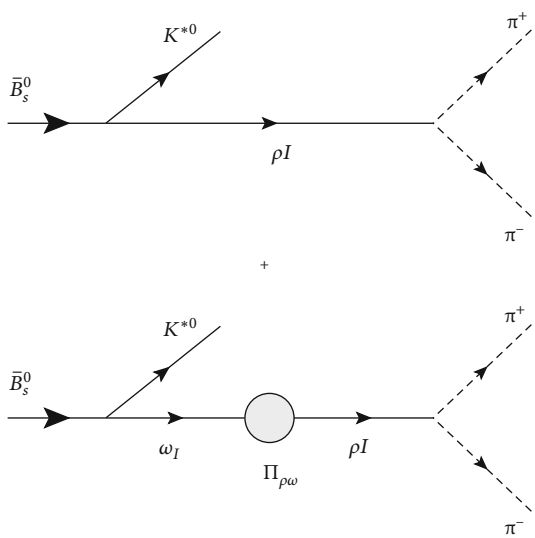

Figure 1: The diagram for the $\bar{B}_s^0 \longrightarrow \pi^+\pi^- K^{*0}$ decay with the $\rho - \omega$ mixing mechanism for the first order of the isospin violation in the isospin representation.

with $\sqrt{s}$ being the invariant masses of the $\pi^+\pi^-$ pairs. The $\rho - \omega$ mixing parameters $\tilde{\Pi}_{\rho\omega}(s) = \mathrm{Re}\,\tilde{\Pi}_{\rho\omega}(m_\omega^2) + \mathrm{Im}\,\tilde{\Pi}_{\rho\omega}(m_\omega^2)$ are [75]

$$\begin{aligned} \mathrm{Re}\,\tilde{\Pi}_{\rho\omega}\left(m_\omega^2\right) &= -4760 \pm 440\ \mathrm{MeV}^2, \\ \mathrm{Im}\,\tilde{\Pi}_{\rho\omega}\left(m_\omega^2\right) &= -6180 \pm 3300\ \mathrm{MeV}^2. \end{aligned} \tag{41}$$

From Eqs. (29), (31), (38), and (39), one has

$$re^{i\delta_i}e^{i\phi} = \frac{\tilde{\Pi}_{\rho\omega}p_\omega^i + s_\omega p_\rho^i}{\tilde{\Pi}_{\rho\omega}t_\omega^i + s_\omega t_\rho^i}, \tag{42}$$

defining [25, 76]

$$\begin{aligned} \frac{p_\omega^i}{t_\rho^i} &\equiv r' e^{i\left(\delta_q^i + \phi\right)}, \\ \frac{t_\omega^i}{t_\rho^i} &\equiv \alpha e^{i\delta_\alpha^i}, \\ \frac{p_\rho^i}{p_\omega^i} &\equiv \beta e^{i\delta_\beta^i}, \end{aligned} \tag{43}$$

where δ_α^i, δ_β^i, and δ_q^i are the strong phases of the decay process $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^- K^{*0}$ from the three kinds of polarizations, respectively. One finds the following expression from Eqs. (42) and (43):

$$re^{i\delta_i} = r' e^{i\delta_q^i}\frac{\tilde{\Pi}_{\rho\omega} + \beta e^{i\delta_\beta^i}s_\omega}{\tilde{\Pi}_{\rho\omega}\alpha e^{i\delta_\alpha^i} + s_\omega}. \tag{44}$$

$\alpha e^{i\delta_\alpha^i}$, $\beta e^{i\delta_\beta^i}$, and $r' e^{i\delta_q^i}$ will be calculated in the perturbative QCD approach. In order to obtain the CP violating asymmetry in Eq. (34), A_{CP}, $\sin\phi$, and $\cos\phi$ are needed. ϕ is determined by the CKM matrix elements. In the Wolfenstein parametrization [77], the weak phase ϕ comes from [$V_{tb}V_{td}^*/V_{ub}V_{ud}^*$]. One has

$$\sin\phi = \frac{\eta}{\sqrt{\left[\rho(1-\rho) - \eta^2\right]^2 + \eta^2}}, \tag{45}$$

$$\cos\phi = \frac{\rho(1-\rho) - \eta^2}{\sqrt{\left[\rho(1-\rho) - \eta^2\right]^2 + \eta^2}}, \tag{46}$$

where the same result has been used for $b \longrightarrow d$ transition from Ref. [28, 78].

3.2. Calculation Details. We can decompose the decay amplitudes for the decay processes $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$ in terms of the tree and penguin contributions depending on the CKM matrix elements of $V_{ub}V_{ud}^*$ and $V_{tb}V_{td}^*$. From Eqs. (34), (42), and (43), in the leading order to obtain the formulas of the *CP* violation, we need to calculate the amplitudes t_ρ, p_ρ, t_ω, and p_ω in the perturbative QCD approach. The relevant function can be found in the Appendix from the perturbative QCD approach.

In the pQCD, there are eight types of the leading order Feynman diagrams contributing to $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$ decays, which are shown in Figure 2. The first row is for the emission-type diagrams, where the first two diagrams in Figures 2(a) and 2(b) are called factorizable emission diagrams and the last two diagrams in Figures 2(c) and 2(d) are called nonfactorizable emission diagrams [68, 79]. The second row is for the annihilation-type diagrams, where the first two diagrams in Figures 2(e) and 2(f) are called factorizable annihilation diagrams and the last two diagrams in Figures 2(g) and 2(h) are called nonfactorizable annihilation diagrams [62, 80]. The relevant decay amplitudes can be easily obtained by these hard gluon exchange diagrams, and the Lorenz structures of the mesons wave functions. Through calculating these diagrams, the formulas of $\bar{B}_s^0 \longrightarrow \rho K^{*0}$ or $\bar{B}_s^0 \longrightarrow \omega K^{*0}$ are similar to those of $B \longrightarrow \phi K^*$ and $B_s \longrightarrow K^{*-}K^{*+}$ [79, 81]. We just need to replace some corresponding Wilson coefficients, wave functions, and corresponding parameters.

With the Hamiltonian Equation (1), depending on CKM matrix elements of $V_{ub}V_{ud}^*$ and $V_{tb}V_{td}^*$, the tree dominant decay amplitudes $A^{(i)}$ for $\bar{B}_s^0 \longrightarrow \rho K^{*0}$ in pQCD can be written as

$$\sqrt{2}A^{(i)}\left(\bar{B}_s^0 \longrightarrow \rho^0 K^{*0}\right) = V_{ub}V_{ud}^*T_\rho^i - V_{tb}V_{td}^*P_\rho^i, \tag{47}$$

where the superscript i denote the different helicity amplitudes L, N, and T. The longitudinal $t_{\rho(\omega)}^0$ and transverse $t_{\rho(\omega)}^\pm$ of helicity amplitudes satisfy relationship from Eq. (27). The amplitudes of the tree and penguin diagrams can be written as $T_\rho^i = t_\rho^i/V_{ub}V_{ud}^*$ and $P_\rho^i = p_\rho^i/V_{tb}V_{td}^*$,

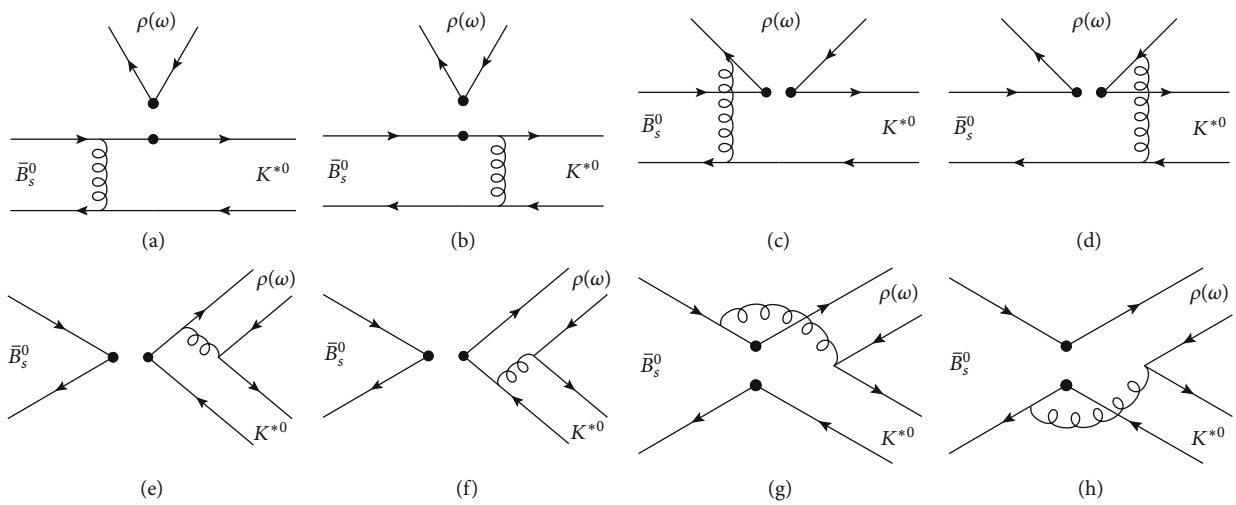

Figure 2: Leading order Feynman diagrams for $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$.

respectively. The formula for the tree level amplitude is

$$T_\rho^i = \frac{G_F}{\sqrt{2}}\left\{f_\rho F^{LL,i}_{B_s\longrightarrow K^*}[a_2] + M^{LL,i}_{B_s\longrightarrow K^*}[C_2]\right\}, \tag{48}$$

where f_ρ refers to the decay constant of ρ meson. The penguin level amplitudes are expressed in the following:

$$\begin{aligned} P_\rho^i = -\frac{G_F}{\sqrt{2}}\Bigg\{ & f_\rho F^{LL,i}_{B_s\longrightarrow K^*}\left[-a_4 + \frac{3}{2}a_7 + \frac{3}{2}a_9 + \frac{1}{2}a_{10}\right] \\ & - M^{LR,i}_{B_s\longrightarrow K^*}\left[-C_5 + \frac{1}{2}C_7\right] + M^{LL,i}_{B_s\longrightarrow K^*}\left[-C_3 + \frac{1}{2}C_9 + \frac{3}{2}C_{10}\right] \\ & - M^{SP,i}_{B_s\longrightarrow K^*}\left[\frac{3}{2}C_8\right] + f_{B_s}F^{LL,i}_{\text{ann}}\left[-a_4 + \frac{1}{2}a_{10}\right] \\ & - f_{B_s}F^{SP,i}_{ann}\left[-a_6 + \frac{1}{2}a_8\right] + M^{LL,i}_{ann}\left[-C_3 + \frac{1}{2}C_9\right] \\ & - M^{LR,i}_{ann}\left[-C_5 + \frac{1}{2}C_7\right]\Bigg\}. \end{aligned} \tag{49}$$

The tree dominant decay amplitude for $\bar{B}_s^0 \longrightarrow \omega K^{*0}$ can be written as

$$\sqrt{2}A^i\left(\bar{B}_s^0 \longrightarrow \omega K^{*0}\right) = V_{ub}V_{ud}^* T_\omega^i - V_{tb}V_{td}^* P_\omega^i, \tag{50}$$

where $T_\omega^i = t_\omega^i / V_{ub}V_{us}^*$ and $P_\omega^i = p_\omega^i / V_{tb}V_{ts}^*$ refer to the tree and penguin amplitude, respectively. We can give the tree level contribution in the following:

$$T_\omega^i = \frac{G_F}{\sqrt{2}}\left\{f_\omega F^{LL,i}_{B_s\longrightarrow K^*}[a_2] + M^{LL,i}_{B_s\longrightarrow K^*}[C_2]\right\}, \tag{51}$$

where f_ω refers to the decay constant of ω meson. The penguin level contribution is given as follows:

$$\begin{aligned} P_\omega^i = \frac{G_F}{\sqrt{2}}\Bigg\{ & f_\omega F^{LL,i}_{B_s\longrightarrow K^*}\left[2a_3 + a_4 + 2a_5 + \frac{1}{2}a_7 + \frac{1}{2}a_9 - \frac{1}{2}a_{10}\right] \\ & - M^{LR,i}_{B_s\longrightarrow K^*}\left[C_5 - \frac{1}{2}C_7\right] + M^{LL,i}_{B_s\longrightarrow K^*}\left[C_3 + 2C_4 - \frac{1}{2}C_9 + \frac{1}{2}C_{10}\right] \\ & - M^{SP,i}_{B_s\longrightarrow K^*}\left[2C_6 + \frac{1}{2}C_8\right] + f_{B_s}F^{LL,i}_{ann}\left[a_4 - \frac{1}{2}a_{10}\right] \\ & - f_{B_s}F^{SP,i}_{ann}\left[a_6 - \frac{1}{2}a_8\right] + M^{LL,i}_{ann}\left[C_3 - \frac{1}{2}C_9\right] - M^{LR,i}_{ann}\left[C_5 - \frac{1}{2}C_7\right]\Bigg\} \end{aligned} \tag{52}$$

Based on the definition of Eq. (43), we can get

$$\alpha e^{i\delta_\alpha^i} = \frac{t_\omega^i}{t_\rho^i}, \tag{53}$$

$$\beta e^{i\delta_\beta^i} = \frac{p_\rho^i}{p_\omega^i}, \tag{54}$$

$$r' e^{i\delta_q^i} = \frac{P_\omega^i}{T_\rho^i} \times \left|\frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*}\right|, \tag{55}$$

where

$$\left|\frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*}\right| = \frac{\sqrt{[\rho(1-\rho)-\eta^2]^2 + \eta^2}}{(1-\lambda^2/2)(\rho^2+\eta^2)}. \tag{56}$$

From above equations, the new strong phases δ_α^i, δ_β^i, and δ_q^i are obtained from the tree and penguin diagram contributions by the $\rho-\omega$ interference. Substituting Eqs. (53), (54), and (55) into (44), we can obtain total strong phase δ_i in the framework of the pQCD. Then in combination with Eqs. (45) and (46), the CP violating asymmetry can be obtained.

4. Branching Ratio of $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$

Based on the relationship of Eqs. (23) and (28), we can calculate the decay rates for the processes of $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$ by using the following expression:

$$\Gamma = \frac{P_c}{8\pi M_{B_s}^2}\left(|H_0|^2 + |H_+|^2 + |H_-|^2\right), \tag{57}$$

where

$$P_c = \frac{\sqrt{\left[M_{B_s} - \left(M_{\rho/\omega} + M_{K^{*0}}\right)^2\right]\left[M_{B_s} - \left(M_{\rho/\omega} - M_{K^{*0}}\right)^2\right]}}{2M_{B_s}} \tag{58}$$

is the c.m. momentum of the product particle and $H_i(i = 0, +, -)$ are the helicity amplitudes.

In this case, we take into account the $\rho - \omega$ mixing contribution to the branching ratio, since we are working on the first order of isospin violation. The derivation is straightforward, and we can explicitly express the branching ratio for the processes $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$ [28, 82]:

$$BR\left(\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}\right) = \frac{\tau_{B_s^0}P_c}{8\pi M_{B_s}^2}\left(\left|H_{\rho\omega 0}\right|^2 + \left|H_{\rho\omega +}\right|^2 + \left|H_{\rho\omega -}\right|^2\right), \tag{59}$$

where $\tau_{B_s^0}$ is the lifetime of the B_s meson and

$$\begin{aligned} H_{\rho\omega i(i=0,+,-)} = &\left(|V_{ub}V_{ud}^*|T_\rho^i - |V_{tb}V_{td}^*|P_\rho^i\right) \\ &+ \left(|V_{ub}V_{ud}^*|T_\omega^i - |V_{tb}V_{td}^*|P_\omega^i\right)\frac{\tilde{\Pi}_{\rho\omega}}{\left(s_\rho - M_\omega^2\right) + \mathrm{i}M_\omega\Gamma_\omega} \end{aligned} \tag{60}$$

take into account the helicity amplitudes of the ρ meson and ω meson contribution involved in the tree and penguin diagrams.

5. Input Parameters

The CKM matrix, which elements are determined from experiments, can be expressed in terms of the Wolfenstein parameters A, ρ, λ, and η [77, 83]:

$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - \mathrm{i}\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - \mathrm{i}\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{61}$$

where $\mathcal{O}(\lambda^4)$ corrections are neglected. The latest values for the parameters in the CKM matrix are [84]

$$\lambda = 0.22650 \pm 0.00048, A = 0.790^{+0.017}_{-0.012}, \tag{62}$$

$$\bar{\rho} = 0.141^{+0.016}_{-0.017}, \bar{\eta} = 0.357 \pm 0.011, \tag{63}$$

where

$$\bar{\rho} = \rho\left(1 - \frac{\lambda^2}{2}\right), \bar{\eta} = \eta\left(1 - \frac{\lambda^2}{2}\right). \tag{64}$$

From Eqs. (63) and (64), we have

$$0.127 < \rho < 0.161, 0.355 < \eta < 0.378. \tag{65}$$

The other parameters and the corresponding references are listed in Table 1.

6. The Numerical Results of *CP* Violation and Branching Ratio

6.1. CP Violation via $\rho - \omega$ Mixing in $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$. We have investigated the CP violating asymmetry, A_{CP}, for the $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ of the three-body decay process in the perturbative QCD. The numerical results of the CP violating asymmetry are shown for the $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ decay process in Figure 3. It is found that the CP violation can be enhanced via $\rho - \omega$ mixing for the decay channel $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ when the invariant mass of $\pi^+\pi^-$ pair is in the vicinity of the m_ω resonance within perturbative QCD scheme.

The CP violating asymmetry depends on the weak phase difference ϕ from CKM matrix elements and the strong phase difference δ in the Eq. (34). The CKM matrix elements, which relate to $\bar{\rho}$, A, $\bar{\eta}$, and λ, are given in Eq. (63). The uncertainties due to the CKM matrix elements are mostly from ρ and η since λ is well determined. Hence, we take the central value of $\lambda = 0.226$ in Eq. (65). In the numerical calculations for the $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ decay process, we use ρ, η, and $\lambda = 0.226$ which vary among the limiting values. The numerical results are shown from Figure 3 with the different parameter values of CKM matrix elements. The solid line, dotted line, and dashed line correspond to the maximum, middle, and minimum CKM matrix element for the decay channel of $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$, respectively. We find that the numerical results of the CP violation are not sensitive to the CKM matrix elements for the different values of ρ and η. In Figure 3, we show the plot of CP violation as a function of $\sqrt{s}$ in the perturbative QCD. From the figure, one can see the CP violation parameter is dependent on $\sqrt{s}$ and changes rapidly by the $\rho - \omega$ mixing mechanism when the invariant mass of $\pi^+\pi^-$ pair is in the vicinity of the m_ω resonance. From the numerical results, it is found that the CP violating asymmetry is large and ranges from -50.19% to 43.02% via the $\rho - \omega$ mixing mechanism for the process. The maximum CP

Table 1: Input parameters.

Parameters	Input data	References
Fermi constant (in GeV^{-2})	$G_F = 1.16638 \times 10^{-5}$.	[84]
	$M_{B_s^0} = 5366.88$, $\tau_{B_s^0} = 1.515 \times 10^{-12} s$.	
	$M_{\rho^0(770)} = 775.26$, $\Gamma_{\rho^0(770)} = 149.1$.	
Masses and decay widths (in MeV)	$M_{\omega(782)} = 782.65$, $\Gamma_{\omega(782)} = 8.49$.	[84]
	$M_\pi = 139.57$, $M_{K*} = 895.55$.	
	$f_\rho = 215.6 \pm 5.9$, $f_\rho^T = 165 \pm 9$.	
Decay constants (in MeV)	$f_\omega = 196.5 \pm 4.8$, $f_\omega^T = 145 \pm 10$.	[65, 85, 86]
	$f_{K^*} = 217 \pm 5$, $f_{K^*}^T = 185 \pm 10$.	

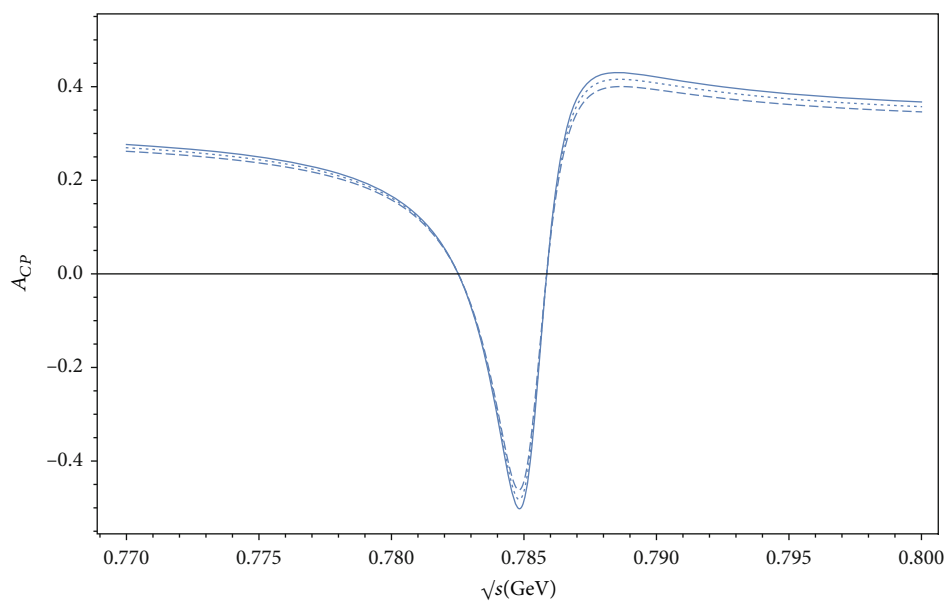

Figure 3: The CP violation, A_{cp}, as a function of $\sqrt{s}$ for different CKM matrix elements. The solid line, dotted line, and dashed line correspond to the maximum, middle, and minimum CKM matrix elements for the decay channel of $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$, respectively.

violating parameter can reach -48.22$^{+1.97}_{-2.04}$% for the decay channel of $\bar{B}_s^0 \longrightarrow \pi^+\pi^-K^{*0}$ in the case of (ρ, η). This error corresponds to the CKM parameters.

From Eq. (34), one can find that the CP violating parameter is related to r and $\sin\delta$. In Figures 4 and 5, we show the plots of $\sin\delta_0$ ($\sin\delta_+$ and $\sin\delta_-$) and r_0 (r_+ and r_-) as a function of $\sqrt{s}$, respectively. We can see that the $\rho - \omega$ mixing mechanism produces a large $\sin\delta_0$ ($\sin\delta_+$ and $\sin\delta_-$) in the vicinity of the ω resonance. As can be seen from Figure 4, the plots vary sharply in the cases of $\sin\delta_0$, $\sin\delta_+$ and $\sin\delta_-$ in the range of the resonance. Meanwhile, $\sin\delta_+$ and $\sin\delta_-$ change weakly compared with the $\sin\delta_0$. It can be seen from Figure 5 that r_0, r_+, and r_- change more rapidly when the $\pi^+\pi^-$ pairs in the vicinity of the ω resonance. Since amplitude A_T is a small quantity, it contributes so little to the transverse H_+ and H_- of the helicity amplitudes in which they are almost equal in Eq. (27). So, we can see that the two curves $\sin(\delta_+)$ and (δ_-) of Figure 4 are the different in the region of resonance but almost the same in other regions.

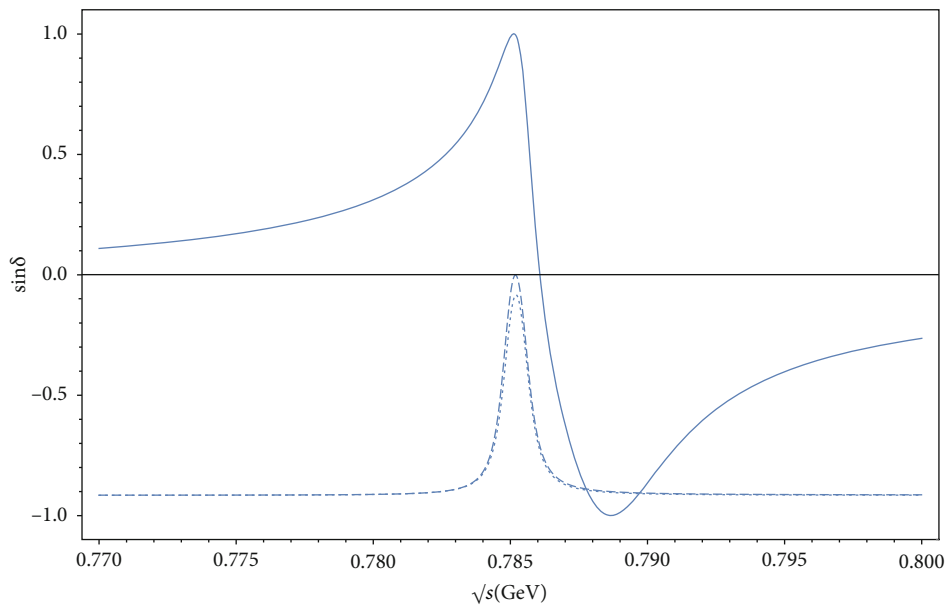

Figure 4: Plot of sin δ as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements for $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$. The solid line, dotted line, and dashed line correspond to sin δ_0, sin δ_+, and sin δ_-, respectively.

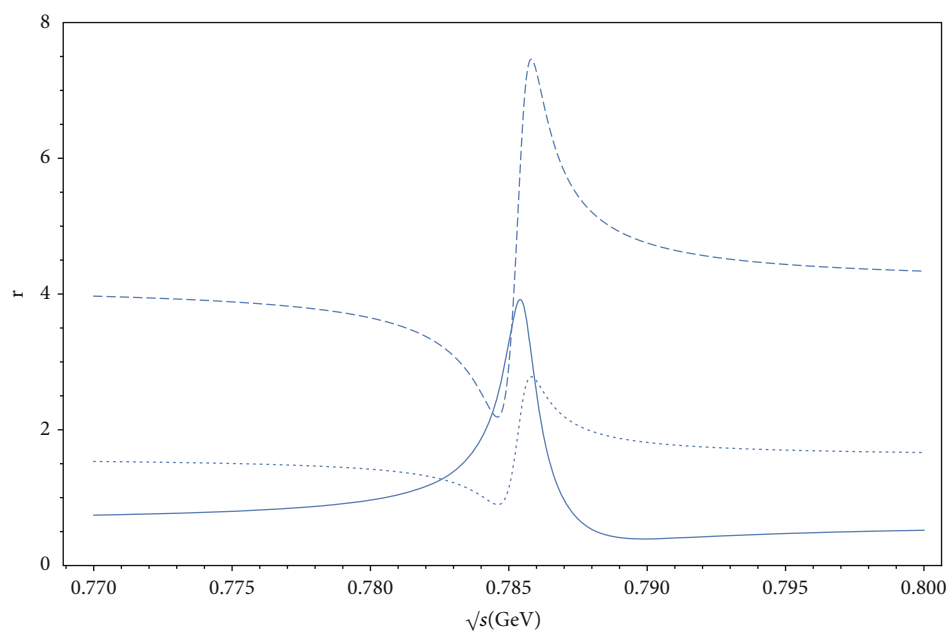

Figure 5: Plot of r as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements for $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$. The solid line, dotted line, and dashed line correspond to r_0, r_+, and r_-, respectively.

We have shown that the $\rho-\omega$ mixing does enhance the direct CP violating asymmetry and provide a mechanism for large CP violation in the perturbative QCD factorization scheme. In other words, it is important to see whether it is possible to observe this large CP violating asymmetry in the experiments. This depends on the branching ratio for the decay channel of $\bar{B}_s^0 \longrightarrow \rho^0 K^{*0}$. We will study this problem in the next section.

6.2. Branching Ratio via $\rho-\omega$ Mixing in $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$. In the pQCD, we calculate the value of the branching ratio via $\rho-\omega$ mixing mechanism for the decay channel $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$. The numerical result is shown for the decay process in Figure 6. Based on a reasonable parameter range, we obtain the maximum branching ratio of $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$ as $(3.05^{+0.25}_{-0.20}) \times 10^{-7}$, which is consistent with the result

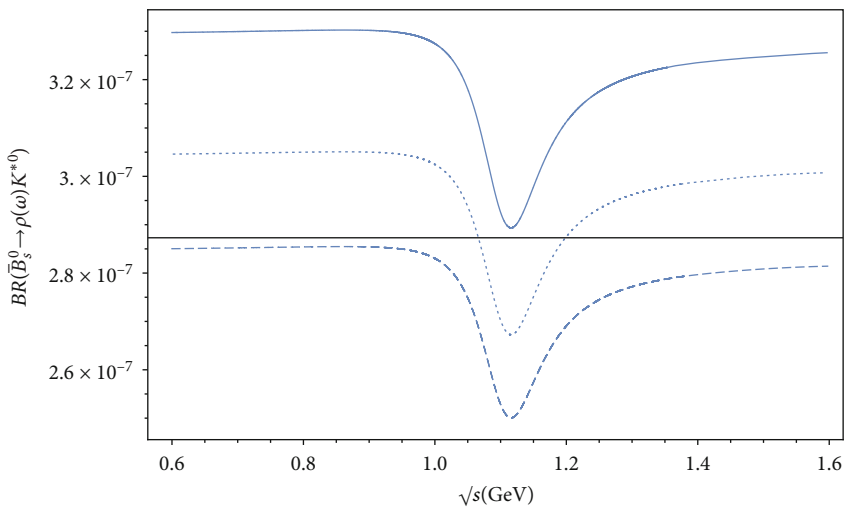

FIGURE 6: The branching ratio, $BR(\bar{B}_s^0 \longrightarrow \rho(\omega)K^{*0})$, as a function of $\sqrt{s}$ for the different CKM matrix elements. The solid line, dotted line, and dashed line correspond to the maximum, middle, and minimum CKM matrix elements for the decay channel of $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$, respectively.

in [81, 87]. The error comes from CKM parameters. On the one hand, although we calculate the branching ratio due to $\rho-\omega$ mixing in the pQCD factorization scheme, we find that the contribution of $\rho-\omega$ mixing to the branching ratio of $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$ is small and can be neglected. This is because branching ratio is proportional to the contribution of $|H_{\rho\omega i(i=0,+,-)}|^2$ in Eq. (59). The square of the $\rho-\omega$ mixing parameter $|\tilde{\Pi}_{\rho\omega}|^2$ is about 10^{-6} GeV^2, which leads to large suppression for the contribution of the branching ratio of $\bar{B}_s^0 \longrightarrow \omega K^{*0}$ and the effect of the width Γ_ρ of the resonances ρ from s_ρ in Eq. (60). On the other hand, the $\rho-\omega$ mixing mechanism produces new strong phase differences. In $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$ decay, we take into account the $\rho-\omega$ mixing contribution to the branching ratio, and we find that the two decays interfere with each other to generate a new resonance region around 1.1 GeV. By expanding the factor $\tilde{\Pi}_{\rho\omega}/(s_\rho - M_\omega^2) + \mathrm{i}M_\omega\Gamma_\omega$ from Eq. (60), the denominator $(s_\rho - M_\omega^2) + \mathrm{i}M_\omega\Gamma_\omega$ produces a new propagator $s-(M_\rho^2 + M_\omega^2) + \mathrm{i}(M_\rho\Gamma_\rho + M_\omega\Gamma_\omega)$. This is the reason why the dips of the curves in Figure 6 does not appear in the region of resonance but around 1.1 GeV ($\sqrt{M_\rho^2 + M_\omega^2} \approx 1.1$ GeV).

The Large Hadron Collider (LHC) is a proton-proton collider that has started at the European Organization for Nuclear Research (CERN). With the designed center-of-mass energy 14 TeV and luminosity $L = 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$, the LHC provides a high energy frontier at TeV-level scale and an opportunity to further improve the consistency test for the CKM matrix. LHCb is a dedicated heavy flavor physics experiments and one of the main projects of LHC. Its main goal is to search for indirect evidence of new physics in CP violation and rare decays in the interactions of beauty and charm hadrons systems, by looking for the effects of new particles in decay processes that are precisely predicted in the SM. Such studies can help us to comprehend the matter-antimatter asymmetry of the universe. Recently, the LHCb collaboration found clear evidence for direct CP violation in some three-body decay channels of B meson. Large CP violation is obtained for the decay channels of $B^\pm \longrightarrow \pi^\pm\pi^+\pi^-$ in the localized phase spaces region $m^2_{\pi^+\pi^-\mathrm{low}} < 0.4$ GeV^2 and $m^2_{\pi^+\pi^-\mathrm{high}} > 15$ GeV^2 [7, 88]. A zoom of the $\pi^+\pi^-$ invariant mass from the $B^+ \longrightarrow \pi^+\pi^+\pi^-$ decay process is shown the region $m^2_{\pi^+\pi^-\mathrm{low}} < 1$ GeV^2 zone in Ref. [88]. In addition, the branching ratio of $\bar{B}_s^0 \longrightarrow \pi^+\pi^-\phi$ is probed in the $\pi^+\pi^-$ invariant mass range $400 < m(\pi^+\pi^-) < 1600$ MeV/c^2 [89]. In the next years, we expect the LHCb collaboration to collect date for detecting our prediction of CP violation from the $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ decay process when the invariant mass of $\pi^+\pi^-$ is in the vicinity of the ω resonance.

At the LHC, the b hadrons come from pp collisions. The possible asymmetry between the numbers of the b hadrons H_b and those of their antiparticles $\bar{H}_b$ has been studied by using the intrinsic heavy quark model and the Lund string fragmentation model [90, 91]. It has been shown that this asymmetry can only reach values of a few percent. In the following discussion, we will ignore this small asymmetry and give the numbers of $H_b\bar{H}_b$ pairs needed for observing our prediction of the CP violating asymmetries. These numbers depend on both the magnitudes of the CP violating asymmetries and the branching ratios of heavy hadron decays which are model dependent. For one-standard deviation (1σ) signature and three-standard deviation (3σ) signature and the numbers of $H_b\bar{H}_b$ pairs, we need [92–94]

$$N_{H_b\bar{H}_b}(1\sigma) \sim \frac{1}{BR\left(\bar{B}_s^0 \longrightarrow \rho^0 K^{*0}\right)A_{CP}^2}\left(1 - A_{CP}^2\right),$$
$$N_{H_b\bar{H}_b}(3\sigma) \sim \frac{9}{BR\left(\bar{B}_s^0 \longrightarrow \rho^0 K^{*0}\right)A_{CP}^2}\left(1 - A_{CP}^2\right), \quad (66)$$

where A_{CP} is the CP violation in the process of $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$. Now, we can estimate the possibility to observe CP violation. The branching ratio for $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$ is of the order 10^{-7}, and then, the number $N_{H_b\bar{H}_b}(1\sigma)$ is 10^6 for 1σ signature and 10^7 for 3σ signature, theoretically, in order to achieve the current experiments on b hadrons, which can only provide about $10^7 B_s\bar{B}_s$ pairs. Therefore, it is very possible to observe the large CP violation for $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ when the invariant masses of $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance in experiments at the LHC.

7. Summary and Conclusion

In this paper, we have studied the direct CP violation for the decay process of $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$ in perturbative QCD. It has been found that, by using $\rho-\omega$ mixing, the CP violation can be enhanced at the area of ω resonance. There is the resonance effect via $\rho-\omega$ mixing which can produce large strong phase in this decay process. As a result, one can find that the maximum CP violation can reach -50.19% when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the ω resonance. Furthermore, taking $\rho-\omega$ mixing into account, we have calculated the branching ratio of the decays of $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0}$. We have also given the numbers of $B_s\bar{B}_s$ pairs required for observing our prediction of the CP violating asymmetries at the LHC experiments.

The $\rho-\omega$ mixing is a small effect duo to the isospin violation. One can estimate the contributions by comparing the two terms on the right-hand side of Eqs. (38) and (39). However, when the invariant mass squared of the $\pi^+\pi^-$ pair is in the vicinity of the omega, $s_\omega \sim im_\omega\Gamma_\omega$, it becomes comparable with $\tilde{\Pi}_{\rho\omega}$. In other words, the $\rho-\omega$ mixing becomes important in the vicinity of the omega. This is also the reason why we only see large CP violation in the vicinity of ω. At the same time, the mixing parameter has determined the magnitude, and s dependence of the effective $\rho-\omega$ mixing matrix element $\tilde{\Pi}_{\rho\omega}(s)$ fits to $e^+e^- \longrightarrow \pi^+\pi^-$ data in the vicinity of the ω resonance. We can make the Taylor expansion for $\tilde{\Pi}_{\rho\omega}(s)$ at $s = m_w^2$ and ignore the s dependence of $\tilde{\Pi}_{\rho\omega}(s)$ in practice. It will cause a large resonance near invariant masses of the $\pi^+\pi^-$ pairs, and the effect will be negligible in distant regions. In the $\rho-\omega$ mechanism, we will ignore the contribution of the final state interaction between the pions in the resonant regions associated with ρ and ω in the decay process of $\bar{B}_s^0 \longrightarrow \rho^0(\omega)K^{*0} \longrightarrow \pi^+\pi^-K^{*0}$.

In our calculation, there are some uncertainties. The major uncertainties come from the input parameters. In particular, these include the CKM matrix element, the particle mass, the perturbative QCD approach, and the hadronic parameters (decay constants, the wave functions, the shape parameters, etc.). We expect that our predictions will provide useful guidance for future experiments.

Appendix

Related Functions Defined in the Text

In this appendix, we present explicit expressions of the factorizable and nonfactorizable amplitudes in the perturbative QCD [19–21, 60]. The factorizable amplitudes $F^{LL,i}_{B_s\longrightarrow K^*}(a_i)$, $F^{LL,i}_{ann}(a_i)$, and $F^{SP,i}_{ann}(a_i)$ $(i = \mathrm{L, N, T})$ are written as

$$\begin{aligned} f_{M_2}F^{LL,L}_{B_s\longrightarrow K^*}(a_i) = {} & 8\pi C_F M^4_{B_s} f_{M_2}\int_0^1 dx_1dx_3 \\ & \cdot \int_0^\infty b_1db_1b_3db_3\phi_{B_s}(x_1,b_1)\,\{a_i(t_a)E_e(t_a) \\ & \times \left[r_3(1-2x_3)\left(\phi_3^s(x_3)+\phi_3^t(x_3)\right)\right. \\ & \left.+(1+x_3)\phi_3(x_3)\right]h_e(x_1,x_3,b_1,b_3) \\ & +2r_3\phi_3^s(x_3)a_i\left(t_a'\right)E_e\left(t_a'\right)h_e(x_3,x_1,b_3,b_1)\Big\}, \end{aligned} \tag{A.1}$$

$$\begin{aligned} f_{M_2}F^{LL,N}_{B_s\longrightarrow K^*}(a_i) = {} & 8\pi C_F M^4_{B_s} f_{M_2} r_2\int_0^1 dx_1dx_3 \\ & \cdot \int_0^\infty b_1db_1b_3db_3\phi_{B_s}(x_1,b_1)\{h_e(x_1,x_3,b_1,b_3) \\ & \times E_e(t_a)a_i(t_a)[2r_3\phi_3^v(x_3)+r_3x_3(\phi_3^v(x_3) \\ & -\phi_3^a(x_3))+\phi_3^T(x_3)]+r_3[\phi_3^v(x_3) \\ & +\phi_3^a(x_3)]E_e(t_{a'})a_i(t_{a'})h_e(x_3,x_1,b_3,b_1)\}, \end{aligned} \tag{A.2}$$

$$\begin{aligned} f_{M_2}F^{LL,T}_{B_s\longrightarrow K^*}(a_i) = {} & 16\pi C_F M^4_{B_s} f_{M_2} r_2\int_0^1 dx_1dx_3 \\ & \cdot \int_0^\infty b_1db_1b_3db_3\phi_{B_s}(x_1,b_1)\{h_e(x_1,x_3,b_1,b_3) \\ & \times [2r_3\phi_3^v(x_3)-r_3x_3(\phi_3^v(x_3)-\phi_3^a(x_3)) \\ & +\phi_3^T(x_3)]E_e(t_a)a_i(t_a)+r_3[\phi_3^v(x_3) \\ & +\phi_3^a(x_3)]E_e(t_{a'})a_i(t_{a'})h_e(x_3,x_1,b_3,b_1)\}, \end{aligned} \tag{A.3}$$

$$\begin{aligned} f_{B_s}F^{LL,L}_{ann}(a_i) = {} & 8\pi C_F M^4_{B_s} f_{B_s}\int_0^1 dx_2dx_3 \\ & \cdot \int_0^\infty b_2db_2b_3db_3\{a_i(t_c)E_a(t_c)[(x_3-1)\phi_2(x_2)\phi_3(x_3) \\ & -4r_2r_3\phi_2^s(x_2)\phi_3^s(x_3)+2r_2r_3x_3\phi_2^s(x_2)(\phi_3^s(x_3) \\ & -\phi_3^t(x_3))]h_a(x_2,1-x_3,b_2,b_3)+[x_2\phi_2(x_2)\phi_3(x_3) \\ & +2r_2r_3\left(\phi_2^s(x_2)-\phi_2^t(x_2)\right)\phi_3^s(x_3)+2r_2r_3x_2(\phi_2^s(x_2) \\ & +\phi_2^t(x_2))\phi_3^s(x_3)]a_i\left(t_c'\right)E_a\left(t_c'\right)h_a(1-x_3,x_2,b_3,b_2)\}, \end{aligned} \tag{A.4}$$

$$f_{B_s}F_{ann}^{LL,N}(a_i) = -8\pi C_F M_{B_s}^4 f_{B_s} r_2 r_3 \int_0^1 dx_2 dx_3 \cdot \int_0^\infty b_2 db_2 b_3 db_3 \{E_a(t_c) a_i(t_c) h_a(x_2, 1-x_3, b_2, b_3)) \times [x_3(\phi_2^v(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^v(x_3)) + (2-x_3)(\phi_2^v(x_2)\phi_3^v(x_3) + \phi_2^a(x_2)\phi_3^a(x_3))] - h_a(1-x_3, x_2, b_3, b_2)[(1+x_2)(\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^v(x_2)\phi_3^v(x_3)) - (1-x_2)(\phi_2^a(x_2)\phi_3^v(x_3)) + \phi_2^v(x_2)\phi_3^a(x_3)]E_a(t_{c'})a_i(t_{c'})\}, \tag{A.5}$$

$$f_{B_s}F_{ann}^{LL,T}(a_i) = -16\pi C_F M_{B_s}^4 f_{B_s} r_2 r_3 \int_0^1 dx_2 dx_3 \cdot \int_0^\infty b_2 db_2 b_3 db_3 \{[x_3(\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^v(x_2)\phi_3^v(x_3)) + (2-x_3)(\phi_2^a(x_2)\phi_3^v(x_3) + \phi_2^v(x_2)\phi_3^a(x_3))]E_a(t_c)a_i(t_c)h_a(x_2, 1-x_3, b_2, b_3) + h_a(1-x_3, x_2, b_3, b_2)[(1-x_2)(\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^v(x_2)\phi_3^v(x_3)) - (1+x_2)(\phi_2^a(x_2)\phi_3^v(x_3) + \phi_2^v(x_2)\phi_3^a(x_3))]E_a(t_{c'})a_i(t_{c'})\}, \tag{A.6}$$

$$f_{B_s}F_{ann}^{SP,L}(a_i) = 16\pi C_F M_{B_s}^4 f_{B_s} \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \cdot \{[(x_3-1)r_3\phi_2(x_2)(\phi_3^s(x_3) + \phi_3^t(x_3)) - 2r_2\phi_2^s(x_2)\phi_3(x_3)]a_i(t_c)E_a(t_c)h_a(x_2, 1-x_3, b_2, b_3) - [2r_3\phi_2(x_2)\phi_3^s(x_3) + r_2x_2(\phi_2^s(x_2) - \phi_2^t(x_2))\phi_3(x_3)]a_i(t_c')E_a(t_c')h_a(1-x_3, x_2, b_3, b_2)\}, \tag{A.7}$$

$$f_{B_s}F_{ann}^{SP,T}(a_i) = 2f_{B_s}F_{ann}^{SP,N}(a_i) = -32\pi C_F M_{B_s}^4 f_{B_s} \int_0^1 dx_2 dx_3 \cdot \int_0^\infty b_2 db_2 b_3 db_3 \{r_2(\phi_2^a(x_2) + \phi_2^v(x_2))\phi_3^T(x_3) \times E_a(t_c)a_i(t_c)h_a(x_2, 1-x_3, b_2, b_3) + r_3\phi_2^T(x_2) \times (\phi_3^v(x_3) - \phi_3^a(x_3))E_a(t_{c'})a_i(t_{c'})h_a(1-x_3, x_2, b_3, b_2)\}, \tag{A.8}$$

with the color factor $C_F = 4/3$, a_i represents the corresponding Wilson coefficients for the specific decay channels and f_{M_2} and f_{B_s} refer to the decay constants of M_2 (ρ or ω) and $\bar{B}_s^0$ mesons. In the above functions, $r_2(r_3) = M_{V_2}(M_{V_3})/M_{B_s}$ and $\phi_2(\phi_3) = \phi_{\rho/\omega}(\phi_{K^*})$, with M_{B_s} and $M_{V_2}(m_{V_3})$ being the masses of the initial and final states.

The nonfactorizable amplitudes $M_{B_s\longrightarrow K^*}^{LL,i}(a_i)$, $M_{B_s\longrightarrow K^*}^{LR,i}(a_i)$, $M_{B_s\longrightarrow K^*}^{SP,i}(a_i)$, $M_{ann}^{LL,i}(a_i)$, and $M_{ann}^{LR,i}(a_i)$ ($i = \mathrm{L, N, T}$) are written as

$$M_{B_s\longrightarrow K^*}^{LL,L}(a_i) = 32\pi C_F M_{B_s}^4/\sqrt{6}\int_0^1 dx_1 dx_2 dx_3 \cdot \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1)\phi_2(x_2) \times \{[(1-x_2)\phi_3(x_3) - r_3x_3(\phi_3^s(x_3) - \phi_3^t(x_3))]a_i(t_b)E_e'(t_b) \times h_n(x_1, 1-x_2, x_3, b_1, b_2) + h_n(x_1, x_2, x_3, b_1, b_2) \times [r_3x_3(\phi_3^s(x_3) + \phi_3^t(x_3)) - (x_2+x_3)\phi_3(x_3)]a_i(t_b')E_e'(t_b')\}, \tag{A.9}$$

$$M_{B_s\longrightarrow K^*}^{LL,N}(a_i) = 32\pi C_F M_{B_s}^4 r_2/\sqrt{6}\int_0^1 dx_1 dx_2 dx_3 \cdot \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \times \{[x_2(\phi_2^v(x_2) + \phi_2^a(x_2))\phi_3^T(x_3) - 2r_3(x_2+x_3)(\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^v(x_2)\phi_3^v(x_3))] \times h_n(x_1, x_2, x_3, b_1, b_2)E_{e'}(t_{b'})a_i(t_{b'}) + (1-x_2)(\phi_2^v(x_2) + \phi_2^a(x_2))\phi_3^T(x_3) \times E_{e'}(t_b)a_i(t_b)h_n(x_1, 1-x_2, x_3, b_1, b_2)\}, \tag{A.10}$$

$$M_{B_s\longrightarrow K^*}^{LL,T}(a_i) = 64\pi C_F M_{B_s}^4 r_2/\sqrt{6}\int_0^1 dx_1 dx_2 dx_3 \cdot \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1)\{E_{e'}(t_{b'})a_i(t_{b'}) \times [x_2(\phi_2^v(x_2) + \phi_2^a(x_2))\phi_3^T(x_3) - 2r_3(x_2+x_3)(\phi_2^v(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^v(x_3))]h_n(x_1, x_2, x_3, b_1, b_2) + (1-x_2)[\phi_2^v(x_2) + \phi_2^a(x_2)]\phi_3^T(x_3) \times E_{e'}(t_b)a_i(t_b)h_n(x_1, 1-x_2, x_3, b_1, b_2)\}, \tag{A.11}$$

$$M_{B_s\longrightarrow K^*}^{LR,L}(a_i) = 32\pi C_F M_{B_s}^4 r_2/\sqrt{6}\int_0^1 dx_1 dx_2 dx_3 \cdot \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \times \{h_n(x_1, 1-x_2, x_3, b_1, b_2)[(1-x_2)\phi_3(x_3)(\phi_2^s(x_2) + \phi_2^t(x_2)) + r_3x_3(\phi_2^s(x_2) - \phi_2^t(x_2))(\phi_3^s(x_3) + \phi_3^t(x_3)) + (1-x_2)r_3(\phi_2^s(x_2) + \phi_2^t(x_2))(\phi_3^s(x_3) - \phi_3^t(x_3))]a_i(t_b)E_e'(t_b) - h_n(x_1, x_2, x_3, b_1, b_2) \cdot [x_2\phi_3(x_3)(\phi_2^s(x_2) - \phi_2^t(x_2)) + r_3x_2(\phi_2^s(x_2) - \phi_2^t(x_2))(\phi_3^s(x_3) - \phi_3^t(x_3)) + r_3x_3(\phi_2^s(x_2) + \phi_2^t(x_2))(\phi_3^s(x_3) + \phi_3^t(x_3))]a_i(t_b')E_e'(t_b')\}, \tag{A.12}$$

$$M^{LR,T}_{B_s\to K^*}(a_i) = 2M^{LR,N}_{B_s\to K^*}(a_i) = 64\pi C_F M^4_{B_s}/\sqrt{6}\int_0^1 dx_1dx_2dx_3 \cdot \int_0^\infty b_1db_1b_2db_2\phi_{B_s}(x_1,b_1)\times r_3x_3\phi_2^T(x_2)(\phi_3^v(x_3) - \phi_3^a(x_3))\times\{E_{e'}(t_b)a_i(t_b)h_n(x_1,1-x_2,x_3,b_1,b_2) + E_{e'}(t_{b'})a_i(t_{b'})h_n(x_1,x_2,x_3,b_1,b_2)\}, \tag{A.13}$$

$$M^{SP,L}_{B_s\to K^*}(a_i) = 32\pi C_F M^4_{B_s}/\sqrt{6}\int_0^1 dx_1dx_2dx_3 \cdot \int_0^\infty b_1db_1b_2db_2\phi_{B_s}(x_1,b_1)\phi_2(x_2) \times\{[(x_2-x_3-1)\phi_3(x_3)+r_3x_3(\phi_3^s(x_3)+\phi_3^t(x_3))] \times a_i(t_b)E'_e(t_b)h_n(x_1,1-x_2,x_3,b_1,b_2) + a_i(t'_b)E'_e(t'_b)\times[x_2\phi_3(x_3)+r_3x_3(\phi_3^t(x_3) - \phi_3^s(x_3))]h_n(x_1,x_2,x_3,b_1,b_2)\}, \tag{A.14}$$

$$M^{SP,N}_{B_s\to K^*}(a_i) = 32\pi C_F M^4_{B_s}/\sqrt{6}\int_0^1 dx_1dx_2dx_3 \cdot \int_0^\infty b_1db_1b_2db_2\phi_{B_s}(x_1,b_1)r_2\times\{x_2(\phi_2^v(x_2) - \phi_2^a(x_2))\phi_3^T(x_3)E_{e'}(t_{b'})a_i(t_{b'})h_n(x_1,x_2,x_3,b_1,b_2) + h_n(x_1,1-x_2,x_3,b_1,b_2)[(1-x_2)(\phi_2^v(x_2) - \phi_2^a(x_2))\phi_3^T(x_3) - 2r_3(1-x_2+x_3)(\phi_2^v(x_2)\phi_3^v(x_3) - \phi_2^a(x_2)\phi_3^a(x_3))]E_{e'}(t_b)a_i(t_b)\}, \tag{A.15}$$

$$M^{SP,T}_{B_s\to K^*}(a_i) = 64\pi C_F M^4_{B_s}/\sqrt{6}\int_0^1 dx_1dx_2dx_3 \cdot \int_0^\infty b_1db_1b_2db_2\phi_{B_s}(x_1,b_1)r_2\times\{x_2(\phi_2^v(x_2) - \phi_2^a(x_2))\phi_3^T(x_3)E_{e'}(t_{b'})a_i(t_{b'})h_n(x_1,x_2,x_3,b_1,b_2) + h_n(x_1,1-x_2,x_3,b_1,b_2)[(1-x_2)(\phi_2^v(x_2) - \phi_2^a(x_2))\phi_3^T(x_3) - 2r_3(1-x_2+x_3)(\phi_2^v(x_2)\phi_3^a(x_3) - \phi_2^a(x_2)\phi_3^v(x_3))]E_{e'}(t_b)a_i(t_b)\}, \tag{A.16}$$

$$M^{LL,L}_{ann}(a_i) = 32\pi C_F M^4_{B_s}/\sqrt{6}\int_0^1 dx_1dx_2dx_3 \cdot \int_0^\infty b_1db_2b_2db_2\phi_{B_s}(x_1,b_1)\{h_{na}(x_1,x_2,x_3,b_1,b_2) \times[r_2r_3x_3(\phi_2^s(x_2)-\phi_2^t(x_2))(\phi_3^s(x_3)+\phi_3^t(x_3)) - (x_2\phi_2(x_2)\phi_3(x_3)+4r_2r_3\phi_2^s(x_2)\phi_3^s(x_3)) + r_2r_3(1-x_2)(\phi_2^s(x_2)+\phi_2^t(x_2))(\phi_3^s(x_3) - \phi_3^t(x_3))]a_i(t_d)E'_a(t_d) + h'_{na}(x_1,x_2,x_3,b_1,b_2) \times[(1-x_3)\phi_2(x_2)\phi_3(x_3)+(1-x_3)r_2r_3(\phi_2^s(x_2) + \phi_2^t(x_2))(\phi_3^s(x_3)-\phi_3^t(x_3)) + x_2r_2r_3(\phi_2^s(x_2) - \phi_2^t(x_2))(\phi_3^s(x_3)+\phi_3^t(x_3))]a_i(t'_d)E'_a(t'_d)\}, \tag{A.17}$$

$$M^{LL,N}_{ann}(a_i) = -64\pi C_F M^4_{B_s}r_2r_3/\sqrt{6}\int_0^1 dx_1dx_2dx_3 \cdot \int_0^\infty b_1db_2b_2db_2\phi_{B_s}(x_1,b_1)[\phi_2^v(x_2)\phi_3^v(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)]E_{a'}(t_d)a_i(t_d)h_{na}(x_1,x_2,x_3,b_1,b_2), \tag{A.18}$$

$$M^{LL,T}_{ann}(a_i) = -128\pi C_F M^4_{B_s}r_2r_3/\sqrt{6}\int_0^1 dx_1dx_2dx_3 \cdot \int_0^\infty b_1db_2b_2db_2\phi_{B_s}(x_1,b_1)[\phi_2^v(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^v(x_3)]E_{a'}(t_d)a_i(t_d)h_{na}(x_1,x_2,x_3,b_1,b_2), \tag{A.19}$$

$$M^{LR,L}_{ann}(a_i) = 32\pi C_F M^4_{B_s}/\sqrt{6}\int_0^1 dx_1dx_2dx_3 \cdot \int b_1db_1b_2db_2\phi_{B_s}(x_1,b_1)\{h_{na}(x_1,x_2,x_3,b_1,b_2) \times[r_2(x_2-2)(\phi_2^s(x_2)+\phi_2^t(x_2))\phi_3(x_3) + r_3(1+x_3)\phi_2(x_2)(\phi_3^s(x_3)-\phi_3^t(x_3))]a_i(t_d)E'_a(t_d) + h'_{na}(x_1,x_2,x_3,b_1,b_2)[-r_2x_2(\phi_2^s(x_2) + \phi_2^t(x_2))\phi_3(x_3) + r_3(1-x_3)\phi_2(x_2)(\phi_3^s(x_3) - \phi_3^t(x_3))]a_i(t'_d)E'_a(t'_d)\}, \tag{A.20}$$

$$M^{LR,T}_{ann}(a_i) = 2M^{LR,N}_{ann}(a_i) = -64\pi C_F M^4_{B_s}/\sqrt{6}\int_0^1 dx_1dx_2dx_3 \cdot \int b_1db_1b_2db_2\phi_{B_s}(x_1,b_1)\{h_{na}{}'(x_1,x_2,x_3,b_1,b_2) \times[r_2x_2(\phi_2^v(x_2)+\phi_2^a(x_2))\phi_3^T(x_3) - r_3(1-x_3)\phi_2^T(x_2)(\phi_3^v(x_3)-\phi_3^a(x_3))]E_{a'}(t_{d'})a_i(t_{d'}) + [r_2(2-x_2)(\phi_2^v(x_2)+\phi_2^a(x_2))\phi_3^T(x_3) - r_3(1+x_3)\phi_2^T(x_2)(\phi_3^v(x_3)-\phi_3^a(x_3))] \times E_{a'}(t_d)a_i(t_d)h_{na}(x_1,x_2,x_3,b_1,b_2)\}. \tag{A.21}$$

The hard scale t are chosen as the maximum of the virtuality of the internal momentum transition in the hard amplitudes, including $1/b_i$:

$$t_a = \max\{\sqrt{x_3}M_{B_s}, 1/b_1, 1/b_3\}, \tag{A.22}$$

$$t'_a = \max\{\sqrt{x_1}M_{B_s}, 1/b_1, 1/b_3\}, \tag{A.23}$$

$$t_b = \max\left\{\sqrt{x_1x_3}M_{B_s}, \sqrt{|1-x_1-x_2|x_3}M_{B_s}, 1/b_1, 1/b_2\right\}, \tag{A.24}$$

$$t'_b = \max\left\{\sqrt{x_1x_3}M_{B_s}, \sqrt{|x_1-x_2|x_3}M_{B_s}, 1/b_1, 1/b_2\right\}, \tag{A.25}$$

$$t_c = \max\left\{\sqrt{1-x_3}M_{B_s}, 1/b_2, 1/b_3\right\}, \tag{A.26}$$

$$t_c' = \max\left\{\sqrt{x_2}M_{B_s}, 1/b_2, 1/b_3\right\}, \tag{A.27}$$

$$t_d = \max\left\{\sqrt{x_2(1-x_3)}M_{B_s}, \sqrt{1-(1-x_1-x_2)x_3}M_{B_s}, 1/b_1, 1/b_2\right\}, \tag{A.28}$$

$$t_d' = \max\left\{\sqrt{x_2(1-x_3)}M_{B_s}, \sqrt{|x_1-x_2|(1-x_3)}M_{B_s}, 1/b_1, 1/b_2\right\}. \tag{A.29}$$

The function h, coming from the Fourier transform of hard part H, are written as

$$\begin{aligned} h_e(x_1,x_3,b_1,b_3) = {} & \left[\theta(b_1-b_3)I_0\left(\sqrt{x_3}M_{B_s}b_3\right)K_0\left(\sqrt{x_3}M_{B_s}b_1\right)\right. \\ & +\theta(b_3-b_1)I_0\left(\sqrt{x_3}M_{B_s}b_1\right)K_0 \\ & \left.\cdot\left(\sqrt{x_3}M_{B_s}b_3\right)\right]K_0\left(\sqrt{x_1x_3}M_{B_s}b_1\right)S_t(x_3), \end{aligned} \tag{A.30}$$

$$\begin{aligned} h_n(x_1,x_2,x_3,b_1,b_2) = {} & \left[\theta(b_2-b_1)K_0\left(\sqrt{x_1x_3}M_{B_s}b_2\right)I_0\left(\sqrt{x_1x_3}M_{B_s}b_1\right)\right. \\ & \left.+\theta(b_1-b_2)K_0\left(\sqrt{x_1x_3}M_{B_s}b_1\right)I_0\left(\sqrt{x_1x_3}M_{B_s}b_2\right)\right] \\ & \times\begin{cases} \dfrac{i\pi}{2}H_0^{(1)}\left(\sqrt{(x_2-x_1)x_3}M_{B_s}b_2\right), & x_1-x_2<0 \\ K_0\left(\sqrt{(x_1-x_2)x_3}M_{B_s}b_2\right), & x_1-x_2>0 \end{cases}, \end{aligned} \tag{A.31}$$

$$\begin{aligned} h_a(x_2,x_3,b_2,b_3) = {} & \left(\frac{i\pi}{2}\right)^2 S_t(x_3) \\ & \cdot\left[\theta(b_2-b_3)H_0^{(1)}\left(\sqrt{x_3}M_{B_s}b_2\right)J_0\left(\sqrt{x_3}M_{B_s}b_3\right)\right. \\ & \left.+\theta(b_3-b_2)H_0^{(1)}\left(\sqrt{x_3}M_{B_s}b_3\right)J_0\left(\sqrt{x_3}M_{B_s}b_2\right)\right]H_0^{(1)} \\ & \cdot\left(\sqrt{x_2x_3}M_{B_s}b_2\right), \end{aligned} \tag{A.32}$$

$$\begin{aligned} h_{na}(x_1,x_2,x_3,b_1,b_2) = {} & \frac{i\pi}{2}\left[\theta(b_1-b_2)H_0^{(1)}\right. \\ & \cdot\left(\sqrt{x_2(1-x_3)}M_{B_s}b_1\right)J_0\left(\sqrt{x_2(1-x_3)}M_{B_s}b_2\right) \\ & +\theta(b_2-b_1)H_0^{(1)}\left(\sqrt{x_2(1-x_3)}M_{B_s}b_2\right)J_0 \\ & \left.\cdot\left(\sqrt{x_2(1-x_3)}M_{B_s}b_1\right)\right] \\ & \times K_0\left(\sqrt{1-(1-x_1-x_2)x_3}M_{B_s}b_1\right), \end{aligned} \tag{A.33}$$

$$\begin{aligned} & h_{na}'(x_1,x_2,x_3,b_1,b_2) \\ & = \frac{i\pi}{2}\left[\theta(b_1-b_2)H_0^{(1)}\left(\sqrt{x_2(1-x_3)}M_{B_s}b_1\right)J_0\right. \\ & \cdot\left(\sqrt{x_2(1-x_3)}M_{B_s}b_2\right)+\theta(b_2-b_1)H_0^{(1)} \\ & \left.\cdot\left(\sqrt{x_2(1-x_3)}M_{B_s}b_2\right)J_0\left(\sqrt{x_2(1-x_3)}M_{B_s}b_1\right)\right] \\ & \times\begin{cases} \dfrac{i\pi}{2}H_0^{(1)}\left(\sqrt{(x_2-x_1)(1-x_3)}M_{B_s}b_1\right), & x_1-x_2<0 \\ K_0\left(\sqrt{(x_1-x_2)(1-x_3)}M_{B_s}b_1\right), & x_1-x_2>0 \end{cases}, \end{aligned} \tag{A.34}$$

where J_0 and Y_0 are the Bessel function with $H_0^{(1)}(z) = \mathrm{J}_0(z) + i\,\mathrm{Y}_0(z)$.

The threshold resums factor S_t follows the parameterized

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c, \tag{A.35}$$

where the parameter $c = 0.4$ [60].

The evolution factors $E_e^{()}$ and $E_a^{()}$ entering in the expressions for the matrix elements are given by

$$\begin{aligned} & E_e(t) = \alpha_s(t)\exp\left[-S_B(t)-S_{M_3}(t)\right], E'_e(t) = \alpha_s(t)\exp \\ & \cdot\left[-S_B(t)-S_{M_2}(t)-S_{M_3}(t)\right]\Big|_{b_1=b_3}, \end{aligned} \tag{A.36}$$

$$\begin{aligned} & E_a(t) = \alpha_s(t)\exp\left[-S_{M_2}(t)-S_{M_3}(t)\right], E'_a(t) = \alpha_s(t)\exp \\ & \cdot\left[-S_B(t)-S_{M_2}(t)-S_{M_3}(t)\right]\Big|_{b_2=b_3}, \end{aligned} \tag{A.37}$$

in which the Sudakov exponents are defined as

$$S_B(t) = s\left(x_1\frac{M_{B_s}}{\sqrt{2}}, b_1\right) + \frac{5}{3}\int_{1/b_1}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{A.38}$$

$$\begin{aligned} S_{M_2}(t) = {} & s\left(x_2\frac{M_{B_s}}{\sqrt{2}}, b_2\right) + s\left((1-x_2)\frac{M_{B_s}}{\sqrt{2}}, b_2\right) \\ & + 2\int_{1/b_2}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \end{aligned} \tag{A.39}$$

$$\begin{aligned} S_{M_3}(t) = {} & s\left(x_3\frac{M_{B_s}}{\sqrt{2}}, b_3\right) + s\left((1-x_3)\frac{M_{B_s}}{\sqrt{2}}, b_3\right) \\ & + 2\int_{1/b_3}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \end{aligned} \tag{A.40}$$

where $\gamma_q = -\alpha_s/\pi$ is the anomalous dimension of the quark. The explicit form for the function $s(Q,b)$ is

$$\begin{aligned} s(Q,b) = {} & \frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right) - \frac{A^{(1)}}{2\beta_1}\left(\hat{q}-\hat{b}\right) + \frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}}-1\right) \\ & - \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma_E-1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right) \\ & + \frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\ln(2\hat{q})+1}{\hat{q}} - \frac{\ln\left(2\hat{b}\right)+1}{\hat{b}}\right] \\ & + \frac{A^{(1)}\beta_2}{8\beta_1^3}\left[\ln^2(2\hat{q}) - \ln^2\left(2\hat{b}\right)\right], \end{aligned} \tag{A.41}$$

where the variables are defined by

$$\hat{q} \equiv \ln\left[Q/\left(\sqrt{2}\Lambda\right)\right], \hat{b} \equiv \ln\left[1/(b\Lambda)\right], \tag{A.42}$$

and the coefficients $A^{(i)}$ and β_i are

$$\beta_1 = \frac{33 - 2n_f}{12}, \qquad \beta_2 = \frac{153 - 19n_f}{24}, \tag{A.43}$$

$$A^{(1)} = \frac{4}{3}, \qquad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1 \ln\left(\frac{1}{2}e^{\gamma_E}\right), \tag{A.44}$$

where n_f is the number of the quark flavors and γ_E is the Euler constant.

Data Availability

This manuscript has no associated data or the data will not be deposited (no data were used to support this study).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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