

## Research Article

# Electron Number Density and Coherence Length of Boson-Fermion Pair in HTSC

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A Bose-Einstein Condensate (BEC) of a nonzero momentum Cooper pair constitutes a composite boson or simply a boson. Previously, it has been shown that the quantum coherence of the two-component BEC (boson and fermion condensates) is controlled by plasmons where <1% of plasmon energy mediates the charge pairing but most of the plasmon energy is used to overcome the modes that compete against superconductivity such as phonons, charge density waves, antiferromagnetism, and damping effects. The dependence of plasmon frequency on the material of a superconductor reveals that modes within a specific range of frequency enhance superconductivity and therefore affect the critical temperature of a particular superconducting material. Against this background, we study the effect on doping on boson-fermion pairing energy and hence the critical temperature. While most hole doping agents are atoms lighter than copper, many of the electron doping agents are materials heavier than copper. This property defines the doping effect on the plasma frequency. Heavier dopants lower the critical temperature while lighter dopants increase the critical temperature of a superconductor. The number density of electrons is also found to be proportional to the square of critical temperature ( $T_c$ ) while the size of a boson-fermion pair condensate (BFPC) is proportional to  $T_c^{-2/3}$ . The size of a BFPC particle is less than boson-fermion (BF) coherence length by almost an order.

## 1. Introduction

A system that contains Cooper-pair bosons interacting with electrons (fermions) in Bose-Einstein condensate (BEC) state was first proposed by Tolmachev [1]. Different from ultracold atom systems, which form condensates near absolute zero temperature, Cooper-pair bosons together with fermions undergo first-order transition from the nematic (BCS) state to BEC state at  $T_c$ , which is much higher than absolute zero [2, 3]. Recent studies have explored fermion- [4] and atom-mediated [5] pairing between BEC systems. However, Pasupathy et al. [6] have suggested that the strength of pairing in high temperature superconductors is determined by the unusual electronic excitations. Some time back, it was predicted [7] that high temperature superconductivity (HTSC) arises from the coexistence of phonon and plasmon mechanisms. The fact that Cooper pairs play a

key role in HTSC already caters, to a large extent, for the phonon mechanism. Plasmon mediation in electronic systems such as Cooper pairs causes transition to a superconducting state with a relatively high  $T_c$  [8, 9]. Excitation of charges with energy much below that of the Fermi electron—acoustic plasmons—has been observed in both electron- and hole-doped cuprates using resonant inelastic X-ray scattering (RIXS) [10, 11]. The observed acoustic plasmons are predominantly associated with O sites of the copper oxide plane [12]. Plasmons in HT superconductors originate from singularity of long-range Coulomb's interactions in the limit of long wavelength [13]. This scenario depends on a charged particle moving in the electron plasma under the influence of an external field/force. In the long-wavelength plasmon branch where  $q \rightarrow 0$ , Coulomb's screening by the Thomas-Fermi method may be considered [14]. The large charge fluctuations (plasmons) in cuprates contribute

significantly to the pairing mechanism [15] and lead to large and sharp velocity peak at  $k = k_F$ , which persists in the presence of screening [16].

The plasmon dispersions observed in YBCO are anisotropic and quadratic in  $a - b$  plane. Strongly anisotropic dielectric functions have also been observed [17]. While surface-plasmon dispersions at the interface of two media are isotropic, bulk-plasmon dispersions are anisotropic. The bulk-plasmon dispersion anisotropy increases with momentum transfer  $q$  up to its critical value  $q_c = \omega/v_F$  beyond which the anisotropy tends to zero. Here,

$$\omega = \left(\frac{3}{r_s^3}\right)^{1/2} \quad (1)$$

is the plasma frequency.

$$v_F = (3\pi^2 n_s)^{1/3} \quad (2)$$

is the Fermi velocity,  $n_0$  is the electron density.

$$r_s = \left[\frac{3}{(4\pi^2 n_s)}\right]^{1/3} \quad (3)$$

is the bulk radius per electron and  $n_s$  is the number density of electrons per cubic metre [18, 19].

In high temperature superconductors, the number of Cooper pairs is negligible compared to that of free electrons and the bulk plasmon dispersion is almost similar to that in pure metal—free electrons—and follows relation [(19)].

$$E^2 = \omega_p^2 + \left[\frac{3}{5}v_F^2 - \frac{v_F}{3\pi} - \epsilon\right]q^2 + \frac{1}{4}q^4. \quad (4)$$

The term in  $q^2$  has the kinetic energy contribution ( $3v_F^2/5$ ), the exchange interaction contribution ( $v_F/3\pi$ ), and the correlation energy contribution ( $\epsilon$ ). Note that  $\epsilon < v_F/3\pi$ . The term in  $q^4$  is the quasifree electron kinetic energy contribution. The screened of Coulomb's potential is proportional to the critical temperature of HTSC [20].

Recent studies on high temperature superconductors have revealed that the minimum size of a boson-fermion pair condensate (BFPC) is in the order of the Thomas-Fermi length [21], that is

$$r_{s(\min)} = -\frac{1}{k_s} = \left(\frac{4}{a_0} \left(\frac{3n_s}{\pi}\right)^{1/3}\right)^{-1/2}, \quad (5)$$

and the frequency ( $\omega_p$ ) of the plasmonic energy that mediates superconductive coherence is found to be

$$\omega_p = \frac{V_{\text{BFP}}}{\hbar}, \quad (6)$$

where  $\hbar$  is reduced Planck's constant and  $V_{\text{BFP}}$  is the attractive potential of a boson-fermion pair (BFP).

This paper has been organized as follows: the theoretical derivation seeks to establish the relation between, we focused on Coulomb's screening of a Cooper-pair boson as it moves in an electron plasma under phonon mediation. Section 2.1 has dealt with the consequences of the Coulomb screening—BFP coherence length—while plasmon frequency has been discussed in Section 2.2. Finally, the results are discussed in Section 3 and conclusion made in Section 4.

## 2. Theoretical Formulation

**2.1. The Dielectric Function.** The dielectric function  $\epsilon(\mathbf{r}, \mathbf{r}', t - t')$  is in particular a key quantity for superconductivity and is probed often via its Fourier transform  $\epsilon(\mathbf{q}, \omega)$  in the limit of the Thomas-Fermi approximations where the transferred momentum  $|\mathbf{q}| = q \rightarrow 0$  with long-range interactions ( $|\mathbf{r} - \mathbf{r}'| \rightarrow \infty$ ) [22]. The inverse  $\epsilon^{-1}$  of the dielectric function measures the screening of the bare Coulomb's repulsion and can directly indicate reverse screening (antiscreening) spatial regions where the interaction between two electrons is attractive rather than repulsive. In these regions, electrons pair up, giving rise to superconductivity [23]. Introducing  $\mathbf{k}$  dependence is necessary to address the evident anisotropy in HTSC [24, 25], a relationship that has been proved theoretically [26]. Thus, the new general dielectric function becomes

$$\epsilon(\mathbf{q}, \omega, \mathbf{k}) = \epsilon(\mathbf{q}) + \epsilon(\omega) + \epsilon(\mathbf{k}). \quad (7)$$

Here, the dielectric function  $\epsilon(\mathbf{q})$  represents plasmon dispersion,  $\epsilon(\omega)$  describes collective excitation of the Fermi Sea, while  $\epsilon(\mathbf{k})$  represents the Coulomb screening of a boson by fermions during interaction. Both  $\epsilon(\mathbf{q})$  and  $\epsilon(\omega)$  arise from the plasmon dispersion relation in Equation (4). It has also been noted that dielectric screening at shorter distances and at frequencies of the order of the superconducting gap, but small compared to the Fermi energy, can significantly enhance the transition temperature  $T_c$  of an unconventional superconductor [27].

A plasmon is quantum of plasma oscillations and may be excited by passing a charged particle through the plasma [28]. When a Cooper-pair boson is excited (by phonons), it displaces the electrons in its way due to Coulomb's repulsion, thereby exciting a plasmon in the electron plasma. Thus, the boson creates a plasmon, due to Coulomb's interaction, while the electron annihilates it. The first electron in the Cooper pair creates a phonon while the second one creates a strong plasmon as it annihilates the phonon. Thus, the effective induced charge by a single boson is  $+q$  (a hole). The boson is an impurity within the electron plasma with an associated screened Coulomb's potential,  $\psi(r)$ , over distance  $r$ .

The function  $\epsilon(\mathbf{k})$  relates: (i) the density  $\rho_h(k)$  of holes induced in the electron plasma and the charge density  $\rho_b(k)$  of bosons due to the phonon field; and (ii) the screened potential,  $\psi(k)$ , the unscreened potential,  $\psi_b(k)$ , due to

phonons and the unscreened potential,  $\psi_b(k)$ , due to induced holes according to Equation (3):

$$\epsilon(k) = \frac{\psi_b(k)}{\psi(k)} = \frac{\rho_b(k)}{\rho(k)} = 1 - \frac{\rho_h(k)}{\rho(k)}, \quad (8)$$

where  $\psi(k) = \psi_b(k) + \psi_h(k)$  and  $\rho(k) = \rho_h(k) + \rho_b(k)$  are the total electrostatic potential due to the displacement of the fermion gas and the total charge density, respectively. When  $\psi(k) > 0$ , the function  $\epsilon(k)$  has a scattering effect but when  $\psi(k) < 0$ , the interacting particles experience attraction towards each other. Starting with Equation (8), rigorous derivations [28] lead to

$$\epsilon(k) = 1 - \frac{k_s^2}{K^2}, \quad (9)$$

where

$$k_s = \pm \left( \frac{3n_e Q}{2\epsilon_0 \epsilon_F} \right)^{1/2} = \pm \left( \frac{4}{a_0} \left( \frac{3n_e}{\pi} \right)^{1/3} \right)^{1/2}. \quad (10)$$

Here,  $1/k_s$  is the Thomas-Fermi screening length,  $a_0 = 5.29 \times 10^{-11}$  m is the Bohr radius, and  $n_e$  is the electron density. On the other hand, the frequency-dependent part has been determined for a superconductor [29] as

$$\epsilon(\omega) = 1 - \frac{\omega_i^2}{\omega_T^2}, \quad (11)$$

where  $\omega_i = (n_i q^2 / m_i \epsilon_0)^{1/2}$  is the ionic plasma frequency, while  $\omega_T$  is the effective plasma frequency (ionic plus electronic) and  $n_i$  is the number density of ions in a superconductor. Combining Equations (9) and (11) yields assuming that  $\epsilon(\mathbf{q}) < 2$ , we have

$$\epsilon(\omega, \mathbf{k}) = 2 - \frac{\omega_i^2}{\omega_T^2} - \frac{k_s^2}{K^2}. \quad (12)$$

**2.2. Maximum Plasmon Scattering Energy and Size of a BFPC.** In the case of a boson,  $Q = -q$  (the second electron charge that creates the plasmon) and the screened potential  $\psi_0(r)$  is given as

$$\psi_0(r) = -\left(\frac{q}{r}\right) \exp(-k_s r). \quad (13)$$

The total potential energy  $V_{pl}$ , which eventually leads to plasmon scattering, is equal to the screened Coulomb's potential in equation (13) and so

$$V_{pl} = \psi_0(r) = -\frac{q}{r} \exp(-k_s r). \quad (14)$$

The maximum plasmon potential  $V_{pl}^{\max}$  is attained at the nearest point where  $r = r_{\min}$ . Minimizing Equation (14) with respect to  $r$  yields

$$\frac{dV_{pl}}{dr} = \left(\frac{q}{r^2}\right) \exp(-k_s r) + \left(\frac{k_s q}{r}\right) \exp(-k_s r) = 0. \quad (15)$$

Simplifying equation (14) leads to

$$r = \begin{cases} -\frac{1}{k_s}; \text{ minimum long-range limit} \\ \infty; \text{ extreme long-range limit} \end{cases}. \quad (16)$$

In the extreme long-range limit, the Coulomb screening cannot induce boson-fermion pairing. However, at the maximum Coulomb's screening, the coherence length  $r_{\min} = -1/k_s$  is minimum. The minimum coherence length is typically the size of a BFPC. Since  $r_{\min} > 0$ , then substituting for  $k_s < 0$  in equation (10) using equation (16) gives

$$r_{\min} = \zeta = \left[ \frac{a_0}{4} \left( \frac{\pi}{3n_s} \right)^{1/3} \right]^{1/2}, \quad (17)$$

where  $n_s$  is the number density of electrons that can readily take part in superconductive boson fermion pairing. Equation (17) gives the nearest point where maximum plasmon scattering energy and minimum coherence length (size of the BFPC) are attained. Substituting Equation (14) into Equation (17) yields the maximum attractive potential energy,  $V_{pl}^{\max}$ , due to the screening of the boson at  $r = r_{(\min)}$  as

$$V_{pl}^{\max} = k_s q \exp(-1) = q \left( \frac{4}{a_0} \left( \frac{3n_s}{\pi} \right)^{1/3} \right)^{1/2} \exp(1). \quad (18)$$

As the free electron (fermion) moves to annihilate the net plasmon, based on the Pauli exclusion principle, it experiences attraction towards the boson. The plasmon energy  $V_{pl}^{\max}$  is much greater than the attractive potential energy  $V_{BFP}$ . Most of the plasmon energy is used to overcome the competing phonons, charge density waves (CDW), and the damping effect among others. Only a small fraction  $q$  of  $V_{pl}^{\max}$  remains to mediate boson-fermion interaction.

$$V_{BFP} = q V_{pl}^{\max} \quad (19)$$

The process of mediation mainly involves overcoming the Coulomb repulsion between the boson and the fermion.

**2.3. Minimum Plasmon Scattering Energy, Coherence Length, and Electron Number Density.** The energy ( $E$ ) contained in a plasmon of frequency  $\omega$  can be obtained from the Planck-Einstein relation:

$$E = -\hbar\omega, \quad (20)$$

where  $\hbar = h/2\pi$  is the reduced Planck's constant. The negative sign has been used because the energy is attractive. The mediating plasmon is the source of attraction ( $V_{BFP}$ ) between the interacting boson and fermion in condensate state. Therefore, for any coherence to occur

$$\omega = \frac{V_{BFP}}{\hbar}, \quad (21)$$

where  $V_{BFP}$  is the attractive potential energy between a boson and a fermion has been given [22] as

$$V_{BFP} = \begin{cases} -E_k; \text{ maximum } V_{BFP} \\ -\frac{2}{3}E_k; \text{ minimum } V_{BFP} \end{cases}. \quad (22)$$

Here,  $E_k = 2k_B T_c$  is the excitation energy of a single-particle-like BFP condensate in a superconductor of critical temperature,  $T_c$ . Combining the first part of equation (1) and equation (21), then we obtain the coherence length (maximum size of a BFP) as

$$r_{(\max)} = \xi = \left[ \frac{3}{4} \left( \frac{\hbar}{k_B T_c} \right)^2 \right]^{1/3}. \quad (23)$$

Superconducting boson-fermion coherence occurs when the wave vector  $K$  is such that

$$\frac{1}{k_s} < \frac{1}{K} < \xi = \frac{1}{k'_s}, \quad (24)$$

Or

$$k_s > K > k'_s. \quad (25)$$

The limit  $K > k_s$  defines both superconducting and non-superconducting regions where  $1/K < \xi$  while the region  $K < k_s$  does not exist since it implies that  $1/k_s > 1/K$  which is impractical. In the limits of coherence length (minimum plasmon scattering),  $k_s$  can be replaced by  $k'_s$  where  $K > k'_s$  defines a superconducting region and  $K < k'_s$  is a non-superconducting region. All electrons with  $K > k'_s$  can readily take part in superconductive boson-fermion pairing and hence are referred to as superelectrons. Majority of particles in this range of wave vector are Cooper pairs under phonon mediation. Further, we define [29]

$$\frac{(k'_s)^2}{K^2} = \frac{\omega_s^2}{\omega_T^2} = \frac{n_s q^2}{m_s \epsilon_0 \omega_T^2} \quad (26)$$

as the dielectric contribution of the  $s$  electron gas where  $\omega_s = (n_s q^2 / m_s \epsilon_0)^{1/2}$  is the electron plasma frequency in the superconducting range, while  $\omega_T$  is the effective plasma frequency (ionic plus electronic) and  $n_s$  is the number density of superelectrons in a superconductor.

When we substitute Equation (26) into Equation (12), then at the zero of  $\epsilon(\omega, 0)$ ,

$$\omega_T^2 = \frac{q^2}{2\epsilon_0} \left( \frac{n_i}{m_i} + \frac{n_s}{m_s} \right). \quad (27)$$

In cuprates, there exists a range of non-superconducting frequencies ( $0 < \omega_i^2 \leq \omega_0^2$ ) whose energy contribution is not sufficient to mediate the pairing between a boson and a fermion. In the limit  $\omega_i > \omega_0$ , a material exhibits superconductivity.

The mediating plasmon is the source of attraction between the interacting boson and fermion in condensate state, and therefore,

$$V_{BFP} = q\hbar \left[ \frac{1}{2\epsilon_0} \left( \frac{n_i}{m_i} + \frac{n_s}{m_s} \right) \right]^{1/2}. \quad (28)$$

Since  $m_i > m_e$  and  $n_i \sim n_s$ , it is clear that  $n_i/m_i < n_s/m_s$  and Equation (28) yields

$$V_{BFP} = \hbar q \left[ \frac{n_s}{2\epsilon_0 m_s} \right]^{1/2}. \quad (29)$$

Using the maximum value of  $V_{BFP}$  and considering symmetry between holes and electrons, we obtain the maximum number density of charge carriers per unit volume in a superconductor as

$$n_s = 4\epsilon_0 m_s \left[ \frac{2k_B T_c}{\hbar q} \right]^2. \quad (30)$$

### 3. Results and Discussion

From Equation (27), doping has effect on both  $n_i/m_i$  and  $n_s/m_s$ . In the case of hole doping, for example, the ion of the dopant 'sits' at the copper oxide plane and hence the density of ions per unit volume  $n_i$  increases slightly while  $m_i$  decreases. The net effect is that the ratio  $n_i/m_i$  increases. Conversely, the presence of hole lowers the electron density  $n_s$  but it does not change the electron mass  $m_s$  leading to a reduction in the ratio  $n_s/m_s$ .

**3.1. Hole-Doping Effect on the Ion Lattices.** In undoped cuprates,  $\omega_i \leq \omega_0$  where the plasmon energy cannot sufficiently mediate the pairing between electrons and Cooper pairs. During hole doping, the ion of the dopant 'sits' at the copper oxide plane thereby affecting the average mass per ion of the plane. Most of the dopants used in hole doping are lighter than copper ion. Therefore, by hole doping, the effective plasmon frequency  $\omega$  is increased to successfully mediate superconductive pairing. Hole doping involves lighter element (reduce  $m_i$  and increases  $\omega_i$ ), which enhances  $T_c$  of a superconductor. Conversely, electron doping involves heavier elements, resulting to a lower  $\omega$ , an effect that reduces  $T_c$  in optimally hole-doped compounds and increases the energy gap. In the absence of hole doping, electron doping does not induce superconductivity in a cuprate.

Superconductivity in cuprates occurs mainly along the copper oxide planes which consist of two oxygen ions (a.m.u = 16) and one copper ion (a.m.u = 63.5). Note that (1a.m.u =  $1.66 \times 10^{-27}$ kg). Using Equation (12), the average threshold frequency contribution by an ion, in the absence of doping, is obtained as

$$\omega_0 = \left( \frac{n_i \times (1.6 \times 10^{-19})^2}{2(2(16) + (63.5)/3) \times 1.66 \times 10^{-27} \times 8.85 \times 10^{-12}} \right)^{1/2} = 0.165\sqrt{n_i} \text{ Hz}, \quad (31)$$

where  $n_i$  is the number density of ions per unit volume. This is the average threshold plasma frequency contribution by an ion in cuprates. When the oxygen content is increased by 1 atom the new lattice contribution becomes

$$\omega_i = \left( \frac{n_i \times (1.6 \times 10^{-19})^2}{2(3(16) + (63.5)/4) \times 1.66 \times 10^{-27} \times 8.85 \times 10^{-12}} \right)^{1/2} = 0.176\sqrt{n_i} \text{ Hz}. \quad (32)$$

Clearly,  $\omega_i > \omega_0$ , and therefore, superconductivity occurs.

Electron-doping is associated with metals heavier than copper. Hence, increasing copper content (or a metal heavier than copper) by 1 atom yields

$$\omega_i = \left( \frac{n_{is} \times (1.6 \times 10^{-19})^2}{2(2(16) + 2(63.5)/4) \times 1.66 \times 10^{-27} \times 8.85 \times 10^{-12}} \right)^{1/2} = 0.148\sqrt{n_i} \text{ Hz}. \quad (33)$$

In the case of electron doping,  $\omega_i < \omega_0$  and the plasmons cannot mediate boson-fermion coherence. Thus, in the absence of excess oxygen,  $\omega_i < \omega_0$  and electron-doping cannot instigate superconductivity. The electron plasma frequency is

$$\omega_s = \left( \frac{n_s \times (1.6 \times 10^{-19})^2}{9.1 \times 10^{-31} \times 8.85 \times 10^{-12}} \right)^{1/2} = 56.4\sqrt{n_s} \text{ Hz}, \quad (34)$$

where  $n_s$  is the number density of superelectrons. The frequency of electron plasma is much higher than that of lattice. Although the lattice contribution to the effective plasma frequency is very small, it plays a key role in enhancing boson-fermion pairing.

The lattices (copper oxide planes), within which charge pairing occurs, provide the necessary rigidity with some level of flexibility that modulates the plasmonic waves. The lattices modulate the displacement of electrons from their mean positions—amplitude modulation of the plasmonic wave. Lattices with  $\omega_i > \omega_0$  are more flexible while those with  $\omega_i < \omega_0$  are more rigid. Rigidity of the copper oxide plane

TABLE 1: Number density ( $n_{es}$ ) of superelectrons, size ( $\zeta$ ), and coherence length ( $\xi$ ) of BFPC.

Superconductor	$T_c(K)$	$n_{es}$ ( $\times 10^{23} m^{-3}$ )	$\zeta(\text{\AA})$	$\xi(nm)$
Group I				
YBa <sub>2</sub> Cu <sub>3</sub> O <sub>7</sub>	92	7.39	3.85	1.73
Tl <sub>2</sub> Ba <sub>2</sub> CaCu <sub>2</sub> O <sub>8</sub>	108	10.05	3.66	1.55
Bi <sub>2</sub> Sr <sub>2</sub> CaCu <sub>2</sub> O <sub>8</sub>	95	7.78	3.82	1.69
HgBa <sub>2</sub> Ca <sub>2</sub> Cu <sub>3</sub> O <sub>8</sub>	136	15.94	3.39	1.33
Group II				
HgBa <sub>2</sub> CuO <sub>4</sub>	97	8.11	3.79	1.67
LiFeAs	15	0.19	7.07	5.80
BaFe <sub>2</sub> (As <sub>0.7</sub> P <sub>0.3</sub> ) <sub>2</sub>	30	0.78	5.61	3.65
Ca <sub>0.33</sub> Na <sub>0.67</sub> Fe <sub>2</sub> As <sub>2</sub>	34	1.00	5.383	3.36

enhances modulation and is accompanied by large proportions of electron-ion collisions. On the other hand, flexibility of lattices smoothen the modulation by reducing the number of electron-ion collisions and also increases the impact time of the collisions. The latter delays the ‘come-back’ of electrons after displacement from particles’ mean positions and hence a delayed boson-fermion coherence, that will not take place. In a highly rigid system, plasmon energy is lost through a large number of electron-ion collisions, whereas in a highly flexible system, plasmon energy is lost through increased impact time. Accordingly, it is imperative to balance between the rigidity and flexibility of a copper oxide plane through hole doping so that energy loss through any of the two means is minimized as much as possible. Flexibility of a copper oxide plane increases with the oxygen content as rigidity declines. A plane in which rigidity is optimally balanced (optimal hole doping) against flexibility modulates the electron plasma vibrations (plasmons) to yield maximum pairing energy leading to a higher critical temperature of a superconductor.

**3.2. Hole-Doping Effect on the Electron Plasma.** An increase in the hole content increases flexibility of copper oxide planes but lowers the ratio  $n_s/m_s$  as discussed earlier. Note that the plasma frequency is proportional to the number density of superelectrons [18, 19]:

$$\omega_s = \left( \frac{3}{r_s^3} \right)^{1/2} = [4\pi^2 n_s]^{1/2}. \quad (35)$$

Hence, a decline in  $n_e$  leads to a decrease in the plasma frequency and a rise in the ion lattice frequency. Clearly,  $n_s/m_s$  is proportional to the rigidity, while  $n_i/m_i$  is proportional to the flexibility of copper oxide lattices.

**3.3. Coherence length ( $\xi$ ), Size of BFPC ( $\zeta$ ), and the Number Density ( $n_{es}$ ) of Electrons.** The coherence length is the maximum distance over which the boson and the fermion can pair up to form a BFPC. This value is dependent on  $T_c$  of a material, more or less like the size of a BFPC except the

TABLE 2: Comparing  $V_{\text{BFP}}$  and  $V_{\text{pl}}^{\text{max}}$  in superconductors.

Superconductor	$T_c(K)$	$n_s$ ( $\times 10^{23} m^{-3}$ )	$V_{\text{BFP}}$ ( $\times 10^{-21} J$ )	$V_{\text{pl}}^{\text{max}}$ ( $\times 10^{-9} J$ )	$(V_{\text{BFP}}/V_{\text{pl}}^{\text{max}})$ ( $\times 10^{-12}$ )
Group I					
YBa <sub>2</sub> Cu <sub>3</sub> O <sub>7</sub>	92	3.696	2.54	1.01	2.51
Tl <sub>2</sub> Ba <sub>2</sub> CaCu <sub>2</sub> O <sub>8</sub>	108	5.027	2.98	1.06	2.81
Bi <sub>2</sub> Sr <sub>2</sub> CaCu <sub>2</sub> O <sub>8</sub>	95	3.889	2.62	1.01	2.59
HgBa <sub>2</sub> Ca <sub>2</sub> Cu <sub>3</sub> O <sub>8</sub>	136	7.971	3.75	1.14	3.29
Group II					
HgBa <sub>2</sub> CuO <sub>4</sub>	97	4.055	2.68	1.02	2.63
LiFeAs	15	0.097	0.41	0.55	0.75
BaFe <sub>2</sub> (As <sub>0.7</sub> P <sub>0.3</sub> ) <sub>2</sub>	30	0.388	0.83	0.69	1.20
Ca <sub>0.33</sub> Na <sub>0.67</sub> Fe <sub>2</sub> As <sub>2</sub>	34	0.498	0.94	0.72	1.31

magnitude. Table 1 outlines the values of the three quantities in some high temperature superconductors

The critical temperature ( $T_c$ ) of a material is proportional to the number density of plasma electrons. Materials with higher  $T_c$ 's have smaller BFPCs (shorter  $\zeta$ ) and shorter coherence length ( $\xi$ ) than those with lower  $T_c$ 's. The coherence length is larger than the size of a BFPC by almost an order. Reducing the size of a BFPC involves higher energy, which raises  $T_c$  of a superconductor. These results are comparable to those in references [30, 31].

$$\begin{aligned}
 r_{\min} = \zeta &= \left[ \frac{a_0}{4} \left( \frac{\pi}{3n_{es}} \right)^{1/3} \right]^{1/2} r_{s(\max)} \\
 &= \zeta = \left[ \frac{3}{4} \left( \frac{\hbar}{k_B T_c} \right)^2 \right]^{1/3} n_{es} = 2\epsilon_0 m_{es} \left[ \frac{2k_B T_c}{\hbar q} \right]^2.
 \end{aligned} \tag{36}$$

**3.4. Comparison between Maximum Plasmon Scattering Energy ( $V_{\text{pl}}^{\text{max}}$ ) and BFP Plasmon Mediation Energy ( $V_{\text{BFP}}$ ).** This comparison has been done earlier by Mukubwa and Makokha [21]. This paper has divulged much information (such as electron number density) that can facilitate a closer and more precise comparison between the maximum plasmon scattering energy and the BFP plasmon mediation energy (BFP attractive potential) as in Table 2.

The mediation plasmon energy is much smaller than the maximum (total) plasmon energy. Most of the plasmon energy is used to overcome modes that compete against superconductivity such as phonons, charge density waves, antiferromagnetism, damping effects, and magnetization. The plasmon mediation energy is about  $10^{-12}$  of the total plasmon energy.

Other than the phenomenological BFPC model used in this paper to study plasmon mediation of charge pairing, the full density functional theory (DFT) calculations could also be used to reveal the contribution of charge density waves to charge pairing in high temperature superconductors. Prior works on the use of full ab initio calculations have been detailed in references [32, 33].

## 4. Conclusion

The average mass of an ion in the copper oxide plane determines the nature of plasmons that mediate boson-fermion pairing. Hole doping makes the average ionic mass of a copper oxide plane lighter and hence an increased lattice frequency. Plasmonic waves are modulated by copper oxide planes within which charge pairing occurs. At optimal hole doping, rigidity of the planes is well balanced against flexibility yielding a higher critical temperature of the material. The size of a BFPC, which is inversely proportional to  $T_c$ , is less than the coherence length by almost an order.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The author declares no conflicts of interest.

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