Hindawi
Advances in High Energy Physics
Volume 2022, Article ID 9056096, 10 pages
https://doi.org/10.1155/2022/9056096

Research Article

Inflationary Universe from Anomaly-Free $F(R)$-Gravity

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Received 15 October 2021; Revised 27 November 2021; Accepted 27 January 2022; Published 29 March 2022

Academic Editor: Saibal Ray

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By adding a three-dimensional manifold to an eleven-dimensional manifold in supergravity, we obtain the action of $F(R)$-gravity and find that it is anomaly-free. We calculate the scale factor of the inflationary universe in this model and observe that it is related to the slow-roll parameters. The tensor-scalar ratio $(R_{\text{tensor-scalar}})$ is in good agreement with experimental data.

1. Introduction

One of the best models for considering an inflationary universe is suggested by modified theories of gravity like $f(R)$ gravity, whose Lagrangian is a function $f(R)$ of the scalar curvature R in a 4-dimensional universe [1–5]. To date, many models in this subject have been proposed. However, one of the best inflationary models in $f(R)$ gravity is the pioneering Starobinsky $R + R^2$ model, which still remains viable now in contrast to many other models proposed later and produces the best fit to existing observational data. Primordial curvature perturbations and gravitational waves in this model were first calculated in [6]. On the other hand, in these theories, the special property is the existence of the effective cosmological constant in such a way that early-time inflation and late-time cosmic acceleration are naturally unified within a single theory [1].

Some of these $f(R)$ gravity theories, especially a power-law mechanism and Yang-Mills $f(R)$ gravity, give the best fit values compatible with cosmological parameters like the tensor-to-scalar ratio within the permitted ranges derived by the Planck and BICEP2 observations [2]. In some other versions of these theories, a dynamical scalar field (φ) is coupled to gravity ($f(R, \varphi)$) whose dynamics allow for exit from inflation and which gives rise to the correct amount of inflation in agreement with observational data [3]. Also, in some modified gravity theories, F is a generic function of the curvature scalar and the Gauss-Bonnet topological parameter. Cosmological dynamical results which are obtained by two effective masses (lengths) correspond to both of these parameters and work, respectively, at early and very early epochs of the evolution of the universe [4]. Furthermore, modified teleparallel gravity leads to second-order equations and solves the particle horizon problem in a spatially flat cosmic space by providing an initial exponential expansion without resorting to an inflaton particle [5]. For more reviews on the connections between inflation, the dark energy problem, and modified gravity theories, see, for instance, [7].

Now, the question that arises is whether $F(R)$-gravity could be anomaly-free or not? Also, it is required to consider the role of anomaly cancellation in the evolution of the universe during the inflationary epoch. Recently, the exact forms of two-dimensional and four-dimensional anomalies from higher derivative gravity and $F(R)$-gravity have been calculated [8]. On the other hand, the conformal-anomaly driven inflation in $F(R)$-gravity without using the scalar-tensor representation has been studied. It has been shown that in $F(R)$-gravity, the curvature perturbations with their high amplitude which are consistent with observations and are created during inflation [6, 9].

Besides these papers, there are some interesting works which connect M-theory to $F(R)$-gravity and inflation. For example, the first attempt to get $F(R)$ from strings/M-theory is done long ago in [10]. On the other hand, the study of inflation in $F(R)$ with scalars was done in k-essence $f(R)f(R)$ gravity inflation [11, 12]. Also, the reconstruction of $f(\phi)Rf(\phi)R$ and mimetic gravity from viable slow-roll inflation has

been discussed in [13]. And finally, the relation between Geometric Inflation and Dark Energy with Axion $F(R)F(R)$ -gravity has been argued in [14]. These investigations advise us that there may be a relation between generalized gravity theories, M-theory, and inflation. Each gravity theory may have some anomalies which could be removed in a theory with higher dimensions. For example, all anomalies in string theory have been removed in 11-dimensional M-theory. Thus, we can conclude that anomalies in M-theory may be removed in a theory with $11+n$-dimensions. In fact, first, we should consider the origin of M-theory and process of its birth on 11-dimensional manifold in a $11+n$-dimensional manifold and then study its reduction to $F(R)$-gravity in four-dimensional universe. On the other hand, to find the relation between inflation and M-theory and gravity, we have to assume that our universe is located on a manifold which interact and exchange energy and fields with other manifolds in a higher dimensional world. These interactions force to manifolds to become larger and inflate. Motivated by these ideas, we propose a theory which lives on an $(11+3)$-dimensional manifold with two 11-dimensional manifolds and one 3-dimensional manifold. We will show that our universe is a part of an 11-dimensional manifold which is connected with the other 11-dimensional manifold by an extra 3-dimensional manifold. Two 11-dimensional manifolds interact with each other via exchanging fields which move along the 3-dimensional manifold. These fields are the main cause for the appearance of $F(R)$-gravity in the four-dimensional universe.

This paper consists of two main parts. In Section 2, we show that by adding 3-dimensional manifold to an 11-dimensional one, $F(R)$-gravity emerges. In Section 3, we obtain the cosmological parameters like the tensor to scalar ratio and compare with experimental data.

2. Emergence of $F(R)$-Gravity on an $(11+3)$-Dimensional Manifold

In this section, we will assume that our universe has been constructed on a D3-brane. Thus, the action of gravity and matter in a 4-dimensional universe should be equal to the action of a D3-brane (see Figure 1). This means that strings which live on a D3-brane produce gravity and different types of matter. Thus, each string (X^i) can be expanded in terms of curvatures (R), gauge fields (F), and scalars (ϕ) (see Figure 2). According to the Horava-Witten mechanism [15, 16], this D3-brane can be a part of a bigger 11-dimensional manifold on which the action of fields should be anomaly-free (see Figure 3). However, using the relation between strings and fields, we notice that the gauge variation of the action on this manifold is not zero, and some extra anomalies emerge. To remove these anomalies, we have to add an extra 11-dimensional manifold which is connected with the first 11-dimensional manifold by a 3-dimensional manifold. The extra fields which are produced by the interaction of the two manifolds produce $F(R)$-gravity (see Figure 4).

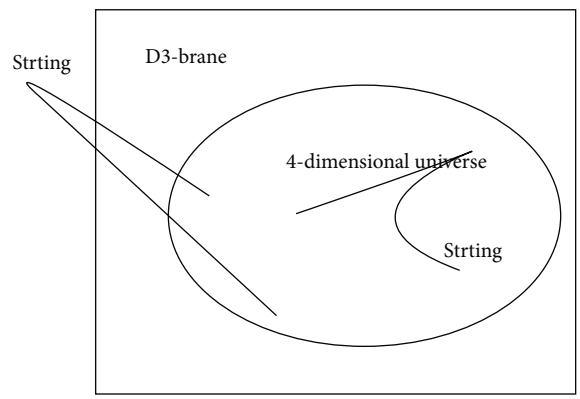

Figure 1: 4-dimensional universe on D3-brane.

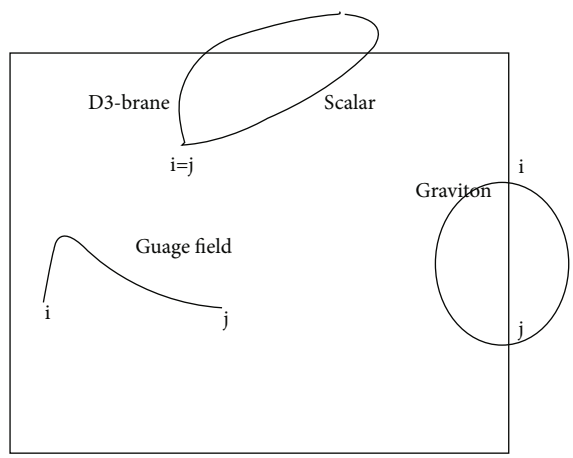

Figure 2: Different shapes of strings produce different types of fields.

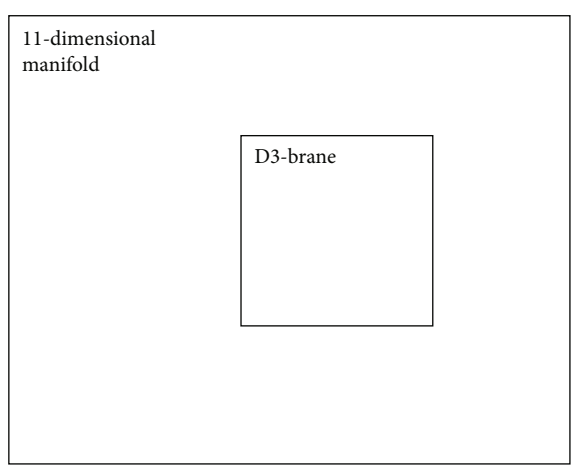

Figure 3: D3-brane is a part of 11-dimensional manifold.

First, let us introduce the action of the D3-brane which is given by [17]:

$$S_{D3} = -T_{D3} \int d^4y \sqrt{-\det\left(\bar{\gamma}_{ab} + 2\pi l_s^2 F_{ab}\right)}, \qquad (1)$$

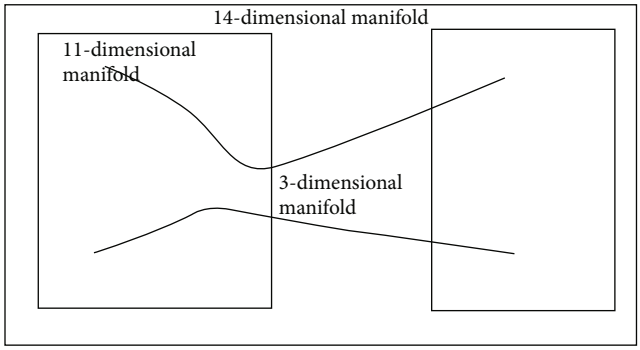

Figure 4: Two 11-dimensional manifolds interact via 3-dimensional manifold in 14-dimensional manifold.

$$\bar{\gamma}_{ab} = g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu, \tag{2}$$

$$F_{ab} = \partial_a A_b - \partial_b A_a, \tag{3}$$

where A_b is the gauge field, F_{ab} is the field strength, X^μ is the string, $g_{\mu\nu}$ is the metric, T_{D3} is the tension, and l_s is the string length. Substituting $X^0 = t$ and doing some mathematical calculations, the action of the D3-brane in equation (1) is given by [17]:

$$S_{D3} = -T_{D3}\int d^4y\sqrt{1 + g_{ij}\partial_a X^i \partial^a X^j - 4\pi^2 l_s^4 F_{ab}F^{ab}}, \tag{4}$$

To construct our universe on a D3-brane, this action should be equal to the action of fields and gravity in a 4-dimensional universe. The action of matter and gravity is given by:

$$S_{\text{Gravity-Matter}} = -\int d^4y\sqrt{-g}\left(R - g_{ab}\partial^a\phi\partial^b\phi - i\bar{\psi}\gamma^a\partial_a\psi + 1\right), \tag{5}$$

where ϕ is the scalar field and ψ is the fermionic field. Putting equation (4) equal to equation (5) and doing some mathematical calculations, we obtain the relation between strings and matter fields:

$$S_{\text{Gravity-Matter}} = S_{D3}, \tag{6}$$

$$\Rightarrow X^i = \int dy^i\sqrt{-1 + 4\pi^2 l_s^4 F_{ij}F^{ij} + \left(\sqrt{-g}\left(g^{ij}R_{ij} - g_{ij}\partial^i\phi\partial^j\phi - i\bar{\psi}\gamma^i\partial_i\psi + 1\right)\right)^2}, \tag{7}$$

where we have assumed $T_{D3} \simeq 1$ and $4\pi^2 l_s^4 \simeq 1$. Using the above relation between matter fields and strings, we can reconsider the Horava-Witten mechanism. We will show that there are also other anomalies that have been ignored in previous considerations. Our goal is to show that by adding a 3-dimensional manifold to 11-dimensional spacetime in the Horava-Witten mechanism, all anomalies can be removed and an action without anomaly can be produced. This action is identical to the action of the modified gravity theory presented in [1–5, 7].

At this stage, we introduce the Horava-Witten mechanism in 11-dimensional spacetime. In this theory, the bosonic part of the action in 11-dimensional supergravity (SUGRA) is given by [15, 16]:

$$S_{\text{Bosonic-SUGRA}} = \frac{1}{\bar{\kappa}^2}\int d^{11}x\sqrt{g}\left(-\frac{1}{2}R - \frac{1}{48}G_{IJKL}G^{IJKL}\right) + S_{CGG}, \tag{8}$$

$$S_{CGG} = -\frac{\sqrt{2}}{3456\bar{\kappa}^2}\int_{M^{11}} d^{11}x\varepsilon^{I_1I_2\cdots I_{11}}C_{I_1I_2I_3}G_{I_4\cdots I_7}G_{I_8\cdots I_{11}}, \tag{9}$$

where $\varepsilon^{I_1I_2\cdots I_k}$ is the rank-k Levi-Civita pseudotensor, and CGG is used to denote the product term of the three-form field $C_{I_1I_2I_3}$ and four-form field G_{IJKL}, which are directly related to the gauge field A^I, field strength F^{IJ}, and Ricci curvature R^{IJ} [16]:

$$G_{IJKL} = -\frac{3}{\sqrt{2}}\frac{\kappa^2}{\lambda^2}\varepsilon\left(x^{11}\right)\left(F_{[IJ}F_{KL]} - R_{[IJ}R_{KL]}\right) + \cdots, \tag{10}$$

$$\delta C_{ABC} = -\frac{\kappa^2}{6\sqrt{2}\lambda^2}\delta\left(x^{11}\right)tr(\varepsilon_C F_{AB} - \varepsilon_C R_{AB}), \tag{11}$$

$$G_{11ABC} = (\partial_{11}C_{ABC} \pm 23 \text{ permutations of the indices 11 and } ABC) + \frac{\kappa^2}{\sqrt{2}\lambda^2}\delta\left(x^{11}\right)\omega_{ABC}, \tag{12}$$

$$\delta\omega_{ABC} = \partial_A tr(\varepsilon F_{BC}) + \text{cyclic permutations of } ABC, \tag{13}$$

$$F^{IJ} = \partial^I A^J - \partial^J A^I, \tag{14}$$

$$R_{IJ} = \partial_I\Gamma^B_{JB} - \partial_J\Gamma^B_{IB} + \Gamma^A_{JB}\Gamma^B_{IA} - \Gamma^A_{IB}\Gamma^B_{JA}, \tag{15}$$

$$\Gamma_{IJK} = \partial_I g_{JK} + \partial_K g_{IJ} - \partial_J g_{IK}, \tag{16}$$

$$G_{IJ} = R_{IJ} - \frac{1}{2}Rg_{IJ}, \tag{17}$$

where ε and ε_C characterize infinitesimal gauge transformations [16]. Here, $\varepsilon(x^{11})$ is 1 for $x^{11} > 0$ and -1 for $x^{11} < 0$ and also $\delta(x^{11}) = 1/2\partial\varepsilon(x^{11})/\partial x^{11}$ is the Dirac delta function. As usual [16], tr is 1/30th of the trace Tr in the adjoint representation for $E_8 \times E_8$. The ellipsis $(\cdots)$ denotes terms that are regular near $x^{11} = 0$ and hence vanish there [16]. It is clear from the above equation that G-fields can be produced by joining F-fields or R-fields. This means that G-fields are produced by joining two strings, each of which produces one gauge field or curvature (R) (see Figure 5).

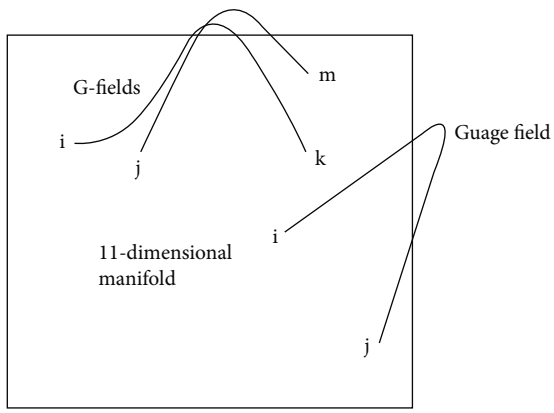

Figure 5: G-fields are produced by joining two strings.

The gauge variation of the CGG term in the action yields the following equation [16]:

$$\begin{aligned}\delta S_{CGG}|_{11} &= -\frac{\sqrt{2}}{3456\bar{\kappa}^2}\int_{M^{11}} d^{11}x\varepsilon^{I_1I_2\cdots I_{11}}\delta C_{I_1I_2I_3}G_{I_4\cdots I_7}G_{I_8\cdots I_{11}} \\ &\approx -\frac{\bar{\kappa}^4}{128\lambda^6}\int_{M^{10}}\Sigma_{n=1}^{5}\left(trF^n - trR^n + tr\left(F^nR^{5-n}\right)\right),\end{aligned} \tag{18}$$

where $trF^n = tr\left(F_{[I_1I_2}..F_{I_{2n-1}I_{2n}]}\right) = \varepsilon^{I_1I_2..I_{2n-1}I_{2n}}F_{I_1I_2}..F_{I_{2n-1}I_{2n}}$ or $trR^n = tr\left(R_{[I_1I_2}..R_{I_{2n-1}I_{2n}]}\right) = \varepsilon^{I_1I_2..I_{2n-1}I_{2n}}R_{I_1I_2}..R_{I_{2n-1}I_{2n}}$. The above terms cancel the anomaly of $(S_{\text{Bosonic-SUGRA}})$ in 11-dimensional manifolds [16]:

$$\delta S_{CGG}|_{11} = -\delta S^{\text{anomaly}}_{\text{Bosonic-SUGRA}}. \tag{19}$$

Thus, S_{CGG} is necessary for anomaly cancellation. Our goal now is to find a good rationale for its inclusion. We also answer the issue of the relation between CGG terms in 11-dimensional supergravity and $F(R)$-gravity. In fact, we suggest a theory in which $F(R)$-gravity terms appear in the supergravity action without being added by hand. To this end, we choose a unified form for all particles and fields by using Nambu-Poisson brackets and properties of string fields (X). Solving equation (7) and assuming that changes of fields with respect to the coordinates be ingnorable, we obtain [18, 19]

$$\begin{aligned}X^I &\approx y^I + y_JF^{IJ} - y_JR^{IJ} + y_J\partial^I\phi\partial^J\phi - iy_J\bar{\psi}\gamma^J\partial^I\psi + \cdots \Rightarrow \left\{X^I, X^J\right\} \\ &= \Sigma_{I,J}\varepsilon^{I'J'}\frac{\partial X^I}{\partial y^{I'}}\frac{\partial X^J}{\partial y^{J'}} = F^{IJ} - R^{IJ} + \partial^I\phi\partial^J\phi - i\bar{\psi}\gamma^J\partial^I\psi + \cdots.\end{aligned} \tag{20}$$

where we introduced the scalar field ϕ. Also, A^I is the gauge field and R is the curvature (R). In fact, the origin of all matter is the same and they are different shapes of strings. In the static state, all strings can be described by a unit vector (X^i). When these strings interact with each other or move, the initial symmetry is broken and fields and particles emerge. Using 4-dimensional instead of 2-dimensional brackets, we may derive the GG term $G_{IJKL}G^{IJKL}$ in supergravity in terms of strings (X) [18, 19]:

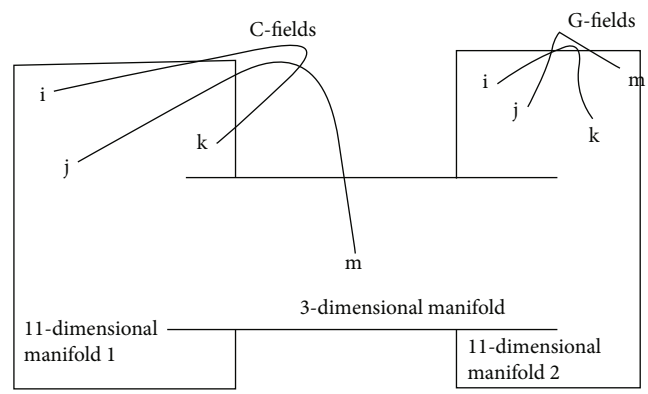

Figure 6: C-fields are produced from G-fields when one end of strings is placed on a 3-dimensional manifold and three ends are placed on an 11-dimensional manifold.

$$\begin{aligned}G^{IJKL} &= \left\{X^I, X^J, X^K, X^L\right\} = \varepsilon^{I'J'K'L'}\frac{\partial X^I}{\partial y^{I'}}\frac{\partial X^J}{\partial y^{J'}}\frac{\partial X^K}{\partial y^{K'}}\frac{\partial X^L}{\partial y^{L'}} \\ &\Downarrow \int d^{11}x\sqrt{g}\left(G_{IJKL}G^{IJKL}\right) = \int d^{11}x\sqrt{g} \\ &\cdot\left(\varepsilon_{I'J'K'L'}\frac{\partial X_I}{\partial y_{I'}}\frac{\partial X_J}{\partial y_{J'}}\frac{\partial X_K}{\partial y_{K'}}\frac{\partial X_L}{\partial y_{L'}}\varepsilon^{I''J''K''L''}\frac{\partial X^I}{\partial y^{I''}}\frac{\partial X^J}{\partial y^{J''}}\frac{\partial X^K}{\partial y^{K''}}\frac{\partial X^L}{\partial y^{L''}}\right).\end{aligned} \tag{21}$$

Equation (21) helps us to extract the CGG terms from the GG terms in supergravity. To this aim, we will add a 3-dimensional manifold to the 11-dimensional manifold which connects it to other 11-dimensional manifold by applying the properties of strings (X) in the Nambu-Poisson brackets [19]:

$$\begin{aligned}X^I &\approx y^I + y_JF^{IJ} - y_JR^{IJ} + y_J\partial^I\phi\partial^J\phi - iy_J\bar{\psi}\gamma^J\partial^I\psi + \cdots \Downarrow \frac{\partial X^{I_5}}{\partial y^{I_5}} \\ &\approx \delta\left(y^{I_5}\right) + \cdots \quad \frac{\partial X^{I_6}}{\partial y^{I_6}} \approx \delta\left(y^{I_6}\right) + \cdots \quad \frac{\partial X^{I_7}}{\partial y^{I_7}} \approx \delta\left(y^{I_7}\right) + \cdots,\end{aligned} \tag{22}$$

$$\int_{M^{N=3}} \longrightarrow \int_{y^{I_5}+y^{I_6}+y^{I_7}} \varepsilon^{I'_5I'_6I'_7}\frac{\partial X^{I_5}}{\partial y^{I'_5}}\frac{\partial X^{I_6}}{\partial y^{I'_6}}\frac{\partial X^{I_7}}{\partial y^{I'_7}} = 1 + \cdots, \tag{23}$$

where ellipses (…) were used to represent higher-order derivatives. The integration is over a 3-dimensional manifold with coordinates $(y^{I_5}, y^{I_6}, y^{I_7})$, and consequently, the integration can be shown by $\int_{y^{I_5}+y^{I_6}+y^{I_7}} = \int dy^{I_5}\int dy^{I_6}\int dy^{I_7})$. This result shows that ignoring fluctuations of strings that leads to the production of fields, the result of integration over each 3-dimensional manifold tends to one. When we add one manifold to another, the integration will be the product of integration over each manifold.

Extending the manifold over additional dimensions extends the integration volume element. By extending the 11-dimensional manifold in equation (21) with the 3-dimensional manifold of equation (23), we get [19]

$$\begin{aligned}&\int_{M^{N=3}}\int_{M^{11}}\sqrt{g}\left(G_{I_1I_2I_3I_4}G^{I_1I_2I_3I_4}\right) \\ &= \int_{M^{11}+y^{I_5}+y^{I_6}+y^{I_7}}\sqrt{g}\varepsilon^{I'_5I'_6I'_7}G_{I_1I_2I_3I_4}G^{I_1I_2I_3I_4}\frac{\partial X^{I_5}}{\partial y^{I'_5}}\frac{\partial X^{I_6}}{\partial y^{I'_6}}\frac{\partial X^{I_7}}{\partial y^{I'_7}},\end{aligned} \tag{24}$$

where we notice that after making the identification [19]:

$$C^{I_5I_6I_7} = \varepsilon^{I'_5I'_6I'_7}\frac{\partial X^{I_5}}{\partial y^{I'_5}}\frac{\partial X^{I_6}}{\partial y^{I'_6}}\frac{\partial X^{I_7}}{\partial y^{I'_7}}, \tag{25}$$

we recover the CGG action in $(11+3)$ dimensions [19]:

$$S_{\mathrm{CGG}}^{N=11+3} = \int_{M^{11}+y^{I_5}+y^{I_6}+y^{I_7}} \sqrt{g}G_{I_1I_2I_3I_4}G^{I_1I_2I_3I_4}C^{I_5I_6I_7}. \tag{26}$$

This equation has three interesting results: (1) the CGG term that appears in the supergravity action is a result of adding a 3-dimensional manifold to an 11-dimensional manifold. This manifold connects the first 11-dimensional manifold to the second one. Combining the 11-dimensional manifold with the 3-dimensional manifold yields 14-dimensional supergravity. The shape of the C-term is now clear in terms of string fields (X^I). It is clear that C-fields are in fact G-fields such that one end of the strings is placed out of the 11-dimensional manifold and on the 3-dimensional manifold and for this reason, C-fields seem to have three ends only (see Figure 6).

To verify that the theory is correct, we should be able to get back the gauge variation of the CGG-action in equation (18) in terms of field strengths and curvature. To this end, using equations (20), (21), (23), and (25), we can calculate the gauge variation of C [19]:

$$\begin{aligned}
X^I &\approx y^I + y_JF^{IJ} - y_JR^{IJ} + y_J\partial^I\phi\partial^J\phi - iy_J\bar{\psi}\gamma^J\partial^I\psi + \cdots \Downarrow \frac{\partial\delta_AX^I}{\partial y^I}\\
&= \delta(y^I)\Downarrow\int_{M^{N=3}+M^{11}}\delta_AC^{I_5I_6I_7}\\
&= \int_{M^{N=3}+M^{11}}\Sigma_{I'_5I'_6I'_7}\varepsilon^{I'_5I'_6I'_7}\delta_A\left(\frac{\partial X^{I_5}}{\partial y^{I'_5}}\frac{\partial X^{I_6}}{\partial y^{I'_6}}\frac{\partial X^{I_7}}{\partial y^{I'_7}}\right)\\
&= \int_{M^{N=3}+M^{10}}\Sigma_{I'_5I'_6}\varepsilon^{I'_5I'_6}\left(\frac{\partial X^{I_5}}{\partial y^{I'_5}}\frac{\partial X^{I_6}}{\partial y^{I'_6}}\right)\\
&= \int_{M^{N=3}+M^{10}}\Big(F^{IJ} - R^{IJ} + \partial^I\phi\partial^J\phi - i\bar{\psi}\gamma^J\partial^I\psi + \cdots.\\
&= \int_{M^{N=3}+M^{10}}\Big(F^{IJ} - R^{IJ} + \partial^I\phi\partial^J\phi - i\bar{\psi}\gamma^J\partial^I\psi + \cdots.\Big).
\end{aligned} \tag{27}$$

where the ellipses (…) represent higher-order derivatives with respect to fields. Using equations (20), (21), (23), (25), and (27), we can calculate the gauge variation of the CGG action in the equation of (26):

$$\delta S_{CGG}^{N=11+3} = \delta\int_{M^{11}+M^{N=3}}\sqrt{g}\varepsilon_{I_1I_2I_3I_4I'_1I'_2I'_3I'_4I_5I_6I_7}\varepsilon^{\bar{I}_5\bar{I}_6\bar{I}_7}\left(\frac{\partial X^{I_5}}{\partial y^{\bar{I}_5}}\frac{\partial X^{I_6}}{\partial y^{\bar{I}_6}}\frac{\partial X^{I_7}}{\partial y^{\bar{I}_7}}\right)G^{I_1I_2I_3I_4}G^{I_1'I_2'I_3'I_4'} = \delta\int_{M^{11}+M^{N=3}}\sqrt{g}\varepsilon_{I_1I_2I_3I_4I'_1I'_2I'_3I'_4I_5I_6I_7}\varepsilon^{\bar{I}_5\bar{I}_6\bar{I}_7}\left(\frac{\partial X^{I_5}}{\partial y^{\bar{I}_5}}\frac{\partial X^{I_6}}{\partial y^{\bar{I}_6}}\frac{\partial X^{I_7}}{\partial y^{\bar{I}_7}}\right), \tag{28}$$

$$0.5\mathrm{in}\times\left(\varepsilon^{\bar{I}_1\bar{I}_2\bar{I}_3\bar{I}_4}\frac{\partial X^{I_1}}{\partial y^{\bar{I}_1}}\frac{\partial X^{I_2}}{\partial y^{\bar{I}_2}}\frac{\partial X^{I_3}}{\partial y^{\bar{I}_3}}\frac{\partial X^{I_4}}{\partial y^{\bar{I}_4}}\right)\left(\varepsilon^{\bar{I}'_1\bar{I}'_2\bar{I}'_3\bar{I}'_4}\frac{\partial X^{I_1'}}{\partial y^{\bar{I}'_1}}\frac{\partial X^{I_2'}}{\partial y^{\bar{I}'_2}}\frac{\partial X^{I_3'}}{\partial y^{\bar{I}'_3}}\frac{\partial X^{I_4'}}{\partial y^{\bar{I}'_4}}\right) = \int_{M^{10}+M^{N=3}}\sqrt{g}\varepsilon_{I_1I_2I_3I_4I'_1I'_2I'_3I'_4I_5I_6}\varepsilon^{\bar{I}_5\bar{I}_6}\left(\frac{\partial X^{I_5}}{\partial y^{\bar{I}_5}}\frac{\partial X^{I_6}}{\partial y^{\bar{I}_6}}\right), \tag{29}$$

$$0.5\mathrm{in}\times\left(\varepsilon^{\bar{I}_1\bar{I}_2\bar{I}_3\bar{I}_4}\frac{\partial X^{I_1}}{\partial y^{\bar{I}_1}}\frac{\partial X^{I_2}}{\partial y^{\bar{I}_2}}\frac{\partial X^{I_3}}{\partial y^{\bar{I}_3}}\frac{\partial X^{I_4}}{\partial y^{\bar{I}_4}}\right)\left(\varepsilon^{\bar{I}'_1\bar{I}'_2\bar{I}'_3\bar{I}'_4}\frac{\partial X^{I_1'}}{\partial y^{\bar{I}'_1}}\frac{\partial X^{I_2'}}{\partial y^{\bar{I}'_2}}\frac{\partial X^{I_3'}}{\partial y^{\bar{I}'_3}}\frac{\partial X^{I_4'}}{\partial y^{\bar{I}'_4}}\right) = \int_{M^{10}+M^{N=3}}\sqrt{g}\varepsilon_{I_1I_2I_3I_4I'_1I'_2I'_3I'_4I_5I_6}\left(\varepsilon^{\bar{I}_4\bar{I}_5}\frac{\partial X^{I_4}}{\partial y^{\bar{I}_4}}\frac{\partial X^{I_5}}{\partial y^{\bar{I}_5}}\right)\left(\varepsilon^{\bar{I}'_4\bar{I}_6}\frac{\partial X^{I_4'}}{\partial y^{\bar{I}'_4}}\frac{\partial X^{I_6}}{\partial y^{\bar{I}_6}}\right), \tag{30}$$

$$\begin{aligned}
&0.5\mathrm{in}\times\left(\varepsilon^{\bar{I}_1\bar{I}_2}\frac{\partial X^{I_1}}{\partial y^{\bar{I}_1}}\frac{\partial X^{I_2}}{\partial y^{\bar{I}_2}}\right)\left(\varepsilon^{\bar{I}'_1\bar{I}'_2}\frac{\partial X^{I'_1}}{\partial y^{\bar{I}'_1}}\frac{\partial X^{I'_2}}{\partial y^{\bar{I}'_2}}\right)\left(\varepsilon^{\bar{I}_3\bar{I}'_3}\frac{\partial X^{I_3}}{\partial \bar{I}^{I_3}}\frac{\partial X^{I'_3}}{\partial y^{\bar{I}'_3}}\right) = \int_{M^{10}+M^{N=3}}\sqrt{g}\Sigma_{n=1}^{5}\left(trF^n - trR^n + trF^nR^{5-n}\right)\\
&+\int_{M^{11}+M^{N=3}}\sqrt{g}\left(\Sigma_{n=1}^{5}\Sigma_{m=0}^{5-n}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}R_{J_1J_2}..R_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2m-1}}\phi\partial^{I_{2m}}\phi\right) - i^m\int_{M^{11}+M^{N=3}}\sqrt{g}\\
&\cdot\left(\Sigma_{n=1}^{5}\Sigma_{m=0}^{5-n}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}R_{J_1J_2}..R_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\right) - i^m\int_{M^{11}+M^{N=3}}\sqrt{g}\\
&\cdot\left(\Sigma_{n=1}^{5}\Sigma_{m=0}^{n-5}\Sigma_{k=0}^{5-n-m}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}R_{J_1J_2}..R_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\times\varepsilon^{I_1I_2..I_{2k-1}I_{2k}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2k-1}}\phi\partial^{I_{2k}}\phi\right)\\
&+\int_{M^{11}+M^{N=3}}\sqrt{g}\left(\Sigma_{n=1}^{5}\Sigma_{m=0}^{5-n}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2m-1}}\phi\partial^{I_{2m}}\phi\right)\\
&-i^m\int_{M^{11}+M^{N=3}}\sqrt{g}\left(\Sigma_{n=1}^{5}\Sigma_{m=0}^{5-n}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\right)\\
&-i^m\int_{M^{11}+M^{N=3}}\sqrt{g}\left(\Sigma_{n=1}^{5}\Sigma_{m=0}^{5-n}\Sigma_{k=0}^{5-n-m}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\times\varepsilon^{I_1I_2..I_{2k-1}I_{2k}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2k-1}}\phi\partial^{I_{2k}}\phi+\cdots\right).
\end{aligned} \tag{31}$$

The first term in equation (31) cancels the anomaly in equation (18). However, the other terms yield the action of modified gravity and matter. In fact, to remove the anomalies, we have to add two extra actions to equation (9), one is related to $F(R)$-gravity, and the second corresponds to matter. We can write

$$S_{SUGRA} = S_{GG} + S_{CGG} + S_{F(R)} + S_{\text{Matter}}, \tag{32}$$

$$S_{GG} = \frac{1}{\bar{\kappa}^2}\int d^{14}x\sqrt{g}\left(-\frac{1}{2}R - \frac{1}{48}G_{IJKL}G^{IJKL}\right), \tag{33}$$

$$S_{CGG} = -\frac{\sqrt{2}}{3456\bar{\kappa}^2}\int_{M^{11}} d^{14}x\varepsilon^{I_1I_2\cdots I_{11}}C_{I_1I_2I_3}G_{I_4\cdots I_7}G_{I_8\cdots I_{11}}, \tag{34}$$

$$S_{F(R)} = \int_{M^{11}+M^{N=3}}\sqrt{g}F(R,\phi,\psi), \tag{35}$$

$$\begin{aligned} S_{\text{Matter}} = &\int_{M^{11}+M^{N=3}}\sqrt{g}\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{n-6}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2m-1}}\phi\partial^{I_{2m}}\phi\right)\\ &- i^m\int_{M^{11}+M^{N=3}}\sqrt{g}\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{6-n}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\right)\\ &- i^m\int_{M^{11}+M^{N=3}}\sqrt{g}\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{6-n}\Sigma_{k=0}^{6-n-m}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\times\varepsilon^{I_1I_2..I_{2k-1}I_{2k}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2k-1}}\phi\partial^{I_{2k}}\phi+\cdots\right), \end{aligned} \tag{36}$$

where $F(R,\phi,\psi)$ can be given by:

$$\begin{aligned} F(R,\phi,\psi) = &\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{n-6}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}R_{J_1J_2}..R_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2m-1}}\phi\partial^{I_{2m}}\phi\right)\\ &- i^m\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{6-n}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}R_{J_1J_2}..R_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\right)\\ &- i^m\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{6-n}\Sigma_{k=0}^{6-n-m}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}R_{J_1J_2}..R_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\times\varepsilon^{I_1I_2..I_{2k-1}I_{2k}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2k-1}}\phi\partial^{I_{2k}}\phi+\cdots\right). \end{aligned} \tag{37}$$

This $F(R,\phi,\psi)$-gravity is now free of any anomaly on the $(11+3)$-dimensional manifold. We can write

$$\delta S_{SUGRA} = \delta S_{GG} + \delta S_{CGG} + \delta S_{F(R)} + \delta S_{\text{Matter}} = 0. \tag{38}$$

The above equation shows that total action is free of any anomaly. This means that any variation in the supergravity action (δS_{GG}) produces some extra terms as new anomalies. These anomalies could be cancelled by variations in the Chern-Simon action (δS_{CGG}). Although, variations of these actions also produce some new anomalies, and these could be removed by variation of the f-gravity action ($\delta S_{F(R)}$). Thus, the f-gravity terms help to remove all anomalies.

In fact, this type of gravity is produced by exchanging gravitons, scalars, and fermions between two 11-dimensional manifolds via one 3-dimensional manifold. The order of curvatures depends on the number of dimensions of the manifolds and also on the number of scalars or fermions which are attached to the curvatures. This result is in good agreement with previous predictions in [8, 9].

3. Inflationary Universe in $F(R,\phi,\psi)$-Gravity

In this section, we construct a 4-dimensional universe on a $(11+3)$-dimensional manifold. We will show that interaction between two 11-dimensional manifolds in this system has a

direct effect on the evolution of the universe during the inflationary epoch. We will obtain the scale factor of the universe in terms of the parameters of the system. Using the action in equation of (36) and the relation between G, C, F, and R -terms in equation (17), we can obtain the following field equations:

$$\begin{aligned}
&R_{ij} - \frac{1}{2}Rg_{ij} + \left(\frac{1}{2}g_{ij}F(R,\phi,\psi) - R_{ij}F'(R,\phi,\psi) + \partial_i\partial_j F'(R,\phi,\psi) - g_{ij}\partial_i\partial^i F'(R,\phi,\psi)\right)\\
&\quad + \left(\frac{1}{2}g_{ij}\left(G_{IJKL}G^{IJKL}\right) - R_{ij}\left(G_{IJKL}G^{IJKL}\right)' + \partial_i\partial_j\left(G_{IJKL}G^{IJKL}\right)' - g_{ij}\partial_i\partial^i\left(G_{IJKL}G^{IJKL}\right)'\right)\\
&\quad + \varepsilon^{I'J'K'}C_{I'J'K'}\left(\frac{1}{2}g_{ij}\left(G_{IJKL}G^{IJKL}\right) - R_{ij}\left(G_{IJKL}G^{IJKL}\right)' + \partial_i\partial_j\left(G_{IJKL}G^{IJKL}\right)' - g_{ij}\partial_i\partial^i\left(G_{IJKL}G^{IJKL}\right)'\right)\\
&\quad + \left(G_{IJKL}G^{IJKL}\right)\left(\frac{1}{2}g_{ij}\left(\varepsilon^{I'J'K'}C_{I'J'K'}\right) - R_{ij}\left(\varepsilon^{I'J'K'}C_{I'J'K'}\right)' + \partial_i\partial_j\left(\varepsilon^{I'J'K'}C_{I'J'K'}\right)' - g_{ij}\partial_i\partial^i\left(\varepsilon^{I'J'K'}C_{I'J'K'}\right)'\right)\\
&\quad + g_{ij}\Big[\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{n-6}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2m-1}}\phi\partial^{I_{2m}}\phi\right)\\
&\quad - i^m\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{6-n}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\right)\\
&\quad - i^m\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{6-n}\Sigma_{k=0}^{6-n-m}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\times\varepsilon^{I_1I_2..I_{2k-1}I_{2k}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2k-1}}\phi\partial^{I_{2k}}\phi+\cdots\right)\Big] = 0,
\end{aligned} \tag{39}$$

where $'$ denotes the derivative with respect to the curvature R and

$$\begin{aligned}
&\left(\frac{\partial}{\partial x_I}\left[\frac{\partial}{\partial(\partial_I\phi)}\right] - \frac{\partial}{\partial\phi}\right)\times\Big[\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{n-6}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2m-1}}\phi\partial^{I_{2m}}\phi\right)\\
&\quad - i^m\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{6-n}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\right)\\
&\quad - i^m\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{6-n}\Sigma_{k=0}^{6-n-m}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\times\varepsilon^{I_1I_2..I_{2k-1}I_{2k}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2k-1}}\phi\partial^{I_{2k}}\phi+\cdots\right)\Big],
\end{aligned} \tag{40}$$

$$\left(\frac{\partial}{\partial x_I}\left[\frac{\partial}{\partial(\partial_I\phi)}\right] - \frac{\partial}{\partial\phi}\right)\times\left[G_{IJKL}G^{IJKL} + \varepsilon^{I'J'K'}C_{I'J'K'}G_{IJKL}G^{IJKL} + F(R,\phi,\psi)\right] = 0, \tag{41}$$

$$\begin{aligned}
&\left(\frac{\partial}{\partial x_I}\left[\frac{\partial}{\partial(\partial_I\psi)}\right] - \frac{\partial}{\partial\psi}\right)\times\Big[\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{n-6}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2m-1}}\phi\partial^{I_{2m}}\phi\right)\\
&\quad - i^m\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{6-n}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\right)\\
&\quad - i^m\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{6-n}\Sigma_{k=0}^{6-n-m}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\times\varepsilon^{I_1I_2..I_{2k-1}I_{2k}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2k-1}}\phi\partial^{I_{2k}}\phi+\cdots\right)\Big],
\end{aligned} \tag{42}$$

$$\left(\frac{\partial}{\partial x_I}\left[\frac{\partial}{\partial(\partial_I\psi)}\right] - \frac{\partial}{\partial\psi}\right)\times\left[G_{IJKL}G^{IJKL} + \varepsilon^{I'J'K'}C_{I'J'K'}G_{IJKL}G^{IJKL} + F(R,\phi,\psi)\right] = 0, \tag{43}$$

$$\begin{aligned}
&\left(\frac{\partial}{\partial x_I}\left[\frac{\partial}{\partial(\partial_I A_J)}\right] - \frac{\partial}{\partial A_I}\right)\times\Big[\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{n-6}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2m-1}}\phi\partial^{I_{2m}}\phi\right)\\
&\quad - i^m\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{6-n}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\right)\\
&\quad - i^m\left(\Sigma_{n=1}^{6}\Sigma_{m=0}^{6-n}\Sigma_{k=0}^{6-n-m}\varepsilon^{J_1J_2..J_{2n-1}J_{2n}}F_{J_1J_2}..F_{J_{2n-1}J_{2n}}\varepsilon^{I_1I_2..I_{2m-1}I_{2m}}\bar{\psi}\gamma^{I_1}\partial^{I_2}\psi..\bar{\psi}\gamma^{2I_m-1}\partial^{I_{2m}}\psi\times\varepsilon^{I_1I_2..I_{2k-1}I_{2k}}\partial^{I_1}\phi\partial^{I_2}\phi..\partial^{I_{2k-1}}\phi\partial^{I_{2k}}\phi+\cdots\right)\Big],
\end{aligned} \tag{44}$$

$$\left(\frac{\partial}{\partial x_I}\left[\frac{\partial}{\partial(\partial_I A_J)}\right] - \frac{\partial}{\partial A_I}\right)\times\left[G_{IJKL}G^{IJKL} + \varepsilon^{I'J'K'}C_{I'J'K'}G_{IJKL}G^{IJKL} + F(R,\phi,\psi)\right] = 0. \tag{45}$$

For a 4-dimensional universe, we have $R = 12H^2 + 6\dot{H}$, where $H = \dot{a}/a$ is the Hubble parameter and a is the scale factor of the universe. Assuming that all fields depend only on time, and using the relation between scalars, gauge fields, curvatures, and strings in equation (17), we can solve equations (39), (41), (43), and (45) simultaneously and obtain

$$, a(t) \approx e^{-\int dtZ(t)}, \tag{46}$$

$$Z(t) \approx \Sigma_{n=1}^{6}\beta_n t_s^{2n}\left(1-\frac{t}{t_s}\right)^{2n}\left(1+\Sigma_{m=0}^{6-n}\alpha_m \ln^{2m} \cdot \left(t_s\left(1-\frac{t}{t_s}\right)\right)\left[1+\Sigma_{k=0}^{12-2n-2m}\lambda_k e^{2\gamma_k k t_s(1-t/t_s)}\right]\right), \tag{47}$$

$$\phi(t) \approx (\psi)^{1/2} \approx \Sigma_{n=1}^{6}\beta_n t_s^{2n}\left(1-\frac{t}{t_s}\right)^{2n}\Sigma_{m=0}^{6-n}\alpha_m \ln^{-2m}\left(\frac{t}{t_s}\right) \times Z^{1/4}(t), \tag{48}$$

$$A^I(t) \approx \varepsilon^I \Sigma_{m=1}^{6}\alpha_m\left(t_s\left(1-\frac{t}{t_s}\right)^{2m}\Sigma_{k=0}^{6-m}\lambda_k' e^{-2\gamma_k{}' k(t/t_s)} \times \ln\left(Z(t)\right)\right), \tag{49}$$

where t_s is time of collision between the two 11-dimensional manifolds. This time is much larger than the present age of the universe ($t_s \lll 13.5Gyr$). The above result shows the coincidence with the birth of the universe ($t = 0$) on the 11-dimensional manifold. The scale factor grows with time, while scalars, fermions, and gauge fields decrease and tend to zero at ($t = t_s$). This is because with the passage of time, the two 11-dimensional manifolds come close to each other and all fields which live on the 3-dimensional manifold between them dissolve into the 11-dimensional manifolds. By the disappearance of these fields, gravitons and $F(R)$-gravity emerge that lead to the expansion and inflation of the universe (see Figure 7).

We can test our model by calculating the magnitude of the slow-roll parameters and the tensor-to-scalar ratio ($R_{\text{tensor-scalar}}$) defined in [20] and compare with previous predictions:

$$H = \frac{\dot{a}}{a} = -\dot{Z} \Rightarrow, \tag{50}$$

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{\ddot{Z}}{\left(\dot{Z}\right)^2}, \tag{51}$$

$$\eta = -\frac{\ddot{H}}{2H\dot{H}} = \frac{\dddot{Z}}{2\dot{Z}\ddot{Z}}, \tag{52}$$

$$R_{\text{tensor-scalar}} = 16\varepsilon = 16\frac{\ddot{Z}}{\left(\dot{Z}\right)^2}. \tag{53}$$

Obviously, during the inflationary epoch, the age of the universe (t) is very smaller with respect to the time of collision between manifolds (t_s) and, as a result, $((t_s - t)^{-n} \ll 1)$ where n is an integer. Using equation (53) and expanding the functions (e^O and ln) by applying the Taylor series, we can derive the following results:

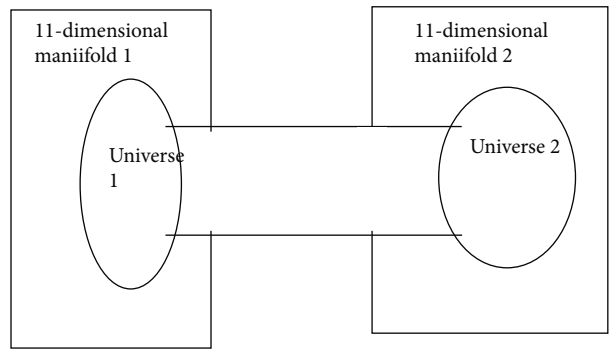

Figure 7: Emergence of 4-dimensional universe on 11-dimensional manifold in 11 + 3-dimensional space-time.

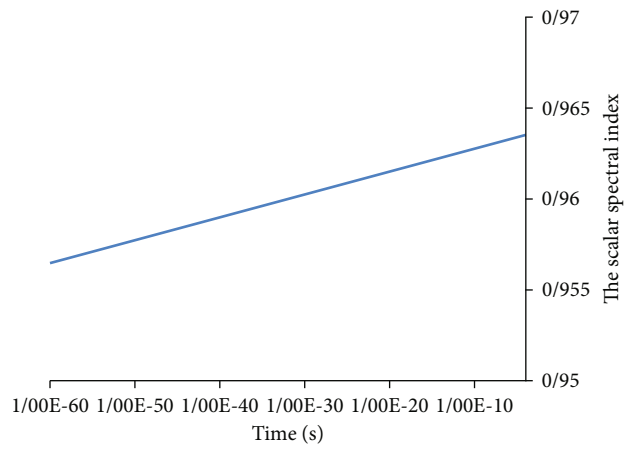

Figure 8: The scalar spectral index in terms of time ($t_s = 33Gyr$).

$$0 \ll t \ll t_s \Rightarrow \eta \approx \Sigma\beta_n(t_s - t)^{-2n} \cdot \left(ln^{-2n}\left(t_s\left(1-\frac{t}{t_s}\right)\right) + O(\text{smallerterms})\right) \ll 1, \tag{54}$$

$$\varepsilon \approx \Sigma\alpha_m(t_s - t)^{-2m}\left(e^{-2\gamma_m m t_s(1-t/t_s)} + O(\text{smallerterms})\right) \ll 1 \Rightarrow R_{tensor-scalar} \ll 1. \tag{55}$$

This equation indicates that the values of the slow-roll parameters and the tensor-to-scalar ratio are very much smaller than the ones which are consistent with the predictions of the experiments in ref [21]. Thus, this type of $F(R, \phi, \psi)$-gravity which is produced due to the interaction of two 11-dimensional manifolds creates the correct values for the experimental parameters and can describe cosmological events.

In Figures 8 and 9, we have presented the scalar spectral index and the tensor to scalar ratio parameters in terms of time. It is clear that for the inflationary age of the universe and the colliding time ($t_s = 33Gyr$), the spectral index is less

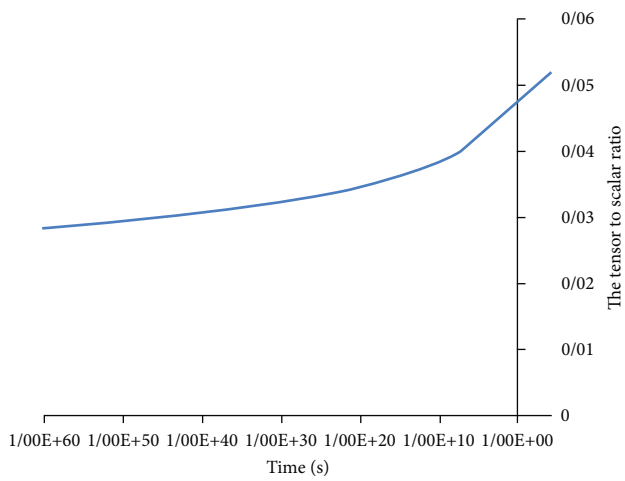

Figure 9: Tensor to scalar ratio parameter in terms of time ($t_s = 33Gyr$).

then or around 0.96 and the tensor to scalar ratio is less than 0.036, which is comparable with results of Planck [22].

4. Summary and Conclusion

Motivated by the successes of modified theories of gravity such as $F(R)$-gravity, in this paper, we have investigated how we could make $F(R)$-gravity anomaly-free. To achieve this, we considered the origin of the action of $F(R)$-gravity on an $(11+3)$-dimensional space-time. However, we found that this was not anomaly-free. So, we proposed a theory which lives on an $(11+3)$-dimensional manifold with two 11-dimensional manifolds and one 3-dimensional manifold. We showed that our universe is a part of one 11-dimensional manifold which is connected with the other 11-dimensional manifold by an extra 3-dimensional manifold. The two 11-dimensional manifolds interact with each other via exchanging fields which move along the 3-dimensional manifold. These fields are the main cause for the appearance of $F(R)$ -gravity in the four-dimensional spacetime. This is because with the passage of time, the two 11-dimensional manifolds come close to each other and all fields which live on the 3-dimensional manifold between them dissolve into the 11-dimensional manifolds. As these fields disappear, gravitons and $F(R)$-gravity emerge that lead to the expansion and inflation of the universe. Hence, the interaction between the two 11-dimensional manifolds has a direct effect on the evolution of the universe during the inflationary era. We have calculated the scale factor of the inflationary universe and examined our model against WMAP and Planck experiments. We have obtained the slow-roll parameters and shown that the tensor-scalar ratio is much smaller than unity ($R_{\text{tensor-scalar}} \lll 1$), which is consistent with experimental observations.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflict of interest.

Authors' Contributions

An arxiv has previously been published [23]. We have made use of data that is publicly available to support this study.

Acknowledgments

Aroonkumar Beesham acknowledges that this work is based on the research supported wholly/in part by the National Research Foundation of South Africa (Grant number: 118511). The work of KB was supported in part by the JSPS KAKENHI Grant Number JP21K03547. The authors are grateful to Professor Sergey Odintsov for bringing several relevant articles to their attention.

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