

## Research Article

# Hadronic Molecule Interpretation of $T_{cc}^+$ and Its Beauty Partners

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Motivated by the latest discovery of a new tetraquark  $T_{cc}^+$  with two charm quarks and two light antiquarks by LHCb Collaboration, we investigated the  $DD^*$  hadronic molecule interpretation of  $T_{cc}^+$ . By calculation, the mass and the decay width of this new structure  $T_{cc}^+$  can be understood in one-meson exchange potential model. The binding energies for these  $DD^*$  hadronic molecules with  $J^P = 1^+$  are around 1 MeV. Besides, we also studied the possible beauty partners  $T_{bb}(10598)$  of hadronic molecule  $T_{cc}^+$ , which may be feasible in future LHCb experiments.

## 1. Introduction

Hadron spectroscopy provides a unique window for us to understand the fundamental strong interactions. In naive quark model, hadrons are established by quark-antiquark pair or three quark objects. However, the quantum chromodynamics (QCD) theory tells us that some exotic states such as multi-quark states or gluon-participated states, which are apart from the conventional configurations, may also be confined into a color-singlet hadron. The earliest evidence of exotic states is the X(3872) discovered by the Belle Collaboration in 2003, which lies above the two open charm meson threshold but has a very narrow decay width ( $\Gamma < 1.2$  MeV) [1]. Interpretation and verification of the special properties of exotic states attracted a lot of attempt from both theoretical and experimental aspects (see the reviews [2–4]). The studies of exotic states are not only to gradually filling in the period table of hadrons but also to enriching our knowledge of QCD color-confining principle.

Very recently, the LHCb Collaboration has reported the first discovery of a new tetraquark  $T_{cc}^+$  with two charm quarks and two light antiquarks in the  $D^0 D^0 \pi^+$  mass spectrum using a proton-proton collision data set corresponding

to an integrated luminosity of  $9fb^{-1}$  [5], where the mass and decay width of  $T_{cc}^+$  are determined as

$$\begin{aligned}\delta_m &= m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0}) = -273 \pm 61 \pm 5_{-14}^{+11} \text{ keV}, \\ \Gamma_{T_{cc}^+} &= 410 \pm 165 \pm 43_{-38}^{+18} \text{ keV}.\end{aligned}\quad (1)$$

And the spin-parity is determined as  $J^P = 1^+$ . Later, the LHCb Collaboration has released a more profound decay analysis [6]; then, the mass and decay width of  $T_{cc}^+$  are updated as

$$\begin{aligned}\delta_m &= m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0}) = -361 \pm 40 \text{ keV}, \\ \Gamma_{T_{cc}^+} &= 47.8 \pm 1.9 \text{ keV}.\end{aligned}\quad (2)$$

The LHCb exotic state  $T_{cc}^+$  has an electrical charge and two charm quantum numbers and thus leads to a strong evidence of least quark content  $[cc\bar{u}\bar{d}]$ . This exotic system has interesting points. First, the two heavy quarks inside the

system have small relative velocities due to its large masses compared to the QCD typical energy scale ( $m_Q \gg \Lambda_{\text{QCD}}$ ). There exists an attractive color force between the color antitriplet heavy quark pair. Similar attraction is produced for the light antiquark pair. From these arguments, diquark models were employed [7–15]. On the other hand, a lower bound state may be produced between two heavy hadrons by exchanging light mesons. Hadronic molecules are also popular choices for the system of two heavy quarks and two light antiquarks [16–22]. In addition, there are other proposals to explain the doubly heavy tetraquarks: compact tetraquarks [23–50], chiral quark model [51–53], constituent quark model [54], and hydrogen-like molecules [55]. The production properties of doubly heavy tetraquarks have been studied in literatures, for example, Refs. [56–59], while the decay properties of doubly heavy tetraquark have been studied in literatures [60–68].

Considering that the LHCb exotic state  $T_{cc}^+$  is near threshold of a pair of charm mesons, we will investigate the  $DD^*$  hadronic molecule interpretation of  $T_{cc}^+$  in this work. From the mass of  $T_{cc}^+$ , it is extremely close to the  $D D^*$  threshold. The binding energy is less than 1 MeV. We will study the mass and decays of doubly charm tetraquarks in one-meson exchange potential (OMEV) model. The effective coupling constants among light mesons and charm mesons are revisited. By power counting, we only consider the leading order contribution from the lightest mesons, i.e., pseudoscalar mesons. Then, the number of parameters is further reduced in OMEV model. By the investigation of the decay channels, it is also possible to hunting for the charge-partners  $T_{cc}^0$  and  $T_{cc}^{++}$  states. As a by-product, we also study the mass spectra of the possible doubly bottomed tetraquarks  $T_{bb}$  and discuss their golden decay channels.

This paper is organized as follows. We give the low energy effective Lagrangian and the effective potential after the Introduction. The OMEV model is employed to extract the effective potential. In Section 3, we present the calculation detail of the mass of the possible ground states of doubly heavy tetraquarks below the heavy meson pair  $HH^*$  threshold. In Section 4, we give the decay amplitude and decay width of the process  $T_{QQ} \rightarrow H + H + \pi$ . We conclude in the end.

## 2. Low Momentum Interaction Effective Theory

In the heavy quark limit  $m_Q \rightarrow \infty$ , the heavy quark behaves like a static point, and the heavy meson dynamics is determined by the degree of freedom of the light quark. In heavy quark spin symmetry, the spin-0 and spin-1 heavy-light mesons are combined into a  $4 \times 4$  matrix

$$H_a = \frac{1 + \not{v}}{2} [H_a^{*\mu} \gamma_\mu + i H_a \gamma_5], \quad (3)$$

where the pseudoscalar and vector heavy-light meson fields are explicitly expressed as  $H_a = (D^0, D^+, D_s^+)$  and  $H_a^* = (D^{*0}, D^{*+}, D_s^{*+})$  for charm sector and  $H_a = (B^-, \bar{B}_s^0, \bar{B}_s^0)$  and  $H_a^* = (B^{*-}, \bar{B}^{*0}, \bar{B}_s^{*0})$  for bottom sector;  $v$  is the velocity of the

heavy quark with the constraint  $\not{v} H_a = H_a$ . Here,  $H_a^{(*)}$  is a triplet in SU(3) flavor symmetry when considering the fact that the masses of light quarks  $u$ ,  $d$ , and  $s$  can be ignored compared with the heavy quark mass  $m_Q$ .

When one considers the exchanging of low energy light mesons between heavy hadrons, it is required to employ the chiral perturbation theory. Using this low energy theory, it becomes easy to separate the long and short range dynamics. The doubly charm tetraquark  $T_{cc}^+$  observed in LHCb experiment is very close to the threshold of  $DD^*$ ; the binding energy is less than 1 MeV if we treat the  $T_{cc}^+$  as a  $DD^*$  bound state. We expect that the low energy expansion converges well. For the decay channel  $T_{cc}^+ \rightarrow D^0 + D^0 + \pi^+$ , the final particles  $D^0 D^0 \pi^+$  will have small velocities and can be treated as nonrelativistic objects. For example, the maximum energy of  $D^0$  in  $T_{cc}^+ \rightarrow D^0 D^0 \pi^+$  is  $E_{D^0}^{\text{max}} = (m_{T_{cc}^+}^2 + m_D^2 - (m_D + m_\pi)^2) / 2m_{T_{cc}^+} = 1867.68$  MeV, which is only 2.84 MeV above the mass of  $D^0$  meson with  $m_{D^0} = 1864.84$  MeV. Similarly, the maximum energy of  $\pi^+$  in  $T_{cc}^+ \rightarrow D^0 D^0 \pi^+$  is  $E_{\pi^+}^{\text{max}} = (m_{T_{cc}^+}^2 + m_\pi^2 - (m_D + m_D)^2) / 2m_{T_{cc}^+} = 144.85$  MeV, which is only 5.279 MeV above the mass of  $\pi^+$  meson with  $m_{\pi^+} = 139.57$  MeV. Thus, we also use the low-energy effective theory to study the  $T_{cc}^+ \rightarrow D^0 + D^0 + \pi^+$  decay properties.

The low momentum interaction effective theory Lagrangian at leading order is written as [69]

$$\mathcal{L}_0 = \frac{f_p^2}{8} \text{Tr} \partial^\mu \Sigma \partial_\mu \Sigma^\dagger - i \text{Tr} \bar{H}_a \gamma_\mu (\delta_{ab} \partial^\mu + i V_{ab}^\mu) H_b + g_p \text{Tr} \bar{H}_a H_b \gamma_\mu \gamma_5 A_{ba}^\mu, \quad (4)$$

where  $V_{ab}^\mu = 1/2(\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger)_{ab}$  and  $A_{ab}^\mu = i/2(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)_{ab}$  and  $\xi = \sqrt{\Sigma}$ .  $\Sigma = \text{Exp}(2iM/f_p)$  is exponentially related to the light pseudoscalar mesons with

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta}{\sqrt{6}} \end{pmatrix}. \quad (5)$$

In principle, the vector and scalar light mesons may also bring new effects in the binding and decay properties of the near-threshold doubly charm tetraquark states in OMEV model, but these effects are expected to be suppressed as  $O(m_\pi^2/m_{V,S}^2)$  according to the power counting rules. Compared to the long distance interaction from pseudoscalar light mesons, the interactions from scalar and vector light mesons are medium and short ranges. On the other hand, one needs to introduce more parameters in the effective theory, and some of them are not well investigated currently. We will address these points in future works.

TABLE 1: Predictions of the masses (MeV) of  $HH^*$  stable hadronic molecules with spin-parity  $J^P = 1^+$ . The uncertainty is from the choice of the effective coupling  $g_\pi = 0.5 \mp 0.1$ .

$T_{cc}$ states	Isospin	Contents	Mass (MeV)	$T_{bb}$ states	Isospin	Contents	Mass (MeV)
$T_{cc}^+$	0	$D^0 D^{*+}, D^{*0} D^+$	$3875.1^{+0.2}_{-0.3}$	$T_{bb}^-$	0	$\bar{B}^0 B^{*-}, \bar{B}^{*0} B^-$	$10598^{+2}_{-3}$
$T_{cc}^0$	0	$D^0 D^{*0}$	$3871.0^{+0.2}_{-0.3}$	$T_{bb}^0$	0	$\bar{B}^0 \bar{B}^{*0}$	$10598^{+2}_{-3}$
$T_{cc}^{++}$	0	$D^+ D^{*+}$	$3879.2^{+0.2}_{-0.3}$	$T_{bb}^{--}$	0	$B^- B^{*-}$	$10598^{+2}_{-3}$
$T_{cc}^{s,+}$	$\frac{1}{2}$	$D^0 D_s^{*+}, D^{*0} D_s^+$	$3974.8^{+0.4}_{-0.5}$	$T_{bb}^{s,-}$	1/2	$\bar{B}_s^0 B_s^{*-}, \bar{B}_s^{*0} B^-$	$10692.4^{+0.3}_{-0.4}$
$T_{cc}^{s,++}$	$\frac{1}{2}$	$D_s^+ D^{*+}, D_s^{*+} D^+$	$3979.3^{+0.2}_{-0.3}$	$T_{bb}^{s,0}$	$\frac{1}{2}$	$\bar{B}_s^0 \bar{B}_s^{*0}, \bar{B}_s^{*0} \bar{B}^0$	$10692.4^{+0.3}_{-0.3}$
$T_{cc}^{ss,++}$	0	$D_s^+ D_s^{*+}$	$4079.6^{+0.2}_{-0.2}$	$T_{bb}^{ss,0}$	0	$\bar{B}_s^0 \bar{B}_s^{*0}$	$10781.7^{+0.3}_{-0.2}$

Using the above Lagrangian, the two-body potential between a vector and a pseudoscalar heavy mesons becomes as [4, 16]

$$V_{HH^*}(\vec{q}) = -\frac{g_P^2}{f_P^2} \frac{\vec{\tau}_1 \cdot \vec{\tau}_2 \vec{\varepsilon}_1 \cdot \vec{q} \vec{\varepsilon}_2^* \cdot \vec{q}}{\vec{q}^2 + \mu^2}. \quad (6)$$

### 3. Doubly Heavy Tetraquark Spectra from $HH^*$ System

In this section, we will investigate the spectra of doubly heavy tetraquark from the possible  $HH^*$  bound states in OMEP model. After implementing the Fourier transformation on the potential in momentum space, the potential in coordinate space can be obtained. Considering the size of the exchanged light meson, the Fourier transformation on the potential with the dipole form factors becomes

$$V_{HH^*}(r) = \int \frac{d^3 \vec{q}}{(2\pi)^3} e^{i\vec{q} \cdot \vec{r}} V_{HH^*}(\vec{q}) \left( \frac{\Lambda^2 - \mu^2}{\Lambda^2 + \vec{q}^2} \right)^2, \quad (7)$$

where a UV cut-off  $\Lambda$  is introduced to regularize the short-distance effects.

In coordinate space, the one-meson exchange potential is then written as [4, 16]

$$V_{HH^*}(r) = -\left( \frac{g_P}{f_P} \right)^2 \gamma_I (C_0 C(r) + S_{12}(\vec{r}) T(r)), \quad (8)$$

where

$$S_{12}(\vec{r}) = 3 \left( \vec{\varepsilon}_1 \cdot \vec{r} \right) \left( \vec{\varepsilon}_2^* \cdot \vec{r} \right) - 1, \quad (9)$$

$$C(r) = \frac{\mu^2}{4\pi} \left( \frac{e^{-\mu r}}{r} - \frac{e^{-\Lambda r}}{r} - \frac{(\Lambda^2 - \mu^2)}{2\Lambda} e^{-\Lambda r} \right), \quad (10)$$

$$T(r) = \frac{1}{4\pi} \left( (3 + 3\mu r + \mu^2 r^2) \frac{e^{-\mu r}}{r^3} - (3 + 3\Lambda r + \Lambda^2 r^2) \frac{e^{-\Lambda r}}{r^3} + \frac{\mu^2 - \Lambda^2}{2} (1 + \Lambda r) \frac{e^{-\Lambda r}}{r} \right). \quad (11)$$

In the potential, the scale is chose as  $\mu^2 = m_P^2 - (m_{D^*} - m_D)^2$  due to the recoil effect from unequal heavy mesons. For the  $DD^*$  interactions,  $\mu^2 < 0$ , and we should replace  $\mu^2$  as  $|\mu^2|$  and take the real parts in the integrals [4]. The form factor parameter  $\Lambda$  is chose as usual  $\Lambda \simeq 1.1$  GeV. A higher UV cut-off  $\Lambda$  may be employed if one includes the medium and short dynamics. When interpreting the doubly charm tetraquark  $T_{cc}^+$  as the possible  $DD^*$  bound states with  $J^P = 1^+$ , the  $T_{cc}^+$  state can be rewritten as

$$|T_{cc}^+\rangle = \frac{|D^0 D^{*+}\rangle \pm |D^+ D^{*0}\rangle}{\sqrt{2}}. \quad (12)$$

For the  $DD^*$  system with  $J^P = 1^+$ , the  $DD^*$  may be in the S-wave state with orbital angular momentum  $\ell = 0$  or D-wave state with orbital angular momentum  $\ell = 2$ . The parameter  $\gamma_I$  is expressed as  $\gamma_I = -2(I(I+1) - I_1(I_1+1) - I_2(I_2+1))/3$  with isospin quantum number  $I_i$  of the hadrons. Consider the above mixing between S-wave and D-wave states, one has the matrix [16]

$$\langle C_0 \rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \langle S_{12}(\vec{r}) \rangle = \begin{pmatrix} 0 & -\sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix}. \quad (13)$$

In nonrelativistic approximation, the  $DD^*$  binding energy  $E$  can be solved by Schrödinger equation

$$\left( -\frac{\hbar^2}{2\mu_R} \nabla^2 + V_{DD^*}(r) \right) \psi(\mathbf{r}) = E \psi(\mathbf{r}), \quad (14)$$

where  $\mu_R = m_D m_{D^*} / (m_D + m_{D^*})$  is the reduced mass [70]. We only focus on the stable bound states with binding energy  $E < 0$ .

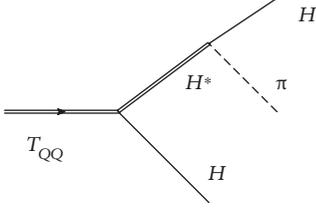


FIGURE 1: Typical Feynman diagram for the  $T_{QQ} \rightarrow HH\pi$  decay.

In the calculation, one needs to input the parameter values. The hadron masses are adopted from PDG [71]:  $m_{D^0} = 1864.84$  MeV,  $m_{D^\pm} = 1869.65$  MeV,  $m_{D^{*0}} = 2006.85$  MeV,  $m_{D^{*\pm}} = 2010.26$  MeV,  $m_{D_s^0} = 1968.34$  MeV,  $m_{D_s^{*\pm}} = 2112.2$  MeV,  $m_{B^0} = 5279.65$  MeV,  $m_{B^\pm} = 5279.34$  MeV,  $m_{B_s^0} = 5366.88$  MeV,  $m_{B_s^\pm} = 5324.65$  MeV,  $m_{B^{*0}} = 5324.65$  MeV,  $m_{B^{*\pm}} = 5415.4$  MeV,  $m_{\pi^\pm} = 139.57$  MeV,  $m_{\pi^0} = 134.977$  MeV,  $m_{K^\pm} = 493.677$  MeV,  $m_{K^0} = 497.611$  MeV, and  $m_\eta = 547.862$  MeV. The effective coupling constants  $g_p$  are needed to extract from experiments or lattice QCD calculations. In flavor SU(3) symmetry, they are approximated to be equal to  $g_\pi$ . For the  $D^* \rightarrow D + \pi$  decays, the Feynman amplitude can be written as

$$iM(D^* \rightarrow D + \pi) = i\lambda_\pi p_\pi^\mu \epsilon_{D^*,\mu}. \quad (15)$$

The data of the decay widths and branching ratios can be inputted from PDG [71]:  $\Gamma_{D^{*\pm}} = 83.4$  keV,  $\Gamma_{D^{*0}} < 2.1$  MeV,  $B(D^{*\pm} \rightarrow D^0\pi^\pm) = 67.7\%$ ,  $B(D^{*\pm} \rightarrow D^\pm\pi^0) = 30.7\%$ , and  $B(D^{*0} \rightarrow D^0\pi^0) = 64.7\%$ . Use the  $D^{*\pm} \rightarrow D^0\pi^\pm$  channel, the effective coupling constant is estimated as  $\lambda_\pi \sim 16.8$ . Use the  $D^{*\pm} \rightarrow D^\pm\pi^0$  channel, the effective coupling constant is estimated as  $\lambda_\pi \sim 11.9$ . While use the  $B(D^{*0} \rightarrow D^0\pi^0) = 64.7\%$  channel and  $\Gamma_{D^{*0}} \approx 60$  keV, one can get  $\lambda_\pi \sim 12.3$ . And the effective coupling constant  $g_\pi$  is estimated from  $\lambda_\pi$  as

$$g_\pi \approx \frac{\lambda_\pi f_\pi}{2\sqrt{m_{D^*}}\sqrt{m_D}} \sim [0.4, 0.6]. \quad (16)$$

We list the numerical results for the  $DD^*$  bound states in Table 1, where the isospin, strange, and beauty partners of hadronic molecule  $T_{cc}^+$  are also given. The binding energies for  $D^0D^{*+}$  and  $D^{*0}D^+$  with  $J^P = 1^+$  are near to 0.6 MeV. The binding energies for strange partners with  $J^P = 1^+$  are close to 1 MeV, while the binding energies for these  $BB^*$  hadronic molecules without strange quantum numbers are around 6 MeV, and then,  $T_{bb}$  states are more stable.

#### 4. $T_{QQ} \rightarrow HH\pi$ Decays

In this section, we will study the decays of  $T_{QQ} \rightarrow HH\pi$ . Here, we only focus on the  $HH^*$  stable hadronic molecules with spin-parity  $J^P = 1^+$  given in Table 1. Consider the fact

that the mass splitting between  $D^*$  and  $D$  mesons is larger than the pion mass, there have  $D^* \rightarrow D + \pi$  decay channels.  $T_{cc} \rightarrow DD\pi$  is also allowed when  $T_{cc}$  is close to or above  $DD^*$  threshold. However, the mass splitting between  $B^*$  and  $B$  mesons is small and less than the pion mass. Thus,  $T_{bb} \rightarrow BB\pi$  channel is forbidden due to the lack of phase space when  $T_{bb}$  is below or close to  $BB^*$  threshold. The LHCb Collaboration has employed the golden channel  $T_{cc}^+ \rightarrow D^0D^+\pi^+$  in the discovery of the first doubly charm tetraquark. The typical Feynman diagram is plotted in Figure 1.

For  $T_{cc}^+ \rightarrow D^0D^+\pi^+$  and  $T_{cc}^+ \rightarrow D^0D^+\pi^0$  processes, the leading-order Feynman amplitudes are similar and can be written as

$$iM(T_{cc}^+ \rightarrow DD\pi) = i\lambda_\pi p_\pi^\mu \frac{-i}{(p_D + p_\pi)^2 - m_{D^*}^2} \cdot \left( -g_{\mu\nu} + \frac{(p_D + p_\pi)_\mu (p_D + p_\pi)_\nu}{(p_D + p_\pi)^2} \right) \cdot im_{T_{cc}} \lambda_{T_{cc}} \epsilon_{T_{cc}}^\nu, \quad (17)$$

where  $\lambda_{T_{cc}}$  is the effective coupling constant for the  $T_{cc} - D - D^*$  vertex.

In general, the three-body partial decay width is written as [72]

$$\frac{d\Gamma}{dsdt} = \frac{1}{(2\pi)^3} \frac{1}{32m_{T_{cc}}^3} |\bar{M}|^2, \quad (18)$$

where  $s$  represents the invariant mass of two heavy charm mesons and  $t$  represents the invariant mass of one heavy charm meson and the pion. For the process  $T_{cc}^+ \rightarrow D^0D^+\pi^0$ , the phase space constraints then read as [73]

$$t_{\min} = (E_2 + E_3)^2 - \left( \sqrt{E_2^2 - m_{D^*}^2} + \sqrt{E_3^2 - m_{\pi^0}^2} \right)^2, \quad (19)$$

$$t_{\max} = (E_2 + E_3)^2 - \left( \sqrt{E_2^2 - m_{D^*}^2} - \sqrt{E_3^2 - m_{\pi^0}^2} \right)^2,$$

$$s_{\min} = (m_{D^*} + m_{D^0})^2, s_{\max} = (m_{T_{cc}} - m_{\pi^0})^2. \quad (20)$$

The energies in the  $s$  rest frame are

$$E_2 = \frac{s - m_{D^0}^2 + m_{D^*}^2}{2\sqrt{s}}, \quad (21)$$

$$E_3 = \frac{m_{T_{cc}}^2 - s - m_{\pi^0}^2}{2\sqrt{s}}.$$

The decay widths of  $T_{QQ} \rightarrow HH\pi$  can be estimated as

$$\Gamma(T_{cc}^+ \rightarrow D^0D^+\pi^0) \approx \left( \frac{\lambda_{T_{cc}}}{6} \right)^2 \left( \frac{g_\pi}{0.4} \right)^2 (50 \text{ keV}), \quad (22)$$

$$\Gamma(T_{cc}^+ \longrightarrow D^0 D^0 \pi^+) \approx \left(\frac{\lambda_{T_{cc}}}{6}\right)^2 \left(\frac{g_\pi}{0.4}\right)^2 (287 \text{ keV}), \quad (23)$$

$$\Gamma(T_{cc}^0 \longrightarrow D^0 D^0 \pi^0) \approx \left(\frac{\lambda_{T_{cc}}}{6}\right)^2 \left(\frac{g_\pi}{0.4}\right)^2 (223 \text{ keV}), \quad (24)$$

$$\Gamma(T_{cc}^{++} \longrightarrow D^0 D^+ \pi^+) \approx \left(\frac{\lambda_{T_{cc}}}{6}\right)^2 \left(\frac{g_\pi}{0.4}\right)^2 (103 \text{ keV}). \quad (25)$$

Currently, it is not easy to determine the value of the effective coupling  $\lambda_{T_{cc}}$ . If we choose  $\lambda_{T_{cc}} = 6$  and  $g_\pi = 0.4$ , the decay width of  $T_{cc}^+$  is estimated as  $\Gamma(T_{cc}^+) \sim 337 \text{ keV}$ , which is consistent to the first round data at LHCb experiment. While we employ the second round LHCb data, the effective coupling  $\lambda_{T_{cc}}$  is extracted as  $\lambda_{T_{cc}} \approx 2.26$ . In this case, the decay width of  $T_{cc}^+$  is then estimated as  $\Gamma(T_{cc}^+) \sim [47.8, 107.6] \text{ keV}$  with the choice of the effective coupling  $g_\pi = [0.4, 0.6]$ . Due to limited phase space,  $T_{cc}^{++}$  does not decay into  $D^+ D^+ \pi^0$ . For the strange partners of  $T_{cc}^+$ , we will discuss its decay properties in future works due to the lack of information of the effective couplings. For the beauty partners  $T_{bb}$ , one can use the  $T_{bb} \longrightarrow BB\gamma$  electromagnetic decay channels to detect due to the limited phase space. The beauty partners  $T_{bb}$  shall be more stable than the  $T_{cc}$  states in turn.

## 5. Conclusion

In this paper, we investigated the mass spectrum and the decay properties of the  $T_{cc}^+$  state first observed at LHCb experiment. Numerical results indicate that the  $T_{cc}^+$  state can be well-understood in the  $DD^*$  hadronic molecule model. As a by-product, the mass spectra of the doubly charm tetraquarks and doubly bottomed tetraquarks in heavy  $HH^*$  hadronic molecule framework are studied, some of which may be hunted in future LHCb experiments. Especially, it is worthwhile to notice the stable  $J^P = 1^+$  doubly bottomed tetraquarks  $T_{bb}^- (10598)$ ,  $T_{bb}^0 (10598)$ , and  $T_{bb}^{--} (10598)$  from the  $T_{bb} \longrightarrow BB\gamma$  decay channels.

## Data Availability

The data used to support the findings of this study are available from the authors upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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