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*Research Article*

# Schwinger-Type Pair Production in Non-SUSY AdS/CFT

**Udit Narayan Chowdhury** [1,2]

[1] *Saha Institute of Nuclear Physics, Block-AF, Sector-1, Salt Lake, Kolkata 700064, India*
[2] *Homi Bhabha National Institute, Training School Complex, Anushakti Nagar, Mumbai 400094, India*

Correspondence should be addressed to Udit Narayan Chowdhury; chowdhury.udit@gmail.com

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We study pair production of particles in the presence of an external electric field in a large $N$ non-supersymmetric Yang-Mills theory using the holographic duality. The dual geometry we consider is asymptotically AdS and is effectively parametrized by two parameters, $u_0$ and $-\sqrt{5/2} < \delta \leq 0$, both of which can be related to the effective mass of quark/antiquark for non-supersymmetric theories. We numerically calculate the interquark potential profile and the effective potential to study pair production and analytically find out the threshold electric field beyond which one gets catastrophic pair creation by studying rectangular Wilson loops using the holographic method. We also find out the critical electric field from DBI analysis of a probe brane. Our initial investigations reveal that the critical electric field necessary for spontaneous pair production increases or decreases w.r.t. its non-supersymmetric value depending on the parameter $\delta$. Ultimately, we find out the pair production rate of particles in the presence of an external electric field by evaluating circular Wilson loops using perturbative methods. From the later investigation, we note the resemblance with our earlier prediction. However, we also see that for and below another certain value of the parameter $\delta$, the pair production rate of particle/antiparticle pairs blows up as the external electric field is taken to zero. We thus infer that the vacuum of the non-SUSY gauge theory is unstable for a range of non-supersymmetric parameter $\delta$ and that the geometry/non-SUSY field theory under consideration has quite different characteristics than earlier reported.

## 1. Introduction

For the last few decades, the AdS/CFT correspondence [1–4] (relating $\mathcal{N}=4$ superconformal Yang-Mills in 4 space-time dimensions to quantum gravity in asymptotic $\mathrm{AdS}_5 \otimes S^5$ spaces), and some of its modifications are one of the exemplary ideas in theoretical physics. AdS/CFT chiefly comes from black hole thermodynamics [5], and type IIB string theory [6] is thus inherently supersymmetric in nature. This is a strong-weak duality meaning; strong coupling in the field theory side corresponds to weak coupling in the quantum gravity side and vice versa. However even after so many years, no trace of supersymmetry has been found by experiments, and again, conformal symmetry is not found quite much in nature. Thus, it is necessary to formulate a modification of AdS/CFT without supersymmetry and conformal symmetry yet respecting its string theory/supergravity origins. Such a solution is obtained in [7, 8]. This solution for D3 branes has two parameters $\delta$ and $u_0$, (i.e.., the field theory dual is a two-parameter deformation over usual $\mathcal{N}=4$ super Yang-Mills) and has certain features which makes it an attractive dual for large $N$ QCD studies via holography [9]. To the best of our knowledge, an explicit interpretation of these two parameters in terms of dual field theory measurables lacks till date. It had been previously reported that [7–9] those include running coupling+confinement in the infrared and absence of both SUSY and conformal symmetry (thus, the two-parameter deformation commented above violates both SUSY and conformal symmetry). Part of what makes AdS/CFT alluring is that when the field theory coupling is high, the corresponding coupling in the quantum gravity side is low, and thus, we are left with classical gravity which is easily computable.

The coupling constant in field theory is usually used as a perturbative parameter, and observables are expressed in a series w.r.t. this parameter; this is called perturbative field theory. However, there are quite some effects in quantum field theory which cannot be explained as such, i.e., nonperturbative effects. Amongst them, the Schwinger effect stands its ground. The vacuum of QED or any gauge theory interacting with charged matter is full of virtual particles and antiparticles (henceforth, $q\bar{q}$). In the presence of an external electric field/gauge field, these particles get the required energy and become real particles. There is no magic involved in this. In realistic situations, the energy of the real $q\bar{q}$ pairs is obtained from the electric field. Schwinger calculated [10] the pair production rate for this process in U(1) gauge theory and obtained

$$\Gamma = \frac{(eE)^3}{(2\pi)^3} e^{-\pi m^2/eE}. \tag{1}$$

The exponential suppression hints that pair productions can be modeled as a tunneling process. Assuming that the virtual $q\bar{q}$ pair has a separation $x$, the potential on a virtual quark in the presence on an external electric field is given as

$$\mathscr{V}_{\text{eff}} = -\frac{\alpha}{x} - eEx + 2m. \tag{2}$$

Imagine this to be the potential barrier though which the $q\bar{q}$ pairs tunnel out in the opposite direction and become real. For $E < m^2/e\alpha$, there exists two zero points of the potential, and $\mathscr{V}_{\text{eff}}$ is positive for intermediate values of $x$. That means there is a potential barrier, and quarks have to tunnel out through them justifying the exponential factor stated above. However for $E > m^2/e\alpha$, the potential becomes negative all along and stops putting up a potential barrier, indicating a catastrophic instability of vacuum where the $q\bar{q}$ are produced spontaneously. The value of electric field for which the potential stops putting up a tunneling barrier is called "critical/threshold electric field" $E_c$.

The Schwinger effect in holographic setting was first calculated in [11] (see [12] for an even earlier work) wherein the pair modified pair production rate was found to be

$$\Gamma \sim \exp\left[-\frac{\sqrt{\lambda}}{2}\left(\sqrt{\frac{E_c}{E}} - \sqrt{\frac{E}{E_c}}\right)^2\right], E_c = \frac{2\pi m^2}{\sqrt{\lambda}}. \tag{3}$$

This formula matches with the one above for low electric field (much lower than $E_c$). For field much higher than $E_c$, we do not see an exponential suppression anymore hinting at catastrophic decay. The chief idea of this work was to place the probe brane at a finite position unlike what is done usually (placing the probe brane at the conformal boundary of AdS) and then to calculate the circular Wilson loop. Another approach was pioneered in [13] which calculated the rectangular Wilson loop for virtual $q\bar{q}$ pair and relate it to interquark potential and then find the critical electric field from the same. Holographic Schwinger effect for confining gauge theories has also been studied in literature [14, 15], and the confinement manifests itself in the presence of another "threshold" electric field, below which pair production does not happen at all. In this work, we want to study the Schwinger effect for non-supersymmetric gauge theories via holographic methods using both of this methods, our chief interest being twofold. On the one hand, we like to see the theoretical effect absence of supersymmetry yields on the value of critical electric field at least for large $N$ Yang-Mills theories (and if such a relation can be reframed to be an indirect experimental evidence towards the presence or absence of supersymmetry in real-world nature). We also like to demonstrate the effect of confinement (as reported earlier for large $N$ non-supersymmetric YM theories via holography) towards holographic Schwinger particle decay and look into exotic results if any. For our purpose, the virtual $q\bar{q}$ pairs are imagined to be endpoint of a string in the boundary. We calculate the rectangular Wilson loop in space-time direction to find out the interquark potential. To account for an external electric field, we add an extra term. We analytically find out the critical electric field from the same. We also plot figures to illustrate the tunneling phenomenon. Next, we move on to finding out the critical electric field from analysis of the DBI action using the fact that the action should be real valued. Then, we move on to finding out the circular Wilson loop. It is impossible to do so without any simplification. We thus expand the expressions to first order of the non-SUSY deformation parameter $(u_0)^4$. Doing so, we explicitly find out the profile of circular Wilson Loop up to the first order of $(u_0)^4$ from which we find out the pair production rate.

This paper is organized as follows: in Section 2, we recap non-SUSY D3 branes and their decoupling limit from supergravity. We also show that the non-SUSY solution goes over to usual AdS when appropriate limits are taken. In Section 3, we show the derivation of pair production in theory with U(1) gauge field coupled to charged matter. Relevant expression for large $N$ gauge theory is also given. In Section 4, we carry on potential analysis of virtual $q\bar{q}$ pairs from which the critical electric field is derived both by analytical and numerical means. In Section 5, we use the DBI action and find out the critical electric field using the fact that the action should be real valued. In Section 6, we use perturbative analysis to find out the profile for circular Wilson loop when the string ends at a finite position ($u_b$). Using this, we find the critical electric field and pair production rate and make some comments about the later. We close this paper with conclusions in Section 7.

## 2. Non-SUSY Dp Branes and Their Decoupling Limit

In this section, we will take a brief recap of non-supersymmetric Dp brane solutions [16] and show how to recover the BPS Dp brane solutions from them. Then, we will state the decoupling limit of non-SUSY D3 branes by analogy with the BPS case and make sure that the decoupling goes over to the BPS brane decoupling limit when SUSY is restored [7]. In addition, we also show by taking suitable coordinate transformation that the decoupled throat geometry is actually identical with two-parameter solution obtained previously by Constable and Myres in which supersymmetry and conformal symmetry are both broken [8]. We start with the action for ten-dimensional type II supergravity which in addition to the string frame metric $g_{\mu\nu}$ has a dilation $\phi$ field and a $(8-p)$ RR from gauge field $F_{[8-p]}$.

$$S=\frac{1}{16\pi G_{10}}\int d^{10}x\sqrt{-\det g_{\mu\nu}}\left[R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{1}{2(8-p)!}F_{[8-p]}^2\right]. \tag{4}$$

We will be looking for solutions of the above using the ansatz.

$$\begin{aligned} ds_{\text{str}}^2 &= e^{2A(r)}\left(-dt^2+dx_1^2+\cdots+dx_p^2\right)\\ &\quad + e^{2B(r)}\left(dr^2+r^2d\Omega_{8-p}^2\right), \end{aligned} \tag{5}$$

$$F_{[8-p]}=\mathrm{QVol}\left(\Omega_{8-p}\right). \tag{6}$$

In the above, the metric has an $\mathrm{ISO(p,1)}\times\mathrm{SO(9-p)}$ isometry and represents a magnetically charged p brane in 10 dimensions with magnetic charge $Q$. It can be shown that the above solution conserves supersymmetry, i.e., saturates the BPS bound if [17]

$$(p+1)B(r)+(7-p)A(r)=0. \tag{7}$$

Solution of equations of (4) compatible with (5)–(7) leads to usual BPS p branes. We will be interested in supergravity solutions which defy the condition (7) and hence donot saturate the BPS bound , thus breaks spacetime supersymmetries. In the rest of the paper, we will be concerned with non-supersymmetric D3 brane solution and thus will consider the case where $p=3$. The non-supersymmetric D3 brane solution is given as

$$\begin{aligned} ds^2 &= \tilde F(\rho)^{-1/2}G(\rho)^{\delta/4}\left[-dt^2+dx_1^2+dx_2^2+dx_3^2\right]\\ &\quad+\tilde F(\rho)^{1/2}G(\rho)^{(1+\delta)/4}\left[\frac{d\rho^2}{G(\rho)}+\rho^2d\Omega_5^2\right],\\ e^{2\phi} &= g_s^2G(\rho)^{\delta},\\ F_{[5]} &= \frac{1}{\sqrt2}(1+\star)\,\mathrm{Q}\,\mathrm{Vol}(\Omega_5). \end{aligned} \tag{8}$$

In the above the functions, $\tilde F(\rho)$ and $G(\rho)$ are given as

$$\tilde F(\rho)=G(\rho)^{\alpha/2}\cosh^2\theta-G(\rho)^{-\beta/2}\sinh^2\theta, \tag{9}$$

$$G(\rho)=1+\frac{\rho_0^4}{\rho^4}. \tag{10}$$

It can be shown that the non-SUSY solution (8) violates condition (7) and thus breaks space-time supersymmetries. In the above, $e^{2\phi}$ is the effective string coupling constant and the solution is characterized by six parameters, i.e., $\alpha$, $\beta$, $\delta$, $\theta$, $\rho_0$, and Q, of which $\rho_0$ has the dimensions of length, $Q$ has dimensions of four volume, and others are dimensionless. One should further note from (9) that the solution given above has a naked singularity at $\rho=0$ and the physical region is given by $\rho>0$. In the context of string theory, one hopes that quantum fluctuations modify the behavior of the solution near the singularity point. As $e^{2\phi}$ is the effective string coupling, for the supergravity description to remain valid, one needs the parameter $\delta$ to be less or equal to zero so as to make the string coupling small. The parameters of the solutions are not all independent but satisfy some consistency relations like

$$\begin{aligned} \alpha &= \beta,\\ \mathrm{Q} &= 2\alpha\rho_0^4\sinh 2\theta,\\ \alpha^2+\delta^2 &= \frac{5}{2}. \end{aligned} \tag{11}$$

In arbitrary dimensions, the solutions and the constraints are a bit complicated and are given in [18]. Just like the BPS D3 brane solution, the non-SUSY solution too is asymptotically flat. One can recover the BPS solution from the non-SUSY solution given above by considering the limits $\rho_0\longrightarrow 0$ and $\theta\longrightarrow\infty$ keeping $\alpha/2\rho_0^4(\cosh^2\theta+\sinh^2\theta)\longrightarrow R^4=\text{fixed}$. Under this scaling, one has $G(\rho)\longrightarrow 1$ and $\tilde F(\rho)\longrightarrow 1+(R^4/\rho^4)$ and $\mathrm{Q}\longrightarrow 4R^4$ under which the standard BPS solution is regained.

The decoupling limit is a low energy limit in which interactions between the bulk theory and theory living on the brane vanish. To work out the decoupling limit and henceforth the throat geometry, one needs to make a change of variables in analogy with the BPS D3 brane.

$$\begin{aligned} \rho &= \alpha' u,\\ \rho_0 &= \alpha' u_0,\\ \alpha\cosh^2\theta &= \frac{\lambda}{\alpha'^2u_0^4},\\ \alpha' &\longrightarrow 0. \end{aligned} \tag{12}$$

In the above, $u$ and $u_0$ have the dimensions of energy and are kept fixed. From (11) and (12), it can be shown that $Q/\alpha'^2\gg 1$ implying that the curvature of space-time in string units must be very small for the supergravity description to be valid. A justification of the above decoupling limit is given explicitly in [7, 18]. Under the above said limit,

$$\begin{aligned} G(\rho) &\longrightarrow G(u)=1+\frac{u_0^4}{u^4}=\text{fixed},\\ \tilde F(\rho) &\longrightarrow \tilde F(u)=\frac{\lambda}{\alpha'^2}F(u). \end{aligned} \tag{13}$$

In the above, $F(u)=1/\alpha u_0^4(G(u)^{\alpha/2}-G(u)^{-\alpha/2})$, and the non-SUSY D3 brane throat geometry in the decoupling limit mentioned above becomes

$$ds^2 = \alpha' \sqrt{\lambda}\left[F(u)^{-1/2}G(u)^{\delta/4}\eta_{\mu\nu}dx^\mu dx^\nu + F(u)^{1/2}G(u)^{(1+\delta)/4}\left(\frac{du^2}{G(u)} + u^2 d\Omega_5^2\right)\right], \tag{14}$$

$$e^{2\phi} = g_s^2 G(u)^\delta. \tag{15}$$

In the above, the space-time coordinates have been rescaled as $(t, x^i) \longrightarrow \sqrt{\lambda}(t, x^i)$, where $\lambda$ is the 't Hooft coupling. In the limit $u_0 \longrightarrow 0$, one has $G(u) \longrightarrow 1$ and $F(u) = 1/\alpha u_0^4[(\alpha u_0^4/u^4) + \mathcal{O}(u_0^8/u^8)] \approx u^4$. In this limit, the non-SUSY throat geometry (14) goes over to the known $AdS_5 \times S^5$, and the effective string coupling becomes constant. To check the relation of solution (14) with that of the previously known one by Constable and Myres [8] which was conjectured to be dual to some non-supersymmetric field theory, one has to rewrite the solution in the Einstein frame.

$$ds_E^2 = \alpha' \sqrt{\lambda}\left[H(u)^{-1/2}G(u)^{\alpha/4}\eta_{\mu\nu}dx^\mu dx^\nu + H(u)^{1/2}G(u)^{(1-\alpha)/4}\left(\frac{du^2}{G(u)} + u^2 d\Omega_5^2\right)\right], \tag{16}$$

$$e^{2\phi} = g_s^2 G(u)^\delta. \tag{17}$$

In the above, the function $H(u)$ is defined by $H(u) = G(u)^{\alpha/2}F(u) = G(u)^\alpha - 1$. Now, one has to make a coordinate transformation like $u = r(1 + (\omega^4/r^4))^{-1/4}$, where $\omega^4 = u_0^4/4$. Under this transformation, $G(u) \longrightarrow (1 + 2(\omega^4/r^4))^2$ and $H(u) \longrightarrow (1 + 2(\omega^4/r^4))^{2\alpha} - 1$. From these relations and (16), one can exactly produce the two-parameter family of solutions as found in [8] in which both supersymmetry and conformal symmetry are broken. The solution in [8] also exhibits QCD-like behavior like running gauge coupling and confinement in the infrared. The geometry (16) exhibits a naked singularity at $u = 0$ and thus should be corrected by stringy corrections which should become dominant at low length scales. Moreover, the proper distance (spatial) from the exterior (say $u = u_b$) to the interior is finite (which says that stringy corrections are a must). In holography, the proper distance is identified with mass of the string hanging from the boundary to the interior [19]. To find the same, we have to choose a gauge of the form: $x_0 = t$, $u = s$, and all others = constant. With this gauge, the mass is given by

$$m = \frac{\sqrt{\lambda}}{2\pi}\int_0^{u_b} du \sqrt{\left(1 + \frac{u_0^4}{u^4}\right)^{(2\delta-3)/4}} = \text{finite and positive for all allowed values of } \delta. \tag{18}$$

The integral can indeed be done in closed form. However, the result is very complicated (hypergeometric functions involved), and it is very difficult to invert $u_b$ in terms of $m$. Thus, we express our results in this work with formula for mass ($m_0$) of $\mathcal{N} = 4$ SYM.

$$m_0 = \frac{\sqrt{\lambda}}{2\pi} u_b. \tag{19}$$

## 3. Pair Production in Presence of External Fields

In this section, we will revisit the concept of pair production in the presence of external electric fields. i.e., the "Schwinger effect." We will demonstrate the effect using Euclidean version of the electromagnetic action [20] and generalize to large $N$ gauge theories. The Euclidean version of U(1) gauge theory coupled to a massive complex scalar field is given by

$$S = \int d^4x \left[\frac{1}{4}F_{\mu\nu}^2 + \left|\left(\partial_\mu + ieA_\mu + iea_\mu^{ex}\right)\phi\right|^2 + m^2|\phi|^2\right]. \tag{20}$$

In the above, $A_\mu$ refers to the dynamical U(1) gauge field and $a_\mu^{ex}$ refers to the external value of (constant) electromagnetic field. The pair production rate, $\Gamma$, can be written as [21]

$$V\Gamma = -2\,\mathrm{Im}\ln\int \mathcal{D}A\mathcal{D}\phi\, e^{-S} = -2\,\mathrm{Im}\ln\int \mathcal{D}A e^{-S_{\mathrm{eff}}}, \tag{21}$$

where $S_{\mathrm{eff}} = 1/4\int d^4x F_{\mu\nu}^2 + \mathrm{tr}\ln[-(\partial_\mu + ieA_\mu + iea_\mu^{ex})^2 + m^2]$. For leading order calculations, one can ignore the coupling of the dynamical gauge field with the scalar field. Thus, the expression above reduces to

$$V\Gamma = -2\ \mathrm{Im}\ \mathrm{tr}\ln\left[-\left(\partial_\mu + iea_\mu^{ex}\right)^2 + m^2\right]. \tag{22}$$

Using the relation, $\mathrm{tr}\ln(A) = -\int_0^\infty (dT/T)\mathrm{tr}e^{-AT}$ and evaluating the trace in position basis, one can rewrite the above expression to

$$V\Gamma = \mathrm{Im}\int_0^\infty \frac{dT}{T}e^{-m^2T/2}\int d^4x \cdot \left\langle x\left|\exp\left[-T\left\{-\left(\partial_\mu + iea_\mu^{ex}\right)^2\right\}\right]\right|x\right\rangle. \tag{23}$$

Note that the integrand under $d^4x$ is synonymous to the path integral of a nonrelativistic particle under the influence of the Hamiltonian $H = 1/2[P_\mu + ea_\mu^{ex}]^2$. Using quantum mechanical path integral representation [22], one can write

$$\begin{aligned} V\Gamma &= \mathrm{Im}\int_0^\infty \frac{dT}{T} e^{-m^2T/2}\int_{x(0)=x(T)} \mathscr{D}x \exp \\ &\cdot \left[-\frac{1}{2}\int_0^T d\tau \dot{x}^2 + ie\oint a_\mu^{ex} dx_\mu\right] \\ &= \mathrm{Im}\int_0^\infty \frac{dT}{T}\int_{x(0)=x(1)} \mathscr{D}x \exp \\ &\cdot \left[-\frac{1}{2T}\int_0^1 d\tau \dot{x}^2 - \frac{m^2T}{2} + ie\oint a_\mu^{ex} dx_\mu\right], \end{aligned} \tag{24}$$

where in the last line, we have rescaled $\tau \longrightarrow 1/T\tau$. We assume $m^2\int_0^1 d\tau\dot{x}^2 \gg 1$ (a condition signifying heavy mass) and note that the integration over $T$ has the form of a modified Bessel function $K_0(x) = \int_0^\infty (dt/t) \exp(-t - (x^2/4t))$ with the asymptotic behavior, $K_0(x) \simeq \sqrt{\pi/2x}e^{-x}$, for large $x$. Thus, the above integral becomes

$$V\Gamma = \mathrm{Im}\int \mathscr{D}x \exp\left[-S_p\right]\frac{1}{m}\sqrt{\frac{2\pi}{T_0}}. \tag{25}$$

In the above, $T_0 = 1/m\sqrt{\int d\tau\dot{x}^2}$ and $S_p = m\sqrt{\int d\tau\dot{x}^2} - ie\oint a_\mu^{ex}dx_\mu$ and $a_1^{ex} = -iEx_0$ (signifying constant electric field of value $E$ in $x_1$ direction, iota comes in due to Euclidean signature). We like to evaluate the above integral by the method of steepest descent. The argument within the exponential is the action for a relativistic particle executing a periodic motion under influence of $a_\mu^{ex}$. The equation of motion for it is given by

$$\frac{1}{\sqrt{\int \dot{x}^2}} m\ddot{x}_\mu = eF_{\mu\nu}^{ex}\dot{x}_\nu. \tag{26}$$

Keeping in mind the periodic boundary conditions $x_\mu(0) = x_\mu(1)$ and $F_{01}^{ex} = E$, one has the following classical solution:

$$\begin{aligned} x_\mu^{cl} &= R(0, 0, \cos 2\pi\tau, \sin 2\pi\tau), \\ \mathrm{R} &= \frac{m}{eE}, \\ S_p^{cl} &= \frac{\pi m^2}{eE}. \end{aligned} \tag{27}$$

Using the above values, one has $1/m\sqrt{2\pi/T_0} = \sqrt{eE}/m$. Thus, decay rate can be approximated as

$$V\Gamma \approx \frac{\sqrt{eE}}{m} e^{-\pi m^2/eE} \tag{28}$$

Ideally one should go around calculating the one loop prefactor and complete the steepest descent process [20, 23], the calculation of which is indeed complicated. The modified prefactor is given by $(eE)^2/(2\pi)^3$. Thus, we see that the pair production rate goes to zero if the external electric field is switched off. In arbitrary coupling, one can no longer neglect the effect of the dynamical fields and one has to include contribution from the Wilson loops.

$$\begin{aligned} V\Gamma = -2\ \mathrm{Im}\int_0^\infty \frac{dT}{T} e^{-m^2T/2}\int \mathscr{D}x \exp\left[-\frac{1}{2T}\int_0^1 d\tau\dot{x}^2\right. \\ \left. + ie\oint a_\mu^{ex}dx_\mu\right]\left\langle \exp\left(ie\oint A_\mu dx_\mu\right)\right\rangle. \end{aligned} \tag{29}$$

The pair production rate gets modified to [15, 20]

$$\Gamma = \frac{(eE)^2}{(2\pi)^3}\sum_{n=1}^\infty \frac{(-1)^{n+1}}{n^2} \exp\left(-n\left(\frac{\pi m^2}{eE} - \frac{e^2}{4}\right)\right). \tag{30}$$

From the above, one can work out that the pair production rate is not exponentially suppressed once the value of electric field exceeds the so-called critical value $E_c = 4\pi m^2/e^3$, beyond which the vacuum becomes unstable.

To implement this argument for AdS/CFT like theories, one faces a number of problems. Firstly, the field theory in those circumstances is a conformal one, and one cannot get a mass term a priori. Moreover in the dual gauge theory, matter fields exist in the adjoint representation of SU(N) gauge group. To evade these issues, one uses the Higgs mechanism to break the symmetry group from $\mathrm{SU(N+1)} \longrightarrow \mathrm{SU(N)} \otimes \mathrm{U(1)}$. Because of this splitting, one has 5 massive W bosons transforming in fundamental representation of SU(N) and interacting with the background Yang-Mills theory. Now, the pair production rate in the presence on an external electric field is given by [15, 24]

$$\Gamma \sim -5\mathrm{N}\int \mathscr{D}x \exp\left(-m\int_0^1 d\tau\sqrt{\dot{x}^2} + i\int_0^1 d\tau a_\mu^{(E)}\dot{x}_\mu\right)\langle W[x]\rangle, \tag{31}$$

where $W[x]$ is the SU(N) Wilson loop and can be calculated by holographic means.

## 4. Pair Production in Non-Supersymmetric Theories via Holography

The ideal way to argue the Schwinger effect [11, 25] is to calculate the expectation value of circular Wilson loops and relate it to the decay rate. However, one can alternatively view the vacuum to be made of virtual $q\bar{q}$ pairs in the presence of an attractive potential and study the influence of an external electric field [13]. This basically amounts to calculating the interquark potential which one does by considering the rectangular Wilson loop. In doing so, one has to make some additional approximations. One considers that the time scale associated with the Wilson loop is much lesser than the length scale. Intuitively, one thinks that the quark antiquark pairs are separated in the far past and unite in the far future. In holography, the Wilson loop is given by following formula [26, 27]:

$$\langle W[\mathscr{C}]\rangle = \frac{1}{\mathrm{Vol}}\int_{\partial X=\mathscr{C}} \mathscr{D}X\mathscr{D}h_{ab}e^{-S[X,h]}. \tag{32}$$

$S[X,h]$ is the Wick rotated action of the fundamental string [6] with endpoints ending at contour $\mathscr{C}$ situated on the probe brane. In the classical limit ($\alpha' \longrightarrow 0$), the extremal value of the string action dominates, and thus, the Wilson loop is the extremal area of string world-sheet ending on the contour. To study the rectangular Wilson loop, we take the quark antiquark dipole to be aligned in the $x_3$ direction. The string action whose on-shell value we are interested with is the Nambu-Goto action $\mathscr{S}_{\mathrm{NG}} = 1/2\pi\alpha' \int dtds\sqrt{\det G_{ab}^{(\mathrm{in})}}$ with $G_{ab}^{(\mathrm{in})} \equiv G_{\mu\nu}(\partial x^\mu/\partial s^a)(\partial x^\nu/\partial s^b)$ which has two diffeomorphism symmetries. We exploit those to choose the following gauge:

$$\begin{aligned} x^0(s,t) &= t,\\ x^3(s,t) &= s,\\ u(s,t) &= u(s),\\ x^{1,2} &= 0,\\ \Theta^i(s,t) &= \text{constant}. \end{aligned} \tag{33}$$

For present purposes, $x_3 \equiv s$ is assumed to range between $[-L,L]$ and temporal direction $x_0 \equiv t$ is ranged between $[-\mathscr{T},\mathscr{T}]$ with the assumption that $\mathscr{T} \gg L$. $2L$ indicates the interquark separation on the probe brane with the boundary condition $u(\pm L) = u_b$, where $u_b$ indicates the position of the probe brane along the holographic direction (see Figure 1). Finally another word about the configuration, it is possible to consider the $q\bar{q}$ pairs at a velocity in the $x_2$ direction. However in the present case where the virtual particles in vacuum are modeled as $q\bar{q}$ dipoles, such a configuration seems hardly sensible. The induced metric as per the above gauge choice reads (14) and (33).

$$\begin{aligned} \frac{1}{\alpha'\sqrt{\lambda}}G_{ab}^{(\mathrm{in})}ds^a ds^b &= -F(u(s))^{-1/2}G(u(s))^{\delta/4}dt^2\\ &\quad + ds^2\left[F(u(s))^{-1/2}G(u(s))^{\delta/4} + F(u(s))^{1/2}\right.\\ &\quad\left.\cdot\, G(u(s))^{(1+\delta)/4}\frac{1}{G(u(s))}\left(\frac{du}{ds}\right)^2\right]. \end{aligned} \tag{34}$$

From the above, we have the determinant of the induced metric to be

$$\begin{aligned} -\det G_{ab}^{(\mathrm{in})} &= \left(\alpha'\sqrt{\lambda}\right)^2\left[G(u(s))^{(2\delta-3)/4}\left(\left(\frac{du}{ds}\right)^2\right.\right.\\ &\quad\left.\left. + G(u(s))^{3/4}F(u(s))^{-1}\right)\right]. \end{aligned} \tag{35}$$

It is not possible to carry on analysis without some simplification. We therefore assume that $(u_0/u)^4 \ll 1$, and with the mentioned, simplify the area, i.e., on-shell Nambu-Goto action, to

$$\begin{aligned} \mathscr{S}_{ng} &= \frac{1}{2\pi\alpha'}\int_{-\mathscr{T}/2}^{\mathscr{T}/2}dt\int_{-L}^{L}ds\sqrt{-\det G_{ab}^{(\mathrm{in})}}\\ &= \frac{\sqrt{\lambda}}{2\pi}\mathscr{T}\int_{-L}^{L}ds\sqrt{\left(\frac{du}{ds}\right)^2\left(1 + A\frac{u_0^4}{u^4}\right) + u^4\left(1 + B\frac{u_0^4}{u^4}\right)}, \end{aligned} \tag{36}$$

wherein

$$\begin{aligned} B &= \frac{\delta+1}{2},\\ A &= \frac{2\delta-3}{4},\\ A + \frac{5}{4} &= B. \end{aligned} \tag{37}$$

Crudely speaking, this can be seen as treating the non-SUSY theory as perturbation over the $\mathscr{N}=4$ supersymmetric Yang-Mills. Since the expression (36) does not explicitly depend on the parameter $s$, the corresponding "Hamiltonian," $Q$, is conserved.

$$\begin{aligned} Q &= -\frac{du}{ds}\frac{dL_{ng}}{d(du/ds)} + L_{ng}\\ &= \frac{u^4 + Bu_0^4}{\sqrt{(du/ds)^2\left(1 + A\left(u_0^4/u^4\right)\right) + u^4\left(1 + B\left(u_0^4/u^4\right)\right)}}. \end{aligned} \tag{38}$$

$A$ is indicated in [28]; the fundamental string is assumed to carry charges at two of its endpoints and is otherwise symmetric about its origin. From the above expression, we see that $du/ds$ has both positive and negative signs. Appealing to its symmetric nature, there exists a point, namely, turning point (with string parameter $s_t$), such that

$$\left(\frac{du}{ds}\right)(u_t) = 0. \tag{39}$$

Using the above expression in (38), the value of the conserved Hamiltonian is found in terms of the turning point

$$Q = \sqrt{u_t^4 + Bu_0^4}. \tag{40}$$

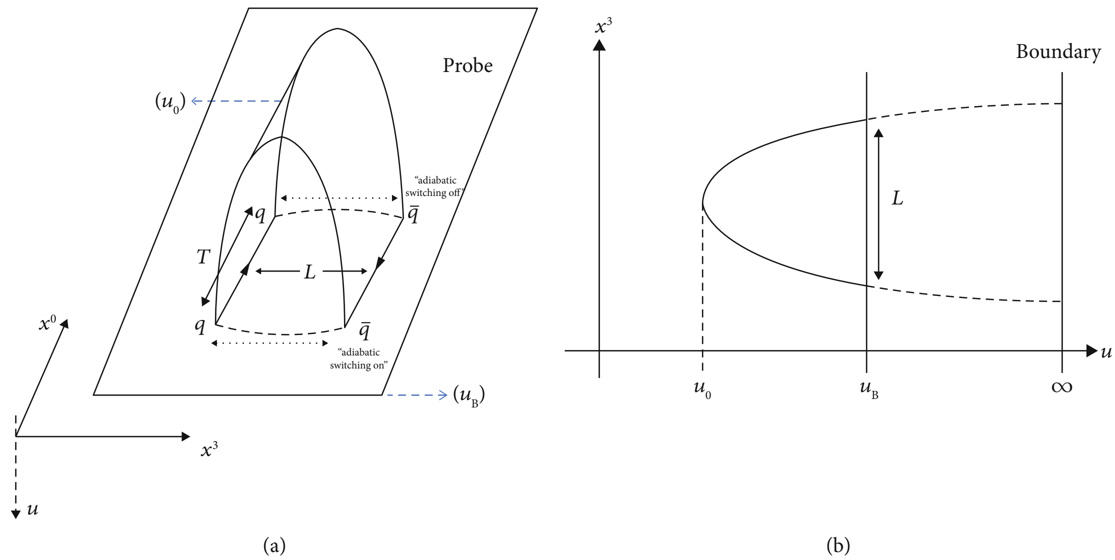


FIGURE 1: This figure illustrates the setup used. The probe brane is placed at a finite position ($u_b$) on the holographic direction as in (b). On the probe brane, the placement of the Wilson loop is shown in (a); the arrows indicate the contour of the loop (not the propagation of the string). For adiabatic interactions, one can neglect the effects of the dotted lines and the string profile becomes static.

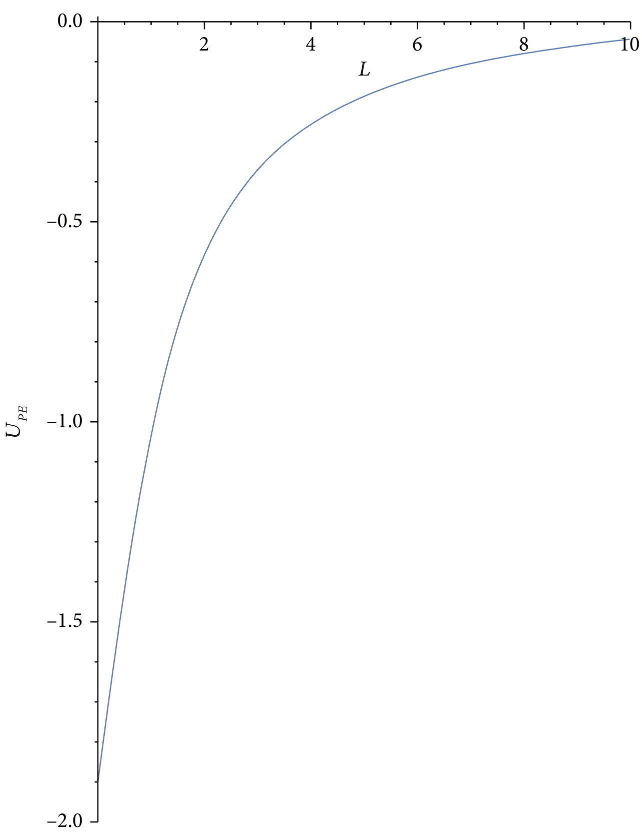


FIGURE 2: This is the graph of $U_{PE}$ vs. $L$. The rest mass has been duly subtracted. Note that for small values of $L$, the graph is approximately linear and for large $L$ coulombic behavior is mimicked. Deviation from usual coulombic behavior is evident. The values used are $\delta = -0.75$, $u_0/u_b = 0.01$, and $\lambda = 4\pi^2$.

Putting the above value in (38), we get

$$\frac{du}{ds} = u^2 \sqrt{\frac{\left(u^4 - u_t^4\right)\left(u^4 + Bu_0^4\right)}{\left(u_t^4 + Bu_0^4\right)\left(u^4 + Au_0^4\right)}}. \tag{41}$$

The length of the (virtual) dipole can be calculated to be (see Figure 1)

$$L = \int_{-L/2}^{L/2} dx_3 = \sqrt{u_t^4 + Bu_0^4} \int_{u_t}^{u_b} du \frac{\sqrt{u^4 + Au_0^4}}{u^2 \sqrt{\left(u^4 - u_t^4\right)\left(u^4 + Bu_0^4\right)}}. \tag{42}$$

From (41) and (36), we can find the on-shell value of interquark potential.

$$\mathcal{U}_{PE+SE} = \frac{\mathcal{S}_{ng}}{\mathcal{T}} = \frac{\sqrt{\lambda}}{2\pi} \int_{u_t}^{u_b} du \sqrt{\left(u^4 + Au_0^4\right)\left(u^4 + Bu_0^4\right)} \frac{1}{u^2 \sqrt{u^4 - u_t^4}}. \tag{43}$$

Notice from (42) that when $u_t \longrightarrow u_b$, the value of the interquark separation becomes small. But as said earlier, we are in an approximation where $(u_0/u)^4 \ll 1$. Thus, the calculations in this section are trustable for large interquark separation. Now, the expression in (43) (see Figure 2 for the plot) does not take the presence of an external electric field into account. Thus, we define an effective potential as

$$\mathcal{V}_{\text{eff}} = \mathcal{U}_{PE+SE} - E.L = (1 - r)E_c.L + G(u_t(L)). \tag{44}$$

In the above, we have assumed the presence on a critical electric field $E_c$, above which the effective interquark force becomes repulsive for all values of the interquark separation.

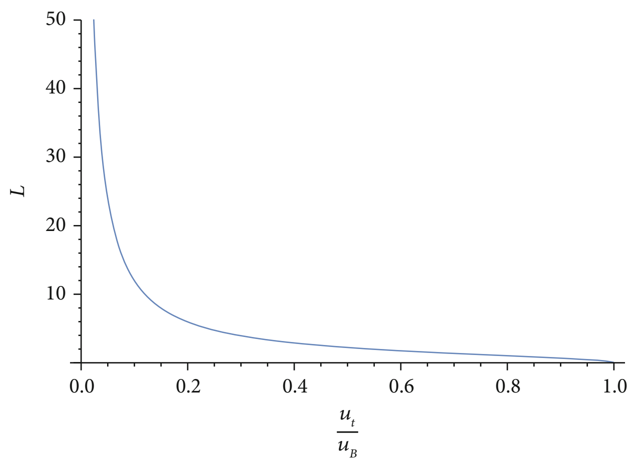


FIGURE 3: This is the graph of $L$ vs. $u_t$. Note that the function is an isomorphism. The values used are $\delta = -0.75$ and $u_0/u_b = 0.01$.

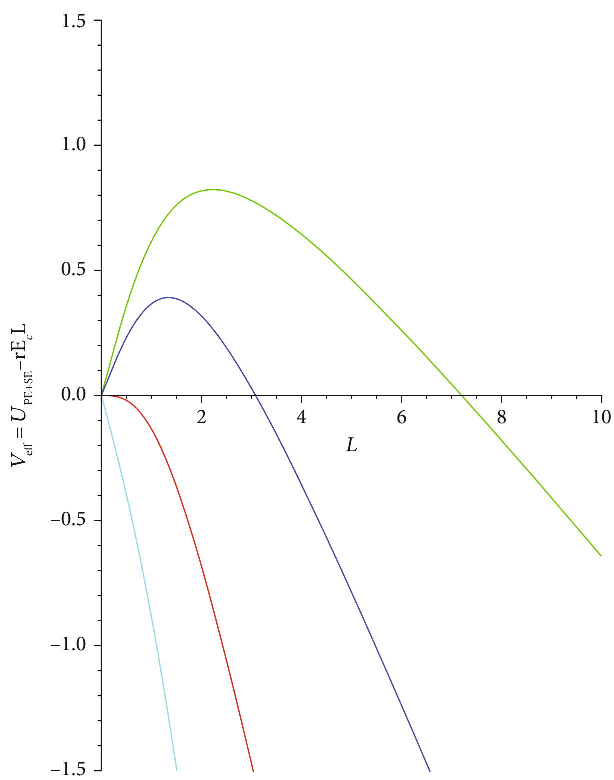


FIGURE 4: The plot indicates the effective potential (in the presence of external electric field) vs. the interquark separation. Imagine this to be then potential through which $q\bar{q}$ tunnels out. The green line indicates $r = 0.25$ and blue line for $r = 0.75$. The parameter $r$ is the ratio of the applied field to its threshold value. The red line which exhibits the threshold behavior, i.e., no potential barrier, stands for $r = 1.0$ and cyan for $r = 1.75$ shows catastrophic decay of vacuum. Note that at the threshold/critical value, the slope of the potential vanishes at $L = 0$ and is negative for nonzero value of $L$ which is precisely the conditions we have used to analytically find out the value of $E_c$. Also note that none of the above plots exhibit confining behavior as earlier reported in literature. The values used are $\delta = -0.75$, $u_0/u_b = 0.01$, and $\lambda = 4\pi^2$.

The quantity $G(u_t)$ is

$$G(u_t) = \mathscr{U}_{PE+SE} - E_c L = \int_{u_t}^{u_b} du \frac{\sqrt{u^4 + Au_0^4}}{u^2\sqrt{u^4 - u_t^4}} \cdot \left[\frac{\sqrt{\lambda}}{2\pi}\sqrt{u^4 + Bu_0^4} - E_c \frac{\sqrt{u_t^4 + Bu_0^4}}{\sqrt{u^4 + Bu_0^4}}\right]. \tag{45}$$

The parameter $r$ is the ratio of applied electric field to its critical value. The slope of the effective potential is given as

$$\frac{d\mathscr{V}_{\text{eff}}}{dL} = (1 - r)E_c + \frac{du_t}{dL}\frac{dG(u_t)}{du_t}. \tag{46}$$

We now proceed to find the value of the critical electric field. Note that at $u_t = u_b$, the interquark separation (42) and the interquark potential (43) vanish (see Figure 3). At the critical value of the electric field $r = 1$, the $1^{st}$ term of (46) ceases to contribute, and the behavior of the interquark force will be completely governed by the second term of (46). Criticality demands that the potential ceases to put up a tunneling barrier for all values of interquark separation (see red line in Figure 4). Given that $G(u_t(L))$ vanishes at $L = 0$, we need to show that $G(u_t(L))$ is a monotonically decreasing function with respect to $L$ whose slope vanishes at $L = 0$ (this is because critical electric field is the least one for which pair production happens spontaneously). From (42), we have

$$\frac{dL}{du_t} = -\frac{\sqrt{u_t^4 + Au_0^4}}{u_t^2\sqrt{\left(u_t^4 + \epsilon\right)^4 - u_t^4}} + 2\int_{u_t+\epsilon}^{u_b} du \frac{u_t^3}{u^2} \frac{\sqrt{\left(u^4 + Au_0^4\right)\left(u^4 + Bu_0^4\right)}}{\sqrt{\left(u^4 - u_t^4\right)^3\left(u_t^4 + Bu_0^4\right)}}. \tag{47}$$

Similarly, we have from (45)

$$\begin{aligned}\frac{dG}{du_t}(u_t) &= -\frac{\sqrt{u_t^4 + Au_0^4}}{u_t^2\sqrt{(u_t + \epsilon)^4 - u_t^4}}\left[\frac{\sqrt{\lambda}}{2\pi}\sqrt{u_t^4 + Bu_0^4} - E_c\right] \\ &\quad + 2u_t^3\int_{u_t+\epsilon}^{u_b} du \frac{\sqrt{\left(u^4 + Au_0^4\right)\left(u^4 + Bu_0^4\right)}}{u^2\left(\sqrt{u^4 - u_t^4}\right)^3} \cdot \left[\frac{\sqrt{\lambda}}{2\pi} - \frac{E_c}{\sqrt{u_t^4 + Bu_0^4}}\right] \\ &= \left[\frac{\sqrt{\lambda}}{2\pi}\sqrt{u_t^4 + Bu_0^4} - E_c\right]\frac{dL}{du_t}.\end{aligned} \tag{48}$$

Thus, we get

$$\frac{d\mathscr{V}_{\text{eff}}}{dL} = (1 - r)E_c + \left[\frac{\sqrt{\lambda}}{2\pi}\sqrt{u_t^4 + Bu_0^4} - E_c\right]. \tag{49}$$

At threshold condition, the slope of the potential should be zero at when interquark separation vanishes, i.e., $u_t = u_b$. Implementing the same in (49), we get

$$E_c = \frac{\sqrt{\lambda}}{2\pi} u_b^2 \sqrt{1 + \frac{\delta + 1}{2} \frac{u_0^4}{u_b^4}} = \frac{2\pi}{\sqrt{\lambda}} m_0^2 \sqrt{1 + \frac{\lambda^2}{32\pi^4} (\delta + 1) \frac{u_0^4}{m_0^4}}. \tag{50}$$

We thus have

$$\frac{d\mathcal{V}_{\text{eff}}}{dL} = (1 - r) E_c + \left[ \frac{\sqrt{\lambda}}{2\pi} \sqrt{u_t^4 + B u_0^4} - \frac{\sqrt{\lambda}}{2\pi} \sqrt{u_b^4 + B u_0^4} \right]. \tag{51}$$

From Figure 3, we see that $L$ increases as $u_t$ decreases using which we can say from (51) that $d\mathcal{V}_{\text{eff}}/dL$ is a monotonically decreasing function of $L$ at $r = 1$. It can be easily understood that from $r > 1$, the effective potential is totally repulsive. Thus, we establish the existence of a critical electric field with value given by (50). We see that as $\delta$ switches over $-1$, the critical electric field increases and decreases, respectively, compared to the supersymmetric value. Not even that, just at $\delta = -1$, the critical field has the same value as that of the supersymmetric theory. Will this kind of behavior remain when one considers higher orders? How much of the calculation in this section should be trusted for small values of interquark separation? The answer to this question will be found in the next section.

It so happens that analytical solutions to (42), (43), and (44) cannot be found out in a closed from via Mathematica. Thus, we resort to numerical methods. Some plots to illustrate the situation are given.

## 5. DBI Analysis of Critical Electric Field

In this section, we look to find out the critical electric field from analysis of the DBI action of the probe brane in the presence of an external electric field. In due course, we will also answer the question raised in Section 4.

As earlier, we imagine the probe brane situated at $u = u_b$ (see Figure 1) in the holographic dual with an electric field switched on at the brane position. The DBI action is given as

$$\mathcal{S}_{\text{DBI}} = \frac{1}{(2\pi)^3 g_s \alpha'} \int_{u=u_b} d^4x \sqrt{-\det \left( P[g]_{\mu\nu} + B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} \right)}. \tag{52}$$

In the above, $P[g]_{\mu\nu}$ is the pullback of the curved metric on the probe brane, $B_{\mu\nu}$ is the NS 2-form which is zero in the present case, and $F_{\mu\nu}$ is the Faraday tensor which we set to the value $F_{03} = E$, to indicate the presence of an external electric field. Evaluating the above from (14), we have

$$P[g]_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} = \begin{pmatrix} -\alpha' \sqrt{\lambda} F(u_b)^{-1/2} G(u_b)^{\delta/4} & 0 & 0 & 2\pi\alpha' E \\ 0 & \alpha' \sqrt{\lambda} F(u_b)^{-1/2} G(u_b)^{\delta/4} & 0 & 0 \\ 0 & 0 & \alpha' \sqrt{\lambda} F(u_b)^{-1/2} G(u_b)^{\delta/4} & 0 \\ -2\pi\alpha' E & 0 & 0 & \alpha' \sqrt{\lambda} F(u_b)^{-1/2} G(u_b)^{\delta/4} \end{pmatrix}. \tag{53}$$

Thus, the DBI action becomes

$$\mathcal{S}_{\text{DBI}} = \frac{\alpha' \lambda}{(2\pi)^3 g_s} \int_{u=u_b} d^4x F(u_b)^{-1/2} G(u_b)^{\delta/4} \cdot \sqrt{F(u_b)^{-1} G(u_b)^{\delta/4} - \left( \frac{2\pi E}{\sqrt{\lambda}} \right)^2}. \tag{54}$$

Thus, we see that (54) is not real for all values of the external electric field and there is an upper limit of the same. This limiting value is nothing but the critical electric field is

$$E_c = \frac{\sqrt{\lambda}}{2\pi} F(u_b)^{-1/2} G(u_b)^{\delta/4}. \tag{55}$$

The functions $F(u)$ and $G(u)$ have been defined before in (9). One can check that up to $\mathcal{O}(u_0/u_b)^4$, (55) reduces to (50). However in finding (55), we have refrained from using perturbations of any sort, and thus, (55) is the exact value. Let us check the behavior of it with respect to the parameter $\delta$.

We see from Figure 5 that the critical electric field is the same as that of its supersymmetric cousin somewhere around $\delta = -0.85$, which matches more or less with our perturbative analysis in the last section. For values of $\delta > 0.85$, the critical electric field is greater than the supersymmetric counterpart, and for $\delta < 0.85$, the critical field is lesser. Thus, the question raised in the last section is answered in the affirmative. The calculation in Section 4 will not be affected drastically for small values of interquark separation. (This is because it is the small separation behavior that decides the critical value.)

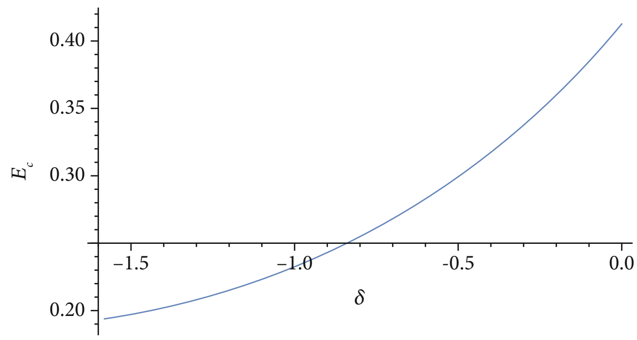


FIGURE 5: This is the plot of critical electric field $E_c$ vs. the non-supersymmetric parameter $\delta$. In this figure, we have set $\sqrt{\lambda}/2\pi = 1$ and $u_0/u_b = 1/0.5$. According to this values, the supersymmetric critical field would have been 0.25.

## 6. Holographic Pair Production Rate for Non-Supersymmetric Theories

In this section, we calculate the pair production rate by using the method of circular Wilson loops. As indicated earlier in (31) and (32), to find the pair production rate, we need to find the on-shell value of the Nambu-Goto action with string endpoints ending on a circular contour at the probe brane ($u = u_b$). For pure AdS, the calculation of the same has been presented in [11, 15, 29]. However, it is not possible to find exact solutions to the relevant equation of motions for the present case (14). Thus, we will resort to perturbative treatments like that of [30] to calculate the circular Wilson loop, and hence, the decay rate to first order of the non-SUSY deformation parameter ($u_0^4$). Since the metric (14) enjoys circular symmetry, we start by making an ansatz.

$$\begin{aligned} x^0 &= r(\sigma)\cos\tau, \\ x^3 &= r(\sigma)\sin\tau, \\ u &= u(\sigma). \end{aligned} \tag{56}$$

In the above, all other coordinates have been put to be constants as circular symmetry would imply. The parameter $\tau$ ranges from $(0, 2\pi)$ while the parameter $\sigma$ is still arbitrary. There exists a diffeomorphism invariance of the Nambu-Goto action with which we can set $u = u(\sigma)$ to a function of our choosing. Putting the ansatz (56) in (14), we have the induced metric to be

$$\begin{aligned} ds^2 = \alpha'\sqrt{\lambda}\Bigg[&\left(F(u)^{-1/2}G(u)^{\delta/4}\left(\frac{dr}{d\sigma}\right)^2 \right. \\ &\left. + F(u)^{1/2}G(u)^{(\delta-3)/4}\left(\frac{du}{d\sigma}\right)^2\right)d\sigma^2 \\ &+ r^2F(u)^{-1/2}G(u)^{\delta/4}d\tau^2\Bigg]. \end{aligned} \tag{57}$$

From the above, one can get the Nambu-Goto action to be of the form

$$\begin{aligned} \mathcal{S}_{ng} = &\frac{\left(\alpha'\sqrt{\lambda}\right)}{2\pi\alpha'}\int_0^{2\pi}d\tau\int_0^{\sigma_b}d\sigma \\ &\cdot\sqrt{r^2G(u)^{\delta/2}\left(F(u)^{-1}\left(r'\right)^2 + G(u)^{-3/4}\left(u'\right)^2\right)}. \end{aligned} \tag{58}$$

For purposes of calculation, we expand the function $F(u)$ and $G(u)$ in their leading order to the non-supersymmetric deformation parameter, and we have

$$\begin{aligned} \mathcal{S}_{ng} &= \sqrt{\lambda}\int_0^{\sigma_b}d\sigma\sqrt{r^2\left(1+\frac{\delta}{2}\frac{u_0^4}{u^4}\right)\left(\left(r'\right)^2u^4\left(1+\frac{1}{2}\frac{u_0^4}{u^4}\right)+\left(u'\right)^2\left(1-\frac{3}{4}\frac{u_0^4}{u^4}\right)\right)+\mathcal{O}\left(\frac{u_o^8}{u^8}\right)} \\ &\approx \sqrt{\lambda}\int_0^{\sigma_b}d\sigma\sqrt{r^2\left(\left(r'\right)^2u^4\left(1+A\frac{u_0^4}{u^4}\right)+\left(u'\right)^2\left(1+B\frac{u_0^4}{u^4}\right)\right)}, \end{aligned} \tag{59}$$

wherein

$$\begin{aligned} B &= \frac{\delta+1}{2}, \\ A &= \frac{2\delta-3}{4}, \\ A + \frac{5}{4} &= B. \end{aligned} \tag{60}$$

The above binomial expansion and all the others that follow are simply treating the non-SUSY theory as a perturbation over the regular $\mathcal{N} = 4$ SYM. In this paper, we limit ourselves to the first-order perturbations. Recall that we still had one diffeomorphism invariance left as mentioned before, with the help of which we set $du(\sigma)/d\sigma = 1$. Thus, (59) is simplified to

$$\mathcal{S}_{ng} = \sqrt{\lambda}\int_{u_t}^{u_b}du\sqrt{r^2\left(\frac{dr}{du}\right)^2\left(u^4+Au_0^4\right)+\frac{r^2}{u^4}\left(u^4+Bu_0^4\right)}. \tag{61}$$

We would like to find out the function $r = r(u)$ which extremizes (61). Extremizing the same, one has to encounter the equation

$$\begin{aligned} &u^4\left(u^4+Au_0^4\right)\left(\frac{d\rho}{du}\right)^2\left(2\left(u^4+Bu_0^4\right)-u^7\frac{d\rho}{du}\right)\\ &\quad-4\rho\left(u^3\frac{d\rho}{du}\left(ABu_0^8+3Bu^4u_0^4+2u^8\right)-\left(u^4+Bu_0^4\right)^2\right)\\ &\quad-2u^4\left(u^4+Au_0^4\right)\left(u^4+Bu_0^4\right)\rho\frac{d^2\rho}{du^2}=0, \end{aligned} \tag{62}$$

where $\rho = r^2$. The above equation is very hard to solve in closed form. Thus, we adopt perturbative techniques like that of [24]. To do so, we decompose the solution to (62) as $\rho = \rho_0 + u_0^4\rho_1$ in which $\rho_0 = -1/u^2$, and $\rho_1$ indicates the perturbation. From (62), the equation for $\rho_1$ to the leading order of $u_0^4$ is

$$2u^2\left(6(B-A)+2u^7\frac{d\rho_1}{du}+u^8\frac{d^2\rho_1}{du^2}\right)=0. \tag{63}$$

One can check that the above is solved by

$$\rho_1(u)=\frac{A-B}{5u^6}+K'. \tag{64}$$

Thus, the full solution is

$$\begin{aligned} r^2(u)=\rho(u)&=u_0^4K'-\frac{1}{u^2}+\frac{A-B}{5u^6}\frac{u_0^4}{u^6}\\ &\equiv K-\frac{1}{u^2}+\frac{A-B}{5}\frac{u_0^4}{u^6}\\ &=K-\frac{1}{u^2}-\frac{1}{4}\frac{u_0^4}{u^6}, \end{aligned} \tag{65}$$

where a redefinition of constant has been made. Now, it is time to relate the constant $K$ to physical parameters. At $u = u_b$, the value of $r$ is the radius of the Wilson loop $R$. Thus, we have

$$K=R^2+\frac{1}{u_b^2}-\frac{A-B}{5}\frac{u_0^4}{u_b^6}. \tag{66}$$

From the above, we can also find the value of the turning point $u_t$, since at the turning point, the radius $r(u_t)=0$. Thus, the equation which determines the turning point is

$$K=\frac{1}{u_t^2}\left(1-\frac{A-B}{5}\frac{u_0^4}{u_t^4}\right). \tag{67}$$

Now, we proceed to calculate the on-shell value of the Nambu-Goto action (61) on the solution (65). We have

$$\begin{aligned} \mathcal{S}_{ng}&=\sqrt{\lambda}\int_{u_t}^{u_b}du\sqrt{\frac{1}{4}\left(\frac{d(r^2)}{du}\right)^2\left(u^4+Au_0^4\right)+\left(r^2\right)\left(1+B\frac{u_0^4}{u^4}\right)}\\ &=\sqrt{\lambda}\int_{u_t}^{u_b}du\sqrt{\frac{1}{4}\cdot\frac{4}{u^6}\left(1-\frac{3}{5}(A-B)\frac{u_0^4}{u^4}\right)^2\left(u^4+Au_0^4\right)+\left(1+B\frac{u_0^4}{u^4}\right)\left(K-\frac{1}{u^2}+\frac{A-B}{5}\frac{u_0^4}{u^6}\right)}\\ &=\sqrt{\lambda}\int_{u_t}^{u_b}du\sqrt{K+KB\frac{u_0^4}{u^4}+\mathcal{O}\left(u_0^8\right)}. \end{aligned} \tag{68}$$

We neglect the $\mathcal{O}(u_0^8)$ term in the above. The integral of the remaining part cannot be done in closed form by using Mathematica. So we resort to perturbative methods again and write

$$\begin{aligned} \mathcal{S}_{ng}&=\sqrt{\lambda}\int_{u_t}^{u_b}du\sqrt{K}\left[1+\frac{B}{2}\frac{u_0^4}{u^4}+\mathcal{O}\left(u_0^8\right)\right]\\ &\approx\sqrt{\lambda}\left(\left[\sqrt{K}u\right]_{u_t}^{u_b}-\left[\frac{2B\sqrt{K}u_0^4}{u^3}\right]_{u_t}^{u_b}\right). \end{aligned} \tag{69}$$

So far so good, however, the reader may agree that working with (69) is still daunting given that we now have to substitute the highly nonlinear relations (66) and (67) into it. Happily, there is a way out of this mess. Recall that our theme has been to work in the leading order of $u_0^4$ and the last two terms of (69) come with a $u_0^4$ of their own. Thus to leading order, we may substitute the usual AdS relations (relating K to $u_t$ and $u_b$) in the last term of (69), but use the non-SUSY relations (66) and (67) in the first term of the same. Doing so, we have

$$\begin{aligned} \mathcal{S}_{ng}=\sqrt{\lambda}\Bigg(&\sqrt{\left(R^2u_b^2+1\right)-\frac{A-B}{5}\frac{u_0^4}{u_b^4}}\\ &-\sqrt{1-\frac{A-B}{5}\frac{u_0^4}{u_t^4}}+u_0^4\left[\frac{2B}{u_t^4}-\frac{2B\sqrt{R^2u_b^2+1}}{u_b^4}\right]\Bigg) \end{aligned}$$

$$\approx \sqrt{\lambda}\left(\sqrt{\left(R^2u_b^2+1\right)-\frac{A-B}{5}\frac{u_0^4}{u_b^4}} - \sqrt{1-\frac{A-B}{5}\frac{u_0^4}{u_b^4}\left(R^2u_b^2+1\right)^2} + \frac{u_0^4}{u_b^4}\left[2B\left(R^2u_b^2+1\right)^2-2B\sqrt{R^2u_b^2+1}\right]\right)$$
$$\approx \sqrt{\lambda}\left(\sqrt{\left(R^2u_b^2+1\right)}-1+\frac{u_0^4}{u_b^4}\left[\frac{A+19B}{10}\left(R^2u_b^2+1\right)^2 - \frac{A-B}{10\sqrt{R^2u_b^2+1}}-2B\sqrt{R^2u_b^2+1}\right]\right). \quad (70)$$

In the second line of the above, we have used the usual AdS relations for the term $1/u_t^4$ to leading order as it is accompanied by a $u_0^4$. Again in the third line, we have used a binomial expansion and retained terms of leading order in $u_0^4/u_b^4$. Now, in the presence of an electric field, the effective action of the string has an extra piece, $S_B = T_0\int d\sigma d\tau B_{\mu\nu}\partial_\sigma x^\mu\partial_\tau x^\nu$. Specializing to constant electric field, the contribution of $S_B$ is a pure boundary term with on-shell value $\pi R^2E$, where $R$ is the radius of the Wilson loop, $E = B_{01}$, and all other components of the $B_{\mu\nu}$ are set to zero. The effective action is given by

$$S_{eff} = S_{ng}+S_B = \sqrt{\lambda}\left(\sqrt{x}-1+\frac{u_0^4}{u_b^4}\left[\frac{A+19B}{10}x^2 - \frac{A-B}{10\sqrt{x}}-2B\sqrt{x}\right]-\mathscr{E}x+\mathscr{E}\right). \quad (71)$$

In the above, $x = R^2u_b^2+1$ and $E = (\sqrt{\lambda}u_b^2/\pi)\mathscr{E}$. Thus, the radius $R(x)$ is a free parameter in expression (71). Following [11, 25], the radius should be set to an extremum of (71). Instead of extremizing w.r.t. $R$, we extremize the action (71) w.r.t. the parameter $y = \sqrt{x}$. Doing so, we find

$$0 = \frac{dS_{\text{eff}}}{dy} = \sqrt{\lambda}\left(1-2\mathscr{E}y+\frac{u_0^4}{u_b^4}\cdot\left[\frac{2(A+19B)}{5}y^3+\frac{A-B}{10y^2}-2B\right]\right). \quad (72)$$

The radius $R$ should be set to be the solution of (72); recall $y = \sqrt{R^2u_b^2+1}$. Thus, the value of $y$ in the above equation is constrained and should always be greater than 1. This is because the radius of the Wilson loop should be a real number. A subtle point is that the range of parameter $y$ should be restricted to half of the real line, because the radius $R$ is nonnegative. The critical electric field $\mathscr{E}_c$ is the one for which the radius $R=0$, i.e., $y=1$. Setting so in the above, we see

$$1-2\mathscr{E}_c+\frac{u_0^4}{u_b^4}\left[\frac{2(A+19B)}{5}+\frac{A-B}{10}-2B\right]=0. \quad (73)$$

Thus,

$$E_c = \frac{\sqrt{\lambda}}{2\pi}u_b^2\left[1+\frac{1}{8}\frac{u_0^4}{u_b^4}(23+24\delta)\right] = \frac{2\pi}{\sqrt{\lambda}}m_0^2\left[1+\frac{\lambda^2}{128\pi^4}(23+24\delta)\frac{u_0^4}{m_0^4}\right]. \quad (74)$$

We see that like (50) and (55), the value of the critical electric field is greater than the supersymmetric value for the value $\delta = -23/24$ and less than the supersymmetric cousin otherwise. Our perturbative analysis has even shown that the value of parameter $\delta$ for which this phase transition occurs is slightly bigger than -1 as can be seen from the nonperturbative DBI analysis. Now to find the expression of the pair production rate, we need to solve (72) for $y$. As can be seen, that is not analytically possible. We thus resort to perturbative treatments again and write

$$y = y_0+\frac{u_0^4}{u_b^4}y_1. \quad (75)$$

$y_0$ is the usual AdS solution, i.e., $u_0=0$ in (72). The value of $y_0$ is $1/2\mathscr{E}$. We put the above relation in the equation in (72) to get up to leading order in $u_0^4/u_b^4$.

$$y_1 = \frac{1}{2\mathscr{E}}\left[\frac{A+19B}{20}\frac{1}{\mathscr{E}^3}+\frac{2(A-B)}{5}\mathscr{E}^2-2B\right]. \quad (76)$$

Now, we put (75) and (76) in (71), i.e., find out the on-shell action. Retaining terms in leading order of $u_0^4/u_b^4$ leads us to

$$S_{\text{eff}}^{\text{on-shell}} = \frac{\sqrt{\lambda}}{2}\left(\frac{1}{2\mathscr{E}}-2+2\mathscr{E}+\frac{u_0^4}{u_b^4}\cdot\left[\frac{A+19B}{80}\frac{1}{\mathscr{E}^4}+\frac{B-A}{10}\mathscr{E}-\frac{2B}{\mathscr{E}}\right]\right) = \frac{\sqrt{\lambda}}{2}\left(\frac{1}{2\mathscr{E}}-2+2\mathscr{E}+\frac{u_0^4}{u_b^4}\cdot\left[\frac{40\delta+39}{320}\frac{1}{\mathscr{E}^4}+\frac{1}{40}\mathscr{E}-\frac{\delta+1}{\mathscr{E}}\right]\right). \quad (77)$$

The pair production rate of quark antiquark pairs per unit volume per unit time is given by the formula $\Gamma \sim e^{-S_{\text{eff}}^{\text{on-shell}}}$. Note that we are using reduced parameter $\mathscr{E} = (\pi/\sqrt{\lambda}u_b^2)E$, in terms of which the pure AdS pair production rate (per unit volume per unit time) is given by [11, 25, 29]

$$\Gamma_{\text{SUSY}} \sim \exp\left[-\frac{\sqrt{\lambda}}{2}\left(\sqrt{\frac{1/2}{\mathscr{E}}}-\sqrt{\frac{\mathscr{E}}{1/2}}\right)^2\right]. \quad (78)$$

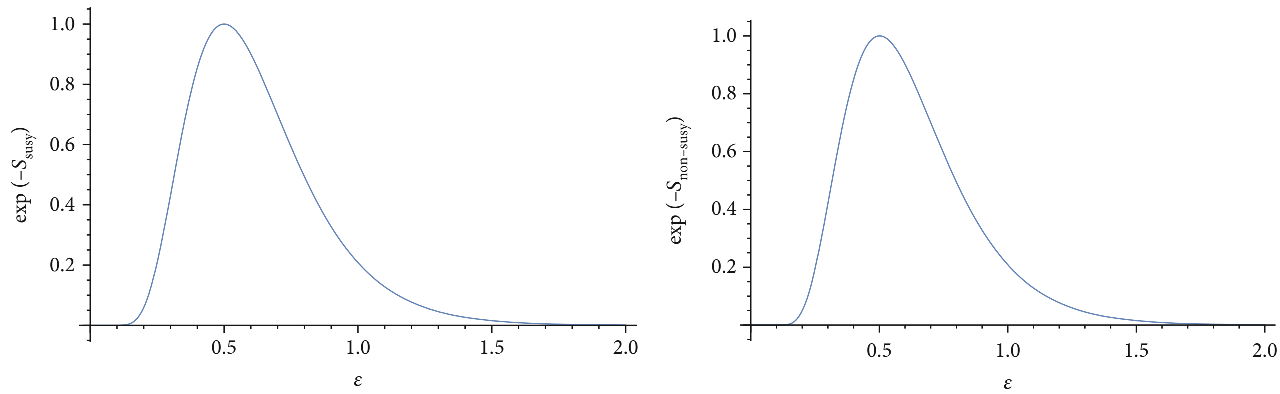


Figure 6: The figure on the left illustrates relation between the pair production rate and applied electric field for pure $\mathcal{N} = 4$ SYM (78). In our units, the critical electric field is at $\mathcal{E}_c = 1/2$. The right figure is for non-supersymmetric case (77) for the value $\delta = -0.75$. Almost no change in the profile is found.

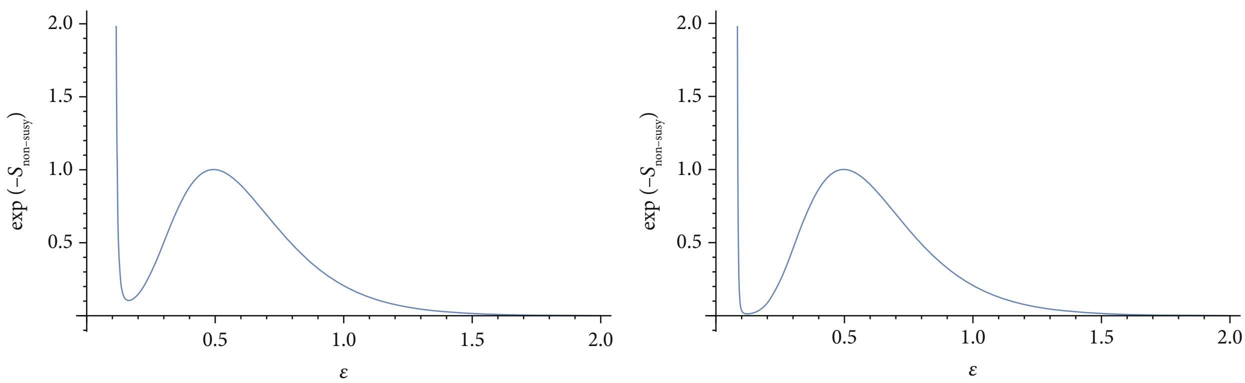


Figure 7: We plot the pair production rate for non-supersymmetric Yang-Mills for the value $\delta = -\sqrt{2}$. On the right, we have $(u_0/u_b)^4 = (1/5)^4$ and on the left $(u_0/u_b)^4 = (1/3.75)^4$.

For the pure AdS/Supersymmetric scenario, the critical electric field is $\mathcal{E}_c = 1/2$, i.e., $E_c = \sqrt{\lambda} u_b^2 / 2\pi$. Now unlike the supersymmetric case, the pair production rate cannot be brought in closed form. We will have to resort to numerical calculations. We present the plots of pair production rate. Computation of the fluctuation prefactor (i.e., the $(eE)^2/(2\pi)^3$ term in (30)) is somewhat an open question in holography, which is the reason we have plotted $e^{-S_{\text{on-shell}}}$ instead of $\Gamma$.

Physical interpretation of the plots: as commented in the caption, no stark contrast is found between the SUSY and non-SUSY case in Figure 6. This is the case when the parameter is greater than (somewhere around) -0.975. The reason for this can be seen from (77). For low electric field, the pair production rate is dominated by the $(u_0^4/u_b^4)((40\delta + 39)/320)(1/\mathcal{E}^4)$ term. Above $\delta = -0.975$, the effective correction at the low electric field limit is positive. At the limit of high electric field limit, the correction of pair production rate due to the non-SUSY deformation parameter is always positive. This is reason that at the high electric field limit, the behavior of non-SUSY pair production rate is same as its supersymmetric cousin for all values of parameters. Startling effects happen when the parameter $\delta < -0.975$, for which the plot is shown in Figure 7. Let us recall that the prefactor of the pair production rate is given by field theoretic calculations to be $(eE)^2/(2\pi)^3$ (see (30)). Although the holographic calculation of the fluctuation prefactor is currently a mystery, it should definitely match with field theoretic calculation for low electric field. For small applied electric fields, the production rate shoots up signaling in *nonperturbative instability of the vacuum*. We say "nonperturbative" because the Schwinger effect is by itself a nonperturbative phenomenon. We see that the limit $\delta = -0.975$ approx. is for more interesting than earlier imagined.

Let us end by writing down the pair production rate per unit spatial volume per unit time for non-SUSY Yang-Mills.

$$\begin{aligned}\Gamma_{\text{non-SUSY}} \approx \exp\Bigg[&-\frac{\sqrt{\lambda}}{2}\Bigg(\frac{2\pi m_0^2}{\sqrt{\lambda}}\frac{1}{E} - 2 + \frac{\sqrt{\lambda}}{2\pi m_0^2}E \\ &+ \frac{\lambda^2 u_0^4}{16\pi^4 m_0^4}\Bigg\{\frac{4\pi^4 m_0^8(40\delta + 39)}{5\lambda^2}\frac{1}{E^4} \\ &+ \frac{\sqrt{\lambda}}{160\pi m_0^2}E - \frac{4\pi m_0^2(\delta + 1)}{\sqrt{\lambda}}\frac{1}{E}\Bigg\}\Bigg)\Bigg].\end{aligned} \tag{79}$$

## 7. Conclusion

In this paper, we have studied pair production (Schwinger effect) in the presence of external electric field for non-SUSY AdS/CFT using three methods in the literature. In Section 4, we have done a potential analysis by calculating rectangular Wilson loops and have analytically calculated the critical electric field below which pair production happens via a tunneling phenomenon (and above which the quark-antiquark potential ceases to put up a potential barrier). We have seen that the critical electric field is higher/lower than its supersymmetric counterpart depending on the value of the non-supersymmetric parameter $\delta$ (in that section, we have used a metric perturbation to ease up the calculation). We have also confirmed the same from the DBI analysis of the critical electric field in Section 5 where no such approximation has been made. As can be seen from the main body of this work, the correction absence of supersymmetry yields on the critical electric field (w.r.t. its supersymmetric value) is neither positive nor negative definite ((50), (55), and (74)). Thus, such a relation cannot be conveniently used to be an indirect evidence for the presence or absence of supersymmetry since the modulus of the correction is parameter dependent. Also note that from Figure 4, no trace of confinement is seen contrary to the earlier findings in the literature (for confinement, some of the plot should have been positive with nonnegative slope all along). Next in Section 6, we have performed the analysis for pair production rate for quark-antiquark pairs using circular Wilson loops. Since the relevant equations are rather impossible to solve, we have resorted to perturbative analysis, which can be thought of as perturbation over $\mathcal{N}=4$ SYM by a supersymmetry breaking term with coupling constant proportional to $(u_0/u_b)^4$ parametrized by $\delta$. We have explicitly found out the profile for circular Wilson loop for non-SUSY AdS/CFT up to the first order of $u_0^4$. To our knowledge, this is the first time such a solution has been obtained. We proceed to find the on-shell value of the Nambu-Goto action on the profile found and relate it to pair production rate. We see that for a regime of allowed value of the parameter $\delta$, the pair production rate shoots up as external electric field decreases towards zero. In stark contrast to confinement (as earlier reported), this signals that the vacuum of the dual non-SUSY gauge theory is nonperturbatively unstable for the regime of the parameter $-\sqrt{5/2} \leq \delta < -39/40$ (approx.) (a confining potential would show the exact opposite). Thus, the field theory dual of the non-SUSY geometry considered here is not confining but is the exact opposite, i.e., nonperturbatively unstable. A relevant question is to find this instability from potential analysis and DBI analysis, something which eludes us at this moment.

### Data Availability

No external data was used in this manuscript.

### Conflicts of Interest

The author declares that there are no conflicts of interest.

## References

[1] J. Maldacena, "The large $N$ limit of superconformal field theories and supergravity," *Advances in Theoretical and Mathematical Physics*, vol. 2, no. 2, pp. 231–252, 1998.

[2] E. Witten, "Anti-de Sitter space and holography," *Advances in Theoretical and Mathematical Physics*, vol. 2, no. 2, pp. 253–291, 1998.

[3] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, "Large $N$ field theories, string theory and gravity," *Physics Reports*, vol. 323, no. 3-4, pp. 183–386, 2000.

[4] M. Ammon and J. Erdmenger, *Gauge/Gravity Duality: Foundations and Applications*, Cambridge University Press, Cambridge, 2015.

[5] P. K. Townsend, "Black holes: lecture notes," https://arxiv.org/abs/gr-qc/9707012.

[6] R. Blumenhagen, D. Lüst, and S. Theisen, *Basic Concepts of String Theory, Theoretical and Mathematical Physics*, Springer, Heidelberg, Germany, 2013.

[7] K. Nayek and S. Roy, "Decoupling limit and throat geometry of non-susy D3 brane," *Physics Letters B*, vol. 766, pp. 192–195, 2017.

[8] N. R. Constable and R. C. Myers, "Exotic scalar states in the AdS/CFT correspondence," *Journal of High Energy Physics*, vol. 1999, no. 11, p. 020, 1999.

[9] K. Nayek and S. Roy, "Some aspects of non-perturbative QCD from non-susy D3 brane of Type IIB string theory," https://arxiv.org/abs/1809.00851.

[10] J. Schwinger, "On gauge invariance and vacuum polarization," *Physics Review*, vol. 82, no. 5, pp. 664–679, 1951.

[11] G. W. Semenoff and K. Zarembo, "Holographic Schwinger effect," *Physical Review Letters*, vol. 107, no. 17, article 171601, 2011.

[12] A. S. Gorsky, K. A. Saraikin, and K. G. Selivanov, "Schwinger type processes via branes and their gravity duals," *Nuclear Physics B*, vol. 628, no. 1-2, pp. 270–294, 2002.

[13] Y. Sato and K. Yoshida, "Potential analysis in holographic Schwinger effect," *Journal of High Energy Physics*, vol. 2013, no. 8, p. 2, 2013.

[14] Y. Sato and K. Yoshida, "Holographic Schwinger effect in confining phase," *Journal of High Energy Physics*, vol. 2013, no. 9, p. 134, 2013.

[15] D. Kawai, Y. Sato, and K. Yoshida, "A holographic description of the Schwinger effect in a confining gauge theory," *International Journal of Modern Physics A: Particles and Fields; Gravitation; Cosmology; Nuclear Physics*, vol. 30, no. 11, article 1530026, 2015.

[16] J. X. Lu and S. Roy, "Static, non-SUSY $p$-branes in diverse dimensions," *Journal of High Energy Physics*, vol. 2005, no. 2, p. 001, 2005.

[17] M. J. Duff, R. R. Khuri, and J. X. Lu, "String solitons," *Physics Reports*, vol. 259, no. 4-5, pp. 213–326, 1995.

[18] K. Nayek and S. Roy, "Decoupling of gravity on non-susy Dp branes," *Journal of High Energy Physics*, vol. 2016, no. 3, p. 102, 2016.

[19] Y. Kinar, E. Schreiber, and J. Sonnenschein, "$Q\bar{Q}$ potential from strings in curved space-time - classical results," *Nuclear Physics B*, vol. 566, no. 1-2, pp. 103–125, 2000.

[20] I. K. Affleck, O. Alvarez, and N. S. Manton, "Pair production at strong coupling in weak external fields," *Nuclear Physics B*, vol. 197, no. 3, pp. 509–519, 1982.

[21] C. Itzykson and J. Zuber, *Quantum Field Theory, International Series in Pure and Applied Physics*, McGraw-Hill, New York, 1980.

[22] C. Schubert, "Perturbative quantum field theory in the string-inspired formalism," *Physics Reports*, vol. 355, no. 2-3, pp. 73–234, 2001.

[23] G. V. Dunne, Q.-H. Wang, H. Gies, and C. Schubert, "Worldline instantons and the fluctuation prefactor," *Physical Review D*, vol. 73, no. 6, article 065028, 2006.

[24] U. N. Chowdhury, "Holographic description of noncommutative Schwinger effect," *Advances in High Energy Physics*, vol. 2022, Article ID 6648322, 16 pages, 2021.

[25] S. Bolognesi, F. Kiefer, and E. Rabinovici, "Comments on critical electric and magnetic fields from holography," *Journal of High Energy Physics*, vol. 2013, no. 1, p. 174, 2013.

[26] J. Maldacena, "Wilson loops in large $N$ field theories," *Physical Review Letters*, vol. 80, no. 22, pp. 4859–4862, 1998.

[27] N. Drukker, D. J. Gross, and H. Ooguri, "Wilson loops and minimal surfaces," *Physical Review D*, vol. 60, no. 12, article 125006, 1999.

[28] S.-J. Rey and J.-T. Yee, "Macroscopic strings as heavy quarks: large-$N$ gauge theory and anti-de Sitter supergravity," *European Physical Journal C: Particles and Fields*, vol. 22, no. 2, pp. 379–394, 2001.

[29] Y. Sato and K. Yoshida, "Holographic description of the Schwinger effect in electric and magnetic field," *Journal of High Energy Physics*, vol. 2013, no. 4, p. 111, 2013.

[30] M. Alishahiha, D. Allahbakhshi, and A. Naseh, "Entanglement thermodynamics," *Journal of High Energy Physics*, vol. 2013, no. 8, p. 102, 2013.

[31] U. N. Chowdhury, "Pair production in non-SuSy AdS/CFT," https://arxiv.org/abs/2206.14021.