

Research Article

Creation Field Cosmological Model with Variable Cosmological Term (Λ) in Bianchi Type III Space-Time

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The paper is devoted to the study of cosmological models with time-varying cosmological term (Λ) in the presence of creation field in the framework of Bianchi type III space-time. To obtain deterministic model of the universe, we have assumed $\Lambda \propto 1/R^3$, where R is the scale factor, for steady state cosmology and creation field, and shear (σ) is proportion to expansion (θ) which leads to $B = C^n$, where B and C are the metric potentials to explain small anisotropic behaviour of the universe and its isotropy. To obtain the results in terms of cosmic time t , we have assumed $n = 2$. The model satisfies conservation equations, and creation field increases with time. The present model is free from singularity, has particle horizon, and provides a natural explanation for inflationary scenario and isotropization. Creation field and Einstein field equations are derived using principle of least action and Lagrangian formulation of variable cosmological term. For illustrative purposes, evolutionary behaviour of some cosmological parameters are shown graphically. The other physical aspects like accelerating behaviour of the model are also discussed. Thus, the model represents not only expanding universe but also accelerating which matches with the results of present-day observations.

1. Introduction

The investigations deal with the physical process and use a model of the universe which is called big bang model, but the big bang models face problems such as the following: (i) singularity problem, (ii) the conservation of energy is violated, (iii) it leads to a very small particle horizon in early epoch of the universe, (iv) it fails to provide a consistent scenario that explains the origin and evolution of the universe, and (v) it has flatness problem.

If a model successfully explains the creation of positive energy matter without violating the conservation of energy, then it becomes necessary to have a negative energy mode that provides a natural way for creation of matter. Hoyle and Narlikar [1] adopted a field theoretic approach for creation of matter by introducing a massless and chargeless scalar field to discuss creation field (C -field) cosmology. There is no big bang-type singularity in C -field cosmology. The creation field cosmological models create more interest

in the study because these models solve a number of outstanding problems of big bang cosmology like singularity, horizon, and flatness problems of the universe [2]. These problems could be solved by (i) quantum gravity models and (ii) inflationary cosmological models. But we do not have complete quantum theory of gravity, and inflationary models do not solve the problem of singularity for creation. Therefore, the C -field cosmology with negative energy field is justified as introduced by Hoyle and Narlikar [1].

Frieman et al. [3] have pointed out that we have entered an era dominated by dark energy (cosmological term). A wide range of observations suggest that cosmological term (Λ) is the most favoured candidate of dark energy representing energy density of vacuum. In Einstein's field equations, the nontrivial role of vacuum generates a cosmological constant (Λ) term which leads to inflationary scenario [4]. The present-day observations of smallness of cosmological constant ($\Lambda_0 \approx 10^{-122}$) support to assume that the cosmological constant is time dependent. Gibbons and Hawking [5] have

investigated that cosmological models with positive cosmological constant lead to de-Sitter space-time asymptotically. Therefore, the cosmological models linking the variation of cosmological constant and having the form of Einstein field equations unchanged have been studied by many authors, viz., [6–10].

In the late eighties, astronomical observations revealed that the predictions of FRW models do not always meet our requirements as was believed earlier (Smoot et al. [11]). Therefore, spatially homogeneous and anisotropic Bianchi space-times (I–IX) have been investigated to study the universe in its early stages of evolution. Recent cosmological observations support the existence of anisotropic universe that approaches to isotropic phase (Land and Maguejo [12]). Among these, Bianchi type III space-time is simple generalisation of Bianchi type I space-time which plays a significant role for the study of the universe.

Recently, Bali [13] investigated creation field cosmological model for dust distribution and time-dependent cosmological term in Bianchi type II space-time.

In this paper, we have investigated creation field cosmological model for dust distribution and cosmological term Λ in the framework of Bianchi type III space-time and discussed inflationary scenario, horizon problem, inflationary parameters, and isotropisation. The model fulfills the requirements as per astronomical observations.

2. Metric and Field Equations

We consider Bianchi type III space-time in orthogonal form represented by the line element

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2x} dy^2 - C^2 dz^2, \quad (1)$$

where A , B , and C are the metric potential functions of t alone.

By the introduction of creation field (C), the modified Einstein field equation is given by Hoyle and Narlikar [1] as

$$R^{ik} - \frac{1}{2} R g^{ik} = 8\pi G \left[\begin{matrix} T^{ik} \\ (m) \end{matrix} + \begin{matrix} T^{ik} \\ (C) \end{matrix} \right] - \Lambda(t) g^{ik}. \quad (2)$$

The energy momentum tensor $\begin{matrix} T^{ik} \\ (m) \end{matrix}$ for perfect fluid distribution and $\begin{matrix} T^{ik} \\ (C) \end{matrix}$ for creation field are given by

$$\begin{matrix} T^{ik} \\ (m) \end{matrix} = (\rho + p) v_i^i v^k - p g^{ik}, \quad (3)$$

$$\begin{matrix} T^{ik} \\ (C) \end{matrix} = -f \left(C^i C^k - \frac{1}{2} g^{ik} C_\alpha C^\alpha \right), \quad (4)$$

where $f > 0$ is the coupling constant between matter and creation field and $C_i = \partial C / \partial x^i$. We note that $\begin{matrix} T^{44} \\ (C) \end{matrix} < 0$ for $f > 0$.

Thus, C -field has negative energy density that produces a repulsive gravitational effect. This repulsive force drives the expansion of the universe. The variation of C (creation field) gives the source equation in the form

$$C_{;k}^k = \frac{n}{f}, \quad (5)$$

where n is the number of net creation events per unit proper 4-volume.

3. The Lagrangian Formulation of Einstein Field Equation with Λ Term

The Lagrangian density (L) is defined by Moffat [14] as

$$L = LR + L_\Lambda + L_M, \quad (6)$$

where

$$\begin{aligned} L_R &= \sqrt{-g} g^{\mu\nu} R_{\mu\nu}, \\ L_\Lambda &= -2\sqrt{-g} [\Lambda + (\Lambda_{;\mu} u^\mu - \alpha\Lambda + \Lambda^2)\phi], \end{aligned} \quad (7)$$

where Λ is the variable cosmological constant treated as dynamical field, ϕ is the Lagrangian multiplier field, $u^\mu = dx^\mu/dt$ is observed for velocity along a world line in a space-time, α is a constant, and L_M is a matter of Lagrangian density. A variation of L with respect to ϕ and Λ yields the equation of motion as

$$\Lambda_{;\mu} u^\mu - \alpha\Lambda + \Lambda^2 = 0, \quad (8)$$

$$\frac{1}{\sqrt{-g}} (\sqrt{-g} u^\mu \phi)_{;\mu} + (\alpha - 2\Lambda)\phi - 1 = 0. \quad (9)$$

Variation of L with respect to $g^{\mu\nu}$ and using (8) gives

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -k T_{\mu\nu}. \quad (10)$$

The cosmological constant enters through the vacuum energy density.

$$T_{\mu\nu} = -\rho V g_{\mu\nu} = -\frac{\Lambda V}{8\pi G} g_{\mu\nu}. \quad (11)$$

Λ has small value $< 10^{-46}$ Gyr. By the introduction of C -field given by Hoyle and Narlikar [1], the Einstein field equation leads to equations (2) with (3) and (4).

The coordinates are assumed to be comoving so that $v^1 = 0 = v^2 = v^3$ and $v^4 = 1$.

The modified Einstein's field equation (2) for dust distribution and the space-time (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = 8\pi G \left(\frac{1}{2} f C_4^2 \right) + \Lambda(t), \quad (12)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = 8\pi G \left(\frac{1}{2} f C_4^2 \right) + \Lambda(t), \quad (13)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = 8\pi G \left(\frac{1}{2} f C_4^2 \right) + \Lambda(t), \quad (14)$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = 8\pi G \left(\rho - \frac{1}{2} f C_4^2 \right) + \Lambda(t), \quad (15)$$

$$\frac{A_4}{A} - \frac{B_4}{B} = 0, \quad (16)$$

where suffix 4 after A , B , and C indicates partial derivative with respect to cosmic time t .

Following Hoyle and Narlikar [1], we have used $p=0$ (dust fluid distribution). The source equation of C -field $C_{;i}^i = n/f$ leads to $C = t$ for large r . Thus, $C_4 = 1$. Equations (12)–(15) lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = 4\pi G f + \Lambda(t), \quad (17)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = 4\pi G f + \Lambda(t), \quad (18)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = 4\pi G f + \Lambda(t), \quad (19)$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = 8\pi G \rho - 4\pi G f + \Lambda(t). \quad (20)$$

Equation (16) leads to

$$A = \alpha B, \quad (21)$$

where α is a constant. For deterministic models of the universe, we assume two conditions:

- (i) Shear (σ) \propto expansion (θ) as investigated by Thorne [15]
- (ii) $\Lambda(t)\alpha 1/R^3$ as investigated by Hoyle et al. [16] for steady state and creation field cosmology, R being the scale factor

The assumption $\sigma/\theta = \text{constant}$ is given by Thorne [15] as per astronomical observations. Thorne [15] has investigated that the current observations of velocity redshift relation for extragalactic sources suggest that the Hubble expansion of the universe is isotropic to within $\sim 30\%$. Also, redshift studies place the limit $\sigma/H \leq 0.30$ in the neighbourhood of our galaxy. By contrast, cosmic microwave background measured isotropy is within 3%, at least around the celestial equator as mentioned by Partridge and Wilkinson [17]. In our model, $\sigma/\theta \leq 0.30$ which is very small in large-scale structure of the universe. The shear (σ) and expansion (θ) for metric (1) are given by

$$\sigma = \frac{1}{\sqrt{3}} \left(\frac{B_4}{B} - \frac{C_4}{C} \right), \quad (22)$$

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = 2 \frac{B_4}{B} + \frac{C_4}{C} \quad \text{as} \quad \frac{B_4}{B} = \frac{C_4}{C}.$$

Using $\sigma \propto \theta$, we have

$$B = C^n, \quad (23)$$

where n is a constant. The spatial volume R^3 is given as

$$R^3 = ABC = \alpha C^{2n+1}, \quad (24)$$

and $\Lambda \propto 1/R^3$.

$$\text{leads to } \Lambda = \frac{\gamma}{C^{2n+1}}, \text{ where } \gamma = \frac{l}{\alpha}. \quad (25)$$

Using (21), (23), and (25) in (17), we have

$$2C_{44} + \frac{2n^2}{n+1} \frac{C_4^2}{C} = \frac{2KC}{n+1} + \frac{2\gamma}{n+1} C^{-2n}, \quad (26)$$

where $K = 4\pi G f$.

To get deterministic solution, we assume

$$C_4 = f(C). \quad (27)$$

Thus, $C_{44} = f f'$, $f' = df/dC$.

Now, equation (26) leads to

$$2f f' + \frac{2n^2}{n+1} \frac{f^2}{C} = \frac{2KC}{n+1} + \frac{2\gamma}{n+1} C^{-2n}. \quad (28)$$

Thus, we have

$$\frac{d}{dC} (f^2) + \frac{2n^2}{n+1} \frac{f^2}{C} = \frac{2KC}{n+1} + \frac{2\gamma}{n+1} C^{-2n}, \quad (29)$$

which leads to

$$f^2 = \frac{K}{n^2 + n + 1} C^2 + \frac{2\gamma}{1-n} C^{(1-n-2n^2)/(n+1)} + m C^{-2n^2/(n+1)}, \quad (30)$$

where m is a constant of integration.

To get deterministic model of the universe in terms of cosmic time t , we assume $n=2$ and $m=0$.

Thus, equation (30) for $n=2$ and $m=0$ leads to

$$\left(\frac{dC}{dt} \right)^2 = f^2 = \frac{K}{7} C^2 - 2\gamma C^{-3}, \quad (31)$$

which leads to

$$\frac{dC}{\sqrt{(K/7)C^2 - 2\gamma C^{-3}}} = dt. \quad (32)$$

That is,

$$\frac{C^{3/2} dC}{\sqrt{(K/7)C^5 - 2\gamma}} = dt. \quad (33)$$

Therefore, we have

$$C^{5/2} = \beta \text{Cosh}(t). \quad (34)$$

That is,

$$\begin{aligned} C &= \beta^{2/5} \text{Cosh}^{2/5}(t), \\ B &= C^2 = \beta^{4/5} \text{Cosh}^{4/5}(t), \\ A^2 &= \alpha^2 B^2 = \alpha^2 \beta^{8/5} \text{Cosh}^{8/5}(t), \end{aligned} \quad (35)$$

where $\beta^2 = 14\gamma/K$, $l = 5/2\sqrt{K/7}$, and m is a constant of integration.

We can choose $l = 1$ without loss of generality.

Thus, metric (1) leads to

$$\begin{aligned} ds^2 &= dt^2 - \alpha^2 \beta^{8/5} \text{Cosh}^{8/5}(t) dx^2 - \beta^{8/5} e^{2x} \text{Cosh}^{8/5}(t) dy^2 \\ &\quad - \beta^{4/5} \text{Cosh}^{4/5}(t) dz^2. \end{aligned} \quad (36)$$

4. Physical and Geometrical Aspects

The matter density (ρ), spatial volume (R^3), shear (σ), expansion (θ), Hubble parameter (H), the cosmological term (Λ), and deceleration parameter (q) for model (36) are given by

$$\begin{aligned} 8\pi G\rho &= \left(\frac{32}{25} + K\right) - \left(\frac{32}{25} + \frac{2}{\alpha^2 \beta^2}\right) \text{sech}^2(t), \\ R^3 &= ABC = \alpha \beta^2 \text{Cosh}^2(t), \\ \sigma &= \frac{1}{\sqrt{3}} \left(\frac{B_4}{B} - \frac{C_4}{C}\right) = \frac{2}{5\sqrt{3}} \tanh(t), \\ \theta &= \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \left(2\frac{B_4}{B} + \frac{C_4}{C}\right) \text{ as } \frac{A_4}{A} = \frac{B_4}{B}, \\ &= 5\frac{C_4}{C} = 2 \tanh(t), \\ H &= \frac{\theta}{3} = \frac{2}{3} \tanh(t). \end{aligned} \quad (37)$$

$\sigma/H = \sqrt{3}/5 \cong 0.3 = 30\%$ which agrees with the result as investigated by Thorne [15].

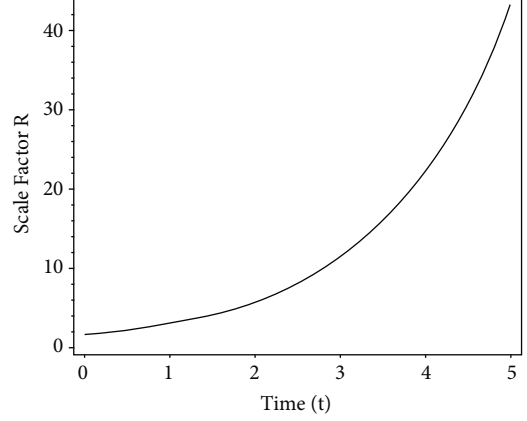


FIGURE 1: Scale factor with time.

$$\begin{aligned} \Lambda &= \frac{\gamma}{C^{2n+1}} = \frac{\gamma}{C^5} \text{ as } n = 2, \\ &= \frac{\gamma}{\beta^2 \text{Cosh}^2(t)}, \\ q &= -\frac{R_{44}/R}{R_4^2/R^2} = -\frac{3}{2} \text{Coth}^2(t) + \frac{1}{2} < 0. \end{aligned} \quad (38)$$

The reality condition $\rho > 0$ leads to

$$\frac{2}{\alpha^2 \beta^2} < \left(\frac{32}{25} + \frac{2}{\alpha^2 \beta^2}\right) \tanh^2(t) + K. \quad (39)$$

The creation rate is determined by the conservation equation

$$\left(8\pi G T_i^j + \Lambda g_i^j\right)_{;j} = 0, \quad (40)$$

which leads to

$$\begin{aligned} \frac{d}{dt} C_4^2 + 2\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) C_4^2 &= \frac{2\rho_4}{f} \\ &+ \frac{2\rho}{f} \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{\Lambda_4}{K}, \end{aligned} \quad (41)$$

where $4\pi Gf = K$.

Thus, we have

$$\frac{d}{dt} C_4^2 + 4 \tanh(t) C_4^2 = \frac{2\rho_4}{f} + \frac{2\rho}{f} \left(2 \tanh(t) + \frac{2\gamma l}{\beta^2 k} \text{sech}^2(t) \tanh(t)\right), \quad (42)$$

which leads to

$$C_4^2 \text{Cosh}^4(t) = \left(\frac{16}{25K} + \frac{1}{2}\right) \text{Cosh}^4(t). \quad (43)$$

Thus, $C_4^2 = 1$, where $(16/25K) + (1/2) = 1$.

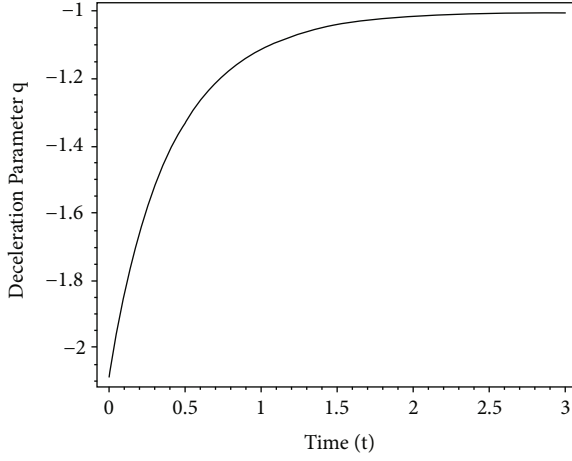


FIGURE 2: Deceleration parameter with time.

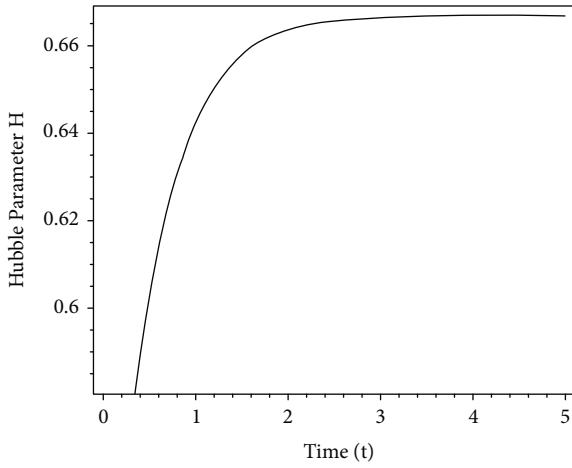


FIGURE 3: Hubble parameter with time.

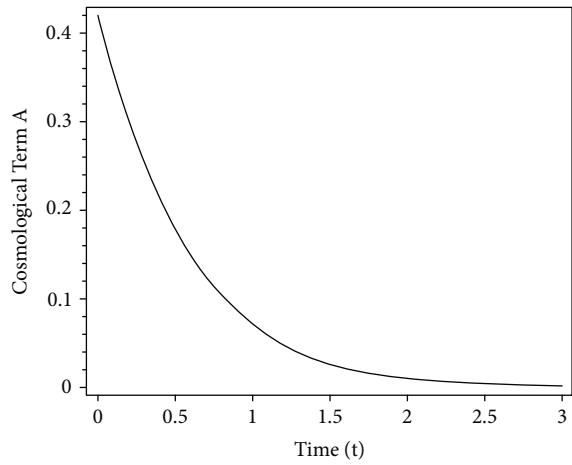


FIGURE 4: Cosmological term with time.

Therefore, $C = t$ and $f = 8l^2/25\pi G > 0$ which matches with the result as investigated by Hoyle and Narlikar [1] in creation field cosmology.

In anisotropic models, the directional Hubble parameters $H_1, H_2,$ and H_3 are always different. We define average Hubble parameter $H = 1/3(H_1 + H_2 + H_3)$ and calculate $\sigma/H = 0.3 = 30\%$ which agrees with the result as investigated by Thorne [15]. Crawford and Aguiar [18] have shown that if the parameter $\tilde{A}^2 = \sigma^2/3H^2 = 0.01$ is almost zero, where σ is the shear and \tilde{A} is the anisotropic parameter, then we find isotropic phase (Figures 1, 2, 3 and 4).

5. Discussion and Conclusion

The spatial volume for model (36) increases with time exponentially, thus representing inflationary universe. The matter density is positive which satisfies reality condition. The model also satisfies the condition $\tilde{A}^2 = \sigma^2/3H^2 = 0.01$ which is very small, as pointed out by Crawford and Aguiar [18]. Thus, the model leads to isotropic phase. The cosmological term $\Lambda \sim 1/(1+t^4)$. The shear (σ) is 0 at the initial stage and at the late time; i.e., at $t = \infty$, it leads to 1. But $\sigma/\theta = 0.3 = 30\%$ which is small and agrees with the result as investigated by Thorne [15]. We have studied the matter dominated stage for creation field cosmology. But the creation always exists in the beginning and end. This is also applicable to radiation dominated stage, i.e., $\rho = 3p$. The deceleration parameter $q < 0$ indicates that the model shows accelerating universe throughout. The cosmological parameter (Λ) helps to solve the problems of steady state theory as mentioned by Hoyle et al. [16]. The unit of time in the figure is indicated in gigayear. The model represents expanding and accelerating universe which agrees with the result obtained by Riess et al. [19] and Perlmutter et al. [20] as deceleration parameter $q < 0$.

The coordinate distance $\gamma_H(t)$ to the horizon is the maximum distance a null ray could have travelled at time t starting from the infinite past. Thus,

$$\gamma_H = \int_{-\infty}^{\tau} \frac{d\tau}{R^3(\tau)}. \quad (44)$$

We can extend the proper time τ to $-\infty$ in the past because of the nonsingular nature of model (36). Therefore,

$$\gamma_H = \int_0^{\tau} \frac{d\tau}{\alpha\beta^3 \cosh^2 t}. \quad (45)$$

Taking $l = 1, m = 0$.

$$= \int_0^{\tau} \frac{1}{\alpha\beta^2} \text{Sech}^2 dt, \quad (46)$$

$$\frac{1}{\alpha\beta^2} [\text{tanht}]_0^{\tau} = \text{finite},$$

which shows that model (36) has particle horizon.

Data Availability

We believe that ensuring that the data underlying the findings of a paper are publicly available wherever possible—as open as possible and as close as necessary—will help ensure that our work described in an article can potentially be replicated. We therefore firmly support and endorse the FAIR Guiding Principles for scientific data management and stewardship—that of findability, accessibility, interoperability, and reusability. There are many benefits to sharing data—it increases not only both the utility and reliability of our work but also its impact and visibility and our profile and credibility as a researcher.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

- [1] F. Hoyle and J. V. Narlikar, “On the avoidance of singularities in C-field cosmology,” *Proceedings of the Royal Society of London, Mathematical and Physical Sciences*, vol. 278, p. 465, 1964.
- [2] J. V. Narlikar and T. Padmanabhan, “Creation field cosmology: a possible solution to singularity, horizon and flatness problem,” *Physical Review D*, vol. 39, p. 1928, 1985.
- [3] J. A. Friemann, M. S. Turner, and D. Huterer, “Dark energy and the accelerating universe,” *Annual Review of Astronomy and Astrophysics*, vol. 46, no. 1, pp. 385–432, 2008.
- [4] E. Abers and B. W. Lee, “Gauge theories,” *Physics Reports*, vol. 9, no. 1, pp. 1–2, 1973.
- [5] G. Gibbons and S. W. Hawking, “Cosmological event horizons, thermodynamics, and particle creation,” *Physical Review D*, vol. 15, pp. 2738–2751, 1977.
- [6] M. S. Berman, “Cosmological models with variable gravitational and cosmological “constants”,” *General Relativity and Gravitation*, vol. 23, no. 4, pp. 465–469, 1991.
- [7] R. Bali and S. Jain, “Bianchi type V magnetized string dust cosmological model with cosmological term in general relativity,” *International Journal of Modern Physics D: Gravitation; Astrophysics and Cosmology*, vol. 16, p. 11, 2007.
- [8] W. Chen and Y. S. Wu, “Implications of a cosmological constant varying as R^{-2} ,” *Physical Review D*, vol. 41, no. 2, pp. 695–698, 1990.
- [9] R. Bali and S. Saraf, “C-field cosmological model for barotropic fluid distribution with varying Λ in FRW space time,” *International Journal of Theoretical Physics*, vol. 52, no. 5, pp. 1645–1653, 2013.
- [10] A. Beesham, “Cosmological models with a variable cosmological term and bulk viscous models,” *Physical Review D*, vol. 48, no. 8, pp. 3539–3543, 1993.
- [11] G. F. Smoot, C. L. Bennett, A. Kogut et al., “Structure in the COBE differential microwave radiometer first-year maps,” *The Astrophysical Journal*, vol. 396, p. 11, 1992.
- [12] K. Land and J. Maguejo, “Examination of evidence for a preferred axis in the cosmic radiation anisotropy,” *Physical Review Letters*, vol. 95, no. 7, article 071301, 2005.
- [13] R. Bali, “Creation field cosmological models for dust distribution and time-dependent cosmological term (Λ) in Bianchi type II space-time,” *International Journal of Geometric Methods in Modern Physics*, vol. 19, no. 14, article 2250233, 2022.
- [14] Moffat, “Lagrangian formulation of a solution of cosmological constant problem,” 1996, <http://arxiv.org/abs/astro-ph/9608202v>.
- [15] K. S. Thorne, “Primordial element formation, primordial magnetic fields, and the isotropy of the universe,” *The Astrophysical Journal*, vol. 148, p. 51, 1967.
- [16] F. Hoyle, G. Burbidge, and J. V. Narlikar, “On the Hubble constant and the cosmological constant,” *Monthly Notices of the Royal Astronomical Society*, vol. 286, no. 1, pp. 173–182, 1997.
- [17] B. Partridge and D. T. Wilkinson, “Isotropy and homogeneity of the universe from measurements of the cosmic microwave background,” *Physical Review Letters*, vol. 18, no. 14, pp. 557–559, 1967.
- [18] P. Crawford and P. Aguiar, “Cosmology 2000,” in *Conference Proc. NASA ADS*, pp. 69–76, Northern Ireland, 2000.
- [19] A. G. Riess, A. V. Filippenko, P. Challis et al., “Observational evidence from supernovae for an accelerating universe and a cosmological constant,” *Astronomy Journal*, vol. 116, no. 3, pp. 1009–1038, 1998.
- [20] S. Perlmutter, G. Aldering, G. Goldhaber et al., “Measurements of Ω and Λ from 42 high-redshift supernovae,” *The Astrophysical Journal*, vol. 517, no. 2, pp. 565–586, 1999.