Hindawi
Advances in Multimedia
Volume 2022, Article ID 1861890, 7 pages
https://doi.org/10.1155/2022/1861890

Research Article

Muscle Strength Weighting Based on Deep Learning and Wavelet Packet and Muscle Fatigue Analysis Based on L-Z Complexity

Zhizhong Liu

Anhui Polytechnic of Industry & Trade, Huainan, Anhui 232007, China

Correspondence should be addressed to Zhizhong Liu; 201608581136@stu.yznu.edu.cn

Received 12 February 2022; Revised 2 April 2022; Accepted 22 April 2022; Published 17 May 2022

Academic Editor: Qiangyi Li

By studying the muscle sound signal of biceps brachii and gastrocnemius muscle, we try to find out the relationship between muscle force and load and the characteristic parameters of fatigue stage, so as to guide the exercise training well, ten healthy male college students were selected to perform static contraction experiments under different loads (0 lbs, 10 lbs....maximum load), and weight-bearing heel-lifting fatigue experiment. The relationship between load and muscle strength was analyzed by wavelet packet weighting and the L-Z complexity was used to analyze the muscle acoustic signal in the fatigue process. It has been verified that the L-Z complexity of the gastrocnemius muscle acoustic signal gradually decreases from the maximum in the early stage, relatively stable in the middle stage, and decreases again in the later stage of the weight-bearing heel-lifting exercise. The wavelet packet weighting algorithm makes the muscle strength and the weight-bearing well in line with the linear relationship, and the application of muscle strength map can better reflect the load of muscle. The L-Z complexity reflects the changes in muscle fiber recruitment during muscle fatigue and contraction to a certain extent, and provides a scientific basis for judging the fatigue state.

1. Introduction

The detection and evaluation of exercise-induced muscle fatigue is of great significance to clinical diagnosis, rehabilitation medicine and sports medicine. SEMG is a bioelectrical signal recorded from the muscle surface during the activity of the neuromuscular system. Because its amplitude and frequency are closely related to the functional activities of the neuromuscular system, sEMG has the advantages of non-invasive, real-time and multi-target measurement, and has a good application prospect and important research significance in the diagnosis of muscle fatigue. At present, most studies on sEMG in fatigue process focus on static shrinkage (isometric shrinkage). Median frequency MF and average power frequency MPF based on Fourier transform are the most widely used indicators at present. However, because Fourier transform is the frequency domain analysis of linear time-invariant signal, sEMG signal has the basic characteristics of typical unsteady signal for the relative displacement between the sEMG detection electrode and the muscle under dynamic contraction condition, as well as the changes of the length and thickness of the muscle during contraction. Moreover, the localization contradiction between the resolution of Fourier transform in time domain and frequency domain also limits the application of Fourier transform. Therefore, using Fourier transform to analyze sEMG has great limitations.

With the continuous progress and development of medicine and technology, there is an inseparable connection between the important values in human life and the human muscle state. Muscle state analysis is mainly used in rehabilitation medicine, sports science, occupational disease prevention and other fields [1]. See Figure 1, with the gradual development of sports, the training level of trainers needs to be gradually improved and strictly required, but it is difficult to ensure scientific and reasonable training through human subjective feelings. Excellent sports performance requires the guidance and assistance of physiological science and sports science theory. Nowadays, scientific training and scientific diagnosis have become important means. Muscle fatigue is one of the important indicators for assessing and diagnosing muscle state, which is mainly manifested in the

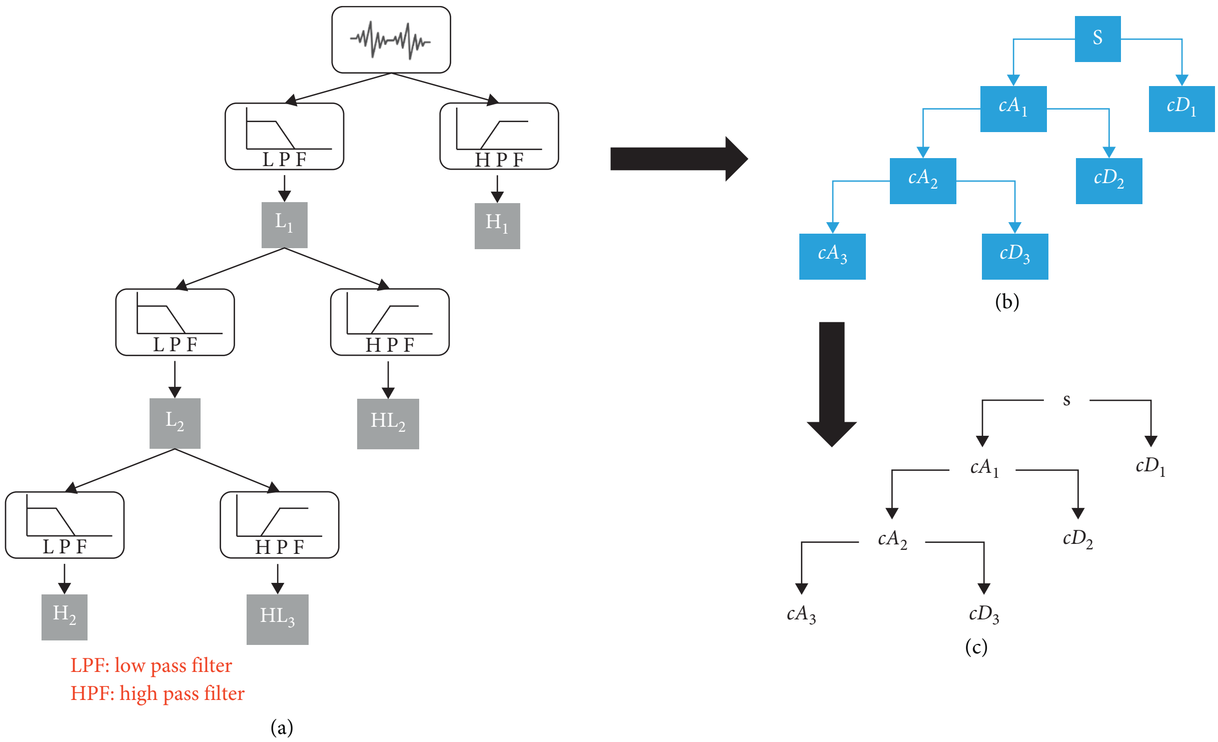

Figure 1: Muscle strength stratification. (a) Single decomposition, (b) Coefficient structure and (c) Wavelet decomposition tree.

reduction or decrease of human muscles' functional power during muscle contraction [2]. At present, muscle fatigue research based on biomedical signals has gradually been applied. Among them, electromyographic (EMG) signals, brain electrical signals and muscle acoustic signals are the most widely used biomedical signals. EMG signal is the most common biological signal in the current research on muscle fatigue, which describes the electrophysiological characteristics of muscle. This method is guided by electrodes to record the bioelectric signal of muscle contraction. The correlation between the EMG signal and the state and function of the muscle has a high potential value for the assessment and diagnosis of muscle fatigue in the field of sports and rehabilitation medicine [3]. There are two types of EMG signals. One type is needle EMG signals, which are invasive. EMG signals are obtained by invading the needle electrode into the muscle. This method will cause discomfort and muscle damage to the subject, which is only used in clinical practice. The other is surface EMG signal. As a one-dimensional time series signal, it can also reflect the characteristics of the neuromuscular system during muscle activity, which has the advantages of non-damaging and good locality [4].

2. Literature Review

Kim et al. initially explored the connection between the generation method of muscle sound signal and muscle movement mechanism in the research based on muscle sound signal, laying a good foundation for subsequent researchers to study muscle sound signal [5]. Filli et al. conducted a passive stretch reflex test in 10 patients with brain injury in a study based on muscle sound signals to assess spasticity in patients with brain injury, and recorded muscle sound signal and EMG signal of the lateral femoral muscle (stimulant) and half of the leg muscle (Antagonist), and proposed a new method for evaluating spasticity by muscle sound signal combined with EMG signal. The MMG ratio expressed in fractional form of myogenic oscillation shows that the degree of myogenic spasm increases as the value approaches 0 [6]. Sun et al. studied lower limb muscle atrophy based on muscle sound signals and EMG signals. The subjects were 20 healthy adults, 10 were elderly (average age 65 ± 5 years), and 10 were young people (average age 23 ± 4 years). The test method is to perform different intensities of thigh movements, record muscle sound signals and EMG signals at the same time, and process and analyze the data obtained [7]. Aslani et al. found in a study based on muscle sound signals that the frequency of muscle sound signals is in the range [2,100] Hz, and its energy is mainly concentrated in the frequency range [5,50] Hz, showing the connection between muscle strength and the muscle sound signal. As the load increases, the peak value of the muscle sound signal increases. There is a certain correspondence between the muscle sound signal and the muscle action characteristics. [8]. Kurosawa et al. used algorithms such as singular value decomposition and wavelet decomposition to recognize four action modes in pattern recognition based on muscle sound signals, and the recognition accuracy rate was 89.7% [9]. In the research of pattern recognition based on muscle sound signals, Das et al recorded the muscle sound signals at different positions of wrist flexor on the flexor side of forearm, and matched them with the recognition rate of hand movements. The research results showed that the muscle sound signal is different in different acquisition positions of the same muscle, and the sensor recognition effect affects the accuracy of the collection position error of 1-2 cm [10]. In the study of upper limb muscle fatigue based on EMG signal, Moosavi et al. collected EMG signals of athletes' upper limb muscles. The results showed that the average power frequency and center of the surface EMG signals of wrist flexors and extensors decreased significantly with the increase of time under different loads [11]. Zhao and other studies have shown that EMG signals can monitor muscle fatigue during isometric contraction [12].Do et al. proposed that wavelet packet analysis is a more detailed signal analysis and reconstruction method extended from wavelet analysis. Using the time-frequency localization characteristic of wavelet packet transform, the time-varying spectrum analysis of signal can be realized, and the signal can be analyzed in any detail, and it is not sensitive to noise. The wavelet packet analysis is more suitable for the analysis of athletes' non-stable EMG signals. With appropriate wavelet packet transform, emG signals in different functional states can be observed in different scales for their frequency changes and time characteristics [13]. Schimmack and Mercorelli Artificial neural network is a signal processing method that has received widespread attention at present, which imitates the structure of biological neurons and the mechanism of neural information transmission [14]. It is composed of many neurons with nonlinear mapping ability, which are connected with each other by weight coefficient to form an adaptive nonlinear dynamic system. It has the characteristics of parallel computing, distributed storage and adaptive learning. Artificial neural network shows great advantages in dealing with highly nonlinear problems because of its self-organization, self-adaptive learning and excellent fault tolerance, and is widely used in sEMG analysis and processing. The rectus femoris muscle is one of the most important muscles used in alternating centripetal contractions of the lower extremities during full pedal cycling. Hasan et al.; The sEMG of the right rectus femoris muscle was pedal with full force at 8% of the body weight of the subjects during the 60 s, and the recorded sEMG was processed, analyzed and studied by combining wavelet packet analysis and artificial neural network. In order to explore the change law of sEMG and its relationship with output power in the process of rectus femoris fatigue caused by full pedal bicycle in the 1960s, explore the quantitative evaluation method of sEMG for sports muscle fatigue [15].

This paper aims to collect muscle sound signals through advanced acquisition and analysis equipment, and digitally display the size of muscle force, and analyze the characteristics of muscle changes in the process of fatigue exercise, infer the fatigue stage, in order to guide the relevant exercise practice, make sports training digital, visual, more efficient. At present, the analysis of muscle contraction characteristics using muscle acoustic signals is mainly focused on frequency domain and time domain, such as energy, root mean square, integral value, spectrum, and power spectrum. There are also studies of statistical analysis in time domain and frequency domain, such as short-term energy, median frequency, center frequency, etc.

3. Muscle Acoustic Signal Analysis Theory

3.1. Wavelet Packet. Apply the wavelet packet algorithm to first perform wavelet packet decomposition on the signal, then weight a certain part of the wavelet packet node, then perform wavelet packet reconstruction on the weighted wavelet signal, and finally calculate the short-term signal energy of the reconstructed signal as the output of muscle strength. It is very useful for analyzing instantaneous time-varying signals. It effectively extracts information from the signal, performs multi-scale refinement analysis on the function or signal through the operation functions such as scaling and translation, and solves many difficult problems that cannot be solved by Fourier transform.

The wavelet packet was proposed by Coifman, Meyer, Quaker and Wickerhauser (CMQW). The following first introduces the concept of wavelet packet basis. If $\{h_g\}_{k\epsilon z}$ and $\{g_g\}_{k\epsilon z}$ are a set of conjugate mirror filter (QMF), namely:

$$\begin{aligned} &\sum_{n\in Z} h_{n-2k}h_{n-2i} = \delta_H, \\ &\sum_{n\in Z} h_n = \sqrt{2}, \\ &g_k = (-1)^k h_{1-k}, \quad k \in Z. \end{aligned} \tag{1}$$

Then it can be defined that the series of functions $\{u_n(t)\}$ $(n = 0, 1, 2, \ldots)$ satisfies the following equations:

$$\begin{aligned} u_{2n+1}(t) &= \sqrt{2}\sum_{k\in Z} h_k u_n(2t-k), \\ u_{2n+1}(t) &= \sqrt{2}\sum_{k\in Z} g_k u_n(2t-k). \end{aligned} \tag{2}$$

Wavelet packet transform not only decomposes the low-frequency part, but also the high-frequency part, which is a more refined decomposition method than wavelet decomposition. Figure 2 is a schematic diagram of the wavelet packet decomposition of the signal. The decomposition coefficients have a reconstruction relationship: So = S30 + S31+. . .+S37, which means that the original signal So can be reconstructed by the sum of all the decomposition coefficients of the third layer.

3.2. L-Z Complexity. It is generally believed that complexity reflects the rate at which new patterns appear in a time series as the length of the series increases. The greater the complexity, the more new patterns appear in the data within the window length, and the faster the rate of new changes, indicating that the data changes during this period are disorderly and complex. Conversely, the smaller the complexity, the slower the rate of new changes, and the stronger the periodicity of data changes. Therefore, the complexity of the time series can be calculated to describe the changes in the state of the system.

For the study of the complexity of nonlinear time series, the methods of coarse-grained time series generally include

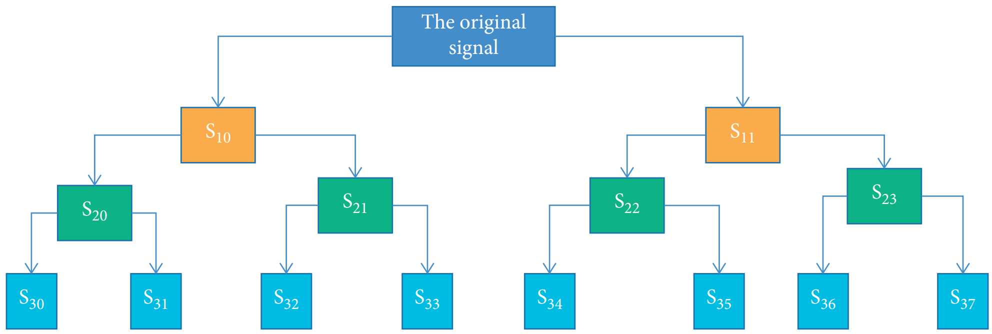

Figure 2: Schematic diagram of wavelet packet decomposition.

the mean method, the first-order difference method, the moving average, and the moving-order difference method. The symbolization method is to convert the signal into a sequence of binary symbols by comparing with a certain reference value. When the signal is less than the reference value, it is symbolized as "0," otherwise it is symbolized as "1."

The reference value of the mean value method is the mean value of this time series, that is, for a certain time series $x(i)$, $i = 1, 2, \ldots, n$, let

$$X_{\mathrm{ave}} = \frac{1}{n}\sum_{i=1}^{n} x(i), \tag{3}$$

X_{ave} is the average value of this time series, $S = (S_1 S_2 \cdots S_n)$ is an empty symbol string with the same length as the time series $i = 1, 2, \ldots, n$. When there is an element in the time series $X_i < X_{\mathrm{ave}}$, the symbol s is taken as "1," otherwise "0." The expression is as (4).

$$s(i) = \left\{ \begin{matrix} 0, & x(i) \le \mathrm{Xave} \\ 1, & x(i) \le \mathrm{Xave} \end{matrix} \right\}. \tag{4}$$

The established symbol sequence is S. This method is not sensitive to noise in dynamic structure analysis because only two states greater than the average value and less than or equal to the average value are considered. And it can only reflect the overall characteristics of the sequence, not the local characteristics [16].

According to the Lempel-Ziv complexity algorithm, it can be seen that the greater the complexity of a symbol sequence, the more operations it adds, the more new patterns it has, the more number of the least mutually different "substrings" required to describe a given symbol sequence, the weaker the periodicity of a given symbol sequence, and the faster the rate of new patterns. Conversely, the more repeated operations, the fewer the new patterns, the stronger the periodicity, and the slower the rate of new patterns appearing [16].

4. Experimental Analysis

4.1. Subject. In order to obtain the muscle acoustic signal of muscle contraction, male volunteer subjects with good mental state and physical health are selected to perform muscle contraction exercises. In this experiment, 10 young males were selected, aged $22.9 + 0.7$ years old, height 172.1 ± 3.9 cm, weight 64.4 ± 8.3 kg. No strenuous exercise was performed within 24 hours before the test. Before the test, the subjects were told about the essentials, the process, and the requirements of the experiment, and the training was carried out before the formal experiment. The subject information table is shown in Table 1.

4.2. Experimental Method

4.2.1. Experiment 1 the Biceps Brachii

(1) Static Load. Action description: Stand up straight with feet shoulder-width apart, straighten chest and close abdomen. Grip the handle tightly with right hand, straighten arm and bend forearm and upper arm more than 90 degrees, with palm facing forward. Inhale, raise right arm at the same time, and quickly lift to the horizontal position. The forearm is at a right angle to the upper arm, the palm is facing the biceps, and the elbow is close to the side of the body. Hold for 5 to 6 seconds. Then slowly put down the dumbbells and inhale at the same time. Change the size of the load and repeat the above actions.

(2) Experiment Procedure. The muscle acoustic signal acquisition experiment was carried out in a quiet room. Before the experiment, first, the test system should be preliminarily tested and set up, and the action essentials should be told to the subjects. The subjects stand upright on a level ground, exert their arms autonomously to find the biceps muscle [17], and determine the maximum vibration position of the biceps: muscle abdomen. Fix the muscle acoustic sensor on the epidermis where the vibration of the two arms is greatest.

4.2.2. Experiment 2 Weight-Bearing Heel-Lifting Fatigue Experiment. Action Description

(1) Sit with your feet shoulder width apart, with the load above your knees and your heels drooping.
(2) Exhale and slowly stand on toes until the highest.
(3) Hold for 1 second, and then slowly lower heels to the lowest point, and inhale at the same time.
(4) Repeat the above action.

Table 1: Subject information.

Number	Age	Height (cm)	Weight (kg)	Upper arm circumference, resting/maximum (cm)	Calf circumference (cm)
1	24	170	62.3	27.3	30.2
1	23	172	61.3	26.0	29.3
3	21	173	60.3	24.3	26.1
4	23	175	61.9	23.5	27.0
5	20	174	62.6	27.0	30.6

Figure 3: Flow chart of the test system.

Experimental procedure: A weight of 20 kg is placed on the front of the thigh, collect the muscle acoustic signal from gastrocnemius muscle movement to fatigue during heel lifting in weight-bearing sitting posture [17]. The muscle acoustic sensor should be attached to the gastrocnemius muscle abdomen.

4.3. Muscle Acoustic Signal Acquisition System. SEMG signals were collected using an EMG recording and analysis system with a sampling frequency of 1 000 Hz. After the experiment, the video frame of the flash moment was used as the synchronization point with the surface EMG zero moment, and the data were synchronized according to the relative delay time of the video and the surface EMG tester.

The frequency of the muscle acoustic signal is generally less than 100 Hz, so the signal acquisition card fully meets the requirements of muscle acoustic signal acquisition. In the signal acquisition process, we choose the sampling rate to be 5000 Hz. In the digital-to-analog conversion process, the 5000 Hz sampling rate is completely satisfied with the muscle acoustic sampling rate, and the signal information loss is very small. After the muscle acoustic signal is amplified and digital-to-analog converted, it is directly input to the terminal computer [18], and the self-developed data acquisition system is used to configure the acquisition card and display and save the data. The flow chart is shown in Figure 3.

In the experiment, there will be noise in signal acquisition, but through computer-recorded signal and spectrum analysis, it is found that the amplitude of the noise relative to the muscle acoustic signal is very small. Before signal processing, the noise is filtered through high-pass and low-pass filters to obtain a large signal-to-noise ratio.

4.4. Analysis of Muscle Acoustic Signal. Each group of muscle acoustic signals collects 10 complete movements. Figure 4 shows the original muscle acoustic signals of 10 complete movements. It can be roughly seen from the figure that the amplitude of the muscle acoustic signal gradually decreases as the movement continues.

For the convenience of research, 3 out of 10 complete movements are intercepted for analysis. Figure 5 shows part of the muscle acoustic signals collected during the biceps curl exercise [19].

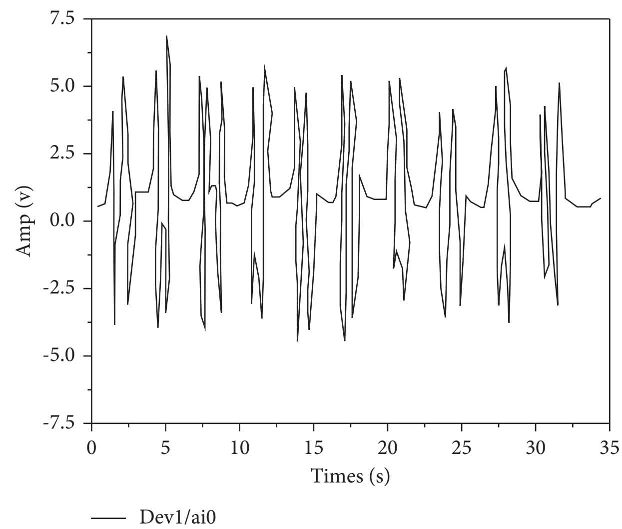

Figure 4: Acoustic signal of biceps brachii during curling exercise.

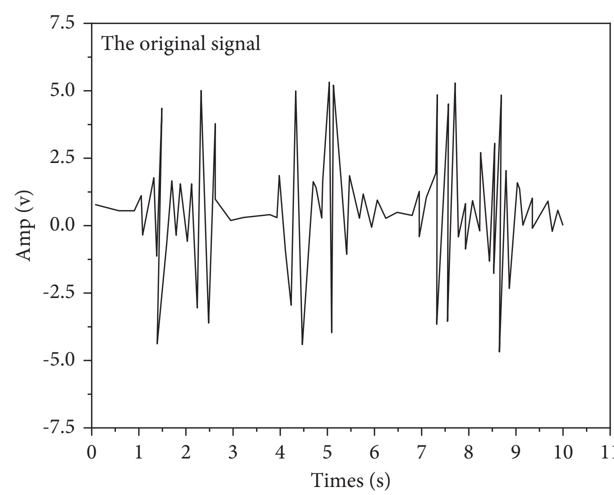

Figure 5: Part of the muscle acoustic signal.

Perform wavelet packet weighting on the muscle acoustic signal of each subject under each load condition and calculate the value of relative muscle strength. The calculation results are shown in Table 2.

It can be seen from Table 2 that the relative muscle strength and load have a tendency to increase at the same

Table 2: Relative muscle strength of different subjects under different loads.

Load (lbs)	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5
0	0.001	0.002	0.012	0.042	0.001
10	0.05	0.003	0.356	0.176	0.002
10	0.13	0.014	0.445	0.268	0.005
20	0.23	0.02	0.753	0.244	0.006
30	0.4	0.03	2.21	0.274	0.003
40	0.61	0.05	3.159	0.496	0.008
50	1.02	0.005	3.412	0.357	0.012
60	0.61	0.07	0.314	0.231	0.017

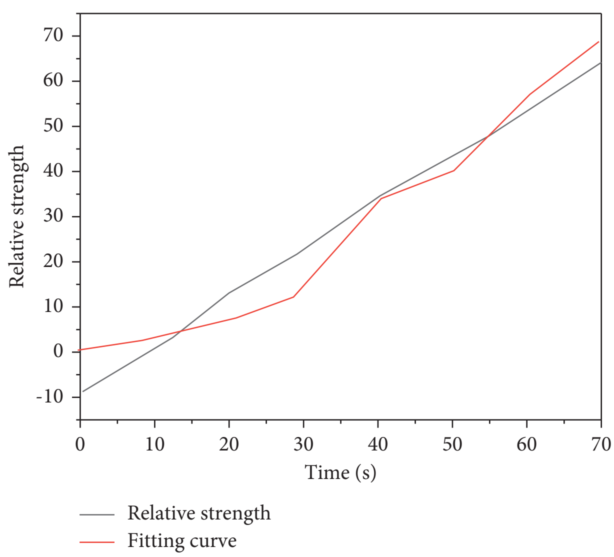

Figure 6: Curve and fitting curve of load-muscle strength.

time. In order to show the relationship between muscle strength and load more clearly [20], the relative muscle strength is normalized to the maximum load. For example: For object 5, normalize to 70, and then perform linear fitting on the normalized data. The result of object 5 is shown in Figure 6.

The calf heel lift test is used to obtain the spectrogram of the gastrocnemius muscle from exercise to fatigue. Through the measurement and statistics of the width of the muscle acoustic spectrum, the law of the change of the spectrum width of the muscle sound signal with the exercise time is obtained. It is found that in the early stage of exercise, the acoustic spectrum bandwidth of gastrocnemius muscle is wide [21–23], indicating that the muscle fibers participating in exercise are in the process of adapting to exercise load and rhythm, and various types of muscle fibers are mobilized to participate in exercise. In the middle of exercise, the muscle enters the adaptation period, the number of muscle fibers participating in exercise remain in a certain range, and the vibration spectrum is relatively stable. At this time, the spectrum bandwidth remains in a relatively stable range. In the later stage of exercise, with the gradual emergence of exercise fatigue, the ability of participating in exercise muscles gradually weakens, showing that the number of muscle fiber vibration is reduced and the spectrum components are reduced, so the muscle acoustic spectrum bandwidth is further reduced [24]. The LZ complexity is applied to verify the characteristics of the muscle acoustic signal changes, the results show that the complexity of the muscle acoustic signal is directly related to the frequency component of the signal, and the amount of frequency component produced is completely equivalent to the type of muscle fiber participating in the exercise. The more types of muscle fibers involved in exercise, the more abundant the frequency components of the spectrum and vice versa.

5. Conclusion

In this paper, the signal which reflects the muscle stress in the collected muscle acoustic signal is weighted by the method of wavelet packet weighting, and then the output of muscle force is calculated by the short-term energy. Wavelet packet weighting can not only provide the fine output state of muscle force at each moment, but also check the change of muscle force in the process of long-term exercise training by calculating the muscle force for a long time. Through the above analysis, the following conclusions can be drawn: in the early stage of exercise, the muscle responds to the load and prepares for it, and the muscle fibers need to be recruited in large quantities to deal with the sudden load exercise, which is quite complicated. With the extension of exercise time, muscles gradually adapt to the load state, and the speed of complexity gradually decreases, enter, and maintain. It remained in a relatively stable state, reflecting that the types of muscle fibers recruited during the adaptation period were relatively stable. At the late stage of exercise, the complexity gradually decreases again, indicating that the muscles have entered the fatigue state, and some types of muscle fibers can no longer participate in exercise, resulting in a continuous decrease in recruited muscle fibers.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Y. Wang, L. Cao, D. Hao et al., “Effects of force load, muscle fatigue and extremely low frequency magnetic stimulation on eeg signals during side arm lateral raise task,” *Physiological Measurement*, vol. 38, no. 5, pp. 745–758, 2017.

[2] H. Sarieddeen, M. M. Mansour, and A. Chehab, “Large mimo detection schemes based on channel puncturing: performance and complexity analysis,” *IEEE Transactions on Communications*, vol. 66, no. 6, pp. 2421–2436, 2018.

[3] B. Zhang and P. Shang, “Complexity and uncertainty analysis of financial stock markets based on entropy of scale exponential spectrum,” *Nonlinear Dynamics*, vol. 98, no. 3, pp. 2147–2170, 2019.

[4] P. Huang, K. Wang, D. Hou, J. Zhang, J. Yu, and G. Zhang, “In situ detection of water quality contamination events based on

signal complexity analysis using online ultraviolet-visible spectral sensor," *Applied Optics*, vol. 56, no. 22, pp. 6317–6323, 2017.

[5] Y. Kim, T. C. Bulea, and H.-S. Park, "Transcutaneous high-frequency alternating current for rapid reversible muscle force reduction below pain threshold," *Journal of Neural Engineering*, vol. 16, no. 6, pp. 066013.1–066013.10, 2019.

[6] L. Filli, D. Kenkel, M. C. Wurnig, and A. Boss, "Diffusional kurtosis mri of the lower leg: changes caused by passive muscle elongation and shortening," *Nmr in Biomedicine*, vol. 29, no. 6, pp. 767–775, 2016.

[7] Y. Sun and Z. Xiong, "An optimal weighted wavelet packet entropy method with application to real-time chatter detection," *IEEE*, vol. 21, no. 4, pp. 2004–2014, 2016.

[8] P. Aslani, S. D. Sommerfeldt, and J. D. Blotter, "Active control of simply supported cylindrical shells using the weighted sum of spatial gradients control metric," *Journal of the Acoustical Society of America*, vol. 143, no. 1, pp. 271–280, 2018.

[9] T. Kurosawa, K. Taniguchi, H. Momose, M. Sakaguchi, M. Kamijo, and A. Nishikawa, "Simultaneous measurement of ear canal movement, electromyography of the masseter muscle and occlusal force for earphone-type occlusal force estimation device development," *Sensors*, vol. 19, no. 15, p. 3441, 2019.

[10] B. Das, S. Pal, and S. Bag, "A combined wavelet packet and hilbert-huang transform for defect detection and modelling of weld strength in friction stir welding process," *Journal of Manufacturing Processes*, vol. 22, pp. 260–268, 2016.

[11] V. Moosavi and Y. Niazi, "Development of hybrid wavelet packet-statistical models (wp-sm) for landslide susceptibility mapping," *Landslides*, vol. 13, no. 1, pp. 97–114, 2016.

[12] M. Zhao, M. Kang, B. Tang, and M. Pecht, "Deep residual networks with dynamically weighted wavelet coefficients for fault diagnosis of planetary gearboxes," *IEEE Transactions on Industrial Electronics*, vol. 65, no. 5, pp. 4290–4300, 2018.

[13] H. T. Do, Z. Xing, N. V. Nguyen, S. S. Li, and T. T. Chu, "Passive-islanding detection method using the wavelet packet transform in grid-connected photovoltaic systems," *IEEE Transactions on Power Electronics*, vol. 31, no. 10, p. 1, 2016.

[14] M. Schimmack and P. Mercorelli, "A structural property of the wavelet packet transform method to localise incoherency of a signal," *Journal of the Franklin Institute*, vol. 356, no. 16, pp. 10123–10137, 2019.

[15] F. S. Hasan, M. J. Zaiter, and R. A. Mohammed, "Design and analysis of a wavelet packet modulation based differential chaos shift keying communication system," *Wireless Personal Communications*, vol. 109, no. 4, pp. 2439–2450, 2019.

[16] J. P. Hoyos Sanchez and P. E. Jojoa Gomez, "Design of dynamic wavelet packet division multiplexing," *IEEE Latin America Transactions*, vol. 15, no. 10, pp. 1834–1839, 2017.

[17] C. N. Efrem and A. D. Panagopoulos, "A framework for weighted-sum energy efficiency maximization in wireless networks," *IEEE Wireless Communications Letters*, vol. 8, no. 1, pp. 153–156, 2019.

[18] S. W. Lin, C. C. Lu, and K. C. Ying, "Minimizing the sum of makespan and total weighted tardiness in a no-wait flowshop," *IEEE Access*, vol. 6, no. 99, p. 1, 2018.

[19] R. Rudek, "The single machine total weighted completion time scheduling problem with the sum-of-processing time based models: strongly np-hard," *Applied Mathematical Modelling*, vol. 50, pp. 314–332, 2017.

[20] R. M'Hallah and A. Alhajraf, "Ant colony systems for the single-machine total weighted earliness tardiness scheduling problem," *Journal of Scheduling*, vol. 19, no. 2, pp. 191–205, 2016.

[21] J.-y. Sohn, S. W. Yoon, and J. Moon, "Pilot reuse strategy maximizing the weighted-sum-rate in massive mimo systems," *IEEE Journal on Selected Areas in Communications*, vol. 35, no. 8, pp. 1728–1740, 2017.

[22] R. A. van Tuijl, C. Harnish, K. Matouš, J. J. C. Remmers, and M. G. D. Geers, "Wavelet based reduced order models for microstructural analyses," *Computational Mechanics*, vol. 63, no. 3, pp. 535–554, 2019.

[23] J. Holm, T. Arildsen, M. Nielsen, and S. L. Nielsen, "Orthonormal, moment preserving boundary wavelet scaling functions in python," *SN Applied Sciences*, vol. 2, no. 12, pp. 2032–2039, 2020.

[24] P. Xie, X. Pang, S. Cheng et al., "Cross-frequency and iso-frequency estimation of functional corticomuscular coupling after stroke," *Cognitive Neurodynamics*, vol. 15, no. 3, pp. 439–451, 2021.