Research Article

Effect of Unit Hydrographs and Rainfall Hyetographs on Critical Rainfall Estimates of Flash Flood

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To obtain critical rainfall (CR) estimates similar to the rainfall value that causes minor basin outlet flooding, and to reduce the flash flood warning missed/false alarm rate, the effect of unit hydrographs (UHs) and rainfall hyetographs on computed threshold rainfall (TR) values was investigated. The Tanjia River basin which is a headwater subbasin of the Greater Huai River basin in China was selected as study basin. Xin'anjiang Model, with subbasins as computation units, was constructed, and time-variant distributed unit hydrographs (TVUHs) were used to route the channel network concentration. Calibrated Xin'anjiang Model was employed to derive the TVUHs and to obtain the maximum critical rainfall duration ($D_{\max}$) of the study basin. Initial soil moisture condition was represented by the antecedent precipitation index (Pa). Rainfall hyetographs characterized by linearly increasing, linearly decreasing, and uniform hyetographs were used. Different combinations of the three hyetographs and UHs including TVUHs and time-invariant unit hydrographs (TIVUHs) were utilized as input to the calibrated Xin'anjiang Model to compute the relationships between TR and Pa (TR-Pa curves) by using trial and error methodology. The computed TR-Pa curves reveal that, for given Pa and UH, the TR corresponding to linearly increasing hyetograph is the minimum one. So, the linearly increasing hyetograph is the optimum hyetograph type for estimating CR. In the linearly increasing hyetograph context, a comparison was performed between TR-Pa curves computed from different UHs. The results show that TR values for different TIVUHs are significantly different and the TR-Pa curve gradient of TVUHs is lower than that of TIVUHs. It is observed that CR corresponds to the combination of linearly increasing hyetograph and TVUHs. The relationship between CR and Pa (CR-Pa curves) and that between CR and duration (D) (CR- D curves) were computed. Warnings for 12 historical flood events were performed. Warning results show that the success rate was 91.67% and that the critical success index (CSI) was 0.91. It is concluded that the combination of linearly increasing hyetograph and TVUHs can provide the CR estimate similar to the minimum rainfall value necessary to cause flash flooding.

1. Introduction

Flooding is the worst weather-related hazard, causing loss of life and excessive property damage [1–3]. In general, flash floods are characterized by their rapid onset, leaving very limited effective response opportunities [3–5]. Flood damage mitigation is provided through a variety of structural and nonstructural methods. A significant nonstructural method is the operation of flood warning systems [1].

Currently, three criteria are used for an expected flooding determination: critical discharge, critical runoff, and critical rainfall (CR). Critical rainfall criterion is used by most flood warning systems [3, 6–9].

Given an initial soil moisture condition and a rainfall duration (D), different hyetographs show the diverse areal rainfall volumes over the study basin necessary to cause minor basin outlet flooding which is defined as threshold rainfall (TR), and the minimum of these TR values is referred to as CR. That is to say, TR is a function of initial soil moisture condition, rainfall duration, and the form of rainfall hyetograph, but CR is a function of only initial soil moisture condition and rainfall duration.

By comparing real-time observed or predicted rainfall volume of a given duration to the CR value, the CR-based flood warning systems decide whether to issue a warning. For early warning, the consequences of under- or

overestimating the CR value are extremely different. Adopting a CR value higher than the rainfall volume that actually produces flood damage leads to missing such events and failure to issue an alarm. Underestimating the CR may instead determine the issue of false alarms [10]. For flood warning systems development, it is important to obtain as accurate as possible CR estimates. The most significant way to reduce missed/false alarm rate is to have the CR estimates be comparable to the minimum rainfall value necessary to cause flooding.

The false warning costs are commonly not only much lower than the avoidable flood loss, but also cannot match up to indirect and/or intangible flood damages such as serious injury or loss of life [11, 12]. So considering that an error will always be present, it is better to underpredict rather than overpredict the CR estimate for safety reasons [10].

Currently, there are three CR value computation methods: inverse, positive, and empirical. The Flash Flood Guidance (FFG) system [6] is representative of an inverse method. The computational process of FFG is divided into three steps. First, the critical discharge value is estimated. In the second step, threshold runoff estimates for various rainfall durations are obtained based on critical discharge and unit hydrograph (UH) peak, which belongs to runoff concentration computation of hydrology. In the third step, the FFGs are obtained based on threshold runoff values where the rainfall vs. runoff curves as a function of soil moisture conditions are needed [13], which belongs to runoff generation computation of hydrology. A significant disadvantage of FFG is that uniform rainfall over rainfall duration is presumed [14].

The empirical methods are based on historical rainfall and streamflow data [15]. Miao et al. [15] proposed an empirical method to determine TR value by using a linear binary classification based on long-term historical rainfall and flood data. Enough flood event data are necessary to derive the binary classification. So this method cannot be implemented in ungauged basins.

The positive methods based on a watershed hydrological model estimate the TR values from critical discharge by trial and error [16, 17]. The hydrological responses of different cumulative rainfall values, for fixed duration, initial soil moisture condition, and hyetograph type, are simulated by calibrated watershed hydrological model. The cumulative rainfall value generating the critical discharge is taken as the TR estimate. The computation process of the positive method has an explicit hydrology theoretical basis, so the disadvantages inherent in the inverse method may be overcome. In this study, the calibrated Xin'anjiang Model with its subbasins being used as computation units [18] is employed to compute TR and CR values.

Montesarchio et al. [3] estimated TR values for the Mignone River cross section using an entropy-based decision approach and a simulation approach based on radar data and rain gauge data. Results show that the TR values computed using various methods are obviously different and that, for the fixed watershed hydrological model, the type of rainfall data source used for model calibration significantly affects the TR estimates.

According to hydrological rainfall-runoff formation theory, for a fixed computing method, the TR estimate is generally a function of initial soil moisture condition, rainfall duration, and hyetograph. The effect of initial soil moisture conditions has been taken into account in the vast majority of current methods [6, 7, 16, 17, 19, 20]. In [6], the rainfall vs. runoff curves were taken as a function of initial soil moisture content to take into account the effect of initial soil moisture conditions on TR estimates. In [3, 7, 16], initial soil moisture conditions were classified into antecedent moisture classes AMC I, AMC II, and AMC III, representing dry soil, moderately saturated soil, and wet soil, according to the total amount of accumulative rain. In [17], the initial soil moisture conditions were taken into account by imposing an initial discharge value of the watershed hydrological model, and the effect of different initial conditions is analyzed by varying the initial discharge in the model simulations. In [19], a probability distributed moisture model was used to estimate the soil moisture content as the initial soil moisture condition of a rainfall-runoff model employed to compute TR values. In [20], both AMC and antecedent precipitation index (API) were utilized to estimate the initial soil conditions.

Given an initial soil moisture condition, for the same rainfall volume, the hydrographs and peak discharge rates may be significantly different when different rainfall hyetographs are adopted [16, 17, 21]. Consequently, for the same initial soil moisture conditions, various hyetograph types result in different TR values [16, 17]. In [17, 21], three synthetic hyetograph types characterized by linearly increasing intensity, decreasing intensity, and linearly increasing-decreasing intensity were employed to investigate the effect of rainfall hyetograph type on the hydrograph at given rainfall volume. In [16], four standards hyetographs including step hyetograph, triangular increasing rate hyetograph, triangular decreasing rate hyetograph, and isosceles triangular hyetograph were used to analyze the effect of rain type on TR estimates. If the hyetograph type corresponding to the minimum rainfall necessary to cause flooding is found, it can be used to directly compute CR value. So determining the hyetograph type corresponding to the minimum rainfall to cause flooding is one objective of this work.

No matter whether an inverse or positive method is used, TR values are always computed by routing surface runoff using the UH method [1, 6, 17, 19, 22]. So deriving the UHs representing the true basin concentration characteristics is a key to calculating the CR estimate matching the minor rainfall value necessary to cause flooding. For more than 75 years since the inception of UH theory was presented by Sherman, it is still one of the most widely used methods for flood prediction and warning system development in gauged basins with observed rainfall and runoff data, but this data-driven traditional approach limits the UH derivation only to gauged watersheds. Synthetic UHs may only be used in basins whose hydrographs have a single peak [23–25]. Geomorphologic UHs, regardless of time-invariant (TIVUH) [26–31] or time-variant (TVUH) [32, 33], do not take the dynamic factor (flow velocity) spatial distribution

into account. Distributed UHs based on a spatially distributed velocity field can adequately take the nonuniformity of basin characteristics into account [34–36]. Formulas defined as a function of rainfall intensity are adopted to compute spatially distributed velocity fields so as to derive TVUHs [18, 37, 38] that can solve to a certain extent the nonlinear problem of runoff concentration.

For a fixed runoff generation computing method, different rainfall-runoff transformation methods may lead to different TR estimates. By doing this work, investigating the effect of UHs on TR estimates and suggesting a reasonable UH used in computing CR value is another study objective.

In this study, three rainfall hyetograph types (linearly increasing, linearly decreasing, and uniform) and two UH types (TIVUHs and TVUHs) are used to investigate the effects of hyetographs and UHs on TR/CR estimates and warning results. The objectives are (1) to explain, for fixed duration and initial conditions, that rainfall hyetographs and UHs significantly affect the TR estimates; (2) to suggest that the rainfall hyetograph type leading to minimum TR estimate and the UH resulting in optimal simulation results should be adopted to compute CR estimates; and (3) to propose a method for CR computation.

Determining TR and CR value is a hydrological problem. The uncertainties of TR and CR estimates related principally to the method (including runoff generation and runoff concentration), parameters, data sources (including rainfall and discharge), and adopted rainfall hyetograph types [3, 16, 17, 21]. In this work, the method based on watershed hydrological model was used. For the fixed study watershed and data sources (observed data), in order to obtain the CR estimate approximate as far as possible with the minimum rainfall value necessary to cause flooding, the appropriate model (Xin'anjiang Model and UHs) and opportunely calibrated parameters were employed. For nonlinear types, no matter increasing or decreasing, the forms of rainfall hyetograph are innumerable. In this study, the linearly change hyetograph types were used. In fact, the rainfall hyetograph corresponding to the minimum rainfall value causing flooding is nonlinear.

This paper consists of six sections: Section 2 introduces the methods used in the study including the watershed hydrological model structure, TR and CR computation methods, and warning system assessment method. Section 3 describes the study basin and model calibration. Section 4 introduces the computation process and results of CR. In Section 5, the application of CR values in warnings historical flood event is performed. Finally, some conclusions are presented in Section 6.

2. Methods

2.1. Watershed Hydrological Model

2.1.1. Structure of Model. The Xin'anjiang Model is employed in this study and the basin is divided into a series of subbasins as computation units. Runoff generation and flow concentration computations are performed within the

subbasins and the runoff from each subbasin is routed to the main basin outlet. The total hydrograph at the main basin outlet is equal to the sum of all subbasin hydrographs.

2.1.2. Rainfall Computation Component. The Kriging interpolation method is used to derive rainfall depth of all cells within subbasins from that of rainfall gauges. Mean areal rainfall values of each subbasin are computed by arithmetic mean method using rainfall depth for all cells within the subbasin.

2.1.3. Runoff Generation Computation Component. A tension water storage capacity curve, also named a parabolic curve, is used to represent the nonuniform distribution of tension water storage capacity and to compute the runoff value. The computational equations are as follows:

$$a = WMM \left[1 - \left(1 - \frac{W}{WM} \right)^{1/1+b} \right], \quad (1)$$

when $a + PE \leq WMM$,

$$R = PE + W - WM + WM \left(1 - \frac{PE + a}{WMM} \right)^{1/1+b}, \quad (2)$$

when $a + PE \geq WMM$,

$$R = PE + W - WM, \quad (3)$$

where PE is the rainfall value, mm; R is the runoff value, mm; W is the mean areal tension water storage, mm; WM is the mean areal tension water storage capacity, mm; WMM is the maximum tension water storage capacity of the watershed, mm; and b is the tension water capacity distribution curve exponent (parabolic curve).

A free water storage capacity curve is used to represent the nonuniform distribution of free water storage capacity over runoff-producing areas and to separate runoff R into surface flow, interflow, and groundwater. Computational equations are as follows:

$$AU = SMM \left[1 - \left(1 - \frac{S}{SM} \right)^{1/1+EX} \right], \quad (4)$$

when $PE + AU \leq SMM$,

$$RS = \left\{ PE - SM + S + SM \left[1 - \frac{(PE + AU)}{SMM} \right]^{1+EX} \right\} FR, \quad (5)$$

$$RI = (PE + S - RS)KI \cdot FR, \quad (6)$$

$$RG = (PE + S - RS)KG \cdot FR,$$

when $PE + AU \geq SMM$,

$$RS = (PE + S - SM)FR, \quad (7)$$

$$\begin{aligned} \text{RI} &= \text{SM} \cdot \text{KI} \cdot \text{FR}, \\ \text{RG} &= \text{SM} \cdot \text{KG} \cdot \text{FR}, \end{aligned} \quad (8)$$

where PE is the rainfall value (equal to excess rainfall R in the runoff-producing area), mm; RS, RI, and RG are surface flow, interflow, and groundwater, respectively, mm; S is the equivalent free water storage over the runoff-producing area, mm; SM is the area mean free water storage capacity (maximum possible deficit of free water storage), mm; EX is the free water storage capacity curve exponent (parabolic curve); SMM is the maximum watershed free water storage capacity, mm; FR is runoff-producing area; KI is the outflow coefficient of free water storage to interflow; and KG is the outflow coefficient of free water storage to groundwater.

Estimating initial soil moisture conditions is necessary for runoff generation computation. In this study, the antecedent precipitation index (Pa), used in the past as a moisture content indicator, is used and calculated at the beginning of every storm event as follows:

$$P_{a,i+1} = P_{i+1} + kP_{a,i}, \quad (9)$$

where $P_{a,i}$ is the antecedent precipitation index, mm; P is daily rainfall, mm; i is day of estimate; and k is depletion constant (in this study, $k = 0.83$ [39]).

Because the tension water storage capacity curve is a parabolic type, the relationship between Pa and W is as follows:

$$P_a = \text{WMM} \left[1 - \left(1 - \frac{W}{\text{WM}} \right)^{1/1+b} \right]. \quad (10)$$

2.1.4. Concentration Computation Components

(1). *Hillside concentration.* The surface flow passing directly into the channel systems is treated as TRS, and the interflow RI and groundwater RG are routed through linear reservoirs into channel systems as TRI and TRG. The computational equations are as follows:

$$\begin{aligned} \text{TRS}(t) &= \text{RS}(t) \cdot U, \\ \text{TRI}(t) &= \text{TRI}(t-1) \cdot \text{CI} + \text{RI}(t) \cdot (1 - \text{CI}) \cdot U, \\ \text{TRG}(t) &= \text{TRG}(t-1) \cdot \text{CG} + \text{RG}(t) \cdot (1 - \text{CG}) \cdot U, \\ \text{TTR}(t) &= \text{TRS}(t) + \text{TRI}(t) + \text{TRG}(t), \end{aligned} \quad (11)$$

where TRS, TRI, and TRG are inflow into channel systems of surface flow, interflow, and groundwater, respectively, m^3/s ; TTR is the total inflow, m^3/s ; U is the transformation coefficient transforming runoff depth to discharge rate, $U = F/(3.6\Delta t)$, where F is the drainage area, km^2 , and Δt is computational time interval, h; CI is the recession constant of interflow storage; CG is the recession constant of groundwater storage.

(2). *Channel network concentration.* The channel network routing within a subbasin is represented by convolution of TTR(t) with a dimensionless UH as follows:

$$Q(t) = \sum_{i=1}^N \text{UH}(i) \cdot \text{TTR}(t - i + 1), \quad (12)$$

where $Q(t)$ is the subbasin outlet discharge rate, m^3/s ; UH is the ordinate of dimensionless unit hydrograph; and N is the number of dimensionless UH time intervals. The method presented by Kong et al. [18] is used to derive the TVUHs of each subbasin.

Given a velocity within a cell of DEM, the travel time through the cell is computed as follows:

$$\begin{aligned} \tau_k &= \frac{L}{V_k}, \\ \text{or } \tau_k &= \frac{\sqrt{2}L}{V_k}, \end{aligned} \quad (13)$$

where τ_k is the travel time within cell k , s; L is cell size, m; and V_k is the velocity within cell k , m/s. The travel time of a cell to subbasin outlet is computed as follows:

$$T_j = \sum_{k=1}^m \tau_k \quad (14)$$

where T_j is the travel time of cell j to the subbasin outlet, s; m is the number of cells along the drainage network to subbasin outlet; τ_k is the travel time within cell k , s.

Based on the travel time of all cells to the subbasin outlet, the S-hydrograph of dimensionless UH can be obtained as follows:

$$S_t = \frac{1}{F} \sum_{j=1}^n A_j, \quad (T_j \leq t), \quad (15)$$

where S_t is the ordinate value of S-hydrograph at time t ; F is the subbasin area, km^2 ; A_j is the area of cell j , km^2 ; and n is the number of cells whose T being less than or equal to t . The ordinate of dimensionless UH is computed as follows:

$$\text{UH}(t) = S_t - S_{t-\Delta t}, \quad (16)$$

where Δt is computational time interval, h.

It can be seen that the key to obtain UH is deriving the velocity of each cell. The fundamental equation form used to compute velocity is as follows:

$$V = kS^{0.5}, \quad (17)$$

where k is a coefficient based on the flow type; Sorrell et al. [40] provide values of k for several flow types; and S is the flow path slope.

To take the effect of excess rainfall intensity on velocity into account, equation (17) is modified as follows:

$$V = kS^{0.5} \left(\frac{i}{i_0} \right)^c, \quad (18)$$

where i is the excess rainfall intensity, mm/h; i_0 is the excess rainfall intensity corresponding to the coefficient k , determined by calibration; and c is a parameter, on the basis of literature [37, 38], with 0.4 being adopted.

Equation (18) can not only take into account the effect of excess rainfall intensity on velocity, but also can employ the values of k provided by Sorrell et al. [40].

By using equation (18), the time-variant spatially distributed velocity fields and TVUHs of different rainfall duration in a rainfall event are derived.

2.1.5. Channel Routing Method. The dynamic Muskingum method presented by Tewolde et al. [41] is used to route channel flow from subbasin outlets to the basin outlet.

2.2. CR Computation Method

2.2.1. TR Computation. TR estimate (mean areal rainfall value) depends on not only D and Pa , but also rainfall hyetograph and computation methods (runoff concentration). For a given Pa , the TR value of D is not unique. A primary goal of this study is to investigate the influence of rainfall hyetographs and runoff concentration computation methods on the TR estimates. Using the watershed hydrological model above, TR values are computed by trial and error.

For a given D (taken as an integer multiple of Δt), Pa , and a combination of hyetograph type and UH, computation steps of TR value are as follows: (1) D is divided into n time intervals; (2) according to the hyetograph, the proportion of each time interval rainfall to total rainfall of D (P) is determined; (3) trial and error: for a given P , running hydrological model to derive discharge hydrograph and comparing peak rate and critical discharge value. The P making the computed peak rate equal or close to critical discharge value is taken as the TR value.

2.2.2. CR Computation. CR is related only to D and Pa . For a given Pa and D combination, the minimum of all TR values is taken as the CR value. The relationship between CR and Pa of different D and the relationship between CR and D of different Pa are obtained, being the basis of flash flood warning.

2.3. Representation of Warning Results. The contingency table (Table 1) shows four possible results for a single flash flood warning, X denotes an event occurred and warning was issued (hits), Y denotes an event occurred but warning was not issued (missed events), Z denotes an event did not occur but warning was issued (false alarms), and W denotes an event did not occur and warning was not issued.

2.4. Assessment of a Warning System. Table 2 shows the statistics of flash flood warning results, and N_X , N_Y , N_Z , and N_W represent the number of X, Y, Z, and W, respectively.

TABLE 1: Two-by-two contingency table of the warning results for an event.

Observations	Forecasts	
	Warning	No warning
Event	X	Y
Nonevent	Z	W

TABLE 2: Two-by-two contingency table for the assessment of a warning system.

Observations	Forecasts	
	Warning	No warning
Event	N_X	N_Y
Nonevent	N_Z	N_W

Four indicators indicating the quality of warning results [2, 19, 42] are used as follows:

$$\begin{aligned}
 \text{HAR} &= \frac{N_X}{N_X + N_Y}, \\
 \text{MAR} &= \frac{N_Y}{N_X + N_Y}, \\
 \text{FAR} &= \frac{N_Z}{N_X + N_Z}, \\
 \text{CSI} &= \frac{N_X}{N_X + N_Y + N_Z},
 \end{aligned} \tag{19}$$

where HAR is the hit rate; MAR is the missed alarm rate; FAR is the false alarm rate; and CSI is a comprehensive critical success index reflecting the quality of warning results and its value goes from 0 to 1. The larger the CSI value is, the better the quality is.

3. Study Area and Model

3.1. Basin and Data. The study area is Tanjia River basin located upstream from the Huai River basin in China with a catchment area of 173 km² and a mean slope of 0.30. There are five rainfall gauges within the basin and a station (Tanjiahe station) at the basin outlet as shown in Figure 1. Using DEM as a data source, ArcGIS is used to derive each cell's slope and flow direction, divide subbasins, and compute subbasin areas. DEM resolution is 30 m × 30 m. Data from 1954 to 2001 provided by Henan Hydrology Bureau are used in this study.

3.2. Model Calibration. In this study, Δt is 0.5 h. Net rainfall value of Δt for TVUHs is 5 mm, 10 mm, 15 mm, 20 mm, 25 mm, and 30 mm, with UHs corresponding to them being UH_{5 mm}, UH_{10 mm}, UH_{15 mm}, UH_{20 mm}, UH_{25 mm}, and UH_{30 mm}, respectively. In the hydrograph simulation, according to the net rainfall value calculated by runoff generation in each Δt , the corresponding UH is selected based on Table 3. The calibrated hydrological model parameter values

FIGURE 1: Map of the study basin with (a) basin location, (b) DEM and rainfall gauges, and (c) subbasins.

TABLE 3: Excess rainfall ranges for TVUHs.

i (mm/h)	$0 < i \leq 15$	$15 < i \leq 25$	$25 < i \leq 35$	$35 < i \leq 45$	$45 < i \leq 55$	$55 < i$
UH	UH _{5mm}	UH _{10mm}	UH _{15mm}	UH _{20mm}	UH _{25mm}	UH _{30mm}

TABLE 4: Parameters of Xin'anjiang Model.

WM (mm)	WUM (mm)	WLM (mm)	B	C	SM (mm)	EX	KI	KG	CI	CG	i_0 (mm/h)
100	20	60	0.3	0.13	30	1.3	0.3	0.4	0.1	0.1	40

have been given by Kong et al. [18] as shown in Table 4. Modeling results show that when TIVUHs are used, the best simulation results can be obtained from UH_{20mm}.

4. CR Computation

4.1. Critical Discharge Determination. It is suggested that two-year return period flow can be used as critical discharge [1, 14, 43–46], so in this study, the two-year return period peak discharge is taken as the critical discharge. According to frequency calculation based on historical hydrological data, the calculated two-year return period peak discharge is 350 m³/s, meaning the critical discharge is 350 m³/s.

4.2. Critical Rainfall Time Duration

4.2.1. Maximum Critical Rainfall Time Duration (D_{max}).

According to rainfall-runoff formation theory, the D_{max} is equal to basin concentration time. When a TIVUH is adopted, the time base of UH can be used as D_{max} . When TVUHs are adopted, because of different net rainfall intensities corresponding to different UH time bases, the UH time base corresponding to the smallest net rainfall intensity can be taken as D_{max} .

Only the rainfall of one Δt with antecedent soil water content being saturated is used as the input to the watershed hydrological model. The hydrographs for six rainfall values of 5 mm, 10 mm, 15 mm, 20 mm, 25 mm, and 30 mm are computed, respectively, as illustrated in Figure 2 (ordinate value is the ratio of discharge at each time to peak rate). Based on the linear assumption of the UH method, the time base of each hydrograph is equal to that of the UH corresponding to the same rainfall value, for example, the time base of the hydrograph for 15 mm is equal to that of UH_{15mm}. Because the time base of UH_{5mm}

FIGURE 2: Hydrographs of different rainfall values.

(24 h) is the maximum one of six UHs, it is taken as the $D_{\max}$ of the study basin.

4.2.2. Other Critical Rainfall Time Duration. In the range of 0 to $D_{\max}$, the integral times of Δt can be taken as the critical rainfall time duration D , such as 1 h, 1.5 h, 2 h, 2.5 h, $\dots$, $D_{\max}$. The relationships of TR-Pa and CR-Pa are analyzed by taking $D = 5$ h as an example in the remaining of this article.

4.3. Computation Results of TR

4.3.1. TR for Uniform Rate Hyetograph

(1) *TR for TIVUHs.* $UH_{5\text{ mm}}$, $UH_{10\text{ mm}}$, $UH_{20\text{ mm}}$, and $UH_{30\text{ mm}}$ are used to analyze the relationship between TR and Pa. The TR-Pa curves of four UHs are illustrated in Figure 3. The computed TR value is small when a UH with a large peak (such as $UH_{30\text{ mm}}$) is used at each Δt , and the computed TR value is large when UH with a small peak (such as $UH_{5\text{ mm}}$) is used at each Δt . It is concluded that UHs influence significantly the TR computational results.

(2) *TR for TVUHs.* The derived TR-Pa curve for TVUHs is illustrated in Figure 3. It can be seen that when Pa is 130 mm (antecedent soil water content being saturated), the TR value of TVUHs is equal to that of $UH_{10\text{ mm}}$, meaning $UH_{10\text{ mm}}$ is used for all Δt . When Pa decreases (for example to 40 mm), the net rainfall values of initial time intervals become less than 7.5 mm because of rainfall loss and $UH_{5\text{ mm}}$ is used, so that TR value of TVUHs is larger than that computed using $UH_{10\text{ mm}}$ at all Δt .

4.3.2. TR for Nonuniform Hyetograph

(1) *Hyetograph types.* For a given combination of Pa and D , the reason a minor rainfall value is necessary to cause flooding is that, for the same rainfall volume, different hyetograph types form different hydrographs and peak

FIGURE 3: TR-Pa curves ($D = 5$ h).

values. Consequently, to form the same peak value, different hyetograph types require different rainfall values. Therefore, for a given combination of Pa and D , different hyetograph types have different TR values. The calculation of CR value is to determine the minimum TR value. If the rainfall hyetograph corresponding to the CR can be determined, the watershed hydrological model can be used to derive directly the CR value by trial and error method.

Three synthetic hyetograph types characterized by linearly increasing rate, linearly decreasing rate, and uniform rate are taken into account in this study. For a linearly increasing hyetograph, the ratios of each time interval rainfall value to total rainfall value are calculated as shown in Figure 4 and Table 5 (only that of $D = 1-5$ h is listed).

(2) *TR for TIVUHs.* $UH_{5\text{ mm}}$ and $UH_{30\text{ mm}}$ are selected to compute the TR-Pa curves of three hyetographs as shown in Figures 5(a) and 5(b), respectively. The effect of Pa on rainfall loss of initial rainfall time intervals is obvious. From the maximum net rainfall intensity point of view, the last time interval of linearly increasing hyetograph is the largest of the three hyetograph types. Therefore, for the same UH, whatever the Pa value, the TR value of increasing hyetograph is the smallest, being consistent with previous research results [16, 17].

(3) *TR for TVUHs.* Using TVUH, TR-Pa curves of three hyetographs are computed as shown in Figure 6. The TR value corresponding to the increasing hyetograph is the smallest. Results show that the TR value for increasing hyetograph is the smallest whether TVUHs or TIVUHs are used.

(4) *Comparison of TR values for different UHs.* For the increasing hyetograph, TR-Pa curves of different UHs are derived as shown in Figure 7. For the same Pa, TR value decreases with the increase of UH peak rate. The gradient of TR with Pa of TVUHs is smaller than that of TIVUHs.

FIGURE 4: Distribution of linearly increasing hyetograph.

TABLE 5: Distribution of linearly increasing hyetograph.

No. of Δt	D (h)								
	1	1.5	2	2.5	3	3.5	4	4.5	5
1	1/4	1/9	1/16	1/25	1/36	1/49	1/64	1/81	1/100
2	3/4	1/3	3/16	3/25	1/12	3/49	1/21	1/27	3/100
3		5/9	5/16	1/5	5/36	5/49	5/64	5/81	1/20
4			7/16	7/25	7/36	1/7	7/64	7/81	7/100
5				9/25	1/4	9/49	9/64	1/9	9/100
6					11/36	11/49	11/64	11/81	11/100
7						13/49	13/64	13/81	13/100
8							15/64	5/27	3/20
9								17/81	17/100
10									19/100
Σ	1	1	1	1	1	1	1	1	1

FIGURE 5: TR-Pa curves of (a) UH_{5mm} and (b) UH_{30mm}.

According to the literature [18], better simulation results can be obtained when the TVUH method is used. Ultimately, the combination of TVUHs and increasing hyetograph is used to compute the TR value which can be taken as the CR value.

4.4. Computation Results of CR Value

4.4.1. CR-Pa Curves. The relationships between CR value and Pa (CR-Pa curves) for different durations (only 1–7 h) are derived as shown in Figure 8.

FIGURE 6: TR-Pa curves of TVUHs for different hyetographs.

FIGURE 7: TR-Pa curves of different UHs.

4.4.2. CR-D Curves. Due to the obvious influence of Pa on CR value, the relationships between CR and D (CR-D curves) are derived, respectively, for Pa being 5 mm, 10 mm, 15 mm, ..., 130 mm as shown in Figure 9 which can be used for real-time flash flood warning.

5. Application of CR Values

5.1. Warning Process. In the process of real-time flash flood warning, the first step is to compute Pa value and choose the CR-D curve. The second step is to compute accumulated rainfall P1 of D1 and compare it with the CR of D1 beginning from the smallest time duration (e.g., D1 = 1 h). If P1 is larger than or equal to CR, a warning should be issued. Otherwise, accumulated rainfall P2 of the next time duration D2 (D2 = 1.5 h) is computed and the comparison between P2 and the CR of D2 is performed to decide whether a warning should be issued.

5.2. Analysis of Warning Results. In this study, 12 historical flood events in Tanjia River basin (including 11 events

whose peak discharge is greater than critical discharge and 1 event whose peak discharge is less than but close to critical discharge) are selected. Pa computation results for each rainfall event are shown in Table 6. According to the Pa of each event, the CR-D relationship is selected. The warning process and warning results are shown in Figure 10 and Table 6. The warning result statistics are shown in Table 7.

On the basis of Table 7, four index values are as follows:

$$\text{HAR} = \frac{N_X}{N_X + N_Y} = \frac{10}{10 + 1} = 91\%,$$

$$\text{MAR} = \frac{N_Y}{N_X + N_Y} = \frac{1}{10 + 1} = 9\%,$$

(20)

$$\text{FAR} = \frac{N_Z}{N_X + N_Z} = \frac{0}{10 + 0} = 0,$$

$$\text{CSI} = \frac{N_X}{N_X + N_Y + N_Z} = \frac{10}{10 + 1 + 0} = 0.91.$$

It can be seen that 10 events can successfully be issued warnings before observed discharge exceeds critical discharge, the lead time ranges from 0.5 h to 2 h, and the hit rate is 91%. One event failed to alarm, for a missed alarm rate of 9%.

The observed peak discharge value of event 19830722 is 354 m³/s, exceeding critical discharge by only 1.1% and a missed alarm does not happen. The observed peak discharge of event 19980803 is 330 m³/s, the difference with critical discharge is only 5.7%, and false alarm did not happen.

The observed peak discharge of event 19890808 is 359 m³/s, exceeding critical discharge by only 1.1%, and a missed alarm event happens. According to rainfall depth measured at each rain gauge, it can be found that although the mean areal rainfall value is only 65 mm, there is a large

FIGURE 8: CR-Pa curves of different durations.

FIGURE 9: CR-D curves of different Pa.

single gauge rainfall value whose rainfall depth is 142 mm (Xindian gauge). A noteworthy disadvantage of CR-based warning method is that the mean areal rainfall is used and the spatial distribution of rainfall is not taken into account.

5.3. Effect of CR Values on Warning Results. Six combinations of UHs and hyetographs are selected as shown in Table 8. CR-Pa curves ($D = 5$ h) of the six combinations are illustrated in Figure 11. Three flood events including 19860715, 19920505, and 19820720 (rainfall values of 5 h and

TABLE 6: Warning results of 12 flood events.

Flood no.	Pa (mm)	D (h)	P (mm)	Warning	Lead time (h)	Results	Status
19820718	50	7	130	Warning	1	Hit	X
19820723	110	3.5	77	Warning	2	Hit	X
19830722	40	6.5	120	Warning	2	Hit	X
19860715	45	5	117	Warning	1	Hit	X
19860722	95	1.5	74	Warning	2	Hit	X
19870713	20	2.5	152	Warning	1.5	Hit	X
19890808	40	4	78	No warning		Missed	Y
19920505	50	5	110	Warning	0.5	Hit	X
19950512	50	3	103	Warning	2	Hit	X
19960717	85	4.5	87	Warning	1	Hit	X
19970630	65	5	113	Warning	1.5	Hit	X
19980803	55	6	103	No warning			W

FIGURE 10: Continued.

FIGURE 10: Continued.

FIGURE 10: Rainfall, critical discharge (CQ), hydrographs observed (Q_o), and comparison between CR and accumulated rainfall (P).

TABLE 7: Statistics of warning results.

Observations	Forecasts	
	Warning	No warning
Event	10	1
Nonevent	0	1

TABLE 9: Results of modeled warning.

Flood no.	Pa (mm)	C1	C2	C3	C4	C5	C6
19860715	45	Y	Y	Y	X	X	X
19920505	50	Y	Y	Y	X	X	X
19820720	130	W	W	W	Z	Z	W

TABLE 8: Combination of UHs and hyetographs.

Combination	Hyetograph	UH
C1	Uniform	UH _{10 mm}
C2	Increasing	UH _{10 mm}
C3	Uniform	UH _{20 mm}
C4	Increasing	UH _{20 mm}
C5	Uniform	TVUHs
C6	Increasing	TVUHs

FIGURE 11: CR-Pa curves of six combinations of UHs and hyetographs.

6 h before time to peak is 69 mm and 76 mm, respectively, with peak discharge being $296 \text{ m}^3/\text{s}$ are used. The modeled warning results of three flood events are shown in Table 9. It can be seen that only the combination of TVUHs and increasing hyetograph has no missed alarms or false alarms.

6. Conclusions

The aim of this work is to investigate the effect of UHs and rainfall hyetographs on TR/CR values of flash flood and to propose a computation method for CR estimates. Using Xin'anjiang Model, the relationships between TR and Pa (TR-Pa curves) of different UHs (TIVUHs and TVUHs) were derived for three rainfall hyetograph types (linearly increasing, linearly decreasing, and uniform). The effect of UHs and hyetographs on TR estimates was investigated. Six combinations of two hyetograph types (linearly increasing and uniform) and three UHs (UH_{10 mm}, UH_{20 mm}, and TVUHs) were used to investigate the effect of UHs and hyetographs on warning results. The combination of linearly increasing hyetograph and TVUHs was determined to be optimum for computing CR estimate. Relationships between CR values and durations (CR-D curves) of different initial soil moisture conditions were derived and used to model warning 12 events with a hit rate of 91% and missed alarm rate being 9%. Conclusions may be made as follows:

- (1) When other conditions are the same, the TR value corresponding to the linearly increasing hyetograph is the smallest. Because CR is the minimum rainfall value necessary to cause flooding, the increasing hyetograph corresponds to CR.
- (2) UH is the most widely accepted tool for routing surface runoff used in TR/CR value estimation. When TIVUHs are used to calculate TR value, the UH selection is very important. The larger the UH peak rate is, the smaller the corresponding TR value is. The smaller the UH peak value is, the larger the corresponding TR value is. So, when using TIVUH, the TR/CR estimate is obviously indeterminate. The

simulation result of historical flood events using TVUHs is better than that using TIVUHs. Therefore, it is proposed to use TVUHs to calculate CR value.

- (3) TR resulted from the combination of TVUHs and linearly increasing hyetograph conforms to the meaning of CR and the value is closer to the minimum rainfall value necessary to cause flooding. Therefore, the TR computed from the combination of TVUHs and linearly increasing hyetograph can be taken directly as the CR value.
- (4) Modeled warning results of 12 historical flood events show that the CR value computed from the combination of TVUHs and linearly increasing hyetograph can lead to near ideal flash flood warning results.

Data Availability

The precipitation and discharge data from 1954 to 2001 used in this research are provided by Henan Hydrology Bureau, China. DEM data in 30 m resolution can be freely accessed from CGIAR-CSI SRTM project on the website <http://srtm.csi.cgiar.org/>.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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