# The Rabi Oscillation in Subdynamic System for Quantum Computing 

Bi Qiao and Gu Jiayin<br>Department of Physics, Science School, Wuhan University of Technology, Wuhan 430070, China<br>Correspondence should be addressed to Bi Qiao; biqiao@gmail.com

Received 16 August 2015; Revised 3 October 2015; Accepted 5 October 2015
Academic Editor: Pavel Kurasov
Copyright © 2015 B. Qiao and G. Jiayin. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.


#### Abstract

A quantum computation for the Rabi oscillation based on quantum dots in the subdynamic system is presented. The working states of the original Rabi oscillation are transformed to the eigenvectors of subdynamic system. Then the dissipation and decoherence of the system are only shown in the change of the eigenvalues as phase errors since the eigenvectors are fixed. This allows both dissipation and decoherence controlling to be easier by only correcting relevant phase errors. This method can be extended to general quantum computation systems.


## 1. Introduction

The Rabi oscillation based on the quantum dot system is an efficient way to realize quantum logic operation [1]. The projected formalism of control rotation gate (CROT) for two qubits is expressed as negative CNOT operation [2]. The physical observation of the Rabi oscillation with $10 \pi$ period has been reported, and the detail analysis of quantum computation based on the Rabi oscillation and relevant mechanism of decoherence and dissipation have been discussed by several works with various methods [3-12]. However, the controlling dissipation and decoherence in the Rabi oscillation system is still an important problem to challenge application of the Rabi oscillation in quantum computation.

On the other hand, subdynamics theory rooted from the old Brussels-Austin school [13-15] and followed by some up-to-date works $[16,17]$ has been found that there is a potential advantage to cancel both decoherence and dissipation in quantum information systems. This is so because the relevant subdynamic kinetic equation (SKE) intertwines with the original Schrödinger equation or Liouville equation by a similarity transformation, which allows to construct the relative ideal subdynamic system in which the eigenvectors are fixed when decoherence or dissipation happens. Therefore the errors will only exist in the phase change of the evolution of projected states which makes easier to cancel the decoherence and dissipation.

Considering this background, in this work, we present a subdynamic proposal [13-20] to realize quantum computing based on the Rabi oscillation. This proposal can provide a relative ideal Rabi oscillation used in quantum computing by controlling both dissipation and decoherence. Below the formalism is firstly introduced.

## 2. Subdynamic Formalism

Let us consider a two-level system described by a Hamiltonian:

$$
\begin{equation*}
\widehat{H}(t)=\omega_{0} \sigma_{00}+\omega_{1} \sigma_{11}+\frac{1}{2} g\left(\sigma_{10} e^{-i \omega_{L} t}+\sigma_{01} e^{+i \omega_{L} t}\right) \tag{1}
\end{equation*}
$$

where $\omega_{0}$ and $\omega_{1}$ is an eigen-frequency of states $|0\rangle$ and $|1\rangle$, respectively; $\sigma_{00}=|0\rangle\langle 0|, \sigma_{11}=|1\rangle\langle 1|, \sigma_{01}=$ $|0\rangle\langle 1|$, and $\sigma_{10}=|1\rangle\langle 0| ; \omega_{L}$ is a frequency of exciting optical field; $g=-\mu E(0)$ is a Rabi frequency between states $|0\rangle$ and $|1\rangle$ jumping, $\mu$ is a dipole moment between $|0\rangle$ and $|1\rangle$ jumping, and $E(t)$ is an envelope of exciting optical field, $E(t)=E(0) \cos \omega_{L} t=(1 / 2) E(0)\left(e^{-i \omega_{L} t}+\right.$ $\left.e^{+i \omega_{L} t}\right)$. Notice the so-called rotation wave approximation has been used to produce above Hamiltonian. Then, for simplicity, here the explicitly time-dependent Hamiltonian can be transformed to the time-independent one by a timedependent unitary transformation $U_{00}(t)=\exp \left(-i \omega_{L} t \sigma_{00}\right)$,
where $\sigma_{00}=|0\rangle\langle 0|$ is a projector [21]. So we obtain a timeindependent Hamiltonian as

$$
\begin{align*}
H & =U_{00}^{-1}(t) \widehat{H}(t) U_{00}(t) \\
& =\omega_{0} \sigma_{00}+\omega_{1} \sigma_{11}+\frac{1}{2} g\left(\sigma_{01}+\sigma_{10}\right)=H_{0}+H_{1} \tag{2}
\end{align*}
$$

where defining

$$
\begin{align*}
& H_{0}=\omega_{0} \sigma_{00}+\omega_{1} \sigma_{11} \\
& H_{1}=\frac{1}{2} g\left(\sigma_{01}+\sigma_{10}\right) . \tag{3}
\end{align*}
$$

When the exciting optical field takes off, $U_{00}(t)=1$, we have $\widehat{H}(t)=H$. So, the corresponding Schrödinger equation for the Hamiltonian in (2) is

$$
\begin{equation*}
i \frac{\partial \psi}{\partial t}=H \psi \tag{4}
\end{equation*}
$$

which can reflect the same dynamics described by the Hamiltonian in (1).

If considering the influence of environment, the total Hamiltonian can be supposed as $H=H_{0}+H_{1}+\Delta H$, where $\Delta H$ comes from the interaction between the system and environment, which may introduce dissipation and decoherence in the system. Then a main frame to construct the subdynamic systems is, firstly to establish a subdynamic equation [13-19], as

$$
\begin{equation*}
i \frac{\partial}{\partial t} \widetilde{\psi}=\Theta \widetilde{\psi} \tag{5}
\end{equation*}
$$

where $\widetilde{\psi}$ is a projected state, $H+\Delta H=\Omega \Theta \Omega^{-1}$, and $\Omega$ is a similarity operator. These parameters and relations can be constructed by the following several rules.
(1) Propose projectors as

$$
\begin{align*}
& P_{n k}=\left|n \otimes \varphi_{k}\right\rangle\left\langle n \otimes \varphi_{k}\right|  \tag{6}\\
& \\
& \quad \text { for } n=0,1, k=0,1,2, \ldots,
\end{align*}
$$

and $Q_{n k}=1-P_{n k}$, where $n$ corresponds to the index of eigenvectors for Hamiltonian in system, $k$ denotes the index to the Hamiltonian in environment, and $\varphi_{k}$ is the $k$ th eigenvector of diagonal part of the interaction Hamiltonian between the system and environment.
(2) Introduce the creation (destruction) operator expressed as

$$
\begin{align*}
& C_{n k} \\
& =P_{n k} H P_{n k} \\
& \quad+P_{n k} \Delta H Q_{n k} \frac{1}{E_{n k}-Q_{n k}(H+\Delta H) Q_{n k}} Q_{n k} \Delta H P_{n k},  \tag{7}\\
& \\
& C_{n k}^{\dagger}=
\end{align*}
$$

(3) Introduce the projected state in the subdynamic system as

$$
\begin{equation*}
\widetilde{\psi}=\Omega^{-1} \psi \tag{8}
\end{equation*}
$$

where $\psi$ is a solution of the original Schrödinger equation and the similarity operator $\Omega$ is defined by

$$
\begin{align*}
\Omega_{n k} & =\left(C_{n k}+P_{n k}\right) \\
\Omega & =\sum_{n=0}^{1} \sum_{k} \Omega_{n k}, \\
\Omega^{-1} & =\sum_{n=0}^{1} \sum_{k} \Omega_{n k}^{-1}  \tag{9}\\
& =\sum_{n=0}^{1} \sum_{k}\left(P_{n k}+D_{n k} C_{n k}\right)^{-1}\left(P_{n k}+D_{n k}\right) .
\end{align*}
$$

(4) The ruling equation is subdynamic Liouville equation:

$$
\begin{equation*}
i \frac{\partial}{\partial t} \widetilde{\rho}=[\Theta, \widetilde{\rho}] \tag{10}
\end{equation*}
$$

where $\Theta$ operator is introduced by

$$
\begin{align*}
\Theta & =\sum_{n=0}^{1} \sum_{k} \Theta_{n k}  \tag{11}\\
& =\sum_{n=0}^{1} \sum_{k}\left(P_{n k} H P_{n k}+P_{n k} \Delta H Q_{n k} C_{n k} P_{n k}\right),
\end{align*}
$$

and the relationship of the above $\Theta$ operator with the original $H+\Delta H$ can be proven by a kind of similarity transformation:

$$
\begin{equation*}
\Theta=\Omega^{-1}(H+\Delta H) \Omega \tag{12}
\end{equation*}
$$

So it is obvious to use (12) and consider (8); one can obtain SKE (5) from the Schrödinger equation or the Liouville equation.

## 3. Rabi Oscillation in Subdynamic Space

Using the Rabi oscillation between $|1\rangle$ (or $|1\rangle\langle 1|$ ) and $|0\rangle$ (or $|0\rangle\langle 0|$ ) as the working states of quantum computation is a reasonable approach of quantum computing based on quantum dot system [11, 22]. In the ideal situation, $\Delta H=0$, the Liouville equation for this two-level system with the Rabi oscillation is expressed by

$$
\begin{align*}
& i U_{11}^{-1}(t) \frac{\partial \rho}{\partial t} U_{11}(t)=U_{11}^{-1}(t)[H, \rho] U_{11}(t) \\
&= U_{11}^{-1}(t) H \rho U_{11}(t)-U_{11}^{-1}(t) \rho H U_{11}(t) \\
&= U_{11}^{-1}(t) U_{11}(t) \widehat{H}(t) U_{11}^{-1}(t) \rho U_{11}(t)  \tag{13}\\
&-U_{11}^{-1}(t) \rho U_{11}(t) \widehat{H}(t) U_{11}^{-1}(t) U_{11}(t) \\
&= \widehat{H}(t) U_{11}^{-1}(t) \rho U_{11}(t)-U_{11}^{-1}(t) \rho U_{11}(t) \widehat{H}(t) \\
&= {[\widehat{H}(t), \rho(t)]=i \frac{\partial U_{11}^{-1}(t) \rho U_{11}(t)}{\partial t}=i \frac{\partial \rho(t)}{\partial t}, }
\end{align*}
$$

where one notices that using (13) one has

$$
\begin{align*}
& \frac{\partial U_{11}^{-1}(t)}{\partial t} \rho U_{11}(t)+U_{11}^{-1}(t) \rho \frac{\partial U_{11}(t)}{\partial t}=0, \\
& \rho=  \tag{14}\\
& \quad U_{11}(t) \rho(t) U_{11}^{-1}(t) \\
& \quad=\rho_{11}|1\rangle\langle 1|+\rho_{00}|0\rangle\langle 0|+\rho_{10}|1\rangle\langle 0| \\
& \quad+\rho_{01}|0\rangle\langle 1| .
\end{align*}
$$

Then, from (13) one gets the time-independent equations of $\langle n| \rho|m\rangle=\rho_{n m}$ as

$$
\begin{align*}
& i \frac{d}{d t} \rho_{11}=\frac{1}{2} g\left(\rho_{01}-\rho_{10}\right) \\
& i \frac{d}{d t} \rho_{00}=-i \frac{d}{d t} \rho_{11}=-\frac{1}{2} g\left(\rho_{01}-\rho_{10}\right)  \tag{15}\\
& i \frac{d}{d t} \rho_{10}=\frac{1}{2} g\left(\rho_{00}-\rho_{11}\right)
\end{align*}
$$

Then by introducing $U, V, W$ as a Bloch vector, that is,

$$
\begin{equation*}
\vec{U}=U \widehat{e}_{1}+V \widehat{e}_{2}+W \widehat{e}_{3} \tag{16}
\end{equation*}
$$

with

$$
\begin{align*}
U & =\rho_{10}+\rho_{01} \\
V & =i\left(\rho_{10}-\rho_{01}\right)  \tag{17}\\
W & =\rho_{11}-\rho_{00}
\end{align*}
$$

where $\widehat{e}_{1}, \widehat{e}_{2}, \widehat{e}_{3}$ is an unit vector, respectively, and $\widehat{e}_{i} \cdot \widehat{e}_{j}=\delta_{i j}$, thus (15) can be changed as

$$
\frac{d}{d t}\left(\begin{array}{c}
U  \tag{18}\\
V \\
W
\end{array}\right)=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & g \\
0 & -g & 0
\end{array}\right)\left(\begin{array}{c}
U \\
V \\
W
\end{array}\right)
$$

Then using the subdynamic similarity operator $\Omega$ above, one gets

$$
\begin{align*}
\Omega^{-1}\left(\begin{array}{c}
\frac{d U}{d t} \\
\frac{d V}{d t} \\
\frac{d W}{d t}
\end{array}\right) & =\Omega^{-1}\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & -0 & g \\
0 & -g & 0
\end{array}\right) \Omega \Omega^{-1}\left(\begin{array}{c}
U \\
V \\
W
\end{array}\right)  \tag{19}\\
\frac{d}{d t}\left(\begin{array}{c}
\widetilde{U} \\
\widetilde{V} \\
\widetilde{W}
\end{array}\right) & =\left(\begin{array}{ccc}
\Theta_{1} & 0 & 0 \\
0 & \Theta_{2} & 0 \\
0 & 0 & \Theta_{3}
\end{array}\right)\left(\begin{array}{c}
\widetilde{U} \\
\widetilde{V} \\
\widetilde{W}
\end{array}\right) \tag{20}
\end{align*}
$$

The solution of (20) is given by

$$
\Omega\left(\begin{array}{c}
\widetilde{U}(t)  \tag{21}\\
\widetilde{V}(t) \\
\widetilde{W}(t)
\end{array}\right)=\Omega\left(\begin{array}{ccc}
-i g & 0 & 0 \\
0 & i g & 0 \\
0 & 0 & 0
\end{array}\right) \Omega^{-1} \Omega\left(\begin{array}{c}
\widetilde{U}(0) \\
\widetilde{V}(0) \\
\widetilde{W}(0)
\end{array}\right)
$$

where

$$
\left(\begin{array}{c}
U  \tag{22}\\
V \\
W
\end{array}\right)=\Omega\left(\begin{array}{c}
\widetilde{U} \\
\widetilde{V} \\
\widetilde{W}
\end{array}\right)=\left(\begin{array}{ccc}
0 & 0 & 1 \\
i & -i & 0 \\
1 & 1 & 0
\end{array}\right)\left(\begin{array}{c}
\widetilde{U} \\
\widetilde{V} \\
\widetilde{W}
\end{array}\right)
$$

so (20) can also give a solution based on the Bloch ball,

$$
\left(\begin{array}{c}
U(t)  \tag{23}\\
V(t) \\
W(t)
\end{array}\right)=\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \theta & -\sin \theta \\
0 & \sin \theta & \cos \theta
\end{array}\right)\left(\begin{array}{c}
U(0) \\
V(0) \\
W(0)
\end{array}\right)
$$

where $\theta$ is defined as a radial rotation angle. In the initial condition (by considering (17) which corresponds to an initial pure state of the Rabi oscillation $\rho_{00}=1$ and else three states, $\rho_{11}, \rho_{01}$, and $\rho_{10}$ as zero), one gets

$$
\left(\begin{array}{c}
U(0)  \tag{24}\\
V(0) \\
W(0)
\end{array}\right)=\left(\begin{array}{c}
0 \\
0 \\
-1
\end{array}\right) .
$$

This allows one, from (23), to get

$$
\left(\begin{array}{c}
U(t)  \tag{25}\\
V(t) \\
W(t)
\end{array}\right)=\left(\begin{array}{c}
0 \\
-\sin \theta(t) \\
-\cos \theta(t)
\end{array}\right)
$$

which reflects the Rabi oscillation in the ideal situation without the interaction from environment.

## 4. Controlling Dissipation and Decoherence

Now let us consider there is an interaction between the system and environment. This can introduce some dissipation and decoherence. For example, an interaction of Liouvillian for the Rabi oscillation system can be specified by

$$
\begin{equation*}
\Delta L(\rho)=\frac{1}{2} \gamma_{10}\left(2 \sigma_{01} \rho \sigma_{10}-\sigma_{11} \rho-\rho \sigma_{11}\right) \tag{26}
\end{equation*}
$$

where $\sigma_{10}$ or $\sigma_{01}$ expressed the same meaning as in (1) and $\gamma_{10}$ is a coupling number reflecting dissipation velocity. So the corresponding master equation for the pseudovector $\boldsymbol{\rho}=$ ( $\rho_{00}, \rho_{11}, \rho_{01}, \rho_{10}$ ) is expressed as

$$
\begin{align*}
& \frac{d}{d t} \rho_{11}=-i \frac{1}{2} g\left(\rho_{01}-\rho_{10}\right)-\gamma_{10} \rho_{11} \\
& \frac{d}{d t} \rho_{00}=-\frac{d}{d t} \rho_{11}=i \frac{1}{2} g\left(\rho_{01}-\rho_{10}\right)+\gamma_{10} \rho_{11}  \tag{27}\\
& \frac{d}{d t} \rho_{10}=-i \delta \rho_{10}-i \frac{1}{2} g\left(\rho_{00}-\rho_{11}\right)-\frac{1}{2} \gamma_{10} \rho_{10}
\end{align*}
$$

then through the subdynamic similarity operator $\Omega$ one obtains

$$
\begin{align*}
& i \frac{d}{d t} \Omega^{-1}\left(\begin{array}{c}
U \\
V \\
W
\end{array}\right) \\
& =\Omega^{-1}\left(\begin{array}{rrr}
-\frac{1}{T_{2}} & -\delta & 0 \\
\delta & -\frac{1}{T_{2}} & g \\
0 & -g & -\frac{1}{T_{1}}
\end{array}\right) \Omega \Omega^{-1}\left(\begin{array}{c}
U \\
V \\
W
\end{array}\right)  \tag{28}\\
& \quad+\Omega^{-1}\left(\begin{array}{c}
0 \\
0 \\
-\frac{1}{T_{1}}
\end{array}\right)
\end{align*}
$$

where we notice that $\delta=\omega_{1}-\omega_{0}-\omega_{L} ; T_{2}=1 / 2 T_{1}+\gamma_{\mathrm{ph}}, T_{1}=$ $1 / \gamma_{10}$ expresses the life time of state $|1\rangle$, and $\gamma_{\mathrm{ph}}$ is a coupling number reflecting pure phase decoherence. Then considering (18) one can define

$$
\begin{align*}
(\mathbf{L}+\Delta \mathbf{L})\left(\begin{array}{c}
U \\
V \\
W
\end{array}\right)= & \left(\begin{array}{ccc}
-\frac{1}{T_{2}} & -\delta & 0 \\
\delta & -\frac{1}{T_{2}} & g \\
0 & -g & -\frac{1}{T_{1}}
\end{array}\right)\left(\begin{array}{c}
U \\
V \\
W
\end{array}\right)  \tag{29}\\
& +\left(\begin{array}{c}
0 \\
0 \\
-\frac{1}{T_{1}}
\end{array}\right)
\end{align*}
$$

This allows us to introduce the subdynamics for $(U, V, W) \rightarrow$ $\boldsymbol{\rho}=\left(\rho_{00}, \rho_{11}, \rho_{22}, \rho_{01}, \rho_{10}\right)$ or $\widetilde{\boldsymbol{\rho}}=\left(\widetilde{\rho}_{00}, \widetilde{\rho}_{11}, \widetilde{\rho}_{22}, \widetilde{\rho}_{01}, \widetilde{\rho}_{10}\right)$ by the above formalism:

$$
\begin{align*}
\frac{d}{d t} \Omega^{-1} \boldsymbol{\rho}(t) & =\Omega^{-1}(\mathbf{L}+\Delta \mathbf{L}) \Omega \Omega^{-1} \boldsymbol{\rho}(t)=\Theta \widetilde{\rho}(t) \\
& =\frac{d}{d t} \widetilde{\rho}(t) \tag{30}
\end{align*}
$$

with

$$
\begin{align*}
\Theta & =\Omega^{-1}(\mathbf{L}+\Delta \mathbf{L}) \Omega \\
& =P(\mathbf{L}+\Delta \mathbf{L}) P+P(\mathbf{L}+\Delta \mathbf{L}) Q C P,  \tag{31}\\
\boldsymbol{\rho}(t) & =\Omega \widetilde{\rho}(t)=(P+C) \widetilde{\rho}(t)=\sum_{n m}\left(P_{n m}+C_{n m}\right) \widetilde{\rho}(t),
\end{align*}
$$

by defining the projector

$$
\begin{equation*}
\left.P=\sum_{n m} P_{n m}=\sum_{n m} \mid \widetilde{\rho}_{n m}\right)\left(\widetilde{\rho}_{n m} \mid,\right. \tag{32}
\end{equation*}
$$

for $n m=00,11,01,10$,
with

$$
\begin{equation*}
Q=1-P, \tag{33}
\end{equation*}
$$

so that $\boldsymbol{\rho}=\left(\rho_{00}, \rho_{11}, \rho_{01}, \rho_{10}\right)$ is expressed by the subdynamic formalism:

$$
\begin{equation*}
\left.\left.\mid \rho_{n m}\right)=\left(P_{n m}+C_{n m}\right) \mid \widetilde{\rho}_{n m}\right) \tag{34}
\end{equation*}
$$

This $\rho_{n m}$ can be measured in the practical system with energy term $l_{n m}$, and then using $\rho_{n m}, l_{n m}$ we can construct the projected states $\widetilde{\rho}_{n m}$ in the subdynamic space by the subdynamic formalism:

$$
\begin{gather*}
\left.\left.\mid \rho_{n m}\right)=\mid \tilde{\rho}_{n m}\right)+Q_{n m} \frac{1}{M_{n n}-Q_{n m}(\mathbf{L}+\Delta \mathbf{L}) Q_{n m}}  \tag{35}\\
\left.\cdot Q_{n n}(\mathbf{L}+\Delta \mathbf{L}) \mid \widetilde{\rho}_{n n}\right)
\end{gather*}
$$

where it is marvelous that whatever the system has $\mathbf{L}$ or $\mathbf{L}+\Delta \mathbf{L}$, the above formalism gives the same $\widetilde{\boldsymbol{\rho}}=$ $\left(\widetilde{\rho}_{00}, \widetilde{\rho}_{11}, \widetilde{\rho}_{22}, \widetilde{\rho}_{01}, \widetilde{\rho}_{10}\right) \rightarrow(\widetilde{U}, \widetilde{V}, \widetilde{W})$ in the subdynamic space; namely, if $\mathbf{L}$ of the computing system changes to $\mathbf{L}+\Delta \mathbf{L}$ then the similarity operator $\Omega$ in (22) changes to

$$
\Omega=\left(\begin{array}{ccc}
-\frac{1}{\delta} g & \frac{\delta}{g} & \frac{\delta}{g}  \tag{36}\\
0 & \frac{1}{g} \sqrt{-\delta^{2}-g^{2}} & -\frac{1}{g} \sqrt{-\delta^{2}-g^{2}} \\
1 & 1 & 1
\end{array}\right)
$$

which, from the changed $(U, V, W)$, still gives the same $(\widetilde{U}, \widetilde{V}, \widetilde{W})$ as

$$
\begin{align*}
& \left(\begin{array}{l}
\widetilde{U} \\
\widetilde{V} \\
\widetilde{W}
\end{array}\right) \\
& =\left(\begin{array}{ccc}
-\delta \frac{g}{\delta^{2}+g^{2}} & 0 & \frac{\delta^{2}}{\delta^{2}+g^{2}} \\
\delta \frac{g}{2 \delta^{2}+2 g^{2}} & \frac{1}{2} \frac{g}{\sqrt{-\delta^{2}-g^{2}}} & \frac{g^{2}}{2 \delta^{2}+2 g^{2}} \\
\frac{1}{2} \delta \frac{g}{\delta^{2}+g^{2}} & -\frac{1}{2} \frac{g}{\sqrt{-\delta^{2}-g^{2}}} & \frac{1}{2} \frac{g^{2}}{\delta^{2}+g^{2}}
\end{array}\right)\left(\begin{array}{c}
U \\
V \\
W
\end{array}\right) . \tag{37}
\end{align*}
$$

Hence this equation group allows one, from the measurement of states $\boldsymbol{\rho}=\left(\rho_{00}, \rho_{11}, \rho_{01}, \rho_{10}\right)$ and energy $l_{n m}$, for $n m=00,11,01,10$, to determine the projected states $\widetilde{\boldsymbol{\rho}}=$ $\left(\widetilde{\rho}_{00}, \widetilde{\rho}_{11}, \widetilde{\rho}_{01}, \widetilde{\rho}_{10}\right)$. These projected states are invariant in the subdynamic space in the sense:

$$
\begin{align*}
\left.\Theta \mid \widetilde{\rho}_{n m}\right)= & \left(\widetilde{\rho}_{n m}|H| \widetilde{\rho}_{n m}\right)+\left(\widetilde{\rho}_{n m}\left|H Q_{n m} C_{n m}\right| \widetilde{\rho}_{n m}\right) \\
= & \left.\theta_{n m} \mid \widetilde{\rho}_{n m}\right), \\
\left.\Theta \mid \widetilde{\rho}_{n m}\right)= & \left(\widetilde{\rho}_{n m}|(H+\Delta H)| \widetilde{\rho}_{n m}\right)  \tag{38}\\
& +\left(\widetilde{\rho}_{n m}\left|(H+\Delta H) Q_{n m} C_{n m}\right| \widetilde{\rho}_{n m}\right) \\
= & \left.\theta_{n m}^{\prime} \mid \widetilde{\rho}_{n m}\right),
\end{align*}
$$

in which both cases have the same eigenvectors $\tilde{\rho}_{n m} \rightarrow$ $(\widetilde{U}, \widetilde{V}, \widetilde{W})$, although the eigenvalues are changed from $\theta_{n m}$ to $\theta_{n m}^{\prime}$. So from (21) one can obtain

$$
\left(\begin{array}{ccc}
-i g & 0 & 0  \tag{39}\\
0 & i g & 0 \\
0 & 0 & 0
\end{array}\right) \longrightarrow\left(\begin{array}{ccc}
-\frac{1}{T_{2}}-\sqrt{-\delta^{2}-g^{2}} & 0 & 0 \\
0 & -\frac{1}{T_{2}}+\sqrt{-\delta^{2}-g^{2}} & 0 \\
0 & 0 & -\frac{1}{2} \frac{g^{2}}{T_{2}\left(g^{2}+\delta^{2}\right)}\left(e^{-i\left(t / T_{2}\right)}-1\right)
\end{array}\right)
$$

where one notices that the evolution $\widetilde{W}$ is given by

$$
\begin{align*}
& i \frac{d}{d t} \widetilde{W}=-\frac{1}{T_{2}} \widetilde{W}-\frac{1}{2} \frac{g^{2}}{T_{2}\left(g^{2}+\delta^{2}\right)} \\
& \begin{aligned}
& \widetilde{W}(t)=-e^{-\int\left(1 / T_{2}\right) d t}\left[\int \frac{1}{2} \frac{g^{2}}{T_{2}\left(g^{2}+\delta^{2}\right)} e^{\int\left(1 / T_{2}\right) d t}+C\right] \\
& \quad=-e^{-i\left(t / T_{2}\right)} \int_{0}^{t} \frac{i}{2} \frac{g^{2}}{T_{2}\left(g^{2}+\delta^{2}\right)} e^{i\left(1 / T_{2}\right) t} d t \\
& \quad=\frac{1}{2} \frac{g^{2}}{T_{2}\left(g^{2}+\delta^{2}\right)}\left(e^{-i\left(t / T_{2}\right)}-1\right)
\end{aligned}
\end{align*}
$$

$$
\left(\begin{array}{c}
\widetilde{U}(t) \\
\widetilde{V}(t) \\
\widetilde{W}(t)
\end{array}\right)=\left(\begin{array}{cc}
e^{i\left(1 / T_{2}+\sqrt{-\delta^{2}-g^{2}}\right)} & 0 \\
0 & e^{i\left(1 / T_{2}-\sqrt{-\delta^{2}-g^{2}}\right)} \\
0 & 0
\end{array}\right.
$$

Hence based on the above subdynamic formalism the physical realizing states in the subdynamic space are related to $(\widetilde{U}, \widetilde{V}, \widetilde{W})$ which can be determined by the states $(U, V, W)$.

The remaining change of the eigenvalue from $\theta_{n m}$ to $\theta_{n m}^{\prime}$, which includes two parts of influence of the dissipation and decoherence, may introduce the relative phase error. However since the eigenvector in the subdynamic space is fixed, the correction procedure only needs to consider phase error; this makes the procedure easier; for instance, suppose the working states in quantum computing process are designed as $\left(\widetilde{\rho}_{00}, \widetilde{\rho}_{11}, \widetilde{\rho}_{01}, \widetilde{\rho}_{10}\right)$; then the phase error correction in the evolution of $(\widetilde{U}(t), \widetilde{V}(t), \widetilde{W}(t))$ is only necessarily considered. This can be given by using (39):
so the correction is to replace $\left(-1 / T_{2} \mp \sqrt{-\delta^{2}-g^{2}}\right)$ by $\mp g^{2}$ and replace $\left((1 / 2)\left(g^{2} / T_{2}\left(g^{2}+\delta^{2}\right)\right)\left(e^{-i\left(t / T_{2}\right)}-1\right)\right)$ by 0 , which enables (41) to return (21). For example, using the quantum three-editing-code method [23] if there are phase errors in $\widetilde{W} \widetilde{W} \widetilde{W}$ for a quantum computing tolerance time $\Delta t$ then one can correct the errors by using
$x y z \quad x \oplus z \quad y \oplus z$ error-qubit

| $\widetilde{W} \widetilde{W} \widetilde{W}$ | $\widetilde{W}$ | $\widetilde{W}$ | 0 |
| :--- | :---: | :---: | :---: |
| $\widetilde{W}^{\prime} \widetilde{W} \widetilde{W}$ | $?$ | $\widetilde{W}$ | 1 |
| $\widetilde{W} \widetilde{W}^{\prime} \widetilde{W}$ | $\widetilde{W}$ | $?$ | 2 |
| $\widetilde{W} \widetilde{W} \widetilde{W}^{\prime}$ | $?$ | $?$ | 3, |

where ? means $\widetilde{W}^{\prime} \oplus \widetilde{W} \neq \widetilde{W}$ because $\widetilde{W}^{\prime}$ has a phase error. Thus by means of ? one can conclude which qubit has error. This process is described by an error point formula: $|x y z \widetilde{W} \widetilde{W}\rangle \rightarrow|x y z, x \oplus z, y \oplus z\rangle$. Then the correction is to replace $\widetilde{W}^{\prime}$ by $\widetilde{W}$ through replacing the relevant eigenvalues after considering change of the eigenvalues. This process can keep the working states $\widetilde{W}$ to be corrected by canceling the time evolution phase error of the states during every error tolerance time interval $\Delta t$ in the subdynamic spaces.

Although the above Rabi oscillation seems to deal with one qubit system, it is not problem to generalize to multiqubits systems. For example, it can be applied into a two-qubit system based on a quantum dot with two excitons in which there are four degenerate energy levels, where $|00\rangle$ is vacuum state of excitons and $|10\rangle$ and $|01\rangle$ are $x$ or $y$ polarization of exciton states, respectively, while $|11\rangle$ is states of two excitons. Thus using $x$ and $y$ polarization light, the CROT for two qubits can be realized as

$$
\begin{align*}
& U_{\text {crot }}(|00\rangle,|01\rangle,|10\rangle,|11\rangle)  \tag{43}\\
& \quad=(|00\rangle,|01\rangle,-|11\rangle,|10\rangle)
\end{align*}
$$

with

$$
U_{\text {crot }}=\left(\begin{array}{cccc}
1 & 0 & 0 & 0  \tag{44}\\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0
\end{array}\right)
$$

which gives an operation on the states based on (17) as

$$
U_{\text {crot }}\left(\begin{array}{c}
U U  \tag{45}\\
V V \\
W W
\end{array}\right)=\left(\begin{array}{c}
U U \\
V V \\
W W^{\prime}
\end{array}\right)
$$

so that $W W$ changes to $W W^{\prime}$. Therefore in the subdynamic space one can perform the CROT operation for two-qubit system ideally; that is,

$$
\begin{align*}
& U_{\text {crot }}\left(\begin{array}{c}
\widetilde{U} \widetilde{U} \\
\widetilde{V} \widetilde{V} \\
\widetilde{W} \widetilde{W}
\end{array}\right) \\
& =U_{\text {crot }}\left(\begin{array}{ccc}
0 & -\frac{1}{2} i & \frac{1}{2} \\
0 & \frac{1}{2} i & \frac{1}{2} \\
1 & 0 & 0
\end{array}\right)\left(\begin{array}{c}
U \\
V \\
W
\end{array}\right)\left(\begin{array}{ccc}
0 & -\frac{1}{2} i & \frac{1}{2} \\
0 & \frac{1}{2} i & \frac{1}{2} \\
1 & 0 & 0
\end{array}\right)\left(\begin{array}{c}
U \\
V \\
W^{\prime}
\end{array}\right)  \tag{46}\\
& =\left(\begin{array}{c}
\widetilde{U} \widetilde{U} \\
\widetilde{V} \widetilde{V} \\
\widetilde{W} \widetilde{W}
\end{array}\right)
\end{align*}
$$

The relevant errors can be corrected using the same approach as the previous subdynamic treatment, as in (41) and Table (42).

## 5. Conclusions and Remarks

The subdynamic system for the quantum computation based on the Rabi oscillation is introduced. The working states of subdynamic system are the eigenvectors of $\Theta=\Omega H \Omega^{-1}$ which is similarity operator of the original Hamiltonian or Liouvillian. These eigenvectors constructed are invariant in the subdynamic space, so they are independent on the interaction part of Hamiltonian or Liouvillian between system and environment. While the dissipation and decoherence induced by the interaction part of Hamiltonian or Liouvillian exist in the change of the eigenvalues, there are only phase errors in the subdynamic space. This makes easier for one to find a procedure to cancel the phase error since the eigenvectors are fixed, such as the quantum three-editingcode method listed in this work. Moreover, these corrections of phase errors contain not only an error induced by decoherence but also an error induced by dissipation; therefore this method has an advantage compared with other approaches which only deal with the decoherence. Furthermore, this method principally can be extended to general quantum computing system without restriction. For example, if the system uses laser pulses instead of a continuous laser, it still can find the subsystem dynamics where the eigenstates are constant. This is generally true for subdynamics.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

## Acknowledgment

The authors thank the support of this work from Wuhan University of Technology.

## References

[1] M. V. Gurudev Dutt, Y. W. Wu, X. Q. Li, D. C. Steel, D. Gammon, and L. J. Sham, CP772, Physics of Semiconductors: 27th International Conference on the Physics of Semiconductors, vol. 32, 2005.
[2] X. Q. Li, Y. W. Wu, D. C. Steel et al., "An all-optical quantum gate in a semiconductor quantum dot", Science, vol. 301, no. 5634, pp. 809-811, 2003.
[3] J. M. Villas-Bôas, E. Ulloa Sergio, and A. O. Govorov, "Decoherence of Rabi oscillations in a single quantum dot," Physical Review Letters, vol. 94, no. 5, Article ID 057404, 2005.
[4] P. Borri, W. Langbein, S. Schneider et al., "Ultralong dephasing time in InGaAs quantum dots," Physical Review Letters, vol. 87, no. 15, Article ID 157401, 2001.
[5] H. Htoon, T. Takagahara, D. Kulik, O. Baklenov, A. L. Holmes Jr., and C. K. Shih, "Interplay of rabi oscillations and quantum interference in semiconductor quantum dots," Physical Review Letters, vol. 88, no. 8, Article ID 087401, 2002.
[6] A. Zrenner, E. Beham, S. Stufler, F. Findeis, M. Bichler, and G. Abstreiter, "Coherent properties of a two-level system based on a quantum-dot photodiode," Nature, vol. 418, no. 6898, pp. 612614, 2002.
[7] H. Kamada, H. Gotoh, J. Temmyo, T. Takagahara, and H. Ando, "Exciton Rabi oscillation in a single quantum dot," Physical Review Letters, vol. 87, no. 24, Article ID 246401, 2001.
[8] P. Borri, W. Langbein, S. Schneider et al., "Rabi oscillations in the excitonic ground-state transition of InGaAs quantum dots," Physical Review B, vol. 66, no. 8, Article ID 081306, 2002.
[9] L. Besombes, J. J. Baumberg, and J. Motohisa, "Coherent spectroscopy of optically gated charged single InGaAs quantum dots," Physical Review Letters, vol. 90, no. 25, Article ID 257402, 2003.
[10] H. J. Zhou, S. D. Liu, Q. Q. Wang, M. S. Zhan, and Q. K. Xue, "Rabi flopping in a V-type three-level system with two orthogonal eigenstates," Acta Physica Sinica, vol. 54, no. 2, pp. 710-714, 2005.
[11] Q. Q. Wang, A. Muller, P. Bianucci et al., "Decoherence processes during optical manipulation of excitonic qubits in semiconductor quantum dots," Physical Review B-Condensed Matter and Materials Physics, vol. 72, no. 3, Article ID 035306, 2005.
[12] L. Allen and J. H. Eberly, Optical Resonance and Two-Level Atoms, Dover Publications, New York, NY, USA, 1987.
[13] I. Antoniou and S. Tasaki, "Generalized spectral decompositions of mixing dynamical systems," International Journal of Quantum Chemistry, vol. 46, no. 3, pp. 425-474, 1993.
[14] T. Petrosky and I. Prigogine, "Alternative formulation of classical and quantum dynamics for nonintegrable systems," Physica A, vol. 175, no. 1, pp. 146-209, 1991.
[15] I. Antoniou, Y. Melnikov, and B. Qiao, "Master equation for a quantum system driven by a strong periodic field in the quasienergy representation," Physica A: Statistical Mechanics and its Applications, vol. 246, no. 1-2, pp. 97-114, 1997.
[16] B. Qiao, H. E. Ruda, M. S. Zhan, and X. H. Zeng, "Kinetic equation, non-perturbative approach and decoherence free subspace for quantum open system," Physica A: Statistical Mechanics and Its Applications, vol. 322, no. 1-4, pp. 345-358, 2003.
[17] B. Qiao, H. E. Ruda, and D. Z. Zhou, "Dynamical equations of quantum information and Gaussian channel," Physica A, vol. 363, no. 2, pp. 198-210, 2006.
[18] Q. Bi, L. Guo, and H. E. Ruda, "Quantum computing in decoher-ence-free subspace constructed by triangulation," Advances in Mathematical Physics, vol. 2010, Article ID 365653, 10 pages, 2010.
[19] B. Qiao and J. Fang, Network Science and Statistical Physics Method, Beijing University Press, Beijing, China, 2011 (Chinese).
[20] B. Qiao and H. E. Ruda, "Quantum computing using entanglement states in a photonic band gap," Journal of Applied Physics, vol. 86, no. 9, pp. 5237-5244, 1999.
[21] G. Mahler and V. A. Weberruß, Quantum Networks, Springer, Berlin, Germany, 1995.
[22] S. Trumm, M. Wesseli, H. J. Krenner et al., "Spin-preserving ultrafast carrier capture and relaxation in InGaAs quantum dots," Applied Physics Letters, vol. 87, no. 15, Article ID 153113, 2005.
[23] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, Cambridge, UK, 2000.


Advances in Operations Research $-$


The Scientific World Journal


Advances in
Decision Sciences
= -


## Hindawi

Submit your manuscripts at
http://www.hindawi.com


Mathematical Problems in Engineering


Journal of Function Spaces
$\underline{=}$



International Journal of Differential Equations 5


