Hindawi Publishing Corporation
Advances in Mathematical Physics
Volume 2015, Article ID 271765, 11 pages
http://dx.doi.org/10.1155/2015/271765

Research Article

On Homogeneous Parameter-Dependent Quadratic Lyapunov Function for Robust H_∞ Filtering Design in Switched Linear Discrete-Time Systems with Polytopic Uncertainties

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Received 23 November 2014; Revised 30 January 2015; Accepted 30 January 2015

Academic Editor: Pavel Kurasov

This paper is concerned with the problem of robust H_∞ filter design for switched linear discrete-time systems with polytopic uncertainties. The condition of being robustly asymptotically stable for uncertain switched system and less conservative H_∞ noise-attenuation level bounds are obtained by homogeneous parameter-dependent quadratic Lyapunov function. Moreover, a more feasible and effective method against the variations of uncertain parameter robust switched linear filter is designed under the given arbitrary switching signal. Lastly, simulation results are used to illustrate the effectiveness of our method.

1. Introduction

Switched systems are a class of hybrid systems that consist of a finite number of subsystems and a logical rule orchestrating switching between the subsystems. Since this class of systems has numerous applications in the control of mechanical systems, the automotive industry, aircraft and air traffic control, switching power converters, and many other fields, the problems of stability analysis and control design for switched systems have received wide attention during the past two decades [1–15]. Reference [2] proposed the H_∞ weight learning law for switched Hopfield neural networks with time-delay under parametric uncertainty. Reference [8] dealt with the delay-dependent exponentially convergent state estimation problem for delayed switched neural networks. Some criteria for exponential stability and asymptotic stability of a class of nonlinear hybrid impulsive and switching systems have been established using switched Lyapunov functions in [9]. Reference [14] investigated the problem of designing a switching compensator for a plant switching amongst a family of given configurations.

On the other hand, within robust control theory scheme, the H_∞ noise-attenuation level is an important index for the influence of external disturbance on system stability [16–19]. However, the uncertainties which generally exist in many practical plants and environments may result in significant changes in robust H_∞ noise-attenuation level. In order to suppress the conservativeness, many new methods have been considered. Among these methods, homogeneous polynomial parameter-dependent quadratic Lyapunov function is one of the most effective methods. The main feature of these functions is that they are quadratic Lyapunov functions whose dependence on the uncertain parameters is expressed as a polynomial homogeneous form. It is firstly introduced to study robust stability of polynomial systems in [20]. Most results have been presented in [21–25]. In [19], homogeneous parameter-dependent quadratic Lyapunov functions were used to establish tightness in robust H_∞ analysis. Reference [21] presented some general results concerning the existence of homogeneous polynomial solutions to parameter-dependent linear matrix inequalities whose coefficients are continuous functions of parameters lying in the unit simplex. Reference [23] investigated the problems of checking robust stability and evaluating robust H_2 performance of uncertain continuous-time linear systems with time-invariant parameters lying in polytropic domains. Reference [25] introduced

Gram-tight forms, that is, forms whose minimum coincides with the lower bound provided by LMI optimizations based on SOS (sum of squares of polynomials) relaxations.

Thus, in order to suppress the influence of uncertainties on the system's robust H_∞ control, the homogeneous parameter-dependent quadratic Lyapunov functions are used to design robust H_∞ filter in this paper. We first consider the system's anti-interference of external disturbance. Along this direction, the robust H_∞ filters for switched linear discrete-time systems are designed. Lastly, through the comparison, we have the fact that homogeneous parameter-dependent quadratic Lyapunov functions can suppress conservativeness which the uncertainties bring.

The rest of this paper is organised as follows. We state the problem formulation in Section 2. The main results are presented in Section 3. Section 4 illustrates the obtained result by numerical examples, which is followed by the conclusion in Section 5.

Notation. $\mathbf{R}^n$ denotes the n-dimension Euclidean space and $\mathbf{R}^{n\times m}$ is the real matrices with dimension $n \times m$; $\mathbf{R}_0^n$ means $\mathbf{R}^n/\{0\}$; the notation $X \geq Y$ (resp., $X > Y$), where X and Y are symmetric matrices, represents the fact that the matrix $X - Y$ is positive semidefinite (resp., positive definite); A^T denotes the transposed matrix of A; $sq(x)$ represents $(x_1^2, \ldots, x_n^2)$ with $x \in \mathbf{R}^n$; $he(X)$ means $X + X^T$ with $X \in \mathbf{R}^{n\times n}$; $x \otimes y$ denotes the Kronecker product of vectors x and y. $\|\cdot\|$ denotes Euclidean norm for vector or the spectral norm of matrices.

2. Problem Statement

Consider a class of uncertain switched linear discrete-time systems which were given in [1]:

$$\begin{aligned} x(k+1) &= A_i(\lambda)\, x(k) + B_i(\lambda)\, \omega(k), \\ y(k) &= C_i(\lambda)\, x(k) + D_i(\lambda)\, \omega(k), \\ z(k) &= H_i(\lambda)\, x(k) + L_i(\lambda)\, \omega(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbf{R}^n$ is state vector, $\omega(k) \in \mathbf{R}^l$ is disturbance input which belongs to $l_2[0, +\infty)$, $y(k)$ is the measurement output, $z(k)$ is objective signal to be attenuated, and i is switching rule, which takes its value in the finite set $\Pi := \{1, \ldots, N\}$. As in [1], the switching signal i is unknown a priori, but its instantaneous value is available in real time. As an arbitrary discrete time k, the switching signal i is dependent on k or $x(k)$ or both or other switching rules. λ is an uncertain parameter vector supposed to satisfy $\lambda \in \Lambda = \{\lambda_j \geq 0, \sum_{j=1}^{s} = 1\}$. The vector λ represents the time-invariant parametric uncertainty which affects linearly the system dynamics. The vector λ can take any value in Λ, but it is known to be constant in time.

The matrices of each subsystem have appropriate dimensions and are assumed to belong to a given convex-bounded polyhedral domain described by s vertices in the ith subsystem, that is,

$$\left(A_i(\lambda)\;\; B_i(\lambda)\;\; C_i(\lambda)\;\; D_i(\lambda)\;\; H_i(\lambda)\;\; L_i(\lambda)\right) \in \Gamma_i, \tag{2}$$

where

$$\begin{aligned} \Gamma_i := \Bigg\{ & A_i(\lambda)\;\; B_i(\lambda)\;\; C_i(\lambda)\;\; D_i(\lambda)\;\; H_i(\lambda)\;\; L_i(\lambda) \\ & = \sum_{j=1}^{s} \lambda_j \left(A_{ij}\;\; B_{ij}\;\; C_{ij}\;\; D_{ij}\;\; H_{ij}\;\; L_{ij}\right); \\ & \lambda_j \geq 0\; \sum_{j=1}^{s} \lambda_j = 1 \Bigg\}. \end{aligned} \tag{3}$$

Hence, we are interested in designing an estimator or filter of the form

$$\begin{aligned} x_f(k+1) &= A_{fi} x_f(k) + B_{fi}\, y(k), \\ z_f(k) &= C_{fi} x_f(k) + D_{fi}\, y(k), \end{aligned} \tag{4}$$

where $x_f(k) \in \mathbf{R}^n$, $z_f(k) \in \mathbf{R}^q$; A_{ij}, B_{ij}, C_{ij} and D_{ij} are the parameterized filter matrices to be determined. The filter with the above structure may be called switched linear filter, in which the switching signal i is also assumed unknown a priori but available in real-time and homogeneous with the switching signal in system (1).

Augmenting the model of (1) to include the state of filter (4), we obtain the filtering error system:

$$\begin{aligned} x_e(k+1) &= A_{ei}(\lambda)\, x_e(k) + B_{ei}(\lambda)\, \omega(k), \\ e(k) &= C_{ei}(\lambda)\, x_e(k) + D_{ei}(\lambda)\, \omega(k), \end{aligned} \tag{5}$$

where

$$\begin{aligned} & x_e(k) = \left(x(k)\;\; x_f(k)\right)^T, \qquad e(k) = z(k) - z_f(k), \\ & A_{ei}(\lambda) = \begin{pmatrix} A_i(\lambda) & 0 \\ B_{fi} C_i(\lambda) & A_{fi} \end{pmatrix}, \\ & B_{ei}(\lambda) = \left(B_i(\lambda)\;\; B_{fi} D_i(\lambda)\right)^T, \\ & C_{ei}(\lambda) = \left(H_i(\lambda) - D_{fi} C_i(\lambda)\;\; -C_{fi}\right), \\ & D_{ei}(\lambda) = L_i(\lambda) - D_{fi} D_i(\lambda). \end{aligned} \tag{6}$$

Based on [1], the robust H_∞ filtering problem addressed in this paper can be formulated as follows: finding a prescribed level of noise attention $\gamma > 0$ and determining a robust switched linear filter (4) such that the filtering error system is robustly asymptotically stable and

$$\|e\|_2 < \gamma^2 \|\omega\|_2. \tag{7}$$

This problem has received a great deal of attention. In order to get a better level of noise attention $\gamma > 0$, we will use homogeneous polynomial functions which have demonstrated nonconservative result for serval problems. In this paper, we extend these methods to design robust switched linear filter.

The following preliminaries are given, which are essential for later developments. We firstly recall the homogeneous polynomial function from Chesi et al. [26].

Definition 1. The function $h : \mathbf{R}^n \to \mathbf{R}$ is a form of degree d in n scalar variables if

$$h(x) = \sum_{q \in L_{n,s}} a_q x^q, \quad (8)$$

where $L_{n,s} = \{q \in N^n : \sum_{i=1}^n q_i = s\}$ and $x \in \mathbf{R}^n$ and $a_q \in \mathbf{R}$ is coefficient of the monomial x^q.

The set of forms of degree s in n scalar variables is defined as

$$\Xi_{n,s} = \{h : \mathbf{R}^n \longrightarrow \mathbf{R} : (8) \text{ holds}\}. \quad (9)$$

Definition 2. The function $f : \mathbf{R}^n \to \mathbf{R}$ is a polynomial of degree less than or equal to s, in n scalar variables, if

$$f(x) = \sum_{i=0}^{s} h_i(x), \quad (10)$$

where $x \in \mathbf{R}^n$ and $h_i \in \Xi_{n,i}$, $i = 1, \ldots, s$.

Definition 3. Consider the vector $x \in \mathbf{R}^n$, $x = [x_1, \ldots, x_n]^T$. The power transformation of degree m is a nonlinear change of coordinates that forms a new vector x^m of all integer powered monomials of degree m that can be made from the original x vector:

$$x_j^m = c_j x_1^{m_{j1}} x_2^{m_{j2}} \cdots x_n^{m_{jn}}, \quad m_{ji} \in 1, \ldots, n,$$
$$\sum_{i=1}^{n} m_{ji} = m, \quad j = 1, \ldots, d_{(n,m)}, \; d_{(n,m)} = \frac{(n+m-1)!}{(n-1)!m!}. \quad (11)$$

Usually we take $c_j = 1$; then, with $m > 0$,

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}^m = \begin{pmatrix} x_1 (x_1, x_2, x_3, \ldots, x_n)^{m-1^T} \\ x_2 (x_2, x_3, \ldots, x_n)^{m-1^T} \\ \vdots \\ x_n (x_n)^{m-1^T} \end{pmatrix}; \quad (12)$$

otherwise

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}^m = 1. \quad (13)$$

For example, $n = 2$, $m = 2$, $\Rightarrow d_{(n,m)} = 3$, and

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Longrightarrow x^2 = \begin{pmatrix} x_1^2 \\ x_1 x_2 \\ x_2^2 \end{pmatrix}. \quad (14)$$

Definition 4. The function $M : \mathbf{R}^n \to \mathbf{R}^{r \times r}$ is a homogeneous parameter-dependent matrix of degree m in n scalar variables if

$$M_{i,j} \in \Xi_{n,m}, \quad \forall i, j = 1, \ldots, r. \quad (15)$$

We denote the set of $r \times r$ homogeneous parameter-dependent matrices of degree m in n scalar variables as

$$\Xi_{n,m,r}^{\#} = \{M : \mathbf{R}^n \longrightarrow \mathbf{R}^{r \times r} : (15) \text{ holds}\} \quad (16)$$

and the set of symmetric matrix forms as

$$\Xi_{n,m,r} = \left\{M \in \Xi_{n,m,r}^{\#} : M(x) = M^T(x) \;\; \forall x \in \mathbf{R}^n\right\}. \quad (17)$$

Definition 5. Let $M \in \Xi_{n,2m,r}$ and $H \in \mathbf{S}^{rd_{(n,m)}}$ such that

$$M(x) = \Phi(H, x^m, r), \quad (18)$$

where $\Phi(H, x^m, r) = (x^m \otimes I_r)^T H (x^m \otimes I_r)$. Then (18) is called a square matricial representation (SMR) of $M(x)$ with respect to $x^m \otimes I_r$. Moreover, H is called a SMR matrix of $M(x)$ with respect to $x^m \otimes I_r$.

Lemma 6 (Chesi et al. [26]). *Let $M \in \Xi_{n,2m,r}$. Then $\mathbf{H} = \{H \in \mathbf{S}^{rd_{(n,m)}}$: (18) holds$\}$ is an affine space. Moreover,*

$$H(M) = \left\{H + L : H \in \mathbf{S}^{rd_{(n,m)}} \text{ satisfies } (18), \; L \in \mathbf{L}_{n,m,r}\right\}, \quad (19)$$

where $\mathbf{L}_{n,m,r}$ is linear space:

$$\mathbf{L}_{n,m,r} = \left\{L \in \mathbf{S}^{rd_{(n,m)}} : \Phi(L, x^m, r) = 0_{r \times r} \;\; \forall x \in \mathbf{R}^n\right\} \quad (20)$$

whose dimension is given by

$$\omega(n, m, r) = \frac{r}{2}\left(\left(d_{(n,m)}\left(r d_{(n,m)} + 1\right)\right) - (r+1) d_{(n,2m)}\right). \quad (21)$$

Lemma 7 (Chesi et al. [26]). *Let $M \in \Xi_{n,s,r}$. Then*

$$M(x) > 0, \quad \forall x \in \gamma_q = \left\{x \in \mathbf{R}^q : x_i \geq 0, \; \sum_{i=1}^{q} x_i = 1\right\} \quad (22)$$

holds if and only if

$$M(sq(x)) > 0, \quad \forall x \in \mathbf{R}_0^n. \quad (23)$$

Lemma 8 (Boyd et al. [27]). *The linear matrix inequality*

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{pmatrix} < 0, \quad (24)$$

where $S_{11} = S_{11}^T$ and $S_{22} = S_{22}^T$, is equivalent to

$$S_{11} < 0, \qquad S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0. \quad (25)$$

3. Main Result

In this section, the sufficient condition for existence of robust H_∞ filter for uncertain switched systems is formulated. For this purpose, we firstly consider the anti-interference of system (1) for disturbance.

Theorem 9. *For a given scalar $\gamma > 0$. Consider system (1); if there exist matrices $P_{ij} > 0$ and matrices R_{ij} such that*

$$\begin{pmatrix} \Psi_{11}\left(P_{\bar{i}j}, R_{ij}\right) & 0 & \Psi_{13}\left(R_{ij}, A_{ij}\right) & \Psi_{14}\left(R_{ij}, B_{ij}\right) \\ * & -I_c & \Psi_{23}\left(C_{ij}\right) & \Psi_{24}\left(D_{ij}\right) \\ * & * & \Psi_{33}\left(P_{ij}\right) & 0 \\ * & * & * & -\gamma^2 I_c \end{pmatrix} + \begin{pmatrix} \mathscr{L}_{11} & \mathscr{L}_{12} & \mathscr{L}_{13} & \mathscr{L}_{14} \\ * & \mathscr{L}_{22} & \mathscr{L}_{23} & \mathscr{L}_{24} \\ * & * & \mathscr{L}_{33} & \mathscr{L}_{34} \\ * & * & * & \mathscr{L}_{44} \end{pmatrix} < 0 \quad i, \bar{i} \in \prod, \ j \in [1, s] \tag{26}$$

with

$$\begin{gathered} \Phi_{P_{\bar{i}}R_i}\left(\lambda^{m+1}\right) = \sum_{j=1}^{s}\lambda_j\left(P_{\bar{i}}\left(\lambda^m\right) - R_i\left(\lambda^m\right) - R_i^T\left(\lambda^m\right)\right); \\ \Phi_{R_iA_i}\left(\lambda^{m+1}\right) = R_i\left(\lambda^m\right)A_i\left(\lambda\right); \\ \Phi_{R_iB_i}\left(\lambda^{m+1}\right) = R_i\left(\lambda^m\right)B_i\left(\lambda\right); \\ \Phi_{C_i}\left(\lambda^{m+1}\right) = \left(\sum_{j=1}^{s}\lambda_j\right)^m C_i\left(\lambda\right); \\ \Phi_{D_i}\left(\lambda^{m+1}\right) = \left(\sum_{j=1}^{s}\lambda_j\right)^m D_i\left(\lambda\right); \\ \Phi_I\left(\lambda^{m+1}\right) = \left(\sum_{j=1}^{s}\lambda_j\right)^{m+1} I; \\ \Phi_{P_i}\left(\lambda^{m+1}\right) = \sum_{j=1}^{s}\lambda_j P_i\left(\lambda^m\right); \\ \Phi_{P_{\bar{i}}R_i}\left(sq\left(\lambda^{m+1}\right)\right) = \varpi^T\left(\lambda^{m+1}\right)\Psi_{11}\left(P_{\bar{i}j}, R_{ij}\right)\varpi\left(\lambda^{m+1}\right); \\ \Phi_{R_iA_i}\left(sq\left(\lambda^{m+1}\right)\right) = \varpi^T\left(\lambda^{m+1}\right)\Psi_{13}\left(R_{ij}, A_{ij}\right)\varpi\left(\lambda^{m+1}\right); \\ \Phi_{R_iB_i}\left(sq\left(\lambda^{m+1}\right)\right) = \varpi^T\left(\lambda^{m+1}\right)\Psi_{14}\left(R_{ij}, B_{ij}\right)\varpi\left(\lambda^{m+1}\right); \\ \Phi_{C_i}\left(sq\left(\lambda^{m+1}\right)\right) = \varpi^T\left(\lambda^{m+1}\right)\Psi_{23}\left(C_{ij}\right)\varpi\left(\lambda^{m+1}\right); \\ \Phi_{D_i}\left(sq\left(\lambda^{m+1}\right)\right) = \varpi^T\left(\lambda^{m+1}\right)\Psi_{24}\left(D_{ij}\right)\varpi\left(\lambda^{m+1}\right); \\ \Phi_{P_i}\left(sq\left(\lambda^{m+1}\right)\right) = \varpi^T\left(\lambda^{m+1}\right)\Psi_{33}\left(P_{ij}\right)\varpi\left(\lambda^{m+1}\right); \\ \Phi_I\left(sq\left(\lambda^{m+1}\right)\right) = \varpi^T\left(\lambda^{m+1}\right)I_c\varpi\left(\lambda^{m+1}\right); \\ \varpi\left(\lambda^{m+1}\right) = \lambda^{m+1} \otimes I; \\ \mathscr{L}_{11} \cdots \mathscr{L}_{44} \in \mathbf{L}_{s,m,n}, \end{gathered} \tag{27}$$

where $\Psi(\cdot)$ is a matrix which is made up of A_{ij}, B_{ij}, C_{ij}, D_{ij}, P_{ij}, R_{ij} and the set $\mathbf{L}_{s,m,n}$ is defined in Lemma 6, then system (1) is robustly asymptotically stable with H_∞ performance γ for any switching signal.

Proof. Consider the following homogeneous parameter-dependent quadratic Lyapunov function:

$$V\left(k, x\left(k\right)\right) = x^T\left(k\right)P_i\left(\lambda^m\right)x\left(k\right). \tag{28}$$

Then, along the trajectory of system (1), we have

$$\begin{aligned} \Delta V &= V\left(k+1, x\left(k+1\right)\right) - V\left(k, x\left(k\right)\right) \\ &= x^T\left(k\right)\left[A_i^T\left(\lambda\right)P_{\bar{i}}\left(\lambda^m\right)A_i\left(\lambda\right) - P_i\left(\lambda^m\right)\right]x\left(k\right) \\ &\quad + 2x^T\left(k\right)\left[A_i^T\left(\lambda\right)P_{\bar{i}}\left(\lambda^m\right)B_i\left(\lambda\right)\right]\omega\left(k\right) \\ &\quad + \omega^T\left(k\right)\left[B_i^T\left(\lambda\right)P_{\bar{i}}\left(\lambda^m\right)B_i\left(\lambda\right)\right]\omega\left(k\right). \end{aligned} \tag{29}$$

When $i = \bar{i}$, the switched system is described by the ith mode. When $i \neq \bar{i}$, it represents the switched system being at the switching times from mode $\bar{i}$ to mode i.

Before and after multiplying inequality (26) by $\{\lambda^{m+1} \otimes I\}^T$ and $\{\lambda^{m+1} \otimes I\}$, we have

$$\begin{pmatrix} \Phi_{P_{\bar{i}}R_i}\left(sq\left(\lambda^{m+1}\right)\right) & 0 & \Phi_{R_iA_i}\left(sq\left(\lambda^{m+1}\right)\right) & \Phi_{R_iB_i}\left(sq\left(\lambda^{m+1}\right)\right) \\ * & -\Phi_I\left(sq\left(\lambda^{m+1}\right)\right) & \Phi_{C_i}\left(sq\left(\lambda^{m+1}\right)\right) & \Phi_{D_i}\left(sq\left(\lambda^{m+1}\right)\right) \\ * & * & \Phi_{P_i}\left(sq\left(\lambda^{m+1}\right)\right) & 0 \\ * & * & * & -\gamma^2\Phi_I\left(sq\left(\lambda^{m+1}\right)\right) \end{pmatrix} < 0. \tag{30}$$

From Lemma 7, one has

$$\begin{pmatrix} \Phi_{P_{\bar{i}}R_i}\left(\lambda^{m+1}\right) & 0 & \Phi_{R_iA_i}\left(\lambda^{m+1}\right) & \Phi_{R_iB_i}\left(\lambda^{m+1}\right) \\ * & -\Phi_I\left(\lambda^{m+1}\right) & \Phi_{C_i}\left(\lambda^{m+1}\right) & \Phi_{D_i}\left(\lambda^{m+1}\right) \\ * & * & \Phi_{P_i}\left(\lambda^{m+1}\right) & 0 \\ * & * & * & -\gamma^2\Phi_I\left(\lambda^{m+1}\right) \end{pmatrix} < 0 \tag{31}$$

which implies

$$\begin{pmatrix} P_{\bar{i}}(\lambda^m) - R_i(\lambda^m) - R_i^T(\lambda^m) & 0 & R_i(\lambda^m) A_i(\lambda) & R_i(\lambda^m) B_i(\lambda) \\ * & -I & C_i(\lambda) & D_i(\lambda) \\ * & * & P_i(\lambda^m) & 0 \\ * & * & * & -\gamma^2 I \end{pmatrix} < 0. \quad (32)$$

Under the disturbance $\omega(k) = 0$, we get

$$\Delta V = x^T(k) \left[A_i^T(\lambda) P_{\bar{i}}(\lambda^m) A_i(\lambda) - P_i(\lambda^m) \right] x(k). \quad (33)$$

If

$$A_i^T(\lambda) P_{\bar{i}}(\lambda^m) A_i(\lambda) - P_i(\lambda^m) < 0, \quad \forall i, \bar{i} \in \Pi, \quad (34)$$

then $\Delta V < 0$. It implies system (1) is robust asymptotic stable.

In terms of Lemma 8, condition (34) is equivalent to

$$\begin{pmatrix} -P_{\bar{i}}(\lambda^m) & P_{\bar{i}}(\lambda^m) A_i(\lambda) \\ * & P_i(\lambda^m) \end{pmatrix} < 0. \quad (35)$$

Since (26), it follows that

$$P_{\bar{i}}(\lambda^m) - R_i(\lambda^m) - R_i^T(\lambda^m) < 0 \quad (36)$$

which implies the matrices $R_i(\lambda^m)$ are nonsingular for each i. Then, we have

$$\left(P_{\bar{i}}(\lambda^m) - R_i(\lambda^m)\right) P_{\bar{i}}^{-1}(\lambda^m) \left(P_{\bar{i}}(\lambda^m) - R_i(\lambda^m)\right)^T \geq 0 \quad (37)$$

which is equivalent to

$$P_{\bar{i}}(\lambda^m) - R_i(\lambda^m) - R_i^T(\lambda^m) \geq -R_i(\lambda^m) P_{\bar{i}}^{-1}(\lambda^m) R_i^T(\lambda^m). \quad (38)$$

Hence, it can be readily established that (32) is equivalent to

$$\begin{pmatrix} -R_i(\lambda^m) P_{\bar{i}}^{-1}(\lambda^m) R_i^T(\lambda^m) & 0 & R_i(\lambda^m) A_i(\lambda) & R_i(\lambda^m) B_i(\lambda) \\ * & -I & C_i(\lambda) & D_i(\lambda) \\ * & * & -P_i(\lambda^m) & 0 \\ * & * & * & -\gamma^2 I \end{pmatrix} < 0. \quad (39)$$

Before and after multiplying the above inequality by $\mathrm{diag}\{-R_i^{-T}(\lambda^m) P_{\bar{i}}(\lambda^m), I, I, I\}$, we obtain

$$\begin{pmatrix} -P_{\bar{i}}(\lambda^m) & 0 & P_{\bar{i}}(\lambda^m) A_i(\lambda) & P_{\bar{i}}(\lambda^m) B_i(\lambda) \\ * & -I & C_i(\lambda) & D_i(\lambda) \\ * & * & -P_i(\lambda^m) & 0 \\ * & * & * & -\gamma^2 I \end{pmatrix} < 0, \quad (40)$$

which implies that (35) holds. Then, the stability of system (1) can be deduced.

On the other hand, let

$$J = \sum_{k=0}^{\infty} \left[y^T(k) y(k) - \gamma^2 \omega^T(k) \omega(k) \right] \quad (41)$$

be performance index to establish the H_∞ performance of system (1). When the initial condition $x(0) = 0$, we have $V(k, x(k))|_{k=0} = 0$ which implies

$$\begin{aligned} J &< \sum_{k=0}^{\infty} \left[y^T(k) y(k) - \gamma^2 \omega^T(k) \omega(k) + \Delta V \right] \\ &= \sum_{k=0}^{\infty} \begin{pmatrix} x^T(k) & \omega^T(k) \end{pmatrix} \begin{pmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{12}^T & \Theta_{22} \end{pmatrix} \begin{pmatrix} x(k) \\ \omega(k) \end{pmatrix}, \end{aligned} \quad (42)$$

where

$$\begin{aligned} \Theta_{11} &= A_i^T(\lambda) P_{\bar{i}}(\lambda^m) A_i(\lambda) - P_i(\lambda^m) + C_i^T(\lambda) C_i(\lambda); \\ \Theta_{12} &= A_i^T(\lambda) P_{\bar{i}}(\lambda^m) B_i(\lambda) + C_i^T(\lambda) D_i(\lambda); \\ \Theta_{22} &= -\gamma^2 I + B_i^T(\lambda) P_{\bar{i}}(\lambda^m) B_i(\lambda) + D_i^T(\lambda) D_i(\lambda). \end{aligned} \quad (43)$$

In terms of Lemma 8, we have $J < 0$ which means that $\|y\|_2 < \gamma^2 \|\omega\|_2$. Then, the proof is completed. □

Remark 10. Within robustly H_∞ performance scheme, the basic idea is to get an upper bound of H_∞ noise-attenuation level. Since model uncertainties may result in significant changes in H_∞ noise-attenuation level, we should effectively reduce this effect. For this purpose, the homogeneous parameter-dependent quadratic Lyapunov function is exploited in Theorem 9.

Remark 11. In Theorem 9, $\Psi(\cdot)$ is a matrix which is made up of $A_{ij}, B_{ij}, C_{ij}, D_{ij}, P_{ij}, R_{ij}$. For example, when $n = 2$ and $m = 1$, condition (26) can be written as

$$\Psi_{11}\left(P_{ij}, R_{ij}\right) = \begin{pmatrix} P_{i1} - R_{i1}^T - R_{i1} & 0 & 0 \\ * & -R_{i1} - R_{i1}^T - R_{i2} - R_{i2}^T + P_{i1} + P_{i2} & 0 \\ * & * & P_{i2} - R_{i2} - R_{i2}^T \end{pmatrix},$$

$$\Psi_{13}\left(R_{ij}, A_{ij}\right) = \begin{pmatrix} R_{i1}A_{i1} & 0 & 0 \\ * & R_{i1}A_{i2} + R_{i2}A_{i1} & 0 \\ * & * & R_{i2}A_{i2} \end{pmatrix},$$

$$\Psi_{14}\left(R_{ij}, B_{ij}\right) = \begin{pmatrix} R_{i1}B_{i1} & 0 & 0 \\ * & R_{i1}B_{i2} + R_{i2}B_{i1} & 0 \\ * & * & R_{i2}B_{i2} \end{pmatrix},$$

$$\Psi_{23}\left(C_{ij}\right) = \begin{pmatrix} C_{i1} & 0 & 0 \\ * & C_{i1} + C_{i2} & 0 \\ * & * & C_{i2} \end{pmatrix},$$

$$\Psi_{24}\left(C_{ij}\right) = \begin{pmatrix} D_{i1} & 0 & 0 \\ * & D_{i1} + D_{i2} & 0 \\ * & * & D_{i2} \end{pmatrix},$$

$$\Psi_{33}\left(P_{ij}\right) = \begin{pmatrix} -P_{i1} & 0 & 0 \\ * & -P_{i1} - P_{i2} & 0 \\ * & * & -P_{i2} \end{pmatrix}. \quad (44)$$

Remark 12. About the calculation of matrices $\mathscr{L}_{11} \cdots \mathscr{L}_{44}$, the following algorithm is presented in [26]:

(1) choose x^m and x^{2m} as in Definition 3 where $x \in \mathbf{R}^n$

(2) set $A = 0_{d_{(n,2m)}d_{(r,2)}\times 3}$ and $b = 0$ and define the variable $\alpha \in \mathbf{R}^{\omega(n,m,r)}$

(3) for $i = 1, \ldots, d_{(n,m)}$ and $j = 1, \ldots, r$

(4) set $c = r(i-1) + j$

(5) for $k = 1, \ldots, d_{(n,m)}$ and $l = \max\{1, j + r(i-k)\}, \ldots, r$

(6) set $d = r(k-1) + l$ and $f = ind((x^m)_i (x^m)_k, x^{2m})$

(7) set $g = ind(y_j y_l, y^2)$ and $a = f d_{r,2} + g$ and $A_{a,1} = A_{a,1} + 1$

(8) if $A_{a,1} = 1$

(9) set $A_{a,2} = c$ and $A_{a,3} = d$

(10) else

(11) set $b = b + 1$ and $G = 0_{rd_{(n,m)}\times rd_{(n,m)}}$ and $G_{c,d} = 1$

(12) set $h = A_{a,2}$ and $p = A_{a,3}$ and $G_{h,p} = G_{h,p} - 1$

(13) set $L = L + \alpha_b G$

(14) endif

(15) endfor

(16) endfor

(17) set $\mathscr{L} = 0.5he(\mathscr{L})$.

Next, the condition for robust H_∞ filter is formulated. For convenience, we define $n = 2$, $m = 1$, and $s = 2$.

Theorem 13. *Given a constant $\gamma > 0$, if there exist matrices $P_{2ij}, y_i, \zeta_{ij}, \overline{A}_{fi}, \overline{B}_{fi}, \overline{C}_{fi}, \overline{D}_{fi}, x_{ij}$ and positive definite matrices P_{1ij}, P_{3ij} and scalar ε_i such that*

$$\begin{pmatrix} \Lambda_{11} & \Lambda_{12} & 0 & \Lambda_{14} & \Lambda_{15} & \Lambda_{16} \\ * & \Lambda_{22} & 0 & \Lambda_{24} & \Lambda_{25} & \Lambda_{26} \\ * & * & \Lambda_{33} & \Lambda_{34} & \Lambda_{35} & \Lambda_{36} \\ * & * & * & \Lambda_{44} & \Lambda_{45} & 0 \\ * & * & * & * & \Lambda_{55} & 0 \\ * & * & * & * & * & \Lambda_{66} \end{pmatrix} + \begin{pmatrix} \mathscr{L}_{11} & \mathscr{L}_{12} & \mathscr{L}_{13} & \mathscr{L}_{14} & \mathscr{L}_{15} & \mathscr{L}_{16} \\ * & \mathscr{L}_{22} & \mathscr{L}_{23} & \mathscr{L}_{24} & \mathscr{L}_{25} & \mathscr{L}_{26} \\ * & * & \mathscr{L}_{33} & \mathscr{L}_{34} & \mathscr{L}_{35} & \mathscr{L}_{36} \\ * & * & * & \mathscr{L}_{44} & \mathscr{L}_{45} & \mathscr{L}_{46} \\ * & * & * & * & \mathscr{L}_{55} & \mathscr{L}_{56} \\ * & * & * & * & * & \mathscr{L}_{66} \end{pmatrix} < 0 \quad (45)$$

with

$$\Lambda_{11} = \begin{pmatrix} P_{1\bar{i}1} - x_{i1} - x_{i1}^T & 0 & 0 \\ 0 & -x_{i1} - x_{i1}^T - x_{i2} - x_{i2}^T + P_{1\bar{i}1} + P_{1\bar{i}2} & 0 \\ 0 & 0 & P_{1\bar{i}2} - x_{i2} - x_{i2}^T \end{pmatrix},$$

$$\Lambda_{12} = \begin{pmatrix} P_{2i1} - \varepsilon_i y_i - \zeta_{i1} & 0 & 0 \\ 0 & P_{2i1} + P_{2i2} - \zeta_{i1} - \zeta_{i2} - 2\varepsilon_i y_i & 0 \\ 0 & 0 & P_{2i1} - \varepsilon_i y_i - \zeta_{i2} \end{pmatrix},$$

$$\Lambda_{14} = \begin{pmatrix} x_{i1}A_{i1} + \varepsilon_i \overline{B}_{fi}C_{i1} & 0 & 0 \\ 0 & x_{i1}A_{i2} + \varepsilon_i \overline{B}_{fi}C_{i1} + x_{i2}A_{i1} + \varepsilon_i \overline{B}_{fi}C_{i2} & 0 \\ 0 & 0 & x_{i2}A_{i2} + \varepsilon_i \overline{B}_{fi}C_{i2} \end{pmatrix},$$

$$\Lambda_{15} = \begin{pmatrix} \varepsilon_i \overline{A}_{fi} & 0 & 0 \\ 0 & 2\varepsilon_i \overline{A}_{fi} & 0 \\ 0 & 0 & \varepsilon_i \overline{A}_{fi} \end{pmatrix},$$

$$\Lambda_{16} = \begin{pmatrix} x_{i1}B_{i1} + \varepsilon_i \overline{B}_{fi}D_{i1} & 0 & 0 \\ 0 & x_{i1}B_{i2} + \varepsilon_i \overline{B}_{fi}D_{i1} + x_{i2}B_{i1} + \varepsilon_i \overline{B}_{fi}D_{i2} & 0 \\ 0 & 0 & x_{i2}A_{i2} + \varepsilon_i \overline{B}_{fi}D_{i2} \end{pmatrix},$$

$$\Lambda_{22} = \begin{pmatrix} P_{3i1} - y_i - y_i^T & 0 & 0 \\ 0 & P_{3i1} + P_{3i2} - 2y_i - 2y_i^T & 0 \\ 0 & 0 & P_{3i2} - y_i - y_i^T \end{pmatrix},$$

$$\Lambda_{24} = \begin{pmatrix} \zeta_{i1}A_{i1} + \overline{B}_{fi}C_{i1} & 0 & 0 \\ 0 & \zeta_{i1}A_{i2} + \overline{B}_{fi}C_{i1} + \zeta_{i2}A_{i1} + \overline{B}_{fi}C_{i2} & 0 \\ 0 & 0 & \zeta_{i2}A_{i2} + \overline{B}_{fi}C_{i2} \end{pmatrix},$$

$$\Lambda_{25} = \begin{pmatrix} \overline{A}_{fi} & 0 & 0 \\ 0 & 2\overline{A}_{fi} & 0 \\ 0 & 0 & \overline{A}_{fi} \end{pmatrix},$$

$$\Lambda_{26} = \begin{pmatrix} \zeta_{i1}B_{i1} + \overline{B}_{fi}D_{i1} & 0 & 0 \\ 0 & \zeta_{i1}B_{i2} + \overline{B}_{fi}D_{i1} + \zeta_{i2}B_{i1} + \overline{B}_{fi}D_{i2} & 0 \\ 0 & 0 & \zeta_{i2}A_{i2} + \overline{B}_{fi}D_{i2} \end{pmatrix},$$

$$\Lambda_{33} = \begin{pmatrix} -I_{1\times1} & 0 & 0 \\ 0 & -2I_{1\times1} & 0 \\ 0 & 0 & -I_{1\times1} \end{pmatrix},$$

$$\Lambda_{34} = \begin{pmatrix} H_{i1} - \overline{D}_{fi}C_{i1} & 0 & 0 \\ 0 & H_{i1} - \overline{D}_{fi}C_{i1} + H_{i2} - \overline{D}_{fi}C_{i2} & 0 \\ 0 & 0 & H_{i2} - \overline{D}_{fi}C_{i2} \end{pmatrix},$$

$$\Lambda_{35} = \begin{pmatrix} -\overline{C}_{fi} & 0 & 0 \\ 0 & -2\overline{C}_{fi} & 0 \\ 0 & 0 & -\overline{C}_{fi} \end{pmatrix},$$

$$\Lambda_{36} = \begin{pmatrix} L_{i1} - \overline{D}_{fi}D_{i1} & 0 & 0 \\ 0 & L_{i1} - \overline{D}_{fi}D_{i1} + L_{i2} - \overline{D}_{fi}D_{i2} & 0 \\ 0 & 0 & L_{i2} - \overline{D}_{fi}D_{i2} \end{pmatrix},$$

$$\Lambda_{44} = \begin{pmatrix} -P_{1i1} & 0 & 0 \\ 0 & -P_{1i1} - P_{1i2} & 0 \\ 0 & 0 & -P_{1i2} \end{pmatrix},$$

$$\Lambda_{45} = \begin{pmatrix} -P_{2i1} & 0 & 0 \\ 0 & -P_{2i1} - P_{2i2} & 0 \\ 0 & 0 & -P_{2i2} \end{pmatrix},$$

$$\Lambda_{55} = \begin{pmatrix} -P_{3i1} & 0 & 0 \\ 0 & -P_{3i1} - P_{3i2} & 0 \\ 0 & 0 & -P_{3i2} \end{pmatrix},$$

$$\Lambda_{66} = \begin{pmatrix} -\gamma^2 I_{1\times1} & 0 & 0 \\ 0 & -2\gamma^2 I_{1\times1} & 0 \\ 0 & 0 & -\gamma^2 I_{1\times1} \end{pmatrix},$$

$$\mathscr{L}_{11} \cdots \mathscr{L}_{66} \in \mathbf{L}_{2,2,2}, \tag{46}$$

then there exists a robust switched linear filter in form of (4) such that, for all admissible uncertainties, the filter error system (5) is robustly asymptotically stable and performance index holds for any nonzero $\omega \in l_2[0,\infty)$, *where* $A_{fi} = y_i^{-1}\overline{A}_{fi}$, $B_{fi} = y_i^{-1}\overline{B}_{fi}$, $C_{fi} = \overline{C}_{fi}$, *and* $D_{fi} = \overline{D}_{fi}$.

Proof. Let

$$P_i(\lambda) = \begin{pmatrix} P_{1i}(\lambda) & P_{2i}(\lambda) \\ * & P_{3i}(\lambda) \end{pmatrix}, \qquad R_i(\lambda) = \begin{pmatrix} x_i(\lambda) & \varepsilon_i y_i \\ \zeta_i(\lambda) & y_i \end{pmatrix}, \tag{47}$$

where

$$\begin{gathered} P_{1i}(sq(\lambda)) = \{\lambda \otimes I\}^T \begin{pmatrix} P_{1i1} & 0 \\ 0 & P_{1i2} \end{pmatrix} \{\lambda \otimes I\}, \\ P_{2i}(sq(\lambda)) = \{\lambda \otimes I\}^T \begin{pmatrix} P_{2i1} & 0 \\ 0 & P_{2i2} \end{pmatrix} \{\lambda \otimes I\}, \\ P_{3i}(sq(\lambda)) = \{\lambda \otimes I\}^T \begin{pmatrix} P_{3i1} & 0 \\ 0 & P_{3i2} \end{pmatrix} \{\lambda \otimes I\}, \\ x_i(sq(\lambda)) = \{\lambda \otimes I\}^T \begin{pmatrix} x_{i1} & 0 \\ 0 & x_{i2} \end{pmatrix} \{\lambda \otimes I\}, \\ \zeta_i(sq(\lambda)) = \{\lambda \otimes I\}^T \begin{pmatrix} \zeta_{i1} & 0 \\ 0 & \zeta_{i2} \end{pmatrix} \{\lambda \otimes I\}. \end{gathered} \tag{48}$$

From Theorem 9, the filter error system is robustly asymptotically stable with a prescribed H_∞ noise-attenuation level bound γ if the following matrix inequality holds:

$$\begin{pmatrix} P_{1i}(\lambda) - x_i(\lambda) - x_i(\lambda)^T & P_{2i}(\lambda) - \varepsilon_i y_i - \zeta_i(\lambda) & 0 & x_i(\lambda) A_i(\lambda) + \varepsilon_i \overline{B}_{fi} C_i(\lambda) & \varepsilon_i \overline{A}_{fi} & x_i(\lambda) B_i(\lambda) + \varepsilon_i \overline{B}_{fi} D_i(\lambda) \\ * & P_{3i} - y_i - y_i^T & 0 & \zeta_i(\lambda) A_i(\lambda) + \overline{B}_{fi} C_i(\lambda) & \overline{A}_{fi} & \zeta_i(\lambda) B_i(\lambda) + \overline{B}_{fi} D_i(\lambda) \\ * & * & -I & H_i(\lambda) - \overline{D}_{fi} C_i(\lambda) & \overline{C}_{fi} & L_i(\lambda) - \overline{D}_{fi} D_i(\lambda) \\ * & * & * & -P_{1i}(\lambda) & -P_{2i}(\lambda) & 0 \\ * & * & * & * & -P_{3i}(\lambda) & 0 \\ * & * & * & * & * & -\gamma^2 I \end{pmatrix} < 0. \tag{49}$$

By (48) and Lemma 7, one can obtain that inequality (45) is equivalent to (49). Thus, if (45) holds, the filter error system is robustly asymptotically stable with an H_∞ noise-attenuation level bound $\gamma > 0$. Then, the proof is completed. □

4. Examples

The following example exhibits the effectiveness and applicability of the proposed method for robust H_∞ filtering problems with polytopic uncertainties.

Consider the following uncertain discrete-time switched linear system (1) consisting of two uncertain subsystems which is given in [1]. There are two groups of vertex matrices in subsystem 1:

$$\begin{gathered} A_{11} = \rho \begin{pmatrix} 0.82 & 0.10 \\ -0.06 & 0.77 \end{pmatrix}, \qquad B_{11} = \rho \begin{pmatrix} 0 \\ 0.1 \end{pmatrix}, \\ C_{11} = \rho (1 \;\; 0), \qquad D_{11} = \rho, \\ H_{11} = \rho (1 \;\; 0), \qquad L_{11} = 0, \\ A_{12} = \rho \begin{pmatrix} 0.82 & 0.10 \\ -0.06 & -0.75 \end{pmatrix}, \qquad B_{12} = \rho \begin{pmatrix} 0 \\ -0.1 \end{pmatrix}, \\ C_{12} = \rho (1 \;\; 0.2), \qquad D_{12} = 0.8\rho, \\ H_{12} = \rho (1 \;\; 0), \qquad L_{12} = 0 \end{gathered} \tag{50}$$

and two groups of vertex matrices in subsystem 2:

$$\begin{gathered} A_{21} = \rho \begin{pmatrix} 0.82 & 0.06 \\ -0.10 & 0.77 \end{pmatrix}, \qquad B_{21} = \rho \begin{pmatrix} 0.1 \\ 0 \end{pmatrix}, \\ C_{21} = \rho (0 \;\; -1), \qquad D_{21} = -\rho, \\ H_{21} = \rho (1 \;\; 0), \qquad L_{21} = 0, \\ A_{22} = \rho \begin{pmatrix} 0.82 & 0.06 \\ -0.10 & -0.75 \end{pmatrix}, \qquad B_{22} = \rho \begin{pmatrix} -0.1 \\ 0 \end{pmatrix}, \\ C_{22} = \rho (0.2 \;\; -1), \qquad D_{22} = -0.8\rho, \\ H_{22} = \rho (1 \;\; 0), \qquad L_{22} = 0. \end{gathered} \tag{51}$$

Moreover, we define the disturbance:

$$\omega(k) = 0.001 e^{-0.003k} \sin(0.002\pi k). \tag{52}$$

Firstly, consider the problem of being robustly asymptotically stable with an H_∞ noise-attenuation for the uncertain switched system which was given in Theorem 9. The different minimum H_∞ noise-attenuation level bounds γ can be obtained by different methods. Table 1 lists the different calculation results. From Table 1, it can be clearly seen that the results which are gotten by homogeneous parameter-dependent quadratic Lyapunov functions are better.

Table 1: Different minimum γ for uncertain switched system.

ρ	1	1.1	1.2
[1]	1.2799	1.6985	6.2048
Theorem 9	1.2799	1.6984	4.0988

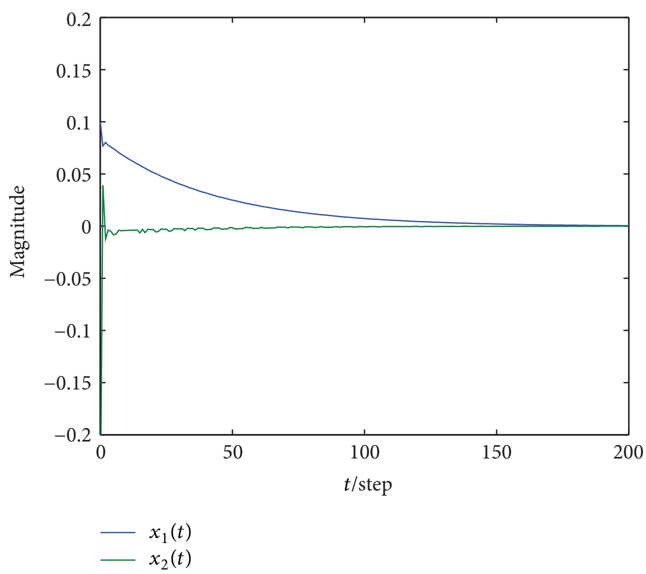

Figure 1: The states responses corresponding to uncertain parameters $\lambda_1 = 0.4$ and $\lambda_2 = 0.6$.

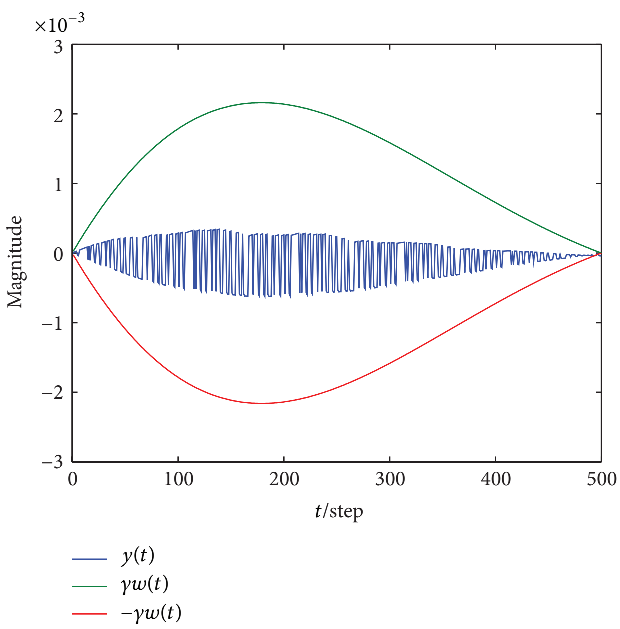

Figure 2: H_∞ noise-attenuation level bound $\gamma = 4.0988$.

In addition, for given $\rho = 1.2$ and initial condition $x(0) = [0.1, -0.3]^T$, Figures 1 and 2 show system (1) is robustly asymptotically stable with an H_∞ noise-attenuation level bound $\gamma = 4.0988$.

Next, we consider the problem of robust H_∞ filtering. In order to get H_∞ noise-attenuation level bound γ, we define

Table 2: Different minimum γ for robust switched linear filter.

ρ	1	1.1	1.2
[1]	0.5375	1.1414	4.4493
Theorem 13	0.5148	1.0826	4.2892

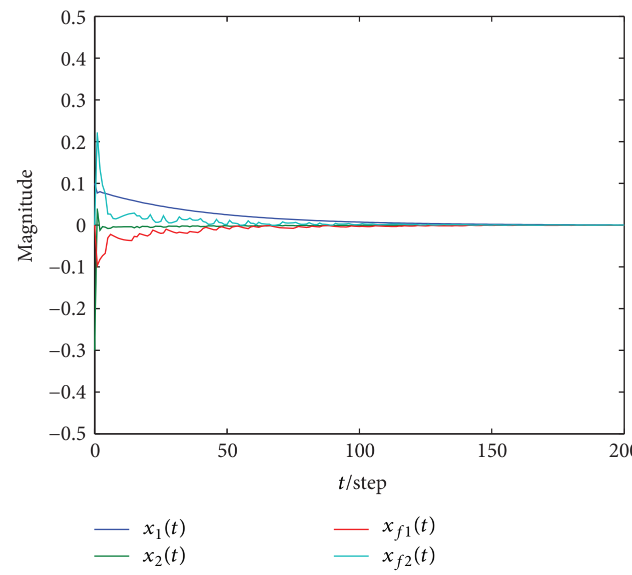

Figure 3: The states responses corresponding to uncertain parameters $\lambda_1 = 0.4$ and $\lambda_2 = 0.6$.

$\varepsilon_1 = \varepsilon_2 = 1$. By solving the corresponding convex optimization problems in Theorem 13, we obtain the minimum H_∞ noise-attenuation level bounds γ. Table 2 lists our results and [1]'s results. Moreover, when $\rho = 1.2$, the admissible filter parameters can be obtained according to Theorem 13 as

$$\begin{gathered} A_{f1} = \begin{pmatrix} 0.8770 & 0.0505 \\ -0.7334 & 0.3356 \end{pmatrix}, \qquad B_{f1} = \begin{pmatrix} -0.0852 \\ -0.1077 \end{pmatrix}, \\ C_{f1} = \begin{pmatrix} -1.2057 & 0.0112 \end{pmatrix}, \\ A_{f2} = \begin{pmatrix} 1.1457 & 0.6743 \\ -1.2492 & -0.8883 \end{pmatrix}, \\ B_{f1} = \begin{pmatrix} -0.2593 \\ 0.5929 \end{pmatrix}, \\ C_{f2} = \begin{pmatrix} -0.8695 & -0.1525 \end{pmatrix}, \\ D_{f1} = -0.0073, \qquad D_{f2} = -0.0045. \end{gathered} \tag{53}$$

By comparison, using homogeneous parameter-dependent quadratic Lyapunov function for the existence of a robust switched linear filter in Theorem 13 is better.

By giving H_∞ noise-attenuation level bound $\gamma = 4.2892$ and initial condition $x_e(0) = [0.1 \;\; -0.3 \;\; 0 \;\; 0]^T$, Figure 3 shows the filtering error system is robustly asymptotically stable and Figure 4 shows the error response of the resulting filtering error system by applying above filter. It is clear that

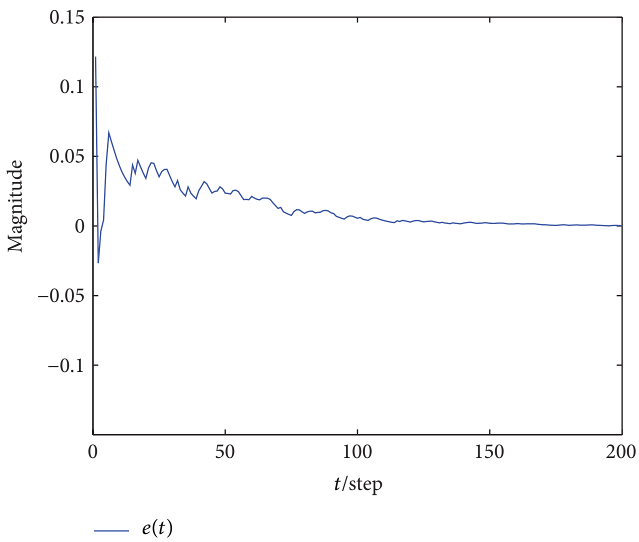

FIGURE 4: Filtering error response corresponding to uncertain parameters $\lambda_1 = 0.4$ and $\lambda_2 = 0.6$.

the method in Theorem 13 is feasible and effective against the variations of uncertain parameter.

5. Conclusions

In this paper, the problems of being robustly asymptotically stable with an H_∞ noise-attenuation level bounds γ and switched linear filter design for uncertain switched linear system are studied by homogeneous parameter-dependent quadratic Lyapunov functions. By using this method, the less conservative H_∞ noise-attenuation level bounds are obtained. Moreover, we also get a more feasible and effective method against the variations of uncertain parameter under the given arbitrary switching signal. Numerical examples illustrate the effectiveness of our method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This project is supported by Major Program of National Natural Science Foundation of China under Grant no. 11190015, National Natural Science Foundation of China under Grant no. 61374006, and Graduate Student Innovation Foundation of Jiangsu province under Grant no. 3208004904.

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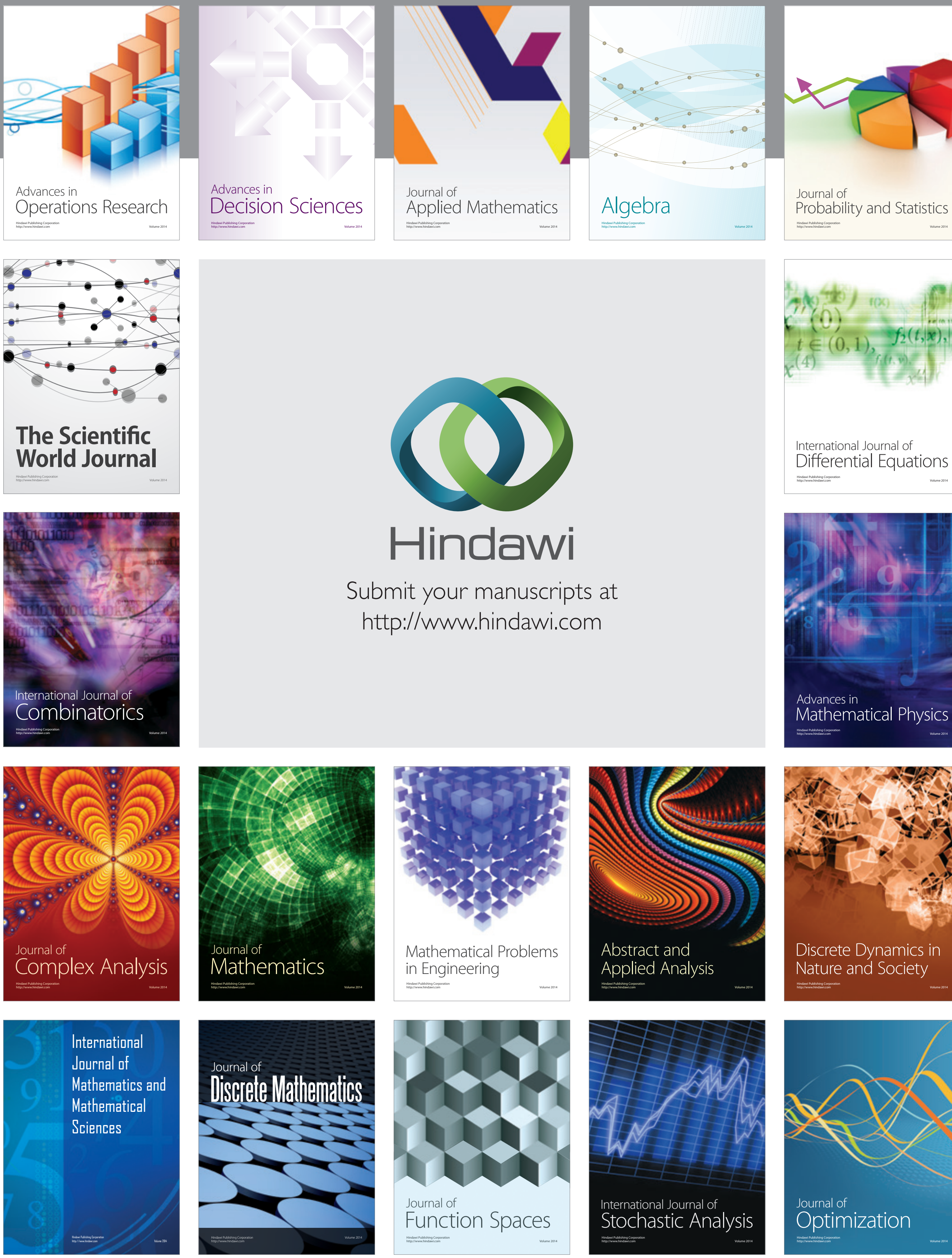
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