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Research Article

The Computation of the Magnitude of the Far Field for an Eccentric Circular Cylinder

Bülent Yılmaz

Department of Mathematics, Faculty of Sciences and Letters, Marmara University, Kadiköy, 34722 Istanbul, Turkey

Correspondence should be addressed to Bülent Yilmaz; bulentyilmaz@marmara.edu.tr

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The specific case of scattering of a plane wave by a two-layered penetrable eccentric circular cylinder has been considered and it is about the validity of the on surface radiation condition method and its applications to the scattering of a plane wave by a two-layered penetrable eccentric circular cylinder. The transformation of the problem of scattering by the eccentric circular cylinder to the problem of scattering by the concentric circular cylinder by using higher order radiation conditions, is observed. Numerical results presented the magnitude of the far field.

1. Introduction

Approximate techniques have been introduced to study the scattering of waves by obstacles. The OSRC method has been devised by Kriegsmann et al. to investigate electromagnetic scattering problems involving cylindrical convex objects [1]. The main concept of this method is the application of a radiation boundary condition (RBC), connecting the field and its normal derivative, directly onto the surface of the scatterer, to determine approximately the surface field or its derivative in terms of the given field.

Two main approaches have been employed to derive RBCs [2, 3]. One of the methods is based on the idea of killing the terms of the expansion of the scattering field satisfying the Helmholtz equation and Sommerfeld radiation condition. An nth-order RBC operator which annihilates the first n terms in the expansion is obtained either on a large circular cylinder enclosing a cylindrical convex object or on a large sphere enclosing a finite convex object, depending on the geometrical dimensions of the problem. These RBCs can be generalized so that they can be used in the OSRC method for constructing the approximate solution of a scattering problem involving an arbitrary convex object.

The second approach which is capable of supplying SRCs of any order on a smooth scattering surface of arbitrary form, thus avoiding the difficulty mentioned above, was introduced by Kriegsmann and Moore [3]. This method is based on an asymptotic expansion, similar to the Luneburg-Kline expansion, made in the neighbourhood of a phase front by assuming that the field has a well defined phase front in the regions of interest. Then the assumption that the surface of the scattering is a phase front yields the SRC of the OSRC method. Although the method of Jones may, in principle, furnish SRCs to any desired order, only the second-order condition has been produced and applied to various problems to examine the predictions of the method. From these investigations it has been observed that, for hard objects, as frequency increases, second-order SRCs begin to fail to account for creeping wave physics adequately.

Considering these results, it has been conjectured that, by introducing higher-order SRCs in the method, creeping wave physics may be modelled more accurately; that is, the approximation of the OSRC method may be improved. In [1, 3–7] only the first- and second-order RBCs have been produced and used in conjunction with the OSRC method. Later [2, 8, 9] third- and fourth-order RBCs have been used to examine whether the use of higher-order SRCs in the OSRC method models creeping wave physics more accurately than a second-order SRC.

In this work we applied the higher-order SRCs to scattering of plane waves by an eccentric penetrable circular cylinder. The results are compared with those of second- and

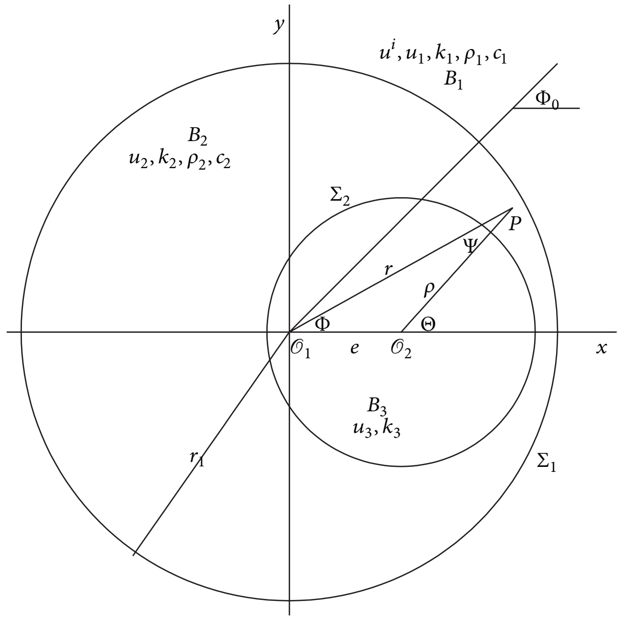

Figure 1

fourth-order SRCs and with those of the scattering of plane waves by a concentric penetrable cylinder [9].

This paper is organized as follows. The formulation of the problem together with the exact and the approximate solutions of these equations with OSRC method is presented and calculated in Sections 2 and 3, respectively. In Section 4 the comparisons and some concluding remarks take place.

2. Formulation and Exact Solution

We denote the outer and the inner layers by $\mathscr{B}_2$, with sound speed c_2 (m/s) and constant density ρ_2 (kg/m^3), and by $\mathscr{B}_3$, with sound speed c_3 (m/s) and constant density ρ_3 (kg/m^3), respectively. The interface between them is the circular cylinder Σ_2 of radius r_2. The radius of the circular cylinder between the exterior region $\mathscr{B}_1$ which is a homogeneous isotropic medium with sound speed c_1 (m/s) and constant density ρ_1 (kg/m^3) and the region $\mathscr{B}_2$ is r_1, $\mathscr{B}_1 = \{r > r_1\}$, $\mathscr{B}_2 = \{r_2 < r < r_1\}$, and $\mathscr{B}_3 = \{r < r_2\}$.

Let us denote by $\mathscr{O}_1$ the center of the outer cylinder Σ_1 and by $\mathscr{O}_2$ the center of the inner cylinder Σ_2 and take $\mathscr{O}_1$ as the origin of the rectangular Cartesian coordinate system (x, y). We suppose that the x-axis passes through the points $\mathscr{O}_1$ and $\mathscr{O}_2$. In addition to this, we consider two polar coordinate systems centered at $\mathscr{O}_1$ and $\mathscr{O}_2$. Radial and angular variables are denoted by (r, ϕ) and (ρ, θ), respectively. ϕ and θ are measured from the positive x-axis. Under these assumptions the geometry of the problem is defined in Figure 1, where r_1, r_2 are the outer and inner cylinder radii, while the distance between the centers of the cylinders is represented by e.

The scattering problem is described by the following equations and boundary conditions:

$$\nabla^2 u_l + k_l^2 u_l = 0 \qquad (x, y) \in \mathscr{B}_l$$
$$\text{for } k_l = \frac{w}{c_l}, \qquad l = 1, 2, 3, \tag{1}$$

$$u_1 + u^i = u_2, \qquad \frac{\partial}{\partial r}\left(u_1 + u^i\right) = \xi_1 \frac{\partial u_2}{\partial r} \quad \text{on } r = r_1, \tag{2}$$

$$u_2 = u_3, \qquad \frac{\partial u_2}{\partial r} = \xi_2 \frac{\partial u_3}{\partial r} \quad \text{on } \rho = r_2, \tag{3}$$

where $\zeta_1 = \rho_1/\rho_2$, $\zeta_2 = \rho_2/\rho_3$, $k_l = w/c_l$ is a wave number, and w is the angular frequency. In addition, at infinity the scattered field u_1 must have the form of a radiating wave; that is, the following Sommerfeld radiation condition must be satisfied:

$$\lim_{r \to \infty} r^{1/2} \left(\frac{\partial u_1}{\partial r} + i k_1 u_1\right) = 0. \tag{4}$$

Since ϕ_0 is the direction of propagation of the incident wave, the propagation vector of the incident wave is assumed to be $\mathbf{p} = (-\cos\phi_0, -\sin\phi_0)$. ϕ, r are the polar coordinates and the assumed time dependence is e^{-iwt}. All the field quantities are then independent of z. If $x = r\cos\phi$, $y = r\sin\phi$ then the incident wave can be expressed as

$$u^i = e^{-ik_1 \mathbf{p}\cdot\mathbf{x}} = e^{ik_1 r\cos(\phi-\phi_0)}$$
$$= \sum_{n=-\infty}^{\infty} J_n\left(k_1 r\right) e^{in(\phi-\phi_0+\pi/2)} \tag{5}$$

[4]. Solutions for u_1, u_2, and u_3 can be expressed in polar coordinates as follows:

$$u_1 = \sum_{n=-\infty}^{\infty} a_n H_n\left(k_1 r\right) e^{in(\phi-\phi_0+\pi/2)}, \tag{6}$$

$$u_2 = \sum_{n=-\infty}^{\infty} \left[b_n J_n\left(k_2\rho\right) + c_n H_n\left(k_2\rho\right)\right] e^{in(\theta-\pi/2)}, \tag{7}$$

$$u_3 = \sum_{n=-\infty}^{\infty} d_n J_n\left(k_3\rho\right) e^{in(\theta-\pi/2)}. \tag{8}$$

u_2 solution is expressed in polar coordinates centered at $\mathscr{O}_2$. In order to be able to use this solution for $r = r_1$ it has to be written first in polar coordinates (r, ϕ). We know that $J_n(k_2\rho)$ and $H_n(k_2\rho)$ can be expressed as follows:

$$J_n\left(k_2\rho\right) e^{in\theta} = \sum_{q=-\infty}^{\infty} J_q\left(k_2 r\right) J_{q-n}\left(k_2 e\right) e^{iq\phi},$$
$$H_n\left(k_2\rho\right) e^{in\theta} = \sum_{q=-\infty}^{\infty} H_q\left(k_2 r\right) J_{q-n}\left(k_2 e\right) e^{iq\phi}. \tag{9}$$

If (9) is inserted in (7) the solution for u_2 in polar coordinates (r, ϕ) takes the following mentioned form:

$$u_2 = \sum_{n=-\infty}^{\infty} b_n \left\{\sum_{q=-\infty}^{\infty} J_{q-n}\left(k_2 e\right) J_q\left(k_2 r\right) e^{iq\phi}\right\} e^{-in\pi/2}$$
$$+ \sum_{n=-\infty}^{\infty} c_n \left\{\sum_{q=-\infty}^{\infty} J_{q-n}\left(k_2 e\right) H_q\left(k_2 r\right) e^{iq\phi}\right\} e^{-in\pi/2}. \tag{10}$$

Now, if (5), (6), and (10) are used under the boundary conditions (2), in order for these conditions to be satisfied

$$J_q(k_1r_1)e^{-iq(\phi_0-\pi/2)}+a_qH_q(k_1r_1)e^{-iq(\phi_0-\pi/2)} = J_q(k_2r_1)\sum_{n=-\infty}^{\infty}b_nJ_{q-n}(k_2e)e^{-in\pi/2} + H_q(k_2r_1)\sum_{n=-\infty}^{\infty}c_nJ_{q-n}(k_2e)e^{-in\pi/2} \quad (11)$$

$$k_1J_q'(k_1r_1)e^{-iq(\phi_0-\pi/2)}+k_1a_qH_q'(k_1r_1)e^{-iq(\phi_0-\pi/2)} = k_2\xi_1J_q'(k_2r_1)\sum_{n=-\infty}^{\infty}b_nJ_{q-n}(k_2e)e^{-in\pi/2} + k_2\xi_1H_q'(k_2r_1)\sum_{n=-\infty}^{\infty}c_nJ_{q-n}(k_2e)e^{-in\pi/2} \quad (12)$$

$$e=0J_{q-n}(k_2e)=\begin{cases}1; & q=n\\ 0; & q\neq n.\end{cases} \quad (13)$$

Thus, if $\phi_0=\pi$ is taken it is seen that in case of $e=0$ or $k_2e\to 0$ (11) and (12) are, respectively, transformed to (2.8) and (2.9) in [9]. Then if (7) and (8) are used with the boundary conditions (3), the requirements for these conditions to be satisfied are

$$b_nJ_n(k_2r_2)+c_nH_n(k_2r_2)=d_nJ_n(k_3r_2), \quad (14)$$

$$k_2b_nJ_n'(k_2r_2)+k_2c_nH_n'(k_2r_2)=\xi_2k_3d_nJ_n'(k_3r_2). \quad (15)$$

Equations (11), (12), (14), and (15) constitute a system of algebraic equations with an infinite dimension for a_n, b_n, c_n, and d_n. First the infinite sums in (11) and (12) are converted to finite sums by choosing a naturel number N_1. The summation is implemented up to N_1. Therefore a system of infinite dimension is approximated by this finite system and is numerically solved. Then the same process is repeated by choosing a greater naturel number N_2. This is repeated until a solution of required precision is obtained. In this process, in order to diminish the load of the numerical calculations, instead of dealing with all (11), (12), (14), and (15) an algebraic system of equations with a finite dimension which includes only b_n's can be derived as follows. By eliminating a_n from (11) and (12) the below mentioned system of equations which includes only b_n and c_n is obtained:

$$\sum_{l=-\infty}^{\infty}\left[\Lambda_l(n)b_l+\Phi_l(n)c_l\right]=\Psi_n. \quad (16)$$

Here the elements of matrices $\Lambda_l(n)$, $\Phi_l(n)$ and vector Ψ_n are defined as follows:

$$\Lambda_l(n)=J_{n-l}(k_2e)e^{-il(\pi/2)+in(\phi_0-\pi/2)}\cdot\left[\frac{J_n(k_2r_1)}{H_n(k_1r_1)}-\frac{k_2\xi_1J_n'(k_2r_1)}{k_1H_n'(k_1r_1)}\right],$$

$$\Phi_l(n)=J_{n-l}(k_2e)e^{-il(\pi/2)+in(\phi_0-\pi/2)}\cdot\left[\frac{H_n(k_2r_1)}{H_n(k_1r_1)}-\frac{k_2\xi_1H_n'(k_2r_1)}{k_1H_n'(k_1r_1)}\right],$$

$$\Psi_n=\left[\frac{J_n(k_1r_1)}{H_n(k_1r_1)}-\frac{J_n'(k_1r_1)}{H_n'(k_1r_1)}\right]. \quad (17)$$

On the other side if d_n is eliminated from (14) and (15) the relation

$$c_n=\frac{\zeta_n}{\eta_n}b_n \quad (18)$$

is obtained. ζ_n and η_n are defined as follows:

$$\zeta_n=\frac{k_3\xi_2J_n'(k_3r_2)J_n(k_2r_2)}{k_2H_n(k_2r_2)H_n'(k_2r_2)}-\frac{J_n(k_3r_2)J_n'(k_2r_2)}{H_n(k_2r_2)H_n'(k_2r_2)}, \quad (19)$$

$$\eta_n=\frac{-k_3\xi_2J_n'(k_3r_2)}{k_2H_n'(k_2r_2)}+\frac{J_n(k_3r_2)}{H_n(k_2r_2)}. \quad (20)$$

If c_n in (16) is replaced by the right hand side of (18), the below system of equations is obtained for b_n:

$$\sum_{l=-\infty}^{\infty}A_l(n)b_l=\Psi_n. \quad (21)$$

Here the elements of the $A_l(n)$ matrix are defined as follows:

$$A_l(n)=\Lambda_l(n)+\Phi_l(n)\frac{\zeta_l}{\eta_l}. \quad (22)$$

By replacing l and n by the elements of the increasing whole number sequence $\{N_1,N_2,N_3,\dots\}$ and by making (21) finite, finite number of b_n is found with the same process as described above. By using b_n's in (18), (14) (or (15)), and (11) (or (12)) the same numbers of c_n, d_n, and a_n's are calculated. Thus, the scattered field is approximated by a finite sum.

3. Approximate Solution

By using on surface radiation condition method, the problem is converted to an internal one together with the boundary conditions of the impedance [9] type on Σ_1, boundary conditions (3) on Σ_2, and (1) (for $l=2,3$). For this reason in the boundary condition of the impedance type [9] θ is replaced by ϕ.

When the incident wave is a plane wave, $u^i(a,\phi)=v^i(\phi)$ and $(1/k_1)(\partial u^i/\partial r)(a,\phi)=w^i(\phi)$

$$v^i(\phi)=\sum_{n=-\infty}^{\infty}J_n(\varepsilon)e^{in(\phi-\phi_0+\pi/2)}, \quad \varepsilon=k_1r_1,$$

$$w^i(\phi)=\sum_{n=-\infty}^{\infty}J_n'(\varepsilon)e^{in(\phi-\phi_0+\pi/2)} \quad (23)$$

are written. Solutions of (1) (for $l = 2, 3$) can be expressed in the following forms:

$$u_2 = \sum_{n=-\infty}^{\infty} \left[\widetilde{b}_n J_n(k_2\rho) + \widetilde{c}_n H_n(k_2\rho)\right] e^{in(\theta-\pi/2)}, \tag{24}$$

$$u_3 = \sum_{n=-\infty}^{\infty} \widetilde{d}_n J_n(k_3\rho) e^{in(\theta-\pi/2)}. \tag{25}$$

The expression of u_2 in (24) is written in (ρ, θ) coordinates. From (10) this solution can be expressed in (ρ, ϕ) coordinates as follows:

$$\begin{aligned} u_2 = &\sum_{n=-\infty}^{\infty} \left[\sum_{q=-\infty}^{\infty} \widetilde{b}_q J_{n-q}(k_2 e) e^{-iq\pi/2}\right] J_n(k_2 r) e^{in\phi} \\ &+ \sum_{n=-\infty}^{\infty} \left[\sum_{q=-\infty}^{\infty} \widetilde{c}_q J_{n-q}(k_2 e) e^{-iq\pi/2}\right] H_n(k_2 r) e^{in\phi}. \end{aligned} \tag{26}$$

When u_2 is taken as given in (26) and (23) are chosen for the incident wave, in order for (2.3) condition and (2.21) of [9] to be satisfied

$$\begin{aligned} &\left\{\Theta^{(l)}(\varepsilon) J_n(k_2 r_1) - \frac{k_2\xi_1}{k_1} J_n'(k_2 r_1)\right\} \\ &\quad \cdot \left\{\sum_{q=-\infty}^{\infty} \widetilde{b}_q J_{n-q}(k_2 e) e^{-iq\pi/2}\right\} \\ &\quad + \left\{\Theta^{(l)}(\varepsilon) H_n(k_2 r_1) - \frac{k_2\xi_1}{k_1} H_n'(k_2 r_1)\right\} \\ &\quad \cdot \left\{\sum_{q=-\infty}^{\infty} \widetilde{c}_q J_{n-q}(k_2 e) e^{-iq\pi/2}\right\} \\ &= \left\{\Theta^{(l)}(\varepsilon) J_n(\varepsilon) - J_n'(\varepsilon)\right\} e^{-in(\phi_0-\pi/2)} \end{aligned} \tag{27}$$

is required. Here the function $\Theta^{(l)}(\varepsilon)$ is defined in (2.32) of [9].

For $e = 0$ two cylinders have the same axis and relation (13) holds. Thus if we take $\phi_0 = \pi$ at the $k_2 e \to 0$ limit we can easily observe that (27) is converted to that of the problem which is the same as in [9]. If solutions (24) and (25) are used under conditions (3) in order for those conditions to be satisfied it is required that

$$\widetilde{b}_n J_n(k_2 r_2) + \widetilde{c}_n H_n(k_2 r_2) = \widetilde{d}_n J_n(k_3 r_2), \tag{28}$$

$$k_2 \widetilde{b}_n J_n'(k_2 r_2) + k_2 \widetilde{c}_n H_n'(k_2 r_2) = \xi_2 k_3 \widetilde{d}_n J_n'(k_3 r_2) \tag{29}$$

hold. As a result (27), (28), and (29) constitute an algebraic system of equations of infinite dimension, for $\widetilde{b}_n$, $\widetilde{c}_n$, and $\widetilde{d}_n$.

We can write (27) as follows:

$$\sum_{q=-\infty}^{\infty} \left[\widetilde{\Lambda}_q(n) \widetilde{b}_l + \widetilde{\Phi}_q(n) \widetilde{c}_l\right] = \widetilde{\Psi}_n. \tag{30}$$

Here $\widetilde{\Lambda}_q(n)$, $\widetilde{\Phi}_q(n)$, and $\widetilde{\Psi}_n$ are defined as follows:

$$\begin{aligned} \widetilde{\Lambda}_q(n) &= J_{n-q}(k_2 e) e^{-iq(\pi/2)+in(\phi_0-\pi/2)} \\ &\quad \cdot \left\{\Theta^{(l,m)}(\varepsilon) J_n(k_2 r_1) - \frac{k_2\xi_1}{k_1} J_n'(k_2 r_1)\right\}, \\ \widetilde{\Phi}_q(n) &= J_{n-q}(k_2 e) e^{-iq(\pi/2)+in(\phi_0-\pi/2)} \\ &\quad \cdot \left\{\Theta^{(l,m)}(\varepsilon) H_n(k_2 r_1) - \frac{k_2\xi_1}{k_1} H_n'(k_2 r_1)\right\} \\ \widetilde{\Psi}_n &= \Theta^{(l,m)}(\varepsilon) J_n(\varepsilon) - J_n'(\varepsilon). \end{aligned} \tag{31}$$

System (30) is the approximation prescribed by the method to system (16) that is derived for the exact solution. It is observed that the replacement of $\Theta^{(m)}(\varepsilon)$ by $H_n'(\varepsilon)/H_n(\varepsilon)$ in (31) yields the exact ones. It is easily seen that the expressions $\widetilde{\Lambda}_q(n)$, $\widetilde{\Phi}_q(n)$, and $\widetilde{\Psi}_n$ are transformed to $H_n^{(2)\prime}(\varepsilon)\Lambda_q(n)$, $H_n^{(2)\prime}(\varepsilon)\Phi_q(n)$, and $H_n^{(2)\prime}(\varepsilon)\Psi_n$. Therefore (30) is transformed to (16). Thus, for the problem under consideration the surface radiation conditions method is equivalent to introducing the approximation $\Theta^{(l)}(\varepsilon) = H_n^{(2)\prime}(\varepsilon)/H_n^{(2)}(\varepsilon)$ and, therefore, this result is independent of the boundary conditions prescribed on the surface of the circular cylinder Σ_1. Hence, the accuracy of the method for the cylinder problems will depend on the accuracy of the approximation in $H_n^{(2)\prime}(\varepsilon)/H_n^{(2)}(\varepsilon)$.

If $\widetilde{d}_n$ is eliminated from (28) and (29) as in the exact solution, the following relation is obtained:

$$\widetilde{c}_n = \frac{\zeta_n}{\eta_n} \widetilde{b}_n, \tag{32}$$

where ζ_n and η_n are given, respectively, by (19) and (20). The system of equations for $\widetilde{b}_n$

$$\sum_{l=-\infty}^{\infty} \widetilde{A}_l(n) \widetilde{b}_l = \widetilde{\Psi}_n \tag{33}$$

is obtained by (32) and (30). Here $\widetilde{A}_l(n)$ is

$$\widetilde{A}_l(n) = \widetilde{\Lambda}_l(n) + \widetilde{\Phi}_l(n) \frac{\zeta_l}{\eta_l}. \tag{34}$$

First $\widetilde{b}_n$ is obtained as a result of this system and then $\widetilde{c}_n$ and $\widetilde{d}_n$ are obtained, respectively, from (32) and (28) or (29). After having calculated $\widetilde{b}_n$ and $\widetilde{c}_n$ from the above relations if at $r = r_1$, namely, over Σ_1, relations $u^t(r_1, \theta) = v_2(\theta)$, $(\partial u^t/\partial r)(a, \theta) = k_2\xi_1 w_2(\theta)$, and $\mathscr{X}(\mathbf{x}, \mathbf{y}) = -(1/4) i H_0^{(2)}(k_1|\mathbf{x} - \mathbf{y}|)$ are used in the integral representation below:

$$\begin{aligned} u^i(\mathbf{x}) + \int_{\Sigma_1} &\left\{\frac{\partial u^t(\mathbf{y})}{\partial n_y} \mathscr{X}(\mathbf{x}, \mathbf{y}) \right. \\ &\left. - u^t(\mathbf{y}) \frac{\partial}{\partial n_y} \mathscr{X}(\mathbf{x}, \mathbf{y})\right\} ds_y = u^t(\mathbf{x}), \quad \mathbf{x} \in \mathscr{B}_1. \end{aligned} \tag{35}$$

The scattered field in any point in the region $\mathscr{B}_1$ is obtained by calculating the following integral [4]:

$$\begin{aligned} u_1(r, \theta) &= \frac{i}{4} \int_0^{2\pi} \left[v_2(\theta') \frac{\partial}{\partial r_1} H_0 \right. \\ &\quad \cdot \left\{ k_1 \left(r^2 + r_1^2 - 2 r r_1 \cos(\theta - \theta')\right)^{1/2} \right\} \end{aligned}$$

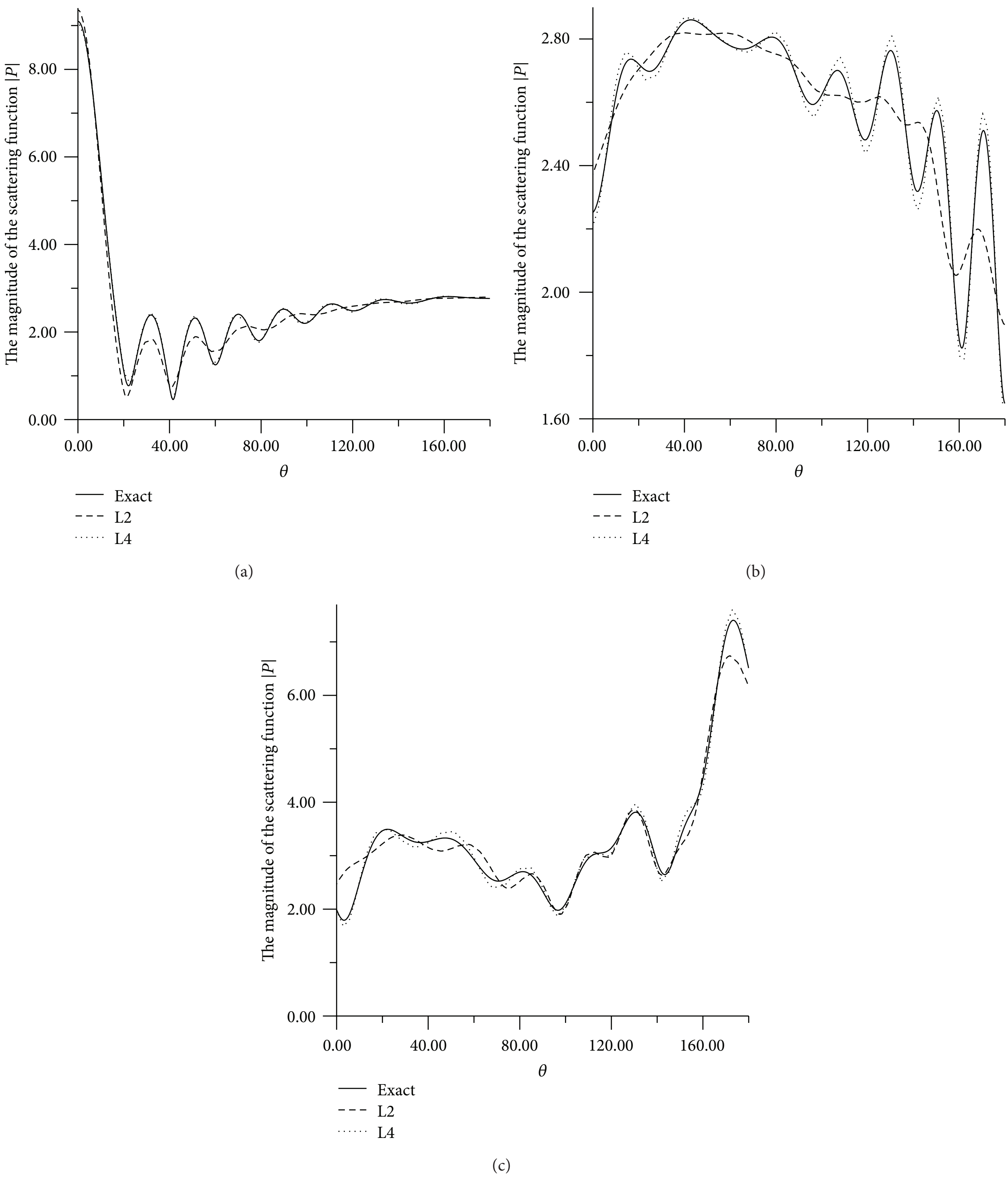

Figure 2

$$- k_2 \xi_1 w_2 \left(\theta'\right) H_0 \cdot \left\{ k_1 \left(r^2 + r_1^2 - 2 r r_1 \cos\left(\theta - \theta'\right) \right)^{1/2} \right\} \Bigg] r_1 d\theta'. \quad (36)$$

Thus the amplitude of the far field is obtained as follows:

$$P(\theta) = \frac{i\varepsilon}{4} \int_0^{2\pi} \left\{ v_2 \left(\theta'\right) i \cos\left(\theta - \theta'\right) - \frac{k_2}{k_1} \xi_1 w_2 \left(\theta'\right) \right\} e^{i\varepsilon \cos(\theta - \theta')} d\theta'. \quad (37)$$

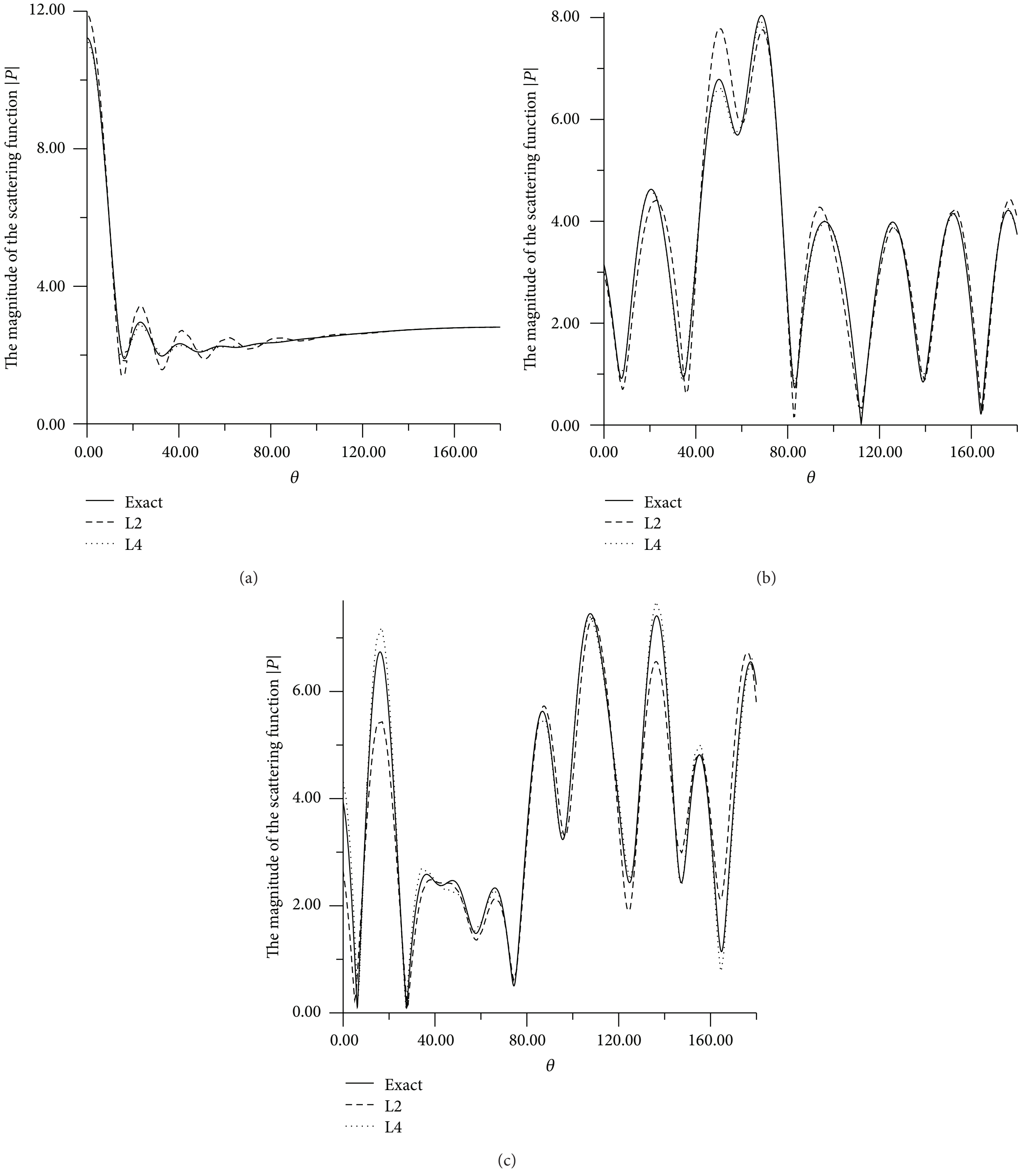

Figure 3

Here if (24) is used and the expressions $\int_0^{2\pi} e^{i\varepsilon\cos(\theta-\theta')+in(\theta'-\pi/2)}d\theta' = 2\pi J_n(\varepsilon)e^{in\theta}$ and $\int_0^{2\pi}\cos(\theta-\theta')e^{i\varepsilon\cos(\theta-\theta')+in(\theta'-\pi/2)}d\theta' = -i2\pi J_n'(\varepsilon)e^{in\theta}$ are considered, after the calculations, the amplitude of the far field is obtained as follows:

$$P(\theta) = \frac{i\varepsilon\pi}{2}\sum_{n=-\infty}^{\infty}\tilde{f}_n e^{in\theta}. \tag{38}$$

Here again $\tilde{f}_n$ is expressed as follows: $\tilde{f}_n = \{J_n'(k_1r_1)J_n(k_2r_1) - (k_2\xi_1/k_1)J_n(k_1r_1)J_n'(k_2r_1)\}\tilde{b}_n + \{J_n'(k_1r_1)H_n(k_2r_1) - (k_2\xi_1/k_1)J_n(k_1r_1)H_n'(k_2r_1)\}\tilde{c}_n$. For the exact solution, the amplitude of the far field is calculated from $P^e(\theta) = \sum_{n=-\infty}^{\infty} a_n e^{in\theta}$. If $\tilde{b}_n$ and $\tilde{c}_n$ are replaced by b_n and c_n, $\tilde{f}_n = \{J_n'(k_1r_1)J_n(k_2r_1) - (k_2\xi_1/k_1)J_n(k_1r_1)J_n'(k_2r_1)\}b_n + \{J_n'(k_1r_1)H_n(k_2r_1) -$

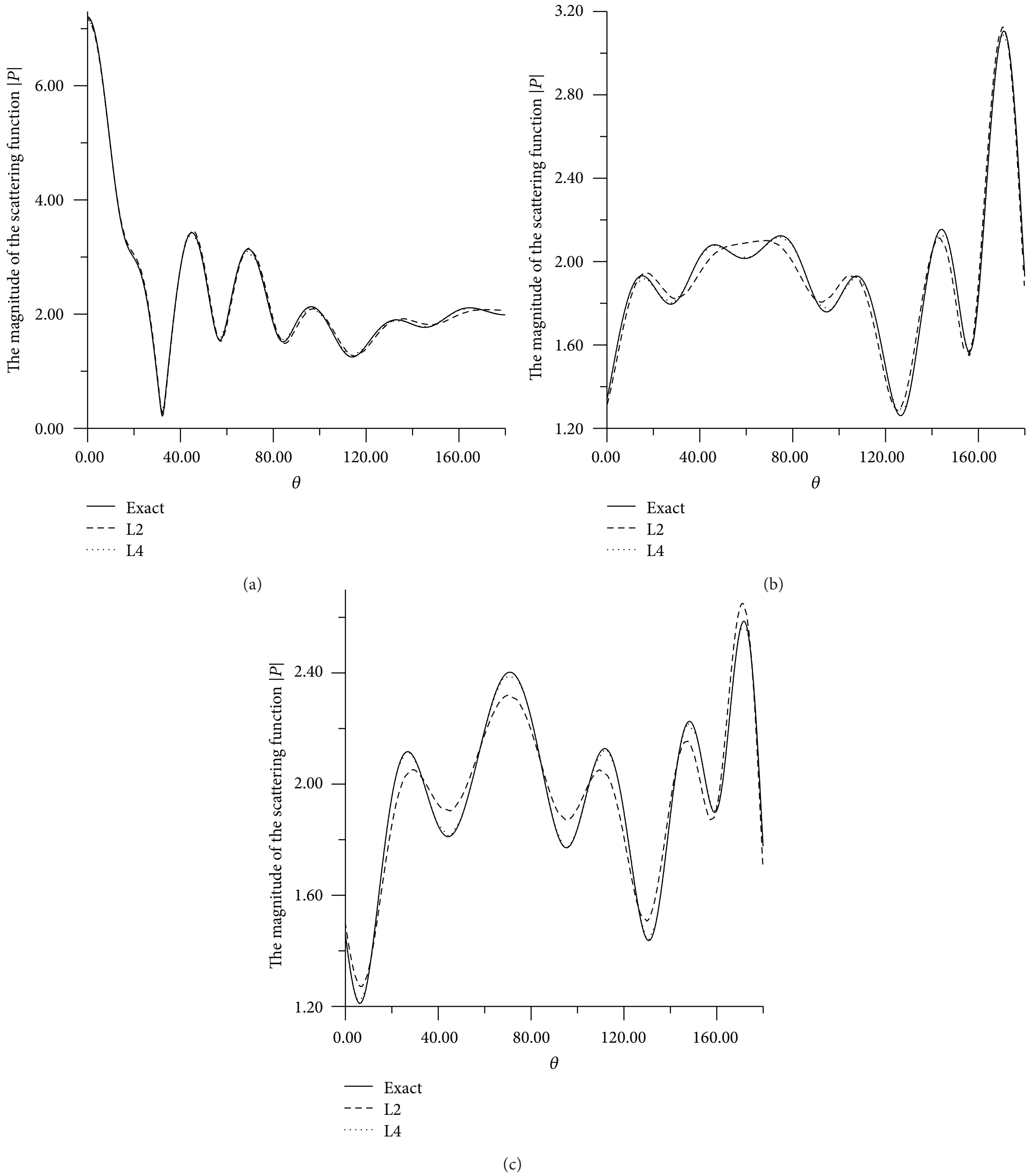

Figure 4

$(k_2\xi_1/k_1)J_n(k_1r_1)H_n'(k_2r_1)\}c_n$ is obtained. Therefore $(i\varepsilon\pi/2)\widetilde{f}_n = a_n$ is obtained.

4. Comparison

Comparisons are made between the exact answer of the problem and the surface radiation condition solutions. It is observed that the introduction of higher-order radiation conditions improves the approximation considerably in comparison with the results obtained by the use of a second-order radiation condition, especially in cases where creeping waves are less pervasive.

Various graphics are generated from analytic solutions obtained from the on surface radiation condition method and from the exact solution of the problem in question in order to examine the consequences of using radiation boundary conditions having an order higher than two in the on surface radiation condition method. For simplicity and readability of graphics only the curves corresponding to solutions of the second and fourth order are shown. Curves corresponding to the third-order condition behave in between those two. In the figures the variations of the modulus of far field, namely, of scattering function P with respect to θ, are given. These studies are done depending on the density of the medium and the velocity of the wave.

From the comparisons made between the exact solution of the problem and the approximate analytic solution obtained by on surface radiation condition method, it can be observed that using the fourth order instead of the second order highly ameliorates the approximation of the method.

(A) Whenever the regions are, respectively, air, water, and a region denser than water from outside to inside the results of the second- and fourth-order surface radiation conditions for the following parameters are depicted together with the exact curve for $r_1 = 10$, $r_2 = 5$, $\varepsilon = 10$, $k_3 = (c_1/c_3)k_1$, $k_2 = (c_1/c_2)k_1$, $\xi_1 = \rho_1/\rho_2$, $\xi_2 = \rho_2/\rho_3$, $\rho_1 = 1.2$, $c_1 = 340$, $\rho_2 = 1000$, $c_2 = 1480$, $\rho_3 = 1200$, and $c_3 = 1600$. As for the graphics of the solutions concerning the far fields and under the parameter values, the variation of the scattered field with $e = 0$ and $\phi_0 = \pi$ is given in Figure 2(a)

Here the attitude of the curve is like in the case of a hard cylinder. With $e = 0.5$ and $\phi_0 = \pi/3$ the scattered field gives acceptable results especially for the fourth order as it is seen in Figure 2(b). As it is seen in Figure 2(c) the scattered field gives good results in shadowed and luminous regions especially for the fourth order when the eccentricity $e = 2.0$ and $\phi_0 = \pi/3$ are taken.

(B) Whenever the regions are, respectively, water, air, and a region denser than air from outside to inside the results of the second- and fourth-order surface radiation conditions for the parameters are $r_1 = 10$, $r_2 = 5$, $\varepsilon = 10$, $k_3 = (c_1/c_3)k_1$, $k_2 = (c_1/c_2)k_1$, $\xi_1 = \rho_1/\rho_2$, $\xi_2 = \rho_2/\rho_3$, $\rho_1 = 1000$, $c_1 = 1480$, $\rho_2 = 1.2$, $c_2 = 340$, $\rho_3 = 1.0$, $c_3 = 280$. As for the graphics of the solutions concerning the far fields and under the parameter values, the variation of the scattered field with $e = 0$ and $\phi_0 = \pi$ is given in Figure 3(a).

Here the attitude of the curve is like in the case of a soft cylinder. In Figures 3(b) and 3(c) the results of the scattered field in shadowed and luminous regions are compared as the eccentricity increases. With $e = 0.5$ and $\phi_0 = \pi/3$ the scattered field gives good results especially for the fourth order as it is seen in Figure 3(b).

As it is seen in Figure 3(c) the scattered field gives good results in shadowed and luminous regions especially for the fourth order when the eccentricity $e = 2.0$ and $\phi_0 = \pi/3$ are taken.

(C) Whenever the regions are, respectively, a region denser than water, water, and air from outside to inside the results of the second- and fourth-order surface radiation conditions for the parameters are, $r_1 = 10$, $r_2 = 5$, $\varepsilon = 10$, $k_3 = (c_1/c_3)k_1$, $k_2 = (c_1/c_2)k_1$, $\xi_1 = \rho_1/\rho_2$, $\xi_2 = \rho_2/\rho_3$, $\rho_1 = 1200$, $c_1 = 1600$, $\rho_2 = 1000$, $c_2 = 1480$, $\rho_3 = 1.2$, $c_3 = 340$.

As for the graphics of the solutions concerning the far fields and under the parameter values, the variation of the scattered field with $e = 0$ and $\phi_0 = \pi$ is given in Figure 4(a). Here also the attitude of the curve is almost like in the case of a soft cylinder.

With $e = 0.5$ and $\phi_0 = \pi/3$ the scattered field gives good results for all orders in both shadowed and luminous regions as it is seen in Figure 4(b).

As it is seen in Figure 4(c) the scattered field gives good results everywhere for the fourth order, when the eccentricity $e = 2.0$ and $\phi_0 = \pi/3$ are taken.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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