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Research Article

Controlling Neimark-Sacker Bifurcation in Delayed Species Model Using Feedback Controller

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Based on the stability and orthogonal polynomial approximation theory, the ordinary, dislocated, enhancing, and random feedback control methods are used to suppress the Neimark-Sacker bifurcation to fixed point in this paper. It is shown that the convergence rate of enhancing feedback control and random feedback control can be faster than those of dislocated and ordinary feedback control. The random feedback control method, which does not require any adjustable control parameters of the model, just only slightly changes the random intensity. Finally, numerical simulations are presented to verify the effectiveness of the proposed controllers.

1. Introduction

The studies of biological models gradually become one of hot spots in nonlinear dynamics. The biological models have great research background and actual significance; therefore, a growing number of researchers have shown great interests in the research of biological models. In many biological models and practical problems, bifurcation and chaos are undesirable behaviors. Thus, we need to control them. In 1976, a population model,

$$x_{n+1} = rx_n\left(1 - x_n\right), \quad r \in [0, 4], \ x \in (0, 1), \tag{1}$$

is given by Ecologist May for the first time. A one-dimensional deterministic delayed population model,

$$x_{n+1} = rx_n\left(1 - x_{n-1}\right), \quad r \in [0, 2.28], \ x \in (0, 1), \tag{2}$$

is investigated by Sun et al. [1].

Recently, the Hopf bifurcation has been given much attention, and those works about bifurcation mainly include the validated existence of bifurcation and its control [2–5]. The aim of bifurcation control is to design a controller to modify the bifurcation properties of a given nonlinear system and then achieve the other desirable dynamical behaviors. OGY feedback control method is studied by Ott et al. [6]. Chen et al. have investigated the feedback control in continuous-time systems [7, 8]. The control of Hopf bifurcation in time-delayed neural network system is investigated by Zhou et al. [9]. Bifurcation analysis and tracking control of an epidemic model with nonlinear incidence rate are investigated by Yi et al. [10]. Wen and Xu studied feedback control of Hopf–Hopf interaction bifurcation with development of torus solutions in high-dimensional maps [11]. Feedback control of bifurcation and chaos in dynamical systems is investigated by Abed and Wang [12]. The Hopf bifurcation control via dynamic state-feedback control is studied by Nguyen and Hong in [13]. Amplitude control of limit cycle from Hopf bifurcation is studied in [14, 15]. Hopf bifurcation control of the system based on washout filter controller is investigated by Wu and Sun [16]. Liu and Xiao have studied complex dynamic behaviors of a discrete-time predator-prey system [17].

However, owing to the uncertain factors of external environment, manufacture, material, and installation, some parameters in practical model are not constant and will be characterized as bound random parameters [18]. The stochastic system can accurately represent the original system better. Therefore the study of stochastic system is more meaningful than deterministic systems. The Hopf bifurcation control is investigated in stochastic system with random parameter [18–20]. It is of interest to examine the stochastic method in biological system and explore its implications.

The rest of this letter is organized as follows. In Section 2, the conditions for the emergence of Neimark-Sacker bifurcation are reviewed. In Section 3, the ordinary, dislocated, enhancing, and random feedback controls for controlling Neimark-Sacker bifurcation are proposed. And numerical simulations are presented to verify the effectiveness of the proposed bifurcation control methods. Finally, conclusions are given in Section 4.

2. Neimark-Sacker Bifurcation

Let us consider the logistic population model [1] for a single species:

$$x_{n+1} = rx_n\left(1 - x_n\right), \tag{3}$$

where x_n stands for the population size at time n and r is the growth rate. In the real environment, the population size is determined not only by the current population size but also by its size in the past. So, we consider

$$x_{n+1} = rx_n\left(1 - x_{n-1}\right), \tag{4}$$

where x_{n-1} stands for the population size at time $n-1$ and r is the growth rate. If we introduce $y_n = x_{n-1}$ in model (4), a two-dimensional discrete-time dynamical model [2] can be rewritten as

$$\begin{aligned} x_{n+1} &= rx_n\left(1 - y_n\right), \\ y_{n+1} &= x_n. \end{aligned} \tag{5}$$

By a simple computation with mathematical software, it is straightforward to obtain the following proposition.

Proposition 1. *(a) For all parameter values, model (5) has one fixed point, $O(0,0)$.*

(b) If $r > 1$, then model (5) has, additionally, a nontrivial positive fixed point, $O^*(x^*, y^*)$, where $x^* = y^* = 1 - 1/r$.

The Jacobian matrix of model (5) evaluated at the fixed point (x, y) is given by

$$J = \begin{pmatrix} r - ry & -rx \\ x & 0 \end{pmatrix}, \tag{6}$$

and the characteristic equation of Jacobian matrix of model (5) can be written as

$$f\left(\lambda\right) = \lambda^2 + p\left(x,y\right)\lambda + q\left(x,y\right) = 0, \tag{7}$$

where $p(x,y) = ry - r$, $q(x,y) = -rx$.

Next, according to the point of view of biology, we study the stability of the nonzero fixed points. Note that the local stability of a fixed point is determined by the modules of eigenvalues of the characteristic equation at the fixed point. From the mathematical software and Lemma 2.2 [17], the following proposition shows the local stability of the fixed point $O^*(x^*, y^*)$.

Proposition 2. *(a) $O^*(x^*, y^*)$ is a sink if $1 < r < 2$.*

(b) $O^*(x^*, y^*)$ is a source if $r > 2$.

(c) $O^*(x^*, y^*)$ is not hyperbolic if $r = 2$.

When the term (c) of Proposition 2 holds, we can obtain that the eigenvalues of the matrix J at the fixed point $O^*(x^*, y^*)$ are a pair of conjugate complex numbers, the modules of which are one. The condition in term (c) of Proposition 2 can be written as the set

$$H_c = \{(r) : r = 2\}; \tag{8}$$

the fixed point $O^*(x^*, y^*)$ can undergo Neimark-Sacker bifurcation when parameters vary in the small neighborhood of H_c. By a simple computation, all eigenvalues of (7) are

$$\lambda_{1,2} = \frac{1}{2} \pm \sqrt{\frac{5}{4} - r}; \tag{9}$$

when $r = r_c = 2$, we can obtain eigenvalues

$$\lambda_{1,2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i. \tag{10}$$

Obviously, the transversality condition, the nondegeneracy condition, and the additional nondegeneracy condition of Neimark-Sacker bifurcation hold (see [3]). Thus, the nontrivial fixed point loses stability in the small neighborhood of H_c. The bifurcation diagram and phase portrait for model (5) are depicted in Figure 1.

3. Neimark-Sacker Bifurcation Control

Without control, model (5) undergoes Neimark-Sacker bifurcations at the point $(0.5025, 0.5025)$ corresponding to the value of bifurcation parameter as $r = 2.01$, as shown in Figure 1. To control the Neimark-Sacker bifurcation to the fixed point, the ordinary, dislocated, enhancing, and random feedback control methods are introduced as shown in the following.

3.1. Ordinary Feedback Control. For the ordinary feedback control, the system variable is often multiplied by a coefficient as the feedback gain, and the feedback gain is added to the right-hand side of the corresponding equation. Let the feedback control input $u = k_1(y_n - y^*)$, and the controlled model is given by

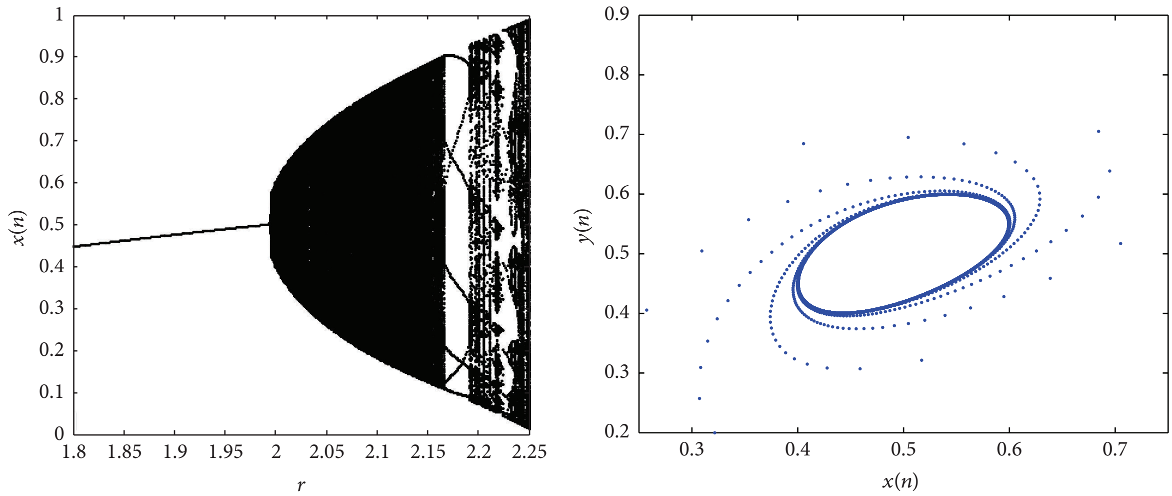
$$\begin{aligned} x_{n+1} &= rx_n\left(1 - y_n\right), \\ y_{n+1} &= x_n + k_1\left(y_n - y^*\right), \end{aligned} \tag{11}$$

where k_1 is the feedback coefficient.

Theorem 3. *The necessary and sufficient condition for the controlled delayed species model (11) to be asymptotically stable at fixed point is $r - 1 > 0$, $2k_1 + r + 1 > 0$, and $k_1 + r - 2 < 0$.*

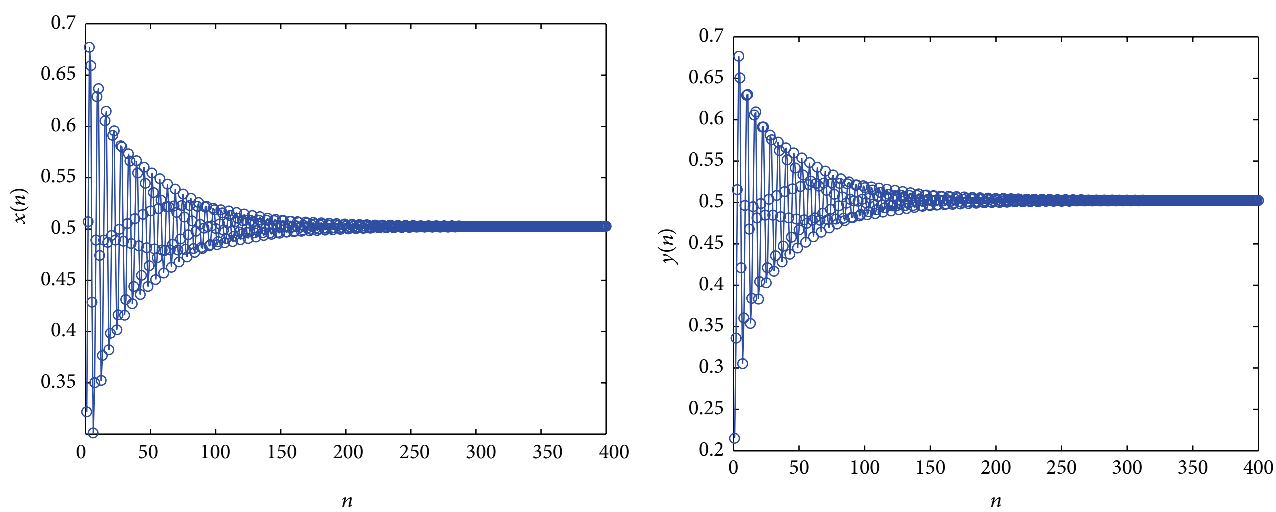
Proof. The Jacobi matrix of model (11) is

$$J = \begin{pmatrix} 1 & -r + 1 \\ 1 & k_1 \end{pmatrix}, \tag{12}$$



(a) (b)

FIGURE 1: (a) Bifurcation diagram of model (5) with bifurcation parameter r covering $[1.8, 2.25]$ and (b) phase portrait corresponding to bifurcation parameter $r = 2.01$. The initial values are $[x(0) = 0.2, y(0) = 0.2]$.



(a) (b)

FIGURE 2: Time history diagram of the controlled delayed species model (11) for $k_1 = -0.05$ and $r = 2.01$; the initial values are $[x(0) = 0.2, y(0) = 0.2]$.

and the characteristic equation of Jacobi matrix J is

$$f(\lambda) = \lambda^2 - (k_1 + 1)\lambda + k_1 + r - 1 = 0; \tag{13}$$

according to Lemma 2.2 [17], the eigenvalues lie inside unit circle if and only if $f(1) > 0$, $f(-1) > 0$, and $q(x^*, y^*) < 1$. By a simple computation, we can obtain the following conditions: (a) $r - 1 > 0$, (b) $2k_1 + r + 1 > 0$, and (c) $k_1 + r - 2 < 0$. Thus, when (a), (b), and (c) hold, the controlled delayed species model (11) will gradually converge to the fixed point.

Numerical simulations are used to investigate the controlled delayed species model (11). From Theorem 3 we conclude that our model (11) will gradually converge to the point $(0.5025, 0.5025)$ for $k_1 \in (-1.505, -0.01)$, when $r = 2.01$. The feedback coefficient is given by $k_1 = -0.05$. The initial values in model (11) are taken as $[x(0) = 0.2, y(0) = 0.2]$. The behaviors of the states $(x(n), y(n))$ of the controlled delayed species model (11) with time n are displayed in Figure 2, respectively. □

3.2. Dislocated Feedback Control. For the dislocated feedback control, a system variable multiplied by a coefficient is added to the right-hand side of another equation. Then, this method is called dislocated feedback control. Let feedback control input $u = k_2(x_n - x^*)$, and the controlled model is given by

$$\begin{aligned} x_{n+1} &= r x_n (1 - y_n), \\ y_{n+1} &= x_n + k_2 (x_n - x^*), \end{aligned} \tag{14}$$

where k_2 is the feedback coefficient.

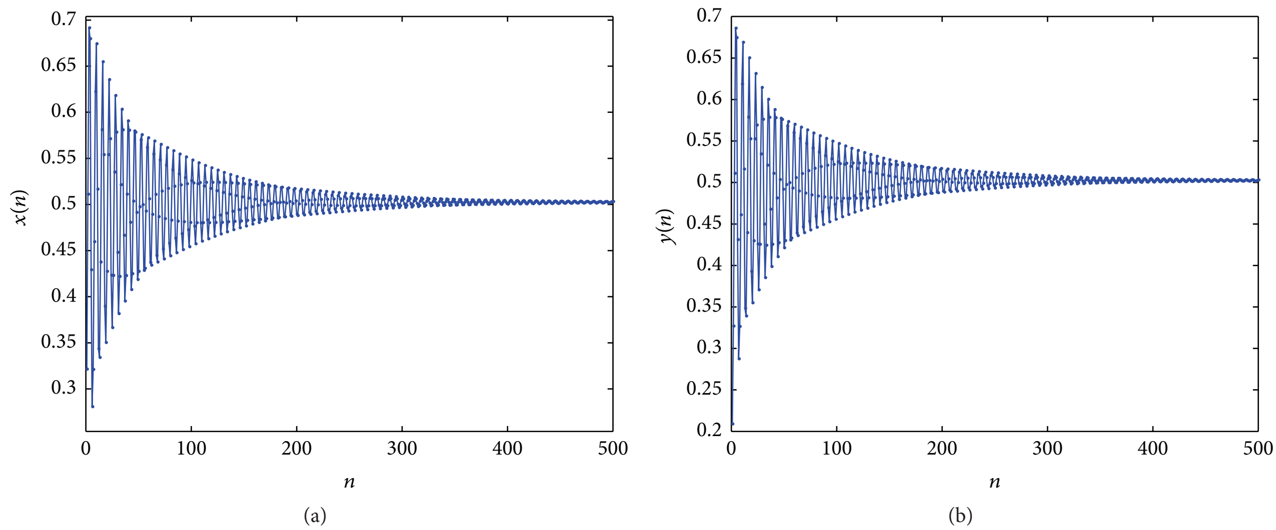


Figure 3: Time history diagram of the controlled delayed species model (14) for $k_2 = -0.03$ and $r = 2.01$. The initial values are $[x(0) = 0.2, y(0) = 0.2]$.

Theorem 4. *The necessary and sufficient condition for the controlled delayed species model (14) to be asymptotically stable at fixed point is* $r + rk_2 - k_2 - 1 > 0$, $rk_2 + r - k_2 + 1 > 0$, *and* $rk_2 + r - k_2 - 2 < 0$.

Proof. The Jacobi matrix of model (14) is

$$J = \begin{pmatrix} 1 & -r+1 \\ 1+k_2 & 0 \end{pmatrix}, \quad (15)$$

and the characteristic equation of Jacobi matrix J is

$$f(\lambda) = \lambda^2 - \lambda + rk_2 + r - k_2 - 1 = 0; \quad (16)$$

according to Lemma 2.2 [17], the eigenvalues lie inside unit circle if and only if $f(1) > 0$, $f(-1) > 0$, and $q(x^*, y^*) < 1$. By a simple computation, we can obtain the following conditions: (a) $r + rk_2 - k_2 - 1 > 0$; (b) $rk_2 + r - k_2 + 1 > 0$; (c) $rk_2 + r - k_2 - 2 < 0$. Thus, when (a), (b), and (c) hold, the controlled delayed species model (14) will gradually converge to the fixed point.

Numerical simulations are used to investigate the controlled delayed species model (14). From Theorem 3, we conclude that our model (14) will gradually converge to the point $(0.5025, 0.5025)$ for $k_2 \in (-1, -0.00990099)$, when $r = 2.01$. The feedback coefficient is given by $k_2 = -0.03$. The initial values in model (14) are taken as $[x(0) = 0.2, y(0) = 0.2]$. The behaviors of the states $(x(n), y(n))$ of the controlled delayed species model (14) with time n are displayed in Figure 3, respectively. □

3.3. Enhancing Feedback Control. For the enhancing feedback control, it is difficult for a complex system to be controlled by only one feedback variable, and in such cases the feedback gain is always very large. So we consider using multiple variables multiplied by a coefficient as the feedback gain. This method is called enhancing feedback control. Let feedback control inputs $u_1 = k_3(x_n - x^*)$, $u_2 = k_3(y_n - y^*)$, and the controlled model is given by

$$\begin{aligned} x_{n+1} &= rx_n(1 - y_n) + k_3(x_n - x^*), \\ y_{n+1} &= x_n + k_3(y_n - y^*), \end{aligned} \quad (17)$$

where k_3 is the feedback coefficient.

Theorem 5. *The necessary and sufficient condition for the controlled delayed species model (17) to be asymptotically stable at equilibrium point is* $-1 - k_3 + k_3^2 + r + (1/r)(k_3 - k_3^2) > 0$, $1+3k_3+k_3^2+r+(1/r)(k_3+k_3^2) > 0$, *and* $k_3+k_3^2+r-(1/r)k_3^2-2 < 0$.

Proof. The Jacobi matrix of system (17) is

$$J = \begin{pmatrix} 1 + k_3\left(1 - \dfrac{1}{r}\right) & -r+1 \\ 1 & k_3 \end{pmatrix}, \quad (18)$$

and the characteristic equation of Jacobi matrix J is

$$\begin{aligned} f(\lambda) = \lambda^2 + \left(-1 - 2k_3 + \frac{k_3}{r}\right)\lambda + k_3 + k_3^2 + r - \frac{k_3^2}{r} \\ - 1 = 0; \end{aligned} \quad (19)$$

according to Lemma 2.2 [17], the eigenvalues lie inside unit circle if and only if $f(1) > 0$, $f(-1) > 0$, and $q(x^*, y^*) < 1$. By a simple computation, we can obtain the following conditions: (a) $-1 - k_3 + k_3^2 + r + (1/r)(k_3 - k_3^2) > 0$; (b) $1+3k_3+k_3^2+r+(1/r)(k_3+k_3^2) > 0$; (c) $k_3+k_3^2+r-(1/r)k_3^2-2 < 0$. Thus, when (a), (b), and (c) hold, the controlled delayed species model (17) will gradually converge to the fixed point.

Numerical simulations are used to investigate the controlled delayed species model (17). From Theorem 3 we conclude that our model (17) will gradually converge to the point $(0.5025, 0.5025)$ for $k_3 \in (-1.980048, -0.010050)$, when

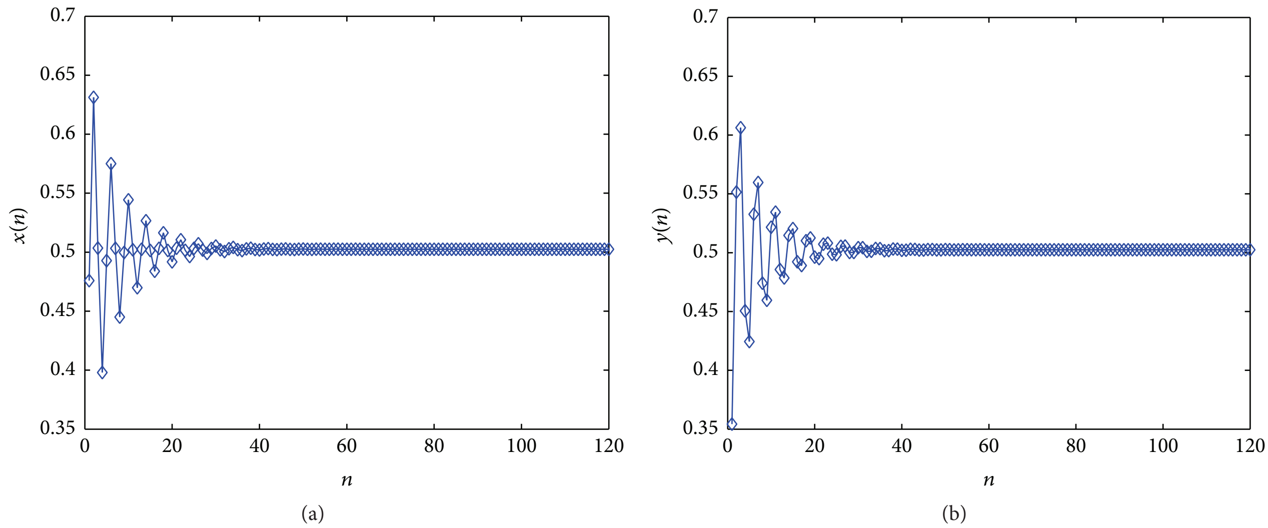


Figure 4: Time history diagram of the controlled delayed species model (17) for $k_3 = -0.41$ and $r = 2.01$. The initial values are $[x(0) = 0.2, y(0) = 0.2]$.

$r = 2.01$. The feedback coefficient is given by $k_3 = -0.41$. The initial values in model (17) are taken as $[x(0) = 0.2, y(0) = 0.2]$. The behaviors of the states $(x(n), y(n))$ of the controlled delayed species model (17) with time n are displayed in Figure 4, respectively. □

3.4. Random Feedback Control. To achieve the control objectives, we need to adjust the control gains for the feedback control methods above. Thus, we use the random feedback control method to control them. If a system variable multiplied by a random coefficient is added to the right-hand side of equation, then this method is called random feedback control. Let $u_n = x_n - x^*$, $v_n = y_n - y^*$; after applying the coordinate transformation, the fixed point is converted to the origin $O(0,0)$; then we have the following system:

$$\begin{aligned} u_{n+1} &= u_n - r u_n v_n - (r-1)(u_n + v_n), \\ v_{n+1} &= u_n. \end{aligned} \tag{20}$$

Let feedback control input $u = k(u_n + v_n)$; k is a random parameter; $k = \alpha k_4 + \beta\delta\xi$. Taking this controller into the right side of the second equation in (20), the controlled delayed species model can be written as

$$\begin{aligned} u_{n+1} &= u_n - r u_n v_n - (r-1)(u_n + v_n), \\ v_{n+1} &= u_n + (\alpha k_4 + \beta\delta\xi) u_n + (\alpha k_4 + \beta\delta\xi) v_n, \end{aligned} \tag{21}$$

where k_4 is the statistic parameter of k, α and β are the input direction of state variable and random variable in the controller, δ is random intensity, and ξ is the random variable defined on nonnegative set integer with the probability density function p_ξ. According to the orthogonal polynomial approximation [18–20] of discrete random function in the Hilbert space, the response of (21) can be expressed by the following series:

$$\begin{aligned} u(n,\xi) &= \sum_{i=0}^{M} u_i(n) C_i(\xi), \\ v(n,\xi) &= \sum_{i=0}^{M} v_i(n) C_i(\xi), \\ u_i(n) &= \sum_{k=0}^{N} p_\xi u(n,\xi) C_i(\xi), \\ v_i(n) &= \sum_{k=0}^{N} p_\xi v(n,\xi) C_i(\xi), \end{aligned} \tag{22}$$

where $C_i(k)$ is the ith Charlier orthogonal polynomial and M is the largest order of the polynomials we have taken. Substituting (22) into (21), we have

$$\begin{aligned} &\sum_{i=0}^{M} u_i(n+1) C_i(\xi) \\ &\quad = \sum_{i=0}^{M} u_i(n) C_i(\xi) \\ &\qquad - r\left(\sum_{i=0}^{M} u_i(n) C_i(\xi)\right)\left(\sum_{i=0}^{M} v_i(n) C_i(\xi)\right) \\ &\qquad - (r-1)\left(\sum_{i=0}^{M} u_i(n) C_i(\xi) + \sum_{i=0}^{M} v_i(n) C_i(\xi)\right), \end{aligned}$$

$$\sum_{i=0}^{M} v_i(n+1) C_i(\xi) = \sum_{i=0}^{M} u_i(n) C_i(\xi) + (\alpha k_4 + \beta\delta\xi) \sum_{i=0}^{M} u_i(n) C_i(\xi) + (\alpha k_4 + \beta\delta\xi) \sum_{i=0}^{M} v_i(n) C_i(\xi). \tag{23}$$

Using the orthogonal polynomial approximation theory, the nonlinear term in the right side of (23) can be further reduced into a linear combination of related single polynomials. It is written as

$$\left(\sum_{i=0}^{M} u_i(n) C_i(k)\right)\left(\sum_{i=0}^{M} v_i(n) C_i(k)\right) = \sum_{i=0}^{2M} X_i(n) C_i(k). \tag{24}$$

We obtained

$$\begin{aligned}
&\sum_{i=0}^{M} u_i(n+1) C_i(\xi) \\
&\quad = \sum_{i=0}^{M} u_i(n) C_i(\xi) - r\sum_{i=0}^{2M} X_i(n) C_i(\xi) \\
&\qquad - (r-1)\left(\sum_{i=0}^{M} u_i(n) C_i(\xi) + \sum_{i=0}^{M} v_i(n) C_i(\xi)\right), \\
&\sum_{i=0}^{M} v_i(n+1) C_i(\xi) \\
&\quad = \sum_{i=0}^{M} u_i(n) C_i(\xi) + (\alpha k_4 + \beta\delta\xi) \sum_{i=0}^{M} u_i(n) C_i(\xi) \\
&\qquad + (\alpha k_4 + \beta\delta\xi) \sum_{i=0}^{M} v_i(n) C_i(\xi).
\end{aligned} \tag{25}$$

The recurrence formula for Charlier polynomial is

$$\xi C_i(\xi) = C_{i+1}(\xi) + (i+\lambda) C_i(\xi) + i\lambda C_{i-1}(\xi). \tag{26}$$

In order to facilitate the numerical analysis of this paper, we take $M = 1$, $\lambda = 1$. Based on the orthogonality of orthogonal polynomials, we can finally obtain the equivalent deterministic equation:

$$\begin{aligned}
&u_0(n+1) \\
&\quad = u_0(n) - r(u_0(n) v_0(n) + u_1(n) v_1(n)) \\
&\qquad - (r-1)(u_0(n) + v_0(n)), \\
&v_0(n+1) \\
&\quad = u_0(n) + \alpha k_4 u_0(n) + \alpha k_4 v_0(n) + \beta\delta u_0(n) \\
&\qquad + \beta\delta v_0(n), \\
&u_1(n+1) \\
&\quad = u_1(n) \\
&\qquad - r(u_0(n) v_1(n) + u_1(n) v_0(n) + u_1(n) v_1(n)) \\
&\qquad - (r-1)(u_1(n) + v_1(n)), \\
&v_1(n+1) \\
&\quad = u_1(n) + \alpha k_4 v_1(n) + \alpha k_4 u_1(n) \\
&\qquad + \beta\delta(2u_1(n) + u_0(n)) + \beta\delta(2v_1(n) + v_0(n)).
\end{aligned} \tag{27}$$

And the ensemble mean response of model (21) is

$$\begin{aligned}
E[u(n,k)] &= \sum_{i=0}^{1} u_i(n) E[P_i(k)] = u_0(n), \\
E[v(n,k)] &= \sum_{i=0}^{1} v_i(n) E[P_i(k)] = v_0(n).
\end{aligned} \tag{28}$$

Here we will first discuss the influence of random feedback controller on the Neimark-Sacker bifurcation. Next we will analyze the Neimark-Sacker bifurcation control with random feedback controller based on deterministic controlled model (27). We suppose that the random intensity of the random controller is from 0 to 0.1. In this section, let $\alpha = -1$, $\beta = 1$, $k_4 = 0.1$, and $\delta = 0.001$, respectively. Owing to the small quantities of intensity δ, the same initial conditions for model (5) and model (27) are given; namely, $u_0 = u(0) = 0.2$, $v_0 = v(0) = 0.2$ and $U(0) = [0.2, 0]^T$, $V(0) = [0.2, 0]^T$. The time history diagrams of the ensemble mean response of controlled delayed species model are shown in Figure 5. By using the same strategy we can discuss the Neimark-Sacker bifurcation control for $\alpha = 1$, $\beta = \pm 1$ and $\alpha = -1$, $\beta = -1$ cases (not reported here).

For the random feedback control, which does not require any adjustable feedback control parameters, we need to slightly change the random intensity of random feedback controller as the random intensity is very small. By means of numerical simulations, we find that the random feedback method to control the Neimark-Sacker bifurcation is available. Next we discuss the influence of the initial values for random feedback control. Taking u_0, v_0 as random initial values covering $[0, 0.5]$, according to numerical simulations, we can find that random feedback control has robustness for

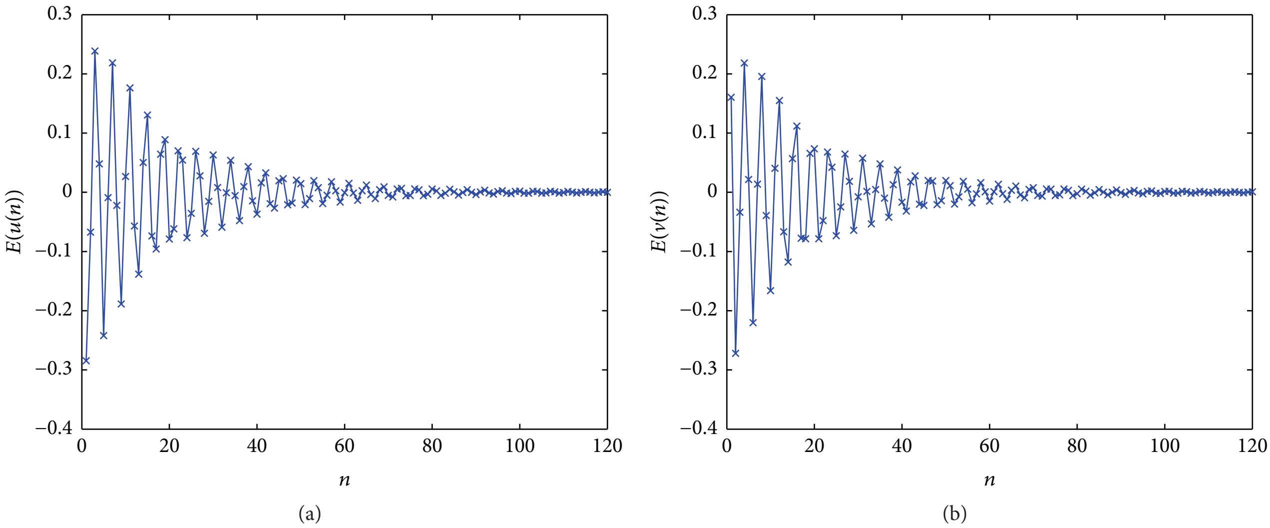


FIGURE 5: Time history diagrams of the controlled delayed species model (21) for $\delta = 0.001$, $k_4 = 0.1$. The initial values are $[u(0) = 0.2, v(0) = 0.2]$.

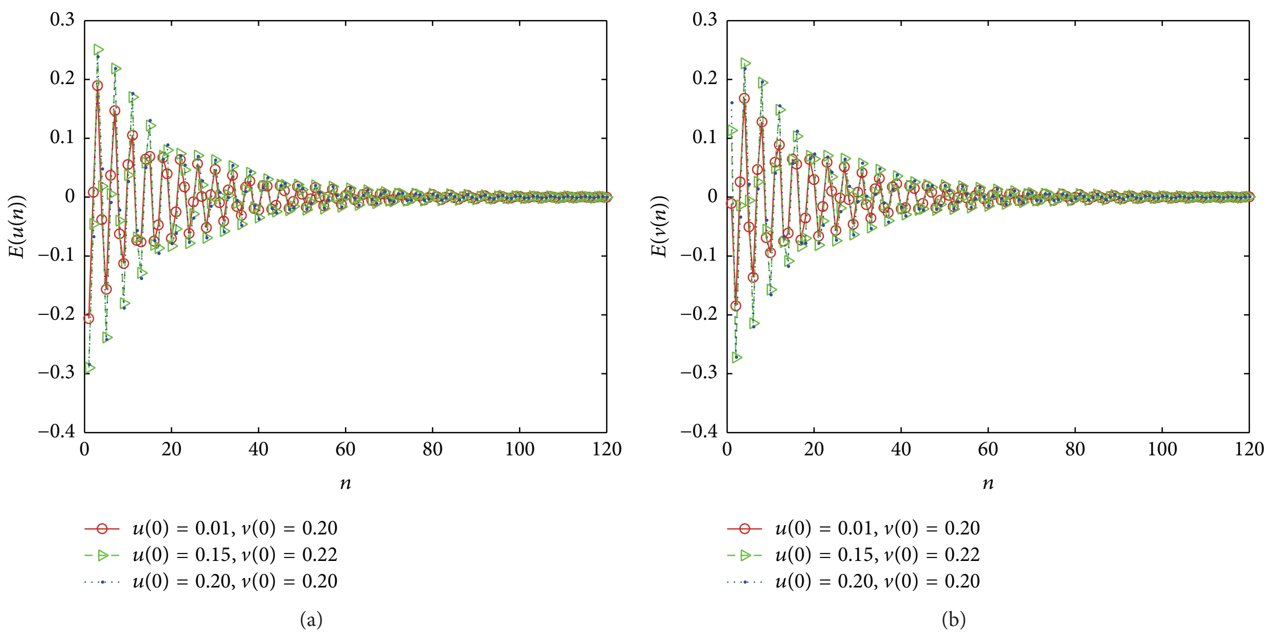


FIGURE 6: Time history diagrams of the controlled delayed species model (21) for $\delta = 0.001$, $k_4 = 0.1$. The initial values $u(0)$ and $v(0)$ take random values covering $[0, 0.5]$.

the random initial values. The time history diagrams of the ensemble mean response of controlled delayed species model (21) are shown in Figure 6.

4. Conclusions

In this paper, the ordinary, dislocated, enhancing, and random feedback control strategies are studied for controlling the Neimark-Sacker bifurcation in the delayed species model. It is found that the convergence rate of variables of enhancing feedback control and random feedback control can be faster than those of dislocated feedback control and ordinary feedback control. From Figure 5 we can find that the effect of the random feedback controller on controlling the Neimark-Sacker bifurcation is available. From Figure 6 we can find that the random feedback controller has robustness for random initial values. Furthermore, numerical simulations are presented to verify the effectiveness of the proposed controllers. Those methods proposed in this paper can be extended to consider other dynamical behaviors such as the double Neimark-Sacker bifurcation and chaos.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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