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Research Article

The Rational Solutions and Quasi-Periodic Wave Solutions as well as Interactions of N-Soliton Solutions for 3 + 1 Dimensional Jimbo-Miwa Equation

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The exact rational solutions, quasi-periodic wave solutions, and N-soliton solutions of 3 + 1 dimensional Jimbo-Miwa equation are acquired, respectively, by using the Hirota method, whereafter the rational solutions are also called algebraic solitary waves solutions and used to describe the squall lines phenomenon and explained possible formation mechanism of the rainstorm formation which occur in the atmosphere, so the study on the rational solutions of soliton equations has potential application value in the atmosphere field; the soliton fission and fusion are described based on the resonant solution which is a special form of the N-soliton solutions. At last, the interactions of the solitons are shown with the aid of N-soliton solutions.

1. Introduction

Some natural phenomena in the physics and in the biology can be depicted by multitudinous nonlinear partial differential equations. Therefore, the solutions of the partial differential equations become the focus points with which we are concerned [1]. There are various ways to get the solutions, such as the Darboux transformation, Bäcklund transformation [2, 3], Inverse scattering transformation, Homogeneous balance method, and Traveling wave solution [4–6]. However, the methods mentioned above cannot be expressing the periodicity of the partial differential equations.

Unlike the above method, the Hirota method [7, 8] plays a crucial role during obtaining the N-soliton solutions by the perturbation and the quasi-periodic wave solutions based on the Riemann theta functions [9]. Hence, it is important to rewrite the partial differential equation into the bilinear forms with the help of the variate transformation. In 2008, Lambert and Springael [10] proposed an explicit way to construct the bilinear forms for the constant coefficient equation.

It is well known that the Hirota method has been widely applied to the 1 + 1 dimensional equations and the 2 + 1 dimensional equations [11–23], but the method is rarely used to the 3 + 1 dimensional equation. As for the 3 + 1 dimensional equation, the quasi-periodic wave solutions happening during the arbitrary two spatial variables x, y, z at one time t or between one spatial variable and the time variable under the other two spatial variables are constants. On the other hand, the rational solutions have attracted more and more attention recently [24–27] because of their graceful structure and potential application value in applied disciplines. The author also applied this kind of rational solutions (also called algebraic solitary wave solutions) to discuss the algebraic Rossby solitary waves and explain the blocking phenomenon which happen in the real atmosphere and ocean [28, 29].

In this paper, we first introduce the well-known 3 + 1 dimensional Jimbo-Miwa equation which has significant efforts in science; it was investigated by Jimbo and Miwa in [30] and its one-soliton solutions were studied by Wazwaz

[31, 32] according to the Tanh-Coth method, and its traveling wave solutions were discussed by Ma and Lee [33] by using rational function transformations. The method provides more systematical and convenient handling of the solution process of nonlinear equations. Lately, we present a brief introduction about the approach and the properties of the Bell-polynomial. Then, the bilinear form of Jimbo-Miwa equation is gained by applying the Bell-polynomial; its rational solutions, quasi-periodic wave solutions, and N-soliton solutions are obtained based on the Hirota method and Riemann theta function. Finally, the resonant solution and the interactions of the N-soliton solutions are given under the Hirota method.

2. The Bell-Polynomial

In order to get the N-soliton solutions of the nonlinear evolutions equations (NLEES), we must get the bilinear form of the NLEES; Lambert et al. connected the Bell-polynomial with the Hirota D operator and give rise to an explicit way to construct the bilinear form to the NLEES. Firstly, we are briefly devoted to the notations of the Bell-polynomial.

The definition of the multidimensional Bell-polynomial is as follows:

$$\begin{aligned} &Y_{n_1x_1n_2x_2\cdots n_rx_r}(g) \\ &\quad\equiv Y_{n_1,n_2,\ldots,n_r}\left(g_{l_1x_1,\ldots,l_rx_r}\left(1\le l_i\le n_i, 0\le i\le r\right)\right) \\ &\quad= e^{-g}\partial_{x_1}^{n_1}\partial_{x_2}^{n_2}\cdots\partial_{x_r}^{n_r}e^{g}, \end{aligned} \tag{1}$$

where g is a C^∞ multivariables' function.

As for a special function g with the variables x, z, we give rise to the following several initial value under the definition of the multivariables Bell-polynomial:

$$\begin{aligned} Y_{x,y}(g) &= g_{x,z}+g_xg_z, \\ Y_{x,2z}(g) &= g_{x,2z}+g_xg_{2z}+2g_{x,z}g_z+g_xg_z^2\cdots. \end{aligned} \tag{2}$$

Then we provide the redefinition of the binary Bell-polynomial as

$$\begin{aligned} &\tau_{n_1x_1,\ldots,n_rx_r}(v,\omega) \\ &\quad\equiv Y_{n_1,\ldots,n_r}(g)\Big|_{g_{l_1x_1,\ldots,l_rx_r}=\begin{cases} v_{l_1x_1,\ldots,l_rx_r} & \sum_{i=1}^r l_i \text{ is odd,} \\ \omega_{l_1x_1,\ldots,l_rx_r} & \sum_{i=1}^r l_i \text{ is even,}\end{cases}} \end{aligned} \tag{3}$$

where v and ω both are the C^∞ functions with the variables $x_1, x_2, \ldots, x_r$. We set out some initial expressions depending on (3) as

$$\begin{aligned} \tau_x(v) &= v_x, \\ \tau_{2x}(v,\omega) &= v_x^2+\omega_{xx}, \\ \tau_{x,y} &= \omega_{x,y}+v_xv_y, \\ \tau_{3x}(v,\omega) &= v_{3x}+3v_x\omega_{2x}+v_x^3, \\ \tau_{2x,y}(v,\omega) &= v_{2x,y}+2v_x\omega_{x,y}+v_x^2v_y+v_y\omega_{2x}\cdots. \end{aligned} \tag{4}$$

The link between the binary Bell-polynomial (3) and the Hirota D-operator can be presented through a transformational identity.

$$\begin{aligned} &\tau_{n_1x_1,\ldots,n_rx_r}\left(v=\ln\frac{F}{G}, \omega=\ln FG\right) \\ &\quad= (F\cdot G)^{-1}D_{x_1}^{n_1}\cdots D_{x_r}^{n_r}F\cdot G, \end{aligned} \tag{5}$$

where the Hirota operator is defined by

$$\begin{aligned} &D_{x_1}^{n_1}\cdots D_{x_r}^{n_r}F\cdot G \\ &\quad= \left(\partial_{x_1}-\partial_{x_1'}\right)^{n_1}\cdots\left(\partial_{x_r}-x_r'\right)F(x_1,\ldots,x_r) \\ &\qquad\times G\left(x_1',\ldots,x_r'\right)\Big|_{x_1'=x_1,\ldots,x_r'=x_r}. \end{aligned} \tag{6}$$

In particular, when $F = G$, (5) can be read as

$$\begin{aligned} F^{-2}D_{x_1}^{n_1}\cdots D_{x_r}^{n_r}F^2 &= \tau_{n_1x_1,\ldots,n_rx_r}(v=0,\omega=2\ln F) \\ &= \begin{cases} 0, & \sum_{i=1}^r n_i \text{ is odd,} \\ P_{n_1x_1,\ldots,n_rx_r}(q), & \sum_{i=1}^r n_i \text{ is even.}\end{cases} \end{aligned} \tag{7}$$

In (7), the Bell-polynomial is equal to the P-polynomial when $\sum_{i=1}^r n_i$ is even, and then we give the first lower order P-polynomial:

$$\begin{aligned} P_{2x}(q) &= q_{2x}, \\ P_{4x}(q) &= q_{4x}+3q_{2x}^2, \\ P_{2x,2y}(q) &= q_{2x,2y}+q_{2x}q_{2y}+2q_{x,y}^2, \\ P_{x,y}(q) &= q_{x,y}, \\ P_{3x,y}(q) &= q_{3x,y}+3q_{x,y}q_{2x}\cdots. \end{aligned} \tag{8}$$

As for the NLEES, we can rewrite them as bilinear from with the aid of the P-polynomial and show the N-soliton solutions and the quasi-periodic wave solutions.

3. The Bilinear Form of the 3 + 1 Dimensional Jimbo-Miwa Equation

The 3 + 1 dimensional Jimbo-Miwa equation is

$$\begin{aligned} &u_{xxxy}+3\left(uu_y\right)_x+3u_{xx}\partial_x^{-1}u_y+3u_xu_y+2u_{yt}-3u_{xz} \\ &\quad= 0. \end{aligned} \tag{9}$$

Letting $u = q_{xx}$ and substituting it into (9) and integrating twice with respect to x yield

$$q_{xxxy}+3q_{xx}q_{xy}+2q_{yt}-3q_{xz}-\lambda=0, \tag{10}$$

where λ is an arbitrary integral constant. So, in terms of the P-polynomial, (10) can be written as

$$P_{3x,y}(q)+2P_{y,t}(q)-3P_{x,z}(q)-\lambda=0. \tag{11}$$

Giving a change of dependent variables

$$q=2\ln F \Longleftrightarrow u=q_{xx}=2(\ln F)_{xx}, \tag{12}$$

then we can acquire the bilinear form of (9) as

$$J\left(D_x, D_y, D_z, D_t\right) \equiv \left(D_{3x}D_y + 2D_yD_t - 3D_xD_z\right) F \cdot F - \lambda F^2 = 0, \tag{13}$$

where the definition of the generalized bilinear D operator is

$$\begin{aligned} D_{p,x}^m D_{p,t}^n F \cdot F &= \left(\frac{\partial}{\partial x} + \alpha_p \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} + \alpha_p \frac{\partial}{\partial t'}\right)^n \\ &\quad \cdot F(x,t) F\left(x', t'\right)\Big|_{x'=x, t'=t} = \sum_{i=0}^{m}\sum_{j=0}^{n} \binom{m}{i}\binom{n}{j} \\ &\quad \cdot \alpha_p^i \alpha_p^j \frac{\partial^{m-i}}{\partial x^{m-i}} \frac{\partial^i}{\partial x'^i} \frac{\partial^{n-j}}{\partial t^{n-j}} \frac{\partial^j}{\partial t'^j} F(x,t) \\ &\quad \cdot F\left(x', t'\right)\Big|_{x'=x, t'=t} \\ &= \sum_{i=0}^{m}\sum_{j=0}^{n} \binom{m}{i}\binom{n}{j} \alpha_p^i \alpha_p^j \frac{\partial^{m+n-i-j} F(x,t)}{\partial x^{m-i} \partial t^{n-j}} \frac{\partial^{i+j} F(x,t)}{\partial x^i \partial t^j}, \\ &\qquad m, n \geq 0, \end{aligned} \tag{14}$$

where α_p^s is calculated as follows:

$$\begin{aligned} \alpha_p^s &= (-1)^{r_p(s)}, \\ s &= r_p(s) \bmod p, \\ D_{2,t}D_{2,y}F \cdot F &= 2F_{y,t}F - 2F_tF_y, \\ D_{2,x}D_{2,z}F \cdot F &= 2F_{x,z}F - 2F_xF_z, \\ D_{2,x}^3 D_{2,y}F \cdot F &= 2F_{3x,y}F - 2F_{3x}F_y - 6F_{2x,y}F_x \\ &\quad + 6F_{x,y}F_{2x}. \end{aligned} \tag{15}$$

Consequently, let $\lambda = 0$; then the linear combination of (13) constructs the 3 + 1 dimensional Jimbo-Miwa equation as

$$\begin{aligned} &\left(D_x^3 D_y + 2D_yD_t - 3D_xD_z\right) F \cdot F \\ &\quad = 2F_{3x,y}F - 2F_{3x}F_y - 6F_{2x,y}F_x + 6F_{x,y}F_{2x} \\ &\qquad + 4F_{y,t}F - 4F_tF_y - 6F_{x,z}F + 6F_xF_z. \end{aligned} \tag{16}$$

4. The Rational Solutions of the 3 + 1 Dimensional Jimbo-Miwa Equation

In this section, we use the symbolic computation with Maple and obtain polynomial solutions whose degree of x is less than 3 and degrees of y, z, and t are less than 2 to the 3 + 1 dimensional Jimbo-Miwa equation:

$$F = \sum_{i=0}^{3}\sum_{s=0}^{2}\sum_{l=0}^{2}\sum_{j=0}^{2} c_{islj} x^i y^s z^l t^j, \tag{17}$$

where c_{ij}'s are constants, and we acquire 36 classes of polynomial solutions to (16). Among the 36 classes of solutions, we enumerate 13 classes of solutions and see Appendix A, where the involved constants c_{islj}'s are arbitrary provided that the solutions are meaningful. We can confirm that there are 13 distinct classes of rational solutions generated from (12) to the 3 + 1 dimensional Jimbo-Miwa equation (9) by considering the transformation of the coefficient c_{islj}; for the detailed expression of rational solutions, see Appendix B.

These above-mentioned solutions are very tedious and difficult to apply in other subjects; here we obtain some reduced form solutions. u_1 can be reduced to

$$\begin{aligned} U_1 &= -\left(147t^4x^4 + 168t^3x^4 + 84t^3x^3 + 90t^2x^4 \right. \\ &\quad - 42t^4x + 48t^2x^3 + 9t^4 + 24tx^4 - 48t^3x + 18t^2x^2 \\ &\quad + 12tx^3 + 3x^4 - 60t^2x + 6t^3 - 24tx + 4t^2 - 2t \\ &\quad \left. - 6x + 1\right)\left(7t^2x^3 + 4tx^3 + 3t^2x + 3tx^2 + x^3 + t^2 \right. \\ &\quad \left. + 2tx + t + x + 1\right)^{-2}, \end{aligned} \tag{18}$$

when $c_{i,s,l,j} = 1 + islj$. The picture of the solution (18) is presented in Figure 1.

In the same way, we obtain reduced form solution of u_5 as follows:

$$\begin{aligned} U_5 &= \left(-0.3888x^4 - 0.5184x^3y - 0.2592x^2y^2 \right. \\ &\quad - 0.2160xy^2 + 0.435456x + 0.577152y \\ &\quad \left. + 1.0368xy + 0.0828y^2 - 0.5184\right)\left(0.36x^3 \right. \\ &\quad \left. + 0.36x^2y + 0.30xy + 0.2016 - 0.72x + 0.24y\right)^{-2}, \end{aligned} \tag{19}$$

when $c_{i,s,l,j} = (1 + i + s + l + j)/100$ and $t = 0.01$. The picture of the solution (19) is presented in Figure 2.

For the solution u_{11}, we have

$$U_{11} = \frac{-(6t+5)^2}{(6xt + 5x + 4)^2}, \tag{20}$$

when $c_{i,s,l,j} = 1 + i + s + l + j$. The picture of the solution (20) is presented in Figure 3.

For the solution u_{12}, we get

$$U_{12} = -\frac{1}{(x+y)^2}, \tag{21}$$

when $c_{i,s,l,j} = 1 + is + lj$. The picture of the solution (21) is presented in Figure 4.

Remark 1. In fact, these above-mentioned rational solutions are greatly different from those common soliton solutions as the form "sech^2." The latter describes that, from the balance between nonlinearity and dispersion, it is possible to have steady waves of a permanent form, which are called classical solitary waves, while, as we know, these solutions which are derived in the paper can be used to describe algebraic solitary waves [28, 29]. During propagation, this kind of solitary

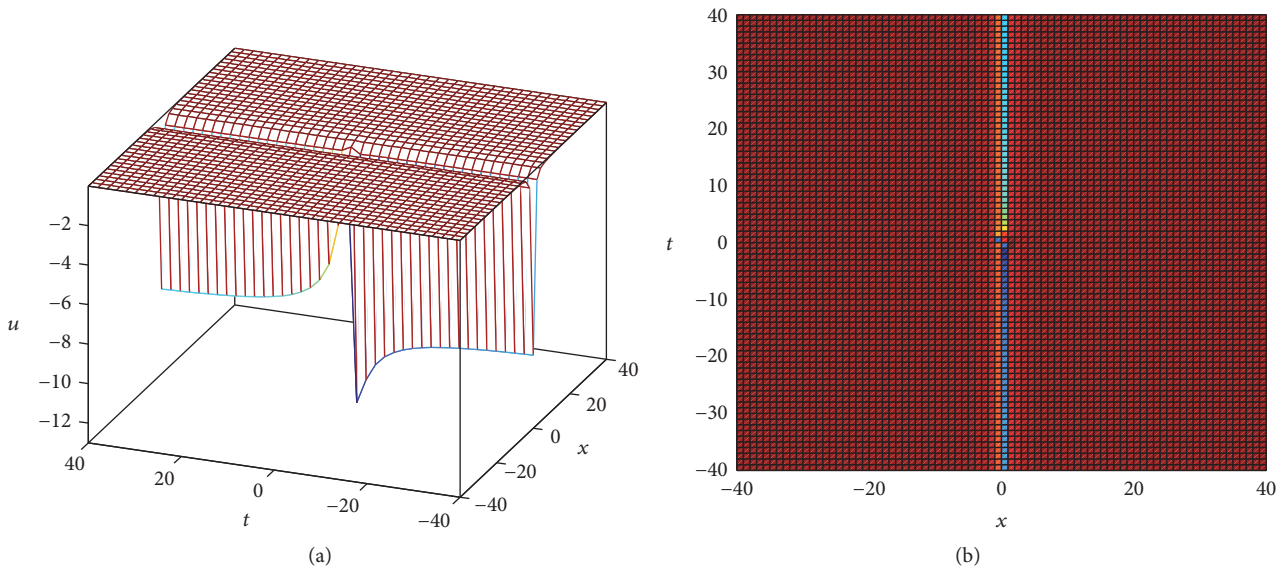

Figure 1: Pictures of (18): 3D plot (a) and density plot (b).

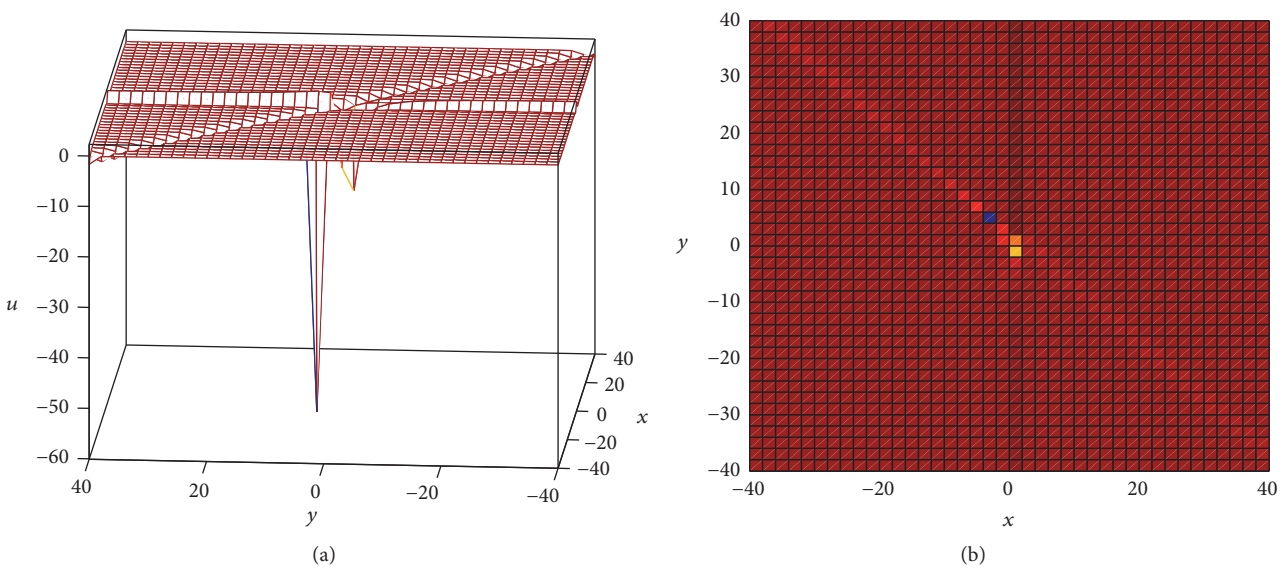

Figure 2: Pictures of (19): 3D plot (a) and density plot (b).

waves will have fission and form an interesting phenomenon: solitary waves in a line that the big amplitude solitary wave is at the front and the small amplitude solitary wave is in subsequent, which is deserved in the real atmosphere in the process of thunderstorm and called squall lines. So the rational solutions of soliton equations are used to explain possible formation mechanism of the rainstorm formation. So the study on the rational solutions of soliton equations has potential application value in the atmosphere field.

5. The Quasi-Periodic Wave Solutions of the 3 + 1 Dimensional Jimbo-Miwa Equation

In this section, we want to get the quasi-periodic wave solution of the 3 + 1 dimensional Jimbo-Miwa equation by applying the Hirota method. Long time ago, the theta functions have been systematically used to construct multiple quasi-periodic solutions [9, 34]. Hence, we let the Riemann function of (9) be

$$F = \sum_{n=-\infty}^{n=\infty} e^{2\pi i n\xi + \pi i n^2 \tau}, \tag{22}$$

where $n \in Z, \tau \in C$, $\operatorname{Im}\tau > 0$, and $\xi = kx + ly + az + wt$; in addition, the parameters k, l, a, w are constant to be determined. Inserting (22) into (13), we have

$$JF \cdot F = J\left(D_x, D_y, D_z, D_t\right) \sum_{n=-\infty}^{n=\infty} e^{2\pi i n\xi + \pi i n^2 \tau} \cdot \sum_{m=-\infty}^{m=\infty} e^{2\pi i m\xi + \pi i m^2 \tau}$$

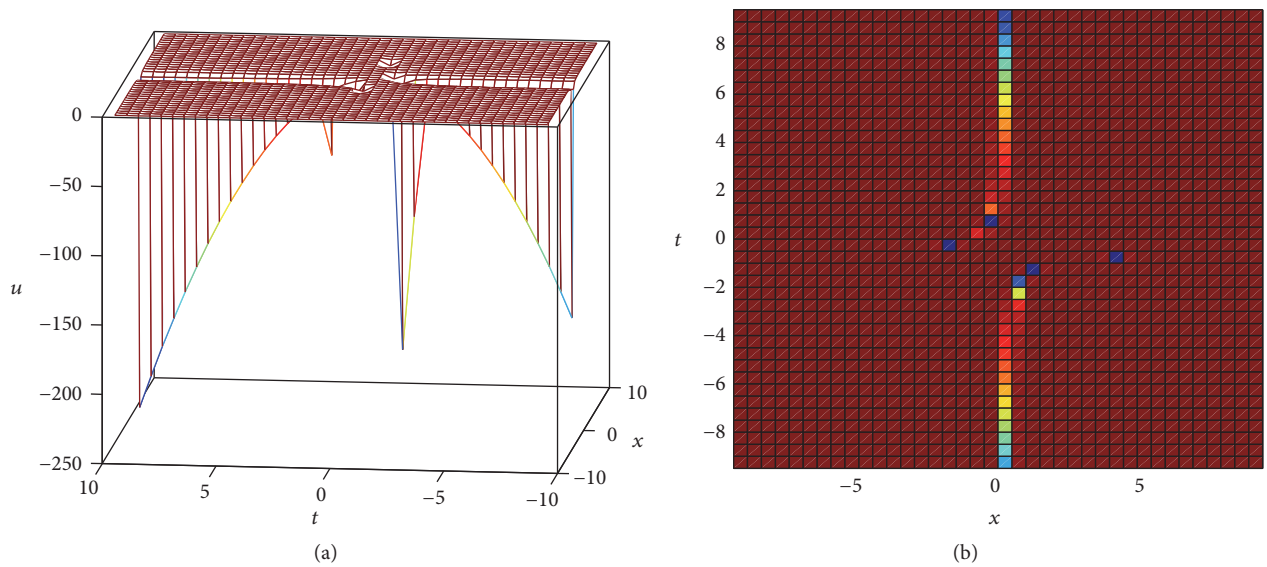

FIGURE 3: Pictures of (20): 3D plot (a) and density plot (b).

$$
\begin{aligned}
&= \sum_{n=-\infty}^{n=\infty} \sum_{m=-\infty}^{m=\infty} J\left(D_x, D_y, D_z, D_t\right) e^{2\pi i n\xi + \pi i n^2 \tau} \\
&\cdot e^{2\pi i m\xi + \pi i m^2 \tau} \\
&= \sum_{n=-\infty}^{n=\infty} \sum_{m=-\infty}^{m=\infty} J\left[2\pi i (n-m)(k,l,a,w)\right] \\
&\cdot e^{2\pi i (n+m)\xi + \pi i (n^2+m^2)\tau} \\
&= \sum_{q=-\infty}^{q=\infty} \left\{ \sum_{n=-\infty}^{n=\infty} J\left[2\pi i (2n-q)(k,l,a,w)\right] \right. \\
&\left. \cdot e^{\pi i (n^2+(q-n)^2)\tau} \right\} e^{2\pi i q\xi} = \sum_{q=-\infty}^{q=\infty} \bar{J}(q) e^{2\pi i q\xi}.
\end{aligned} \tag{23}
$$

Under the calculation of (23), we can denote that

$$
\begin{aligned}
\bar{J}(q) &= \sum_{n=-\infty}^{n=\infty} J\left[2\pi i (2n-q)(k,l,a,w)\right] e^{\pi i (n^2+(q-n)^2)\tau} \\
&= \sum_{h=-\infty}^{h=\infty} J\left(2\pi i B k, 2\pi i B l, 2\pi i B a, 2\pi i B w\right) \\
&\cdot e^{\pi i (n^2+(q-h-2)^2)\tau} \cdot e^{2\pi i (q-1)\tau},
\end{aligned} \tag{24}
$$

where $B = 2h - q + 2$, $q = m + n$, and $h = n - 1$.

From the characters of (24), we can get the following recursion formula:

$$
\bar{J}(q) = \begin{cases} \bar{J}(0) e^{\pi i n q \tau}, & q = 2n, \\ \bar{J}(1) e^{\pi i (2n^2+2n)\tau}, & q = 2n+1. \end{cases} \tag{25}
$$

If we set $\bar{J}(0) = 0$ and $\bar{J}(1) = 0$, it can satisfy (13); that is,

$$
\begin{aligned}
\bar{J}(0) &= \sum_{n=-\infty}^{n=\infty} \left(256\pi^4 n^4 k^3 l - 32\pi^2 n^2 l w + 48\pi^2 n^2 a k \right. \\
&\left. - \lambda\right) e^{2\pi i n^2 \tau} = 0, \\
\bar{J}(1) &= \sum_{n=-\infty}^{n=\infty} \left(16(2n-1)^4 \pi^4 k^3 l - 8\pi^2 (2n-1)^2 l w \right. \\
&\left. + 12\pi^2 (2n-1)^2 a k - \lambda\right) e^{\pi i (2n^2-2n+1)\tau} = 0.
\end{aligned} \tag{26}
$$

With the purpose of computational convenience, we can set

$$
\begin{aligned}
q_{11} &= -\sum_{n=-\infty}^{n=\infty} 32\pi^2 n^2 l e^{2\pi i n^2 \tau}, \\
q_{12} &= \sum_{n=-\infty}^{n=\infty} \left(16(2n-1)^4 \pi^4 k^3 l + 12\pi^2 (2n-1)^2 a k\right) \\
&\cdot e^{\pi i (2n^2-2n+1)\tau}, \\
q_{21} &= \sum_{n=-\infty}^{n=\infty} e^{\pi i (2n^2-2n+1)\tau}, \\
q_{22} &= -\sum_{n=-\infty}^{n=\infty} 8\pi^2 (2n-1)^2 l e^{\pi i (2n^2-2n+1)\tau}, \\
q_{31} &= \sum_{n=-\infty}^{n=\infty} e^{2\pi i n^2 \tau}, \\
q_{13} &= \sum_{n=-\infty}^{n=\infty} \left(256\pi^4 n^4 k^3 l + 48\pi^2 n^2 a k\right) e^{2\pi i n^2 \tau}.
\end{aligned} \tag{27}
$$

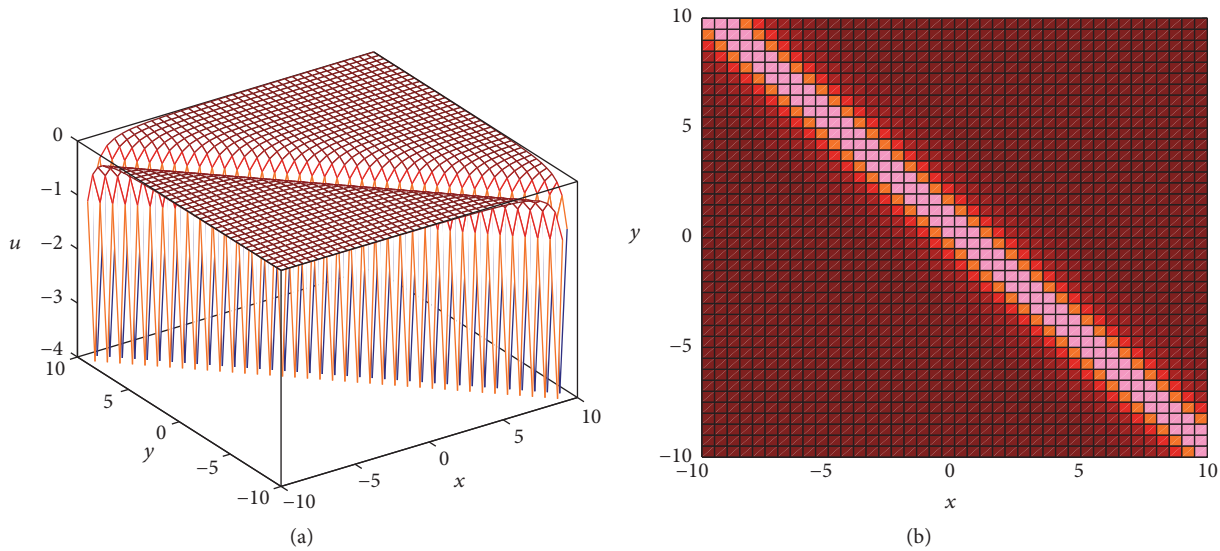

Figure 4: Pictures of (21): 3D plot (a) and density plot (b).

Then (26) can be written as

$$q_{11}w + q_{13} - \lambda q_{31} = 0,$$
$$q_{22}w + q_{12} - \lambda q_{21} = 0. \tag{28}$$

The parameters w, λ can be given by (28) as

$$w = \frac{q_{12}q_{31} - q_{21}q_{13}}{q_{21}q_{11} - q_{31}q_{22}},$$
$$\lambda = \frac{q_{12}q_{11} - q_{22}q_{13}}{q_{21}q_{11} - q_{31}q_{22}}. \tag{29}$$

Therefore, we can obtain the quasi-periodic wave solution of (9) as

$$u = 2(\ln F)_{xx}, \tag{30}$$

where F satisfies (22); meanwhile, w and λ accord with (28). The picture of the quasi-periodic wave solutions of (9) can be shown in Figure 5.

6. The N-Soliton Solutions of the 3 + 1 Dimensional Jimbo-Miwa Equation

In [35], a multiple Exp-function method was proposed to obtain the multiwave solutions and multisoliton solutions. Here we can give rise to the N-soliton solutions of (9) by applying the Hirota direct method and perturbation approach. With the variable transformation $u = 2(\ln F)_{xx}$, (9) has been changed into the bilinear form. We let $\lambda = 0$ and

$$F = 1 + \varepsilon f_1 + \varepsilon^2 f_2 + \varepsilon^3 f_3 + \cdots. \tag{31}$$

Substitute (31) into (13) and compare the power of ε; basing the traditional Hirota method, we can present the N-soliton solution as follows:

$$F = \sum_{\mu_i,\mu_j=0,1} \exp\left\{\sum_{i>j}^{N} A_{ij}\mu_i\mu_j + \sum_{j=1}^{N}\mu_j\eta_j\right\}, \tag{32}$$

where

$$\eta_j = k_j x + l_j y + a_j z + w_j t,$$
$$e^{A_{ij}} = \frac{2(l_j - l_i)(w_i - w_j) + 3(a_i - a_j)(k_i - k_j) + (k_i - k_j)^3(l_j - l_i)}{2(l_j + l_i)(w_i + w_j) - 3(a_i + a_j)(k_i + k_j) + (k_i + k_j)^3(l_j + l_i)}, \tag{33}$$

and then we list the one-soliton solution and the two-soliton solution.

(1) One-Soliton Solution

$$F = 1 + f_1,$$
$$f_1 = e^{\eta},$$
$$\eta = kx + ly + az + wt, \tag{34}$$

where the coefficients k, l, a, w satisfy $2lw + k^3 l - 3ak = 0$, and then the one-soliton solution is

$$u = 2(\ln F)_{xx} = \frac{k^2}{2}\operatorname{sech}^2\frac{\xi}{2}. \tag{35}$$

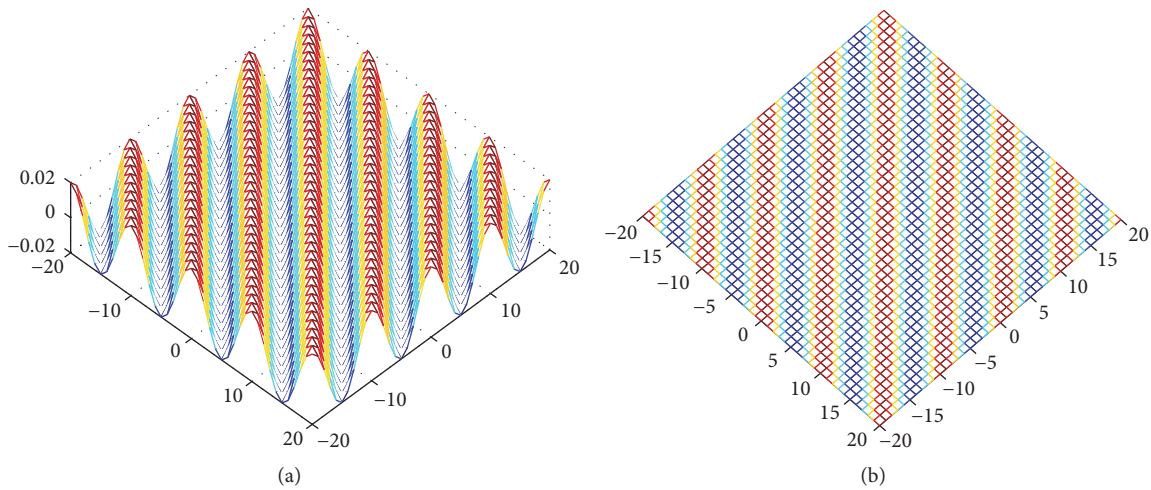

FIGURE 5: One quasi-periodic wave solution to (9) with the parameters $k = 0.1$, $l = 0.1$, $a = 0$, and $\tau = i$. Perspective view of the wave (a); overhead view of the wave (b), with contour plot shown. (The red lines are crests and the blue lines are troughs.)

(2) Two-Soliton Solution

$$F = 1 + f_1 + f_2,$$
$$f_1 = e^{\eta_1} + e^{\eta_2},$$
$$f_2 = e^{\eta_1+\eta_2+A_{12}}, \tag{36}$$

where

$$\eta_1 = k_1 x + l_1 y + a_1 z + w_1 t,$$
$$\eta_2 = k_2 x + l_2 y + a_2 z + w_2 t,$$
$$e^{A_{12}} = \frac{2(l_2 - l_1)(w_1 - w_2) + 3(a_1 - a_2)(k_1 - k_2) + (k_1 - k_2)^3 (l_2 - l_1)}{2(l_2 + l_1)(w_1 + w_2) - 3(a_1 + a_2)(k_1 + k_2) + (k_1 + k_2)^3 (l_2 + l_1)}. \tag{37}$$

Similarly, the coefficients k_i, l_i, a_i, w_i $(i = 1, 2)$ satisfy $2l_i w_i + k_i^3 l_i - 3a_i k_i = 0$, and then the two-soliton solution is

$$u = 2(\ln F)_{xx} = 2\left[\ln(1 + f_1 + f_2)\right]_{xx}. \tag{38}$$

As for the two-soliton solution, when the coefficient $e^{A_{ij}} = 0$, the soliton solution can be changed into the resonant solution whose propagation will be shown in Figure 6.

From Figure 6, we can see that, in Figure 6(a), there happened resonance collision to the two-soliton solution and appeared the third soliton; in Figure 6(b) occurs the soliton fusion whose amplitude has been changed after the resonance collision.

When the coefficient $e^{A_{ij}} \neq 0$, there will happen the soliton pursue collision due to the different soliton speed, as we all know that the wave with the faster speed will catch up with the slower speed wave, then collision of the two-soliton can be happening, and the picture of the pursue collision will be shown in Figure 7.

7. Conclusion

In this paper, we first introduce a 3 + 1 dimensional Jimbo-Miwa equation and get its bilinear form, rational solutions, quasi-periodic wave solutions, and N-soliton solutions based on the Hirota method and the theta function. Afterwards, we analyze the rational solutions and quasi-periodic wave solutions and draw the propagation picture. Finally, we explain the interaction of the N-soliton solutions and get a conclusion that when the coefficient $e^{A_{ij}} = 0$, the soliton solution can be turned into the resonant solution, after the resonance collision, and there will appear the soliton fusion phenomenon; when the coefficient $e^{A_{ij}} \neq 0$, there will appear the pursue collision; that is, the soliton with the faster speed will catch up with the soliton with the slower speed, after the collision, and the two-soliton solution will continue spreading in the previous speed and the direction. Although we acquire the solutions of the 3 + 1 dimensional Jimbo-Miwa equation, the integrability [36] of this equation is not discussed; the problem is worthy of exploring.

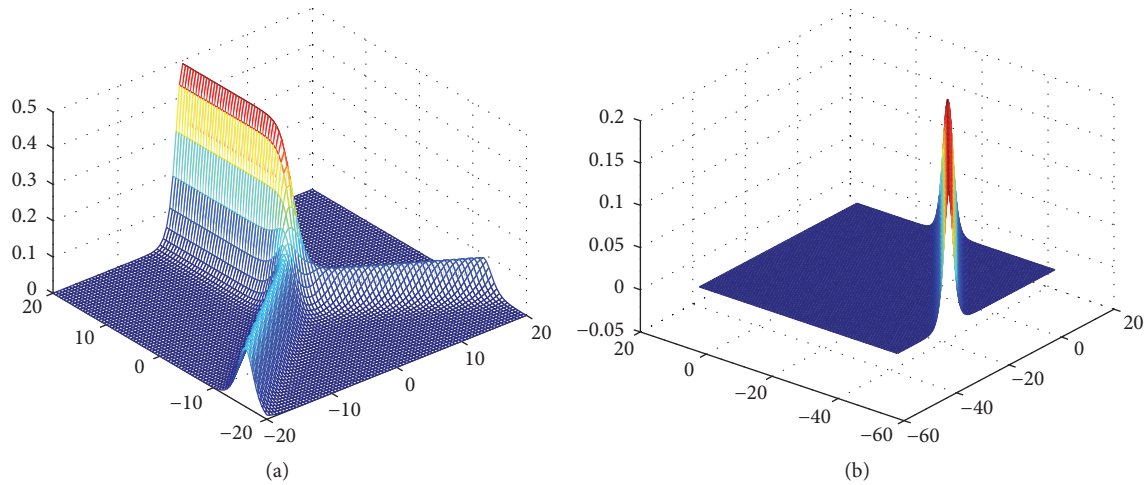

Figure 6: The resonant solution to (9) with the parameters $k_1 = 0.6$, $k_2 = 0.8$, $l_1 = l_2 = 1$, and $a_1 = a_2 = 0$.

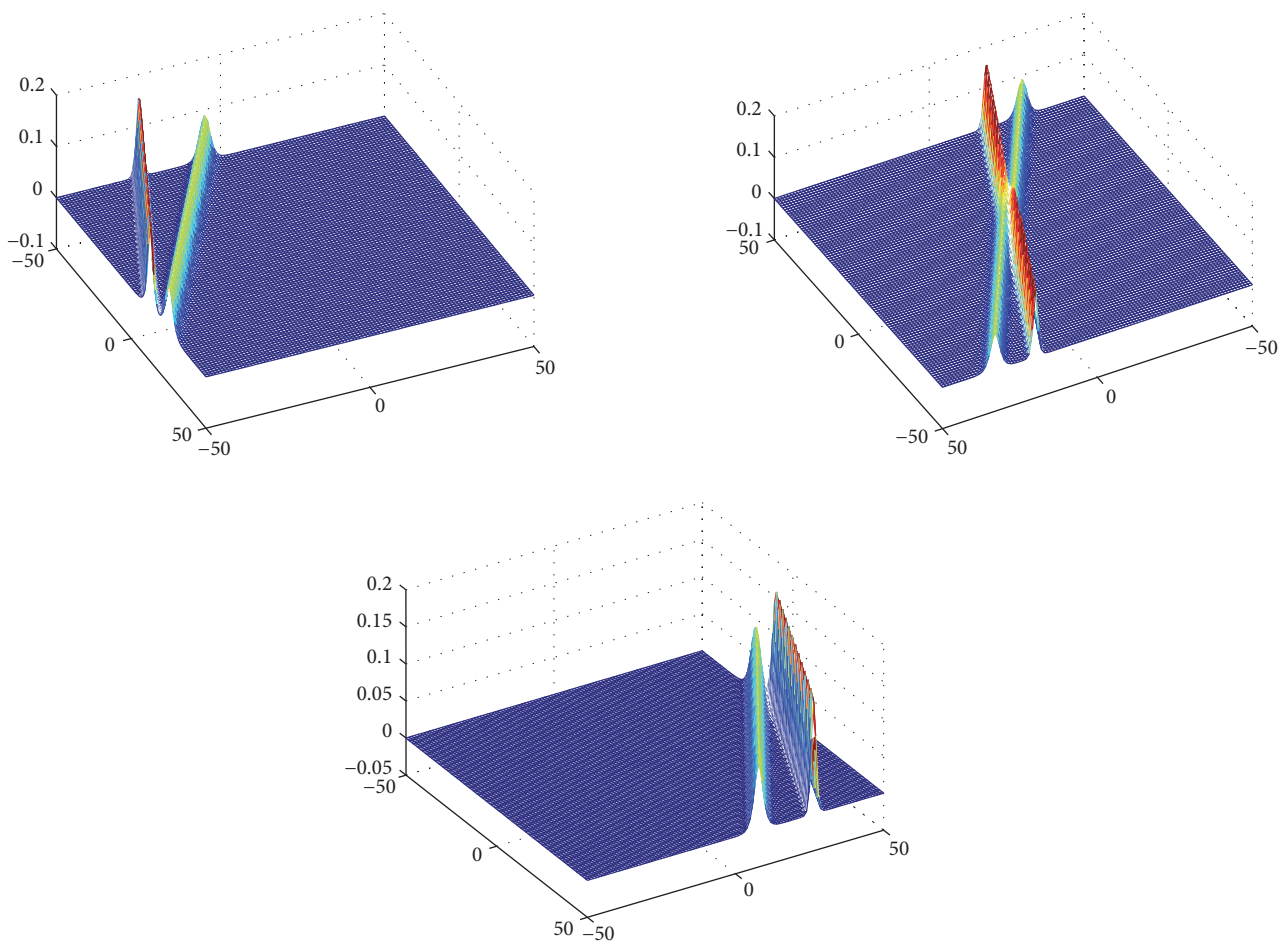

Figure 7: The pursue collision of (35) with $k_1 = 0.6$, $k_2 = 0.8$, $l_1 = 1$, $l_2 = 2$, and $a_1 = a_2 = 0$.

Appendix

A. The Expression of F

$$
\begin{aligned}
F_1 &= t^2x^3yzc_{3,1,1,2} + tx^3yzc_{3,1,1,1} + t^2xyzc_{1,1,1,2} + tx^2yzc_{2,1,1,1} \\
&\quad + x^3yzc_{3,1,1,0} + txyzc_{1,1,1,1} + tyzc_{0,1,1,1} + xyzc_{1,1,1,0} \\
&\quad + yzc_{0,1,1,0} + t^2yzc_{0,1,1,2}, \\
F_2 &= \frac{c_{1,0,1,0}c_{2,1,1,1}zt}{c_{3,1,1,0}} \\
&\quad + \frac{\left(c_{1,1,1,0}{c_{2,1,1,1}}^2 + {c_{1,1,1,1}}^2c_{3,1,1,0} + 6c_{2,1,1,1}{c_{3,1,1,0}}^2\right)yzt}{c_{2,1,1,1}c_{3,1,1,0}} \\
&\quad + tx^2yzc_{2,1,1,1} + c_{0,2,1,1}y^2zt - \frac{c_{1,0,1,0}c_{1,1,1,1}z}{c_{2,1,1,1}} + c_{1,0,1,0}xz \\
&\quad + xyzc_{1,1,1,0} \\
&\quad - \frac{c_{1,1,1,1}\left(c_{1,1,1,0}{c_{2,1,1,1}}^2 + {c_{1,1,1,1}}^2c_{3,1,1,0} + 9c_{2,1,1,1}{c_{3,1,1,0}}^2\right)yz}{{c_{2,1,1,1}}^3} \\
&\quad + x^3yzc_{3,1,1,0} - \frac{c_{0,2,1,1}c_{1,1,1,1}c_{3,1,1,0}y^2z}{{c_{2,1,1,1}}^2} + \frac{c_{0,2,1,1}c_{3,1,1,0}xy^2z}{c_{2,1,1,1}} \\
&\quad + txyzc_{1,1,1,1}, \\
F_3 &= t^2yzc_{0,1,1,2} \\
&\quad - \frac{\left({c_{0,1,1,2}}^2c_{2,1,1,1} - c_{0,1,1,2}c_{1,1,1,1}c_{1,1,1,2} - 3c_{1,1,1,2}{c_{2,1,1,1}}^2\right)yzt}{{c_{1,1,1,2}}^2} \\
&\quad + tx^2yzc_{2,1,1,1} + \frac{c_{0,1,1,2}c_{1,2,1,1}y^2zt}{c_{1,1,1,2}} + c_{1,2,1,1}xy^2zt \\
&\quad + \frac{2c_{0,1,1,2}c_{1,2,1,1}yz^2t}{3c_{2,1,1,1}} + txyzc_{1,1,1,1} + \frac{2c_{1,1,1,2}c_{1,2,1,1}xyz^2t}{3c_{2,1,1,1}} \\
&\quad + t^2xyzc_{1,1,1,2}, \\
F_4 &= t^2yzc_{0,1,1,2} + \frac{c_{1,1,1,2}c_{1,2,1,1}y^2zt^2}{c_{2,1,1,1}} \\
&\quad - \frac{c_{1,0,1,1}\left(c_{0,1,1,2}c_{2,1,1,1} - c_{1,1,1,1}c_{1,1,1,2}\right)zt}{c_{1,1,1,2}c_{2,1,1,1}} \\
&\quad - \frac{\left({c_{0,1,1,2}}^2c_{2,1,1,1} - c_{0,1,1,2}c_{1,1,1,1}c_{1,1,1,2} + 3c_{1,1,1,2}{c_{2,1,1,1}}^2\right)yzt}{{c_{1,1,1,2}}^2} \\
&\quad + txyzc_{1,1,1,1} - \frac{c_{1,2,1,1}\left(c_{0,1,1,2}c_{2,1,1,1} - c_{1,1,1,1}c_{1,1,1,2}\right)y^2zt}{c_{1,1,1,2}c_{2,1,1,1}} \\
&\quad + c_{1,2,1,1}xy^2zt + \frac{c_{1,0,1,1}c_{1,1,1,2}zt^2}{c_{2,1,1,1}} + t^2xyzc_{1,1,1,2} + c_{1,0,1,1}xzt \\
&\quad + tx^2yzc_{2,1,1,1}, \\
F_5 &= -3\frac{c_{0,1,1,1}c_{3,0,1,1}xzt}{c_{2,1,1,1}} + c_{3,0,1,1}x^3zt + tyzc_{0,1,1,1} \\
&\quad + txyzc_{1,1,1,1} + tx^2yzc_{2,1,1,1} + 6c_{3,0,1,1}zt^2 + c_{0,0,1,1}zt, \\
F_6 &= c_{0,0,1,1}zt + tyzc_{0,1,1,1} + tx^2yzc_{2,1,1,1} + c_{0,2,1,1}y^2zt \\
&\quad + \frac{{c_{2,1,1,1}}^2xy^2zt}{3c_{3,0,1,1}} + c_{3,0,1,1}x^3zt - 2c_{2,1,1,1}z^2t - 3c_{3,0,1,1}zt^2, \\
F_7 &= 6c_{3,1,1,1}yzt^2 + \frac{1}{3}\frac{c_{1,0,1,1}c_{2,1,1,1}zt}{c_{3,1,1,1}} + c_{1,0,1,1}xzt + tyzc_{0,1,1,1} \\
&\quad + txyzc_{1,1,1,1} + tx^2yzc_{2,1,1,1} + tx^3yzc_{3,1,1,1} + c_{0,2,1,1}y^2zt \\
&\quad + 3\frac{c_{0,2,1,1}c_{3,1,1,1}xy^2zt}{c_{2,1,1,1}}, \\
F_8 &= 6c_{3,1,1,1}yzt^2 + txyzc_{1,1,1,1} + tx^2yzc_{2,1,1,1} + tx^3yzc_{3,1,1,1} \\
&\quad + \frac{c_{1,2,1,1}c_{2,1,1,1}y^2zt}{3c_{3,1,1,1}} + c_{1,2,1,1}xy^2zt + tyzc_{0,1,1,1}, \\
F_9 &= tx^3yzc_{3,1,1,1} + tx^2yzc_{2,1,1,1} - 3c_{3,1,1,1}yzt^2 + txyzc_{1,1,1,1} \\
&\quad + c_{0,2,1,1}y^2zt + tyzc_{0,1,1,1} + c_{0,0,1,1}zt, \\
F_{10} &= 6c_{3,1,1,1}yzt^2 + \frac{1}{3}\frac{\left(3c_{0,1,1,1}c_{3,0,1,1} + c_{1,0,1,1}c_{2,1,1,1}\right)zt}{c_{3,1,1,1}} \\
&\quad + c_{1,0,1,1}xzt + c_{3,0,1,1}x^3zt + txyzc_{1,1,1,1} + tx^2yzc_{2,1,1,1} \\
&\quad + tx^3yzc_{3,1,1,1} + 6c_{3,0,1,1}zt^2 + tyzc_{0,1,1,1}, \\
F_{11} &= t^2xyzc_{1,1,1,2} + c_{1,0,0,2}xt^2 + c_{1,1,0,2}xyt^2 + c_{1,2,0,2}xy^2t^2 \\
&\quad + c_{1,0,1,2}xzt^2 + \frac{c_{0,0,2,1}c_{1,0,0,2}t}{c_{1,0,2,2}} + c_{1,0,2,2}xz^2t^2 \\
&\quad + \frac{c_{1,0,2,1}c_{1,1,2,2}xyz^2t}{c_{1,0,2,2}} + \frac{c_{1,0,2,1}c_{1,2,2,2}xy^2z^2t}{c_{1,0,2,2}} \\
&\quad + \frac{c_{1,0,2,1}c_{1,1,1,2}xyzt}{c_{1,0,2,2}} + \frac{c_{1,0,2,1}c_{1,2,1,2}xy^2zt}{c_{1,0,2,2}} \\
&\quad + \frac{c_{0,0,2,1}c_{1,1,2,2}yz^2t}{c_{1,0,2,2}} + c_{1,1,2,2}xyz^2t^2 + c_{1,2,2,2}xy^2z^2t^2 \\
&\quad + \frac{c_{0,0,2,1}c_{1,2,2,2}y^2z^2t}{c_{1,0,2,2}} + \frac{c_{0,0,2,1}c_{1,2,1,2}y^2zt}{c_{1,0,2,2}} + \frac{c_{1,0,2,1}c_{1,1,0,2}xyt}{c_{1,0,2,2}} \\
&\quad + \frac{c_{1,0,2,1}c_{1,2,0,2}xy^2t}{c_{1,0,2,2}} + \frac{c_{0,0,2,1}c_{1,1,1,2}yzt}{c_{1,0,2,2}} + \frac{c_{0,0,2,1}c_{1,2,0,2}y^2t}{c_{1,0,2,2}} \\
&\quad + \frac{c_{0,0,2,1}c_{1,1,0,2}yt}{c_{1,0,2,2}} + \frac{c_{0,0,2,1}c_{1,0,1,2}zt}{c_{1,0,2,2}} + \frac{c_{1,0,0,2}c_{1,0,2,1}xt}{c_{1,0,2,2}} \\
&\quad + \frac{c_{1,0,1,2}c_{1,0,2,1}xzt}{c_{1,0,2,2}} + c_{1,0,2,1}xz^2t + c_{0,0,2,1}z^2t + c_{1,2,1,2}xy^2zt^2,
\end{aligned}
$$

$$
\begin{aligned}
F_{12} = {} & \frac{c_{1,0,2,1}c_{1,1,2,2}xyz^2t}{c_{1,0,2,2}} + c_{1,0,2,2}xz^2t^2 + c_{0,1,2,2}yz^2t^2 \\
& + c_{1,1,2,2}xyz^2t^2 + c_{1,0,2,1}xz^2t + c_{1,1,0,2}xyt^2 + c_{0,1,1,2}xzt^2 \\
& + \frac{c_{0,1,2,2}c_{1,1,2,2}c_{1,0,2,1}y^2z^2t}{{c_{1,0,2,2}}^2} + \frac{c_{0,1,2,2}c_{1,0,1,2}c_{1,0,2,1}yzt}{{c_{1,0,2,2}}^2} \\
& + \frac{c_{0,1,2,2}c_{1,0,2,1}c_{1,1,1,2}y^2zt}{{c_{1,0,2,2}}^2} + \frac{c_{1,0,2,1}c_{1,1,1,2}xyzt}{c_{1,0,2,2}} \\
& + \frac{c_{1,0,2,1}c_{1,1,0,2}xyt}{c_{1,0,2,2}} + \frac{c_{1,0,1,2}c_{1,0,2,1}xzt}{c_{1,0,2,2}} + \frac{c_{0,1,2,2}c_{1,1,2,2}y^2z^2t^2}{c_{1,0,2,2}} \\
& + \frac{c_{0,1,2,2}c_{1,0,2,1}c_{1,1,0,2}y^2t}{{c_{1,0,2,2}}^2} + \frac{c_{0,1,2,2}c_{1,0,0,2}c_{1,0,2,1}yt}{{c_{1,0,2,2}}^2} \\
& + \frac{c_{0,1,2,2}c_{1,0,0,2}yt^2}{c_{1,0,2,2}} + \frac{c_{0,1,2,2}c_{1,1,0,2}y^2t^2}{c_{1,0,2,2}} + \frac{c_{1,0,0,2}c_{1,0,2,1}xt}{c_{1,0,2,2}} \\
& + \frac{c_{0,1,2,2}c_{1,0,1,2}yzt^2}{c_{1,0,2,2}} + \frac{c_{0,1,2,2}c_{1,0,2,1}yz^2t}{c_{1,0,2,2}} \\
& + \frac{c_{0,1,2,2}c_{1,1,1,2}y^2zt^2}{c_{1,0,2,2}} + t^2xyzc_{1,1,1,2} + c_{1,0,0,2}xt^2, \\
F_{13} = {} & c_{1,0,1,1}xzt + c_{1,0,0,0}x + c_{1,0,2,1}xz^2t + c_{1,0,0,2}xt^2 \\
& + c_{1,0,1,2}xzt^2 + c_{1,0,2,2}xz^2t^2 + \frac{c_{0,2,2,2}c_{1,0,0,2}y^2t^2}{c_{1,0,2,2}} \\
& + \frac{c_{0,2,2,2}c_{1,0,0,1}y^2t}{c_{1,0,2,2}} + \frac{c_{1,0,0,0}c_{1,1,2,2}xy}{c_{1,0,2,2}} + \frac{c_{1,0,0,0}c_{1,2,2,2}xy^2}{c_{1,0,2,2}} \\
& + \frac{c_{0,2,2,2}c_{1,0,1,0}y^2z}{c_{1,0,2,2}} + c_{1,2,2,2}xy^2z^2t^2 + \frac{c_{1,0,1,0}c_{1,2,2,2}xy^2z}{c_{1,0,2,2}} \\
& + \frac{c_{0,1,2,2}c_{1,0,1,0}yz}{c_{1,0,2,2}} + \frac{c_{0,1,2,2}c_{1,0,1,1}yzt}{c_{1,0,2,2}} + \frac{c_{1,0,1,0}c_{1,1,2,2}xyz}{c_{1,0,2,2}} \\
& + c_{0,1,2,2}yz^2t^2 + \frac{c_{1,0,0,1}c_{1,1,2,2}xyt}{c_{1,0,2,2}} + c_{0,2,2,2}y^2z^2t^2 + c_{1,0,0,1}xt \\
& + \frac{c_{0,1,2,2}c_{1,0,0,0}y}{c_{1,0,2,2}} + \frac{c_{0,2,2,2}c_{1,0,0,0}y^2}{c_{1,0,2,2}} + c_{1,0,1,0}xz \\
& + \frac{c_{0,1,2,2}c_{1,0,0,2}yt^2}{c_{1,0,2,2}} + \frac{c_{0,1,2,2}c_{1,0,1,2}yzt^2}{c_{1,0,2,2}} + \frac{c_{0,1,2,2}c_{1,0,2,1}yz^2t}{c_{1,0,2,2}} \\
& + \frac{c_{0,1,2,2}c_{1,0,0,1}yt}{c_{1,0,2,2}} + c_{1,1,2,2}xyz^2t^2 + \frac{c_{1,0,1,2}c_{1,2,2,2}xy^2zt^2}{c_{1,0,2,2}} \\
& + \frac{c_{1,0,1,1}c_{1,1,2,2}xyzt}{c_{1,0,2,2}} + \frac{c_{1,0,1,1}c_{1,2,2,2}xy^2zt}{c_{1,0,2,2}} \\
& + \frac{c_{1,0,2,1}c_{1,1,2,2}xyz^2t}{c_{1,0,2,2}} + \frac{c_{1,0,2,1}c_{1,2,2,2}xy^2z^2t}{c_{1,0,2,2}} \\
& + \frac{c_{1,0,0,1}c_{1,2,2,2}xy^2t}{c_{1,0,2,2}} + \frac{c_{0,2,2,2}c_{1,0,2,1}y^2z^2t}{c_{1,0,2,2}} \\
& + \frac{c_{1,0,1,2}c_{1,1,2,2}xyzt^2}{c_{1,0,2,2}} + \frac{c_{1,0,0,2}c_{1,1,2,2}xyt^2}{c_{1,0,2,2}} \\
& + \frac{c_{1,0,0,2}c_{1,2,2,2}xy^2t^2}{c_{1,0,2,2}} + \frac{c_{0,2,2,2}c_{1,0,1,2}y^2zt^2}{c_{1,0,2,2}} \\
& + \frac{c_{0,2,2,2}c_{1,0,1,1}y^2zt}{c_{1,0,2,2}}.
\end{aligned}
\tag{A.1}
$$

B. The Rational Solutions of (3 + 1)-Dimensional Jimbo-Miwa Equation

$$
\begin{aligned}
u_1 = {} & -\Big(3t^4x^4{c_{3,1,1,2}}^2 + 6t^3x^4c_{3,1,1,1}c_{3,1,1,2} \\
& + 4t^3x^3c_{2,1,1,1}c_{3,1,1,2} + 6t^2x^4c_{3,1,1,0}c_{3,1,1,2} \\
& + 3x^4{c_{3,1,1,0}}^2 - 6t^4xc_{0,1,1,2}c_{3,1,1,2} \\
& + 4t^2x^3c_{2,1,1,1}c_{3,1,1,1} + 6tx^4c_{3,1,1,0}c_{3,1,1,1} + t^4{c_{1,1,1,2}}^2 \\
& - 6t^3xc_{0,1,1,1}c_{3,1,1,2} - 6t^3xc_{0,1,1,2}c_{3,1,1,1} \\
& + 2t^3xc_{1,1,1,2}c_{2,1,1,1} + 2t^2x^2{c_{2,1,1,1}}^2 \\
& + 4tx^3c_{2,1,1,1}c_{3,1,1,0} + 3t^2x^4{c_{3,1,1,1}}^2 \\
& - 2t^3c_{0,1,1,2}c_{2,1,1,1} + 2t^3c_{1,1,1,1}c_{1,1,1,2} \\
& - 6t^2xc_{0,1,1,0}c_{3,1,1,2} - 6t^2xc_{0,1,1,1}c_{3,1,1,1} + t^2{c_{1,1,1,1}}^2 \\
& + 2t^2xc_{1,1,1,1}c_{2,1,1,1} - 2t^2c_{0,1,1,1}c_{2,1,1,1} \\
& + 2t^2c_{1,1,1,0}c_{1,1,1,2} - 6t^2xc_{0,1,1,2}c_{3,1,1,0} + {c_{1,1,1,0}}^2 \\
& - 6txc_{0,1,1,0}c_{3,1,1,1} - 6txc_{0,1,1,1}c_{3,1,1,0} \\
& + 2txc_{1,1,1,0}c_{2,1,1,1} + 2tc_{1,1,1,0}c_{1,1,1,1} - 6xc_{0,1,1,0}c_{3,1,1,0} \\
& - 2tc_{0,1,1,0}c_{2,1,1,1}\Big)\Big(t^2x^3c_{3,1,1,2} + tx^3c_{3,1,1,1} \\
& + t^2xc_{1,1,1,2} + tx^2c_{2,1,1,1} + x^3c_{3,1,1,0} + t^2c_{0,1,1,2} \\
& + txc_{1,1,1,1} + tc_{0,1,1,1} + xc_{1,1,1,0} + c_{0,1,1,0}\Big)^{-2}, \\
u_2 = {} & -c_{3,1,1,0}{c_{2,1,1,1}}^3\Big(2t^2x^2y^2{c_{2,1,1,1}}^5c_{3,1,1,0} \\
& + 4tx^3y^2{c_{2,1,1,1}}^4{c_{3,1,1,0}}^2 \\
& + 2t^2xy^2c_{1,1,1,1}{c_{2,1,1,1}}^4c_{3,1,1,0} + 3x^4y^2{c_{2,1,1,1}}^3{c_{3,1,1,0}}^3 \\
& - 2t^2y^3c_{0,2,1,1}c_{2,1,1,1}{}^4c_{3,1,1,0} \\
& - 4txy^3c_{0,2,1,1}{c_{2,1,1,1}}^3{c_{3,1,1,0}}^2 - 2t^2y^2c_{1,1,1,0}{c_{2,1,1,1}}^5 \\
& - t^2y^2{c_{1,1,1,1}}^2{c_{2,1,1,1}}^3c_{3,1,1,0} - 12t^2y^2{c_{2,1,1,1}}^4{c_{3,1,1,0}}^2 \\
& - 4txy^2c_{1,1,1,0}{c_{2,1,1,1}}^4c_{3,1,1,0} \\
& - 6txy^2{c_{1,1,1,1}}^2{c_{2,1,1,1}}^2{c_{3,1,1,0}}^2
\end{aligned}
$$

$$
\begin{aligned}
&- 36txy^2 c_{2,1,1,1}{}^3 c_{3,1,1,0}{}^3 \\
&+ 4ty^3 c_{0,2,1,1} c_{1,1,1,1} c_{2,1,1,1}{}^2 c_{3,1,1,0}{}^2 \\
&+ 6xy^3 c_{0,2,1,1} c_{1,1,1,1} c_{2,1,1,1} c_{3,1,1,0}{}^3 \\
&+ y^4 c_{0,2,1,1}{}^2 c_{2,1,1,1} c_{3,1,1,0}{}^3 - 2t^2 y c_{1,0,1,0} c_{2,1,1,1}{}^5 \\
&- 4txy c_{1,0,1,0} c_{2,1,1,1}{}^4 c_{3,1,1,0} \\
&+ 4ty^2 c_{1,1,1,0} c_{1,1,1,1} c_{2,1,1,1}{}^3 c_{3,1,1,0} \\
&+ 2ty^2 c_{1,1,1,1}{}^3 c_{2,1,1,1} c_{3,1,1,0}{}^2 \\
&+ 18ty^2 c_{1,1,1,1} c_{2,1,1,1}{}^2 c_{3,1,1,0}{}^3 \\
&+ 6xy^2 c_{1,1,1,0} c_{1,1,1,1} c_{2,1,1,1}{}^2 c_{3,1,1,0}{}^2 \\
&+ 6xy^2 c_{1,1,1,1}{}^3 c_{3,1,1,0}{}^3 + 54xy^2 c_{1,1,1,1} c_{2,1,1,1} c_{3,1,1,0}{}^4 \\
&+ 2y^3 c_{0,2,1,1} c_{1,1,1,0} c_{2,1,1,1}{}^2 c_{3,1,1,0}{}^2 \\
&+ 4tyc_{1,0,1,0} c_{1,1,1,1} c_{2,1,1,1}{}^3 c_{3,1,1,0} \\
&+ 6xyc_{1,0,1,0} c_{1,1,1,1} c_{2,1,1,1}{}^2 c_{3,1,1,0}{}^2 \\
&+ c_{1,0,1,0}{}^2 c_{2,1,1,1}{}^3 c_{3,1,1,0} \\
&+ 2y^2 c_{0,2,1,1} c_{1,0,1,0} c_{2,1,1,1}{}^2 c_{3,1,1,0}{}^2 \\
&+ y^2 c_{1,1,1,0}{}^2 c_{2,1,1,1}{}^3 c_{3,1,1,0} \\
&+ 2yc_{1,0,1,0} c_{1,1,1,0} c_{2,1,1,1}{}^3 c_{3,1,1,0}\Big) \Big(tx^2 yc_{2,1,1,1}{}^4 c_{3,1,1,0} \\
&+ x^3 yc_{2,1,1,1}{}^3 c_{3,1,1,0}{}^2 + txyc_{1,1,1,1} c_{2,1,1,1}{}^3 c_{3,1,1,0} \\
&+ ty^2 c_{0,2,1,1} c_{2,1,1,1}{}^3 c_{3,1,1,0} + xy^2 c_{0,2,1,1} c_{2,1,1,1}{}^2 c_{3,1,1,0}{}^2 \\
&+ tyc_{1,1,1,0} c_{2,1,1,1}{}^4 + tyc_{1,1,1,1}{}^2 c_{2,1,1,1}{}^2 c_{3,1,1,0} \\
&+ 6tyc_{2,1,1,1}{}^3 c_{3,1,1,0}{}^2 + xyc_{1,1,1,0} c_{2,1,1,1}{}^3 c_{3,1,1,0} \\
&- y^2 c_{0,2,1,1} c_{1,1,1,1} c_{2,1,1,1} c_{3,1,1,0}{}^2 + tc_{1,0,1,0} c_{2,1,1,1}{}^4 \\
&- yc_{1,1,1,1}{}^3 c_{3,1,1,0}{}^2 - yc_{1,1,1,0} c_{1,1,1,1} c_{2,1,1,1}{}^2 c_{3,1,1,0} \\
&- 9yc_{1,1,1,1} c_{2,1,1,1} c_{3,1,1,0}{}^3 - c_{1,0,1,0} c_{1,1,1,1} c_{2,1,1,1}{}^2 c_{3,1,1,0} \\
&+ xc_{1,0,1,0} c_{2,1,1,1}{}^3 c_{3,1,1,0}\Big)^{-2}, \\
u_3 = {} & -c_{1,1,1,2}{}^2 \Big(9t^2 c_{1,1,1,2}{}^4 c_{2,1,1,1}{}^2 \\
&+ 12tzc_{1,1,1,2}{}^4 c_{1,2,1,1} c_{2,1,1,1} + 18txc_{1,1,1,2}{}^3 c_{2,1,1,1}{}^3 \\
&+ 18tyc_{1,1,1,2}{}^3 c_{1,2,1,1} c_{2,1,1,1}{}^2 \\
&+ 18xyc_{1,1,1,2}{}^2 c_{1,2,1,1} c_{2,1,1,1}{}^3 \\
&+ 12xzc_{1,1,1,2}{}^3 c_{1,2,1,1} c_{2,1,1,1}{}^2 + 18x^2 c_{1,1,1,2}{}^2 c_{2,1,1,1}{}^4 \\
&+ 9y^2 c_{1,1,1,2}{}^2 c_{1,2,1,1}{}^2 c_{2,1,1,1}{}^2 \\
&+ 12yzc_{1,1,1,2}{}^3 c_{1,2,1,1}{}^2 c_{2,1,1,1} + 4z^2 c_{1,1,1,2}{}^4 c_{1,2,1,1}{}^2 \\
&- 18tc_{0,1,1,2} c_{1,1,1,2}{}^2 c_{2,1,1,1}{}^3 + 18tc_{1,1,1,1} c_{1,1,1,2}{}^3 c_{2,1,1,1}{}^2 \\
&- 54c_{1,1,1,2} c_{2,1,1,1}{}^5 + 18xc_{1,1,1,1} c_{1,1,1,2}{}^2 c_{2,1,1,1}{}^3 \\
&- 18yc_{0,1,1,2} c_{1,1,1,2} c_{1,2,1,1} c_{2,1,1,1}{}^3 \\
&+ 18yc_{1,1,1,1} c_{1,1,1,2}{}^2 c_{1,2,1,1} c_{2,1,1,1}{}^2 \\
&- 12zc_{0,1,1,2} c_{1,1,1,2}{}^2 c_{1,2,1,1} c_{2,1,1,1}{}^2 \\
&+ 12zc_{1,1,1,1} c_{1,1,1,2}{}^3 c_{1,2,1,1} c_{2,1,1,1} \\
&+ 9c_{1,1,1,1}{}^2 c_{1,1,1,2}{}^2 c_{2,1,1,1}{}^2 \\
&- 18c_{0,1,1,2} c_{1,1,1,1} c_{1,1,1,2} c_{2,1,1,1}{}^3 + 18c_{0,1,1,2}{}^2 c_{2,1,1,1}{}^4\Big) \\
&\cdot \Big(3txc_{1,1,1,2}{}^3 c_{2,1,1,1} + 2xzc_{1,1,1,2}{}^3 c_{1,2,1,1} \\
&+ 3x^2 c_{1,1,1,2}{}^2 c_{2,1,1,1}{}^2 + 3xyc_{1,1,1,2}{}^2 c_{1,2,1,1} c_{2,1,1,1} \\
&+ 3tc_{0,1,1,2} c_{1,1,1,2}{}^2 c_{2,1,1,1} - 3c_{0,1,1,2}{}^2 c_{2,1,1,1}{}^2 \\
&+ 3xc_{1,1,1,1} c_{1,1,1,2}{}^2 c_{2,1,1,1} \\
&+ 3yc_{0,1,1,2} c_{1,1,1,2} c_{1,2,1,1} c_{2,1,1,1} \\
&+ 3c_{0,1,1,2} c_{1,1,1,1} c_{1,1,1,2} c_{2,1,1,1} + 2zc_{0,1,1,2} c_{1,1,1,2}{}^2 c_{1,2,1,1} \\
&+ 9c_{1,1,1,2} c_{2,1,1,1}{}^3\Big)^{-2}, \\
u_4 = {} & -c_{2,1,1,1}{}^2 c_{1,1,1,2}{}^2 \Big(2txy^2 c_{1,1,1,2}{}^3 c_{2,1,1,1} \\
&+ 2x^2 y^2 c_{1,1,1,2}{}^2 c_{2,1,1,1}{}^2 + 2xy^3 c_{1,1,1,2}{}^2 c_{1,2,1,1} c_{2,1,1,1} \\
&+ y^4 c_{1,1,1,2}{}^2 c_{1,2,1,1}{}^2 - 2ty^2 c_{0,1,1,2} c_{1,1,1,2}{}^2 c_{2,1,1,1} \\
&+ 2ty^2 c_{1,1,1,1} c_{1,1,1,2}{}^3 + 2y^2 c_{0,1,1,2}{}^2 c_{2,1,1,1}{}^2 \\
&+ 2xy^2 c_{1,1,1,1} c_{1,1,1,2}{}^2 c_{2,1,1,1} \\
&+ 2y^3 c_{0,1,1,2} c_{1,1,1,2} c_{1,2,1,1} c_{2,1,1,1} \\
&+ 2xyc_{1,0,1,1} c_{1,1,1,2}{}^2 c_{2,1,1,1} + t^2 y^2 c_{1,1,1,2}{}^4 \\
&- 2y^2 c_{0,1,1,2} c_{1,1,1,1} c_{1,1,1,2} c_{2,1,1,1} \\
&+ 2y^2 c_{1,0,1,1} c_{1,1,1,2}{}^2 c_{1,2,1,1} + y^2 c_{1,1,1,1}{}^2 c_{1,1,1,2}{}^2 \\
&+ 6y^2 c_{1,1,1,2} c_{2,1,1,1}{}^3 + 2yc_{0,1,1,2} c_{1,0,1,1} c_{1,1,1,2} c_{2,1,1,1} \\
&+ c_{1,0,1,1}{}^2 c_{1,1,1,2}{}^2\Big) \Big(txyc_{1,1,1,2}{}^3 c_{2,1,1,1} \\
&+ ty^2 c_{1,1,1,2}{}^3 c_{1,2,1,1} + x^2 yc_{1,1,1,2}{}^2 c_{2,1,1,1}{}^2 \\
&+ xy^2 c_{1,1,1,2}{}^2 c_{1,2,1,1} c_{2,1,1,1} + tyc_{0,1,1,2} c_{1,1,1,2}{}^2 c_{2,1,1,1} \\
&+ xyc_{1,1,1,1} c_{1,1,1,2}{}^2 c_{2,1,1,1} \\
&- y^2 c_{0,1,1,2} c_{1,1,1,2} c_{1,2,1,1} c_{2,1,1,1} + tc_{1,0,1,1} c_{1,1,1,2}{}^3
\end{aligned}
$$

$$
\begin{aligned}
&- yc_{0,1,1,2}{}^{2}c_{2,1,1,1}{}^{2} + y^{2}c_{1,1,1,1}c_{1,1,1,2}{}^{2}c_{1,2,1,1} \\
&+ xc_{1,0,1,1}c_{1,1,1,2}{}^{2}c_{2,1,1,1} - 3yc_{1,1,1,2}c_{2,1,1,1}{}^{3} \\
&+ c_{1,0,1,1}c_{1,1,1,1}c_{1,1,1,2}{}^{2} - c_{0,1,1,2}c_{1,0,1,1}c_{1,1,1,2}c_{2,1,1,1} \\
&+ yc_{0,1,1,2}c_{1,1,1,1}c_{1,1,1,2}c_{2,1,1,1}\big)^{-2},
\end{aligned}
$$

$$
\begin{aligned}
u_5 = {} & \big(-3x^{4}c_{2,1,1,1}{}^{2}c_{3,0,1,1}{}^{2} - 4x^{3}yc_{2,1,1,1}{}^{3}c_{3,0,1,1} \\
&- 2xy^{2}c_{1,1,1,1}c_{2,1,1,1}{}^{3} + 36txc_{2,1,1,1}{}^{2}c_{3,0,1,1}{}^{2} \\
&- 2x^{2}y^{2}c_{2,1,1,1}{}^{4} + 12tyc_{2,1,1,1}{}^{3}c_{3,0,1,1} \\
&+ 12xyc_{0,1,1,1}c_{2,1,1,1}{}^{2}c_{3,0,1,1} + 2y^{2}c_{0,1,1,1}c_{2,1,1,1}{}^{3} \\
&- y^{2}c_{1,1,1,1}{}^{2}c_{2,1,1,1}{}^{2} + 6xc_{0,0,1,1}c_{2,1,1,1}{}^{2}c_{3,0,1,1} \\
&+ 2yc_{0,0,1,1}c_{2,1,1,1}{}^{3} + 6tc_{2,1,1,1}c_{3,0,1,1} \\
&- 9c_{0,1,1,1}{}^{2}c_{3,0,1,1}{}^{2}\big)\big(x^{3}c_{2,1,1,1}c_{3,0,1,1} + x^{2}yc_{2,1,1,1}{}^{2} \\
&+ 6yc_{0,1,1,1}c_{1,1,1,1}c_{2,1,1,1}c_{3,0,1,1} - 3xc_{0,1,1,1}c_{3,0,1,1} \\
&+ yc_{0,1,1,1}c_{2,1,1,1} + c_{0,0,1,1}c_{2,1,1,1} + xyc_{1,1,1,1}c_{2,1,1,1}\big)^{-2},
\end{aligned}
$$

$$
\begin{aligned}
u_6 = {} & -\big(27x^{4}c_{3,0,1,1}{}^{4} + 36x^{3}yc_{2,1,1,1}c_{3,0,1,1}{}^{3} \\
&+ 18x^{2}y^{2}c_{2,1,1,1}{}^{2}c_{3,0,1,1}{}^{2} - 18y^{3}c_{0,2,1,1}c_{2,1,1,1}c_{3,0,1,1}{}^{2} \\
&+ 6xy^{3}c_{2,1,1,1}{}^{3}c_{3,0,1,1} + y^{4}c_{2,1,1,1}{}^{4} \\
&- 54xy^{2}c_{0,2,1,1}c_{3,0,1,1}{}^{3} + 162txc_{3,0,1,1}{}^{4} \\
&+ 54tyc_{2,1,1,1}c_{3,0,1,1}{}^{3} - 54xyc_{0,1,1,1}c_{3,0,1,1}{}^{3} \\
&+ 108xzc_{2,1,1,1}c_{3,0,1,1}{}^{3} - 18y^{2}c_{0,1,1,1}c_{2,1,1,1}c_{3,0,1,1}{}^{2} \\
&+ 36yzc_{2,1,1,1}{}^{2}c_{3,0,1,1}{}^{2} - 54xc_{0,0,1,1}c_{3,0,1,1}{}^{3} \\
&- 18yc_{0,0,1,1}c_{2,1,1,1}c_{3,0,1,1}{}^{2}\big)\big(-3x^{3}c_{3,0,1,1}{}^{2} \\
&- xy^{2}c_{2,1,1,1}{}^{2} + 6zc_{2,1,1,1}c_{3,0,1,1} - 3y^{2}c_{0,2,1,1}c_{3,0,1,1} \\
&+ 9tc_{3,0,1,1}{}^{2} - 3yc_{0,1,1,1}c_{3,0,1,1} - 3c_{0,0,1,1}c_{3,0,1,1} \\
&- 3x^{2}yc_{2,1,1,1}c_{3,0,1,1}\big)^{-2},
\end{aligned}
$$

$$
\begin{aligned}
u_7 = {} & 3c_{3,1,1,1}\big(-9x^{4}y^{2}c_{2,1,1,1}{}^{2}c_{3,1,1,1}{}^{3} \\
&- 12x^{3}y^{2}c_{2,1,1,1}{}^{3}c_{3,1,1,1}{}^{2} \\
&+ 18xy^{2}c_{0,1,1,1}c_{2,1,1,1}{}^{2}c_{3,1,1,1}{}^{2} \\
&+ 108txy^{2}c_{2,1,1,1}{}^{2}c_{3,1,1,1}{}^{3} - 6x^{2}y^{2}c_{2,1,1,1}{}^{4}c_{3,1,1,1} \\
&- 27y^{4}c_{0,2,1,1}{}^{2}c_{3,1,1,1}{}^{3} + 36ty^{2}c_{2,1,1,1}{}^{3}c_{3,1,1,1}{}^{2} \\
&- 6xy^{2}c_{1,1,1,1}c_{2,1,1,1}{}^{3}c_{3,1,1,1} \\
&- 18y^{3}c_{0,2,1,1}c_{1,1,1,1}c_{2,1,1,1}c_{3,1,1,1}{}^{2} \\
&+ 6y^{3}c_{0,2,1,1}c_{2,1,1,1}{}^{3}c_{3,1,1,1} + 6y^{2}c_{0,1,1,1}c_{2,1,1,1}{}^{3}c_{3,1,1,1} \\
&- 18y^{2}c_{0,2,1,1}c_{1,0,1,1}c_{2,1,1,1}c_{3,1,1,1}{}^{2} \\
&- 3y^{2}c_{1,1,1,1}{}^{2}c_{2,1,1,1}{}^{2}c_{3,1,1,1} \\
&- 6yc_{1,0,1,1}c_{1,1,1,1}c_{2,1,1,1}{}^{2}c_{3,1,1,1} + 2yc_{1,0,1,1}c_{2,1,1,1}{}^{4} \\
&- 3c_{1,0,1,1}{}^{2}c_{2,1,1,1}{}^{2}c_{3,1,1,1}\big)\big(c_{1,0,1,1}c_{2,1,1,1}{}^{2} \\
&+ 3xc_{1,0,1,1}c_{2,1,1,1}c_{3,1,1,1} + 9xy^{2}c_{0,2,1,1}c_{3,1,1,1}{}^{2} \\
&+ 18tyc_{2,1,1,1}c_{3,1,1,1}{}^{2} + 3xyc_{1,1,1,1}c_{2,1,1,1}c_{3,1,1,1} \\
&+ 3x^{2}yc_{2,1,1,1}{}^{2}c_{3,1,1,1} + 3y^{2}c_{0,2,1,1}c_{2,1,1,1}c_{3,1,1,1} \\
&+ 3yc_{0,1,1,1}c_{2,1,1,1}c_{3,1,1,1} + 3x^{3}yc_{2,1,1,1}c_{3,1,1,1}{}^{2}\big)^{-2},
\end{aligned}
$$

$$
\begin{aligned}
u_8 = {} & 3c_{3,1,1,1}\big(-12x^{3}c_{2,1,1,1}c_{3,1,1,1}{}^{2} - 6x^{2}c_{2,1,1,1}{}^{2}c_{3,1,1,1} \\
&- 3y^{2}c_{1,2,1,1}{}^{2}c_{3,1,1,1} - 6xc_{1,1,1,1}c_{2,1,1,1}c_{3,1,1,1} \\
&+ 108txc_{3,1,1,1}{}^{3} - 9x^{4}c_{3,1,1,1}{}^{3} + 36tc_{2,1,1,1}c_{3,1,1,1}{}^{2} \\
&+ 18xc_{0,1,1,1}c_{3,1,1,1}{}^{2} - 6yc_{1,1,1,1}c_{1,2,1,1}c_{3,1,1,1} \\
&+ 2yc_{1,2,1,1}c_{2,1,1,1}{}^{2} + 6c_{0,1,1,1}c_{2,1,1,1}c_{3,1,1,1} \\
&- 3c_{1,1,1,1}{}^{2}c_{3,1,1,1}\big)\big(3x^{3}c_{3,1,1,1}{}^{2} + 3x^{2}c_{2,1,1,1}c_{3,1,1,1} \\
&+ 3xyc_{1,2,1,1}c_{3,1,1,1} + 18tc_{3,1,1,1}{}^{2} + 3xc_{1,1,1,1}c_{3,1,1,1} \\
&+ yc_{1,2,1,1}c_{2,1,1,1} + 3c_{0,1,1,1}c_{3,1,1,1}\big)^{-2},
\end{aligned}
$$

$$
\begin{aligned}
u_9 = {} & -y\big(3x^{4}yc_{3,1,1,1}{}^{2} + 4x^{3}yc_{2,1,1,1}c_{3,1,1,1} \\
&+ 18txyc_{3,1,1,1}{}^{2} - 6xy^{2}c_{0,2,1,1}c_{3,1,1,1} \\
&+ 6tyc_{2,1,1,1}c_{3,1,1,1} + 2x^{2}yc_{2,1,1,1}{}^{2} - 6xyc_{0,1,1,1}c_{3,1,1,1} \\
&+ 2xyc_{1,1,1,1}c_{2,1,1,1} - 2y^{2}c_{0,2,1,1}c_{2,1,1,1} \\
&- 6xc_{0,0,1,1}c_{3,1,1,1} - 2yc_{0,1,1,1}c_{2,1,1,1} + yc_{1,1,1,1}{}^{2} \\
&- 2c_{0,0,1,1}c_{2,1,1,1}\big)\big(-x^{3}yc_{3,1,1,1} - x^{2}yc_{2,1,1,1} \\
&+ 3tyc_{3,1,1,1} - xyc_{1,1,1,1} - y^{2}c_{0,2,1,1} - yc_{0,1,1,1} \\
&- c_{0,0,1,1}\big)^{-2},
\end{aligned}
$$

$$
\begin{aligned}
u_{10} = {} & 3c_{3,1,1,1}\big(-9x^{4}y^{2}c_{3,1,1,1}{}^{3} - 18x^{4}yc_{3,0,1,1}c_{3,1,1,1}{}^{2} \\
&- 12x^{3}y^{2}c_{2,1,1,1}c_{3,1,1,1}{}^{2} + 108txy^{2}c_{3,1,1,1}{}^{3} \\
&- 9x^{4}c_{3,0,1,1}{}^{2}c_{3,1,1,1} - 6x^{2}y^{2}c_{2,1,1,1}{}^{2}c_{3,1,1,1} \\
&+ 216txyc_{3,0,1,1}c_{3,1,1,1}{}^{2} + 6y^{2}c_{0,1,1,1}c_{2,1,1,1}c_{3,1,1,1} \\
&+ 36ty^{2}c_{2,1,1,1}c_{3,1,1,1}{}^{2} + 18xy^{2}c_{0,1,1,1}c_{3,1,1,1}{}^{2} \\
&- 6xy^{2}c_{1,1,1,1}c_{2,1,1,1}c_{3,1,1,1} - 3c_{1,0,1,1}{}^{2}c_{3,1,1,1}
\end{aligned}
$$

$$
\begin{aligned}
&+ 36tyc_{2,1,1,1}c_{3,0,1,1}c_{3,1,1,1} + 36xyc_{0,1,1,1}c_{3,0,1,1}c_{3,1,1,1} \\
&- 12x^3yc_{2,1,1,1}c_{3,0,1,1}c_{3,1,1,1} - 3y^2{c_{1,1,1,1}}^2c_{3,1,1,1} \\
&+ 18xc_{0,1,1,1}{c_{3,0,1,1}}^2 + 6xc_{1,0,1,1}c_{2,1,1,1}c_{3,0,1,1} \\
&+ 6yc_{0,1,1,1}c_{2,1,1,1}c_{3,0,1,1} - 6yc_{1,0,1,1}c_{1,1,1,1}c_{3,1,1,1} \\
&+ 2yc_{1,0,1,1}{c_{2,1,1,1}}^2 + 108txc_{3,0,1,1}{c_{3,1,1,1}}^2\Big) \\
&\cdot \Big(3x^3y{c_{3,1,1,1}}^2 + 3x^2yc_{2,1,1,1}c_{3,1,1,1} + 18ty{c_{3,1,1,1}}^2 \\
&+ 3xyc_{1,1,1,1}c_{3,1,1,1} + 18tc_{3,0,1,1}c_{3,1,1,1} \\
&+ 3xc_{1,0,1,1}c_{3,1,1,1} + 3yc_{0,1,1,1}c_{3,1,1,1} + 3c_{0,1,1,1}c_{3,0,1,1} \\
&+ 3x^3c_{3,0,1,1}c_{3,1,1,1} + c_{1,0,1,1}c_{2,1,1,1}\Big)^{-2}, \\
u_{11} &= -\left(tc_{1,0,2,2} + c_{1,0,2,1}\right)^2\left(txc_{1,0,2,2} + xc_{1,0,2,1}\right. \\
&\quad \left. + c_{0,0,2,1}\right)^{-2}, \\
u_{12} &= -{c_{1,0,2,2}}^2\left(xc_{1,0,2,2} + yc_{0,1,2,2}\right)^{-2}, \\
u_{13} &= -\left(y^2c_{1,2,2,2} + yc_{1,1,2,2} + c_{1,0,2,2}\right)^2\left(xy^2c_{1,2,2,2}\right. \\
&\quad \left. + xyc_{1,1,2,2} + y^2c_{0,2,2,2} + xc_{1,0,2,2} + yc_{0,1,2,2}\right)^{-2}.
\end{aligned}
\tag{B.1}
$$

Competing Interests

The authors declare that they have no competing interests.

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