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Research Article

Firm Growth Function and Extended-Gibrat's Property

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We analytically show that the logarithmic average sales of firms first follow power-law growth and subsequently follow exponential growth, if the growth-rate distributions of the sales obey the extended-Gibrat's property and Gibrat's law. Here, the extended-Gibrat's property and Gibrat's law are statistically observed in short-term data, which denote the dependence of the growth-rate distributions on the initial values. In the derivation, we analytically show that the parameter of the extended-Gibrat's property is identical to the power-law growth exponent and that it also decides the parameter of the exponential growth. By employing around one million bits of exhaustive sales data of Japanese firms in the ORBIS database, we confirmed our analytic results.

1. Introduction

As in natural science, statistical laws are also frequently observed in social science. In econophysics, the statistical laws observed in the behavior of people and firms and their universality have been thoroughly investigated [1, 2]. The statistical laws observed in various social quantities at a point in time are especially well known. One of the main arguments is power-law distribution [3–23]. The power-law distributions of firm-size variables x_T in calendar year T (e.g., sales, assets, and number of employees) over size threshold x_{th} have been observed in a number of years and countries as follows:

$$P(x_T) \propto {x_T}^{-\mu-1} \quad \text{for } x_T > x_{\text{th}}. \tag{1}$$

Here, $P(x_T)$ is the probability density function (PDF) of x_T. Exponent μ is called Pareto's index [3]. At the same time, mid-sized variables under size threshold x_{th} have also frequently been described by log-normal distribution.

Short-term statistical laws have also been investigated, which were observed in firm-size variables at two successive points in time (x_T, x_{T+1}): (quasi-)inverse symmetry and (non-)Gibrat's law. Inverse symmetry denotes that the system is static and symmetric under the time reversal exchange of variables $x_T \leftrightarrow x_{T+1}$ [24, 25]. Quasi-inverse symmetry means that the system is quasi-static and can be expressed as symmetric under the exchange of variables $x_T \leftrightarrow a{x_{T+1}}^{\theta}$, where a and θ are parameters [26, 27]. Gibrat's law suggests that conditional PDF $Q(R \mid x_T)$ of the growth rate defined by $R = x_{T+1}/x_T$ is independent of initial value x_T [24, 25]. This law is observed in the large-scale range over size threshold x_{th}. Non-Gibrat's law, which reflects the dependence of growth-rate distribution $Q(R \mid x_T)$ on initial value x_T, is observed in the mid-scale range under x_{th} [28–31].

The statistical laws, which are observed in firm-size variables x_T at a point in time, are related to the statistical laws observed in them at two successive points in time (x_T, x_{T+1}). Fujiwara et al. showed that the power law (1) observed in large-sized variables is derived from Gibrat's law [32, 33] under inverse symmetry [24, 25]. The log-normal distribution observed in mid-sized variables is inferred from non-Gibrat's law under inverse symmetry [28–31]. Quasi-statistically varying power-law and log-normal distributions are derived from Gibrat's and non-Gibrat's laws under quasi-inverse symmetry, respectively [26, 27].

At the same time, interesting studies, related to long-term statistical laws, have been reported in empirical data analyses of the network of firms. Miura et al. showed that the number of business connections of firms exponentially increases as they age [34]. Mizuno et al. reported that the sales of firms depend on the number of business connections [35]. Such observations suggest that the sales of firms exponentially grow with age. In this context, the emergence of an early power-law growth is natural for firms that started small. Small firms must grow rapidly to reach the trajectory of exponential growth. This result is probably related to Luttmer's report that the number of employees L grows rapidly in the beginning but soon loses momentum [36]. This can be interpreted as power-law growth. Besides firm-size variables, Petersena et al. observed power-law growth in the progress of researchers defined by the number of publications of scientific papers [37].

In this study, we propose a law of the newly formulated firm growth derived from short-term statistical laws. If there are laws in firm growth [38], they are long-term statistical laws that are different from statistical laws in the short-term. However, these laws might be related. We verify this conjecture in the rest of our paper, which is organized as follows. The law of firm growth must be related to non-Gibrat's and Gibrat's laws that describe the change of the growth-rate distributions that are dependent on the initial values. In the next section, we review Gibrat's law and introduce an extended-Gibrat's property instead of non-Gibrat's law. We also analytically derive firm growth that first obeys the power-law function and subsequently follows an exponential function. After that, using our database, we demonstrate that firm growth is analytically derived and observed in the empirical data. The last section concludes this paper.

2. Results

2.1. Gibrat's Law and Extended-Gibrat's Property. Gibrat's law is represented using conditional PDF $Q(R \mid x_T)$ as follows:

$$Q(R \mid x_T) = Q(R) \quad \text{for } x_T > x_{\text{th}}. \tag{2}$$

Here, $Q(R \mid x_T)$ does not depend on initial value x_T. This Gibrat's law is observed in a range over size threshold x_{th}. On the other hand, under threshold x_{th}, $Q(R \mid x_T)$ does depend on x_T. For the case of sales [31], the positive growth-rate distributions gradually decrease as x_T increases, and the negative growth-rate distributions hardly change. Using conditional PDF $q(r \mid x_T)$ of logarithmic growth rate $r = \log_{10} R$ which is related to $Q(R \mid x_T)$ by $q(r \mid x_T) = \ln 10 R Q(R \mid x_T)$, this property can be expressed as

$$q(r \mid x_T) = \frac{f(r)}{{x_T}^{1/\gamma}} \quad \text{for } r > 0, \tag{3}$$

$$q(r \mid x_T) = g(r) \quad \text{for } r < 0, \tag{4}$$

where γ is a constant. In this paper, we call (3) and (4) the extended-Gibrat's property, which is different from the previously examined non-Gibrat's law [30, 31]. Later, we confirm the extended-Gibrat's property with empirical data.

2.2. Analytical Derivation of Firm Growth. Next, we analytically derive firm growth from the extended-Gibrat's property ((3) and (4)) and from Gibrat's law (2). To estimate firm growth, it is convenient to identify a starting point in time for every firm. We consider a firm's incorporation as its starting point and consider its age from its year of foundation. For simplicity, we examine the average values instead of the distribution of variables and denote the logarithmic average value of the firm-size variable at firm age t as x_t. The foundation year is $t = 1$.

We define the growth rate from x_t to x_{t+1} as

$$R(x_t) = \frac{x_{t+1}}{x_t}. \tag{5}$$

The dependence in growth rate R on x_t is significant in the extended-Gibrat's property. We hypothesize that Gibrat's law and the extended-Gibrat's property, which are observed in two successive calendar years $(T, T+1)$, are also valid in two successive firm ages $(t, t+1)$. Therefore, in the following discussions, calendar year T in $Q(R \mid x_T)$ can be replaced by firm age t.

In extended-Gibrat's property range $x_t < x_{\text{th}}$, using (3) and (4), the average value of r is expressed as

$$\begin{aligned} r(x_t) &= \int_{-\infty}^{\infty} dr\, r q(r \mid x_t) \\ &= \frac{1}{{x_t}^{1/\gamma}} \int_0^{\infty} dr\, r f(r) + \int_{-\infty}^{0} dr\, r g(r) \\ &= \frac{A}{{x_t}^{1/\gamma}} + B, \end{aligned} \tag{6}$$

where integrations $\int_0^{\infty} dr\, r f(r)$ and $\int_{-\infty}^{0} dr\, r g(r)$ are assumed to converge and are denoted as A and B, respectively. In (6), the dependence of $r(x_t)$ on x_t is important. When the second term is negligible compared with the first term in (6), $r(x_t)$ is approximated by

$$\log_{10} r(x_t) = -\frac{1}{\gamma} \log_{10} x_t + \log_{10} \gamma + \frac{1}{\gamma} \log_{10} D. \tag{7}$$

Here, $\log_{10} A = \log_{10} \gamma + (1/\gamma) \log_{10} D$. Equation (7) can be rewritten as follows:

$$r(x_t) = \gamma \left(\frac{D}{x_t} \right)^{1/\gamma}. \tag{8}$$

By combining (5) and (8), we obtain the following recurrence formula:

$$x_{t+1} = x_t 10^{\gamma (D/x_t)^{1/\gamma}}. \tag{9}$$

When t is sufficiently larger than 1, (9) has a solution:

$$x_t = (\ln 10)^{\gamma} D t^{\gamma}. \tag{10}$$

We used Maclaurin expansion as $\ln(1 + 1/t) \sim 1/t$.

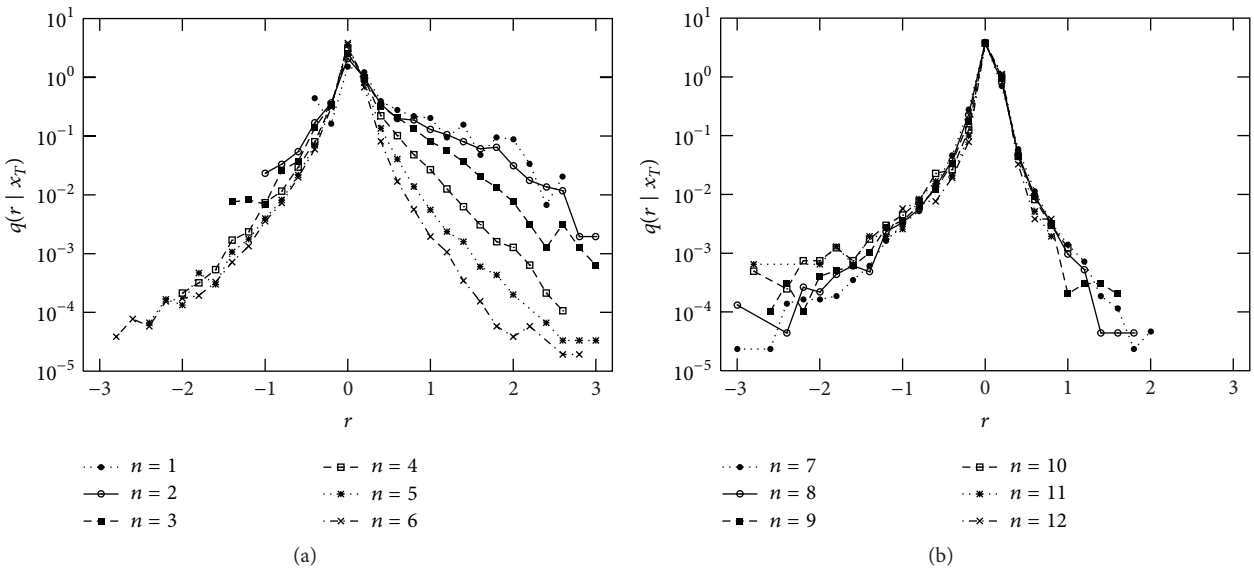

Figure 1: Conditional PDFs $q(r \mid x_T)$ of logarithmical sales growth rate $r = \log_{10} x_{T+1}/x_T$ in logarithmically equal-sized bins $x_T \in [10^{0.5(n-1)}, 10^{0.5n}]$ ((a) $n = 1, 2, \ldots, 6$; (b) $n = 7, 8, \ldots, 12$) of Japanese firms in ORBIS database. Here, x_T and x_{T+1} are sales in consecutive years $T = 2007$ in thousands of US dollars.

In Gibrat's law range $x_t > x_{\text{th}}$, the average value of r is a constant: $\gamma(D/x_{\text{th}})^{1/\gamma}$. Therefore, the recurrence formula takes the following form:

$$x_{t+1} = x_t C, \tag{11}$$

where $C = 10^{\gamma(D/x_{\text{th}})^{1/\gamma}}$ is a constant. Equation (11) has a solution:

$$x_t = C' \exp[\beta t]. \tag{12}$$

In (12), C' is a constant and β is given by

$$\beta = \gamma \left(\frac{D}{x_{\text{th}}}\right)^{1/\gamma} \ln 10. \tag{13}$$

Consequently, using the extended-Gibrat's property and Gibrat's law, we show that the growth of the logarithmic average of sales x_t first follows a power-law function when $t \gg 1$ under x_{th} and next follows an exponential function above x_{th}. In this discussion, the parameter of the extended-Gibrat's property γ is identified by the power-law exponent, and γ also decides the parameter of exponential growth β.

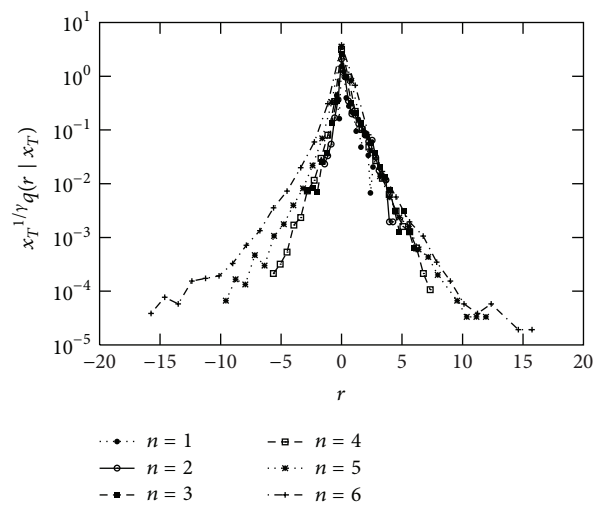

Figure 2: Conditional PDFs $q(r \mid x_T)$ times $x_T^{1/\gamma}$ with $\gamma = 3.3$ in Figure 1(a).

2.3. Data Analysis. We employ the ORBIS database, provided by Bureau van Dijk [39], which contains around 150 million pieces of firm-size data from all over the world. In the database, we analyze around one million pieces of Japanese firm-size data for 2007 and 2008. Since the number of active firms in Japan is estimated to be around one million [40], this database is exhaustive. We denote the sales data of Japanese firms from 2007 and 2008 as (x_T, x_{T+1}) with $T = 2007$.

First, we confirm Gibrat's law (2) and the extended-Gibrat's property ((3) and (4)). Figure 1 depicts the conditional PDFs $q(r \mid x_T)$ of logarithmical sales growth r in logarithmically equal-sized bins $x_T \in [10^{0.5(n-1)}, 10^{0.5n}]$ ($n = 1, 2, \ldots, 12$) in thousands of US dollars. The values (x_T, x_{T+1}) are adjusted using the Consumer Price Index. In Figure 1(b), where $x_T > 10^3$, the growth-rate distributions hardly change as n increases. This corresponds to Gibrat's law (2). On the other hand, in Figure 1(a) where $x_T < 10^3$, the positive growth-rate distributions gradually decrease as n increases, and the negative growth-rate distributions hardly change. The later property corresponds to (4). Figure 2 shows that, for $r > 0$, $x_T^{1/\gamma} q(r \mid x_T)$ hardly changes as n increases when $\gamma = 3.3$. This figure also verifies (3).

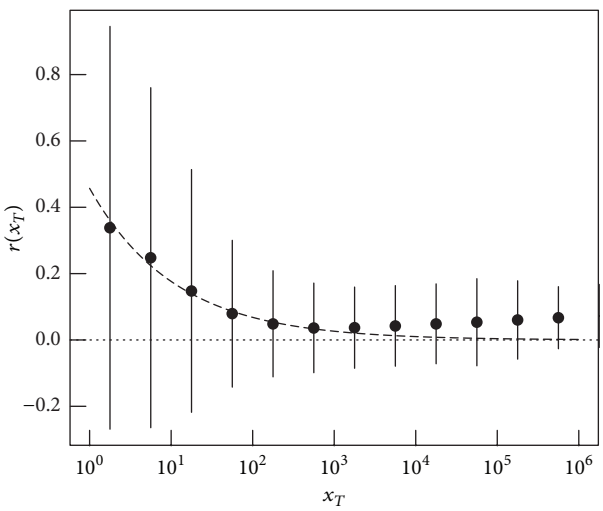

Figure 3: Scatterplot between logarithmical intermediate value $x_T = 10^{0.5(n-1/2)}$ of each bin $[10^{0.5(n-1)}, 10^{0.5n}]$ $(n = 1, 2, \ldots, 12)$ in thousands of US dollars and logarithmic average value $r(x_T)$ of r in each bin in Figure 1. Standard deviations are also depicted as error bars. Dashed line is an optimal exponential curve for $x_T < 10^3$, and dotted line is $r(x_T) = 0$.

Figure 3 shows logarithmic mean value $r(x_T)$ and the standard derivation in each bin $[10^{0.5(n-1)}, 10^{0.5n}]$ $(n = 1, 2, \ldots, 12)$ in Figure 1. In Figure 3, x_T is the logarithmical intermediate value of each bin, where $x_T = 10^{0.5(n-1/2)}$. Average values $r(x_T)$ exponentially decrease as the logarithm of x_T increases in the range under 10^3. This corresponds to the extended-Gibrat's property in the mid-scale range of sales data less than $x_{\text{th}} \sim 10^3$, and (7) is confirmed. On the other hand, average values $r(x_T)$ hardly change in the large-scale range over 10^3. This observation verifies Gibrat's law (2) over a threshold value whose size is $x_{\text{th}} \sim 10^3$.

Second, to verify the analytical derivation of firm growth, we compared the parameters estimated in different ways using empirical data. From Figures 1 and 3, we used $x_{\text{th}} \sim 10^3$. Applying (7) to the data points in Figure 3 under $x_{\text{th}} \sim 10^3$, γ and D are estimated to be 2.42 ± 0.13 and 0.018 ± 0.010, respectively. From these values, (13) gives $\beta = 0.061 \pm 0.031$. On the other hand, in Figure 2, γ is set to 3.3. In Gibrat's law range $x_t > x_{\text{th}}$ of Figure 3, constant average r can be estimated. From this value, β is directly calculated to be 0.11 ± 0.01. Parameters β and γ estimated in one way are very close to the values differently calculated in another way. Using short-term data, we checked the consistency of the analytical derivation of firm growth.

Note that since this derivation is based on average values, we must consider distribution. Figure 3 also provides information about the distribution of the values. In a Gibrat's law range where $x_t > x_{\text{th}} \sim 10^3$, the standard deviation of logarithmic average values $r(x_t)$ hardly changes. However, in the extended-Gibrat's property range where $x_t < x_{\text{th}} \sim 10^3$, the standard deviation becomes larger as x_t becomes smaller. This indicates that exponent γ of the early power-law growth deviates widely until the exponential growth begins, whereas parameter β of the exponential growth does not deviate as much as γ.

In the analytical derivation of firm growth, we assumed that the shapes of Figures 1, 2, and 3 resemble other years that are not $T = 2007$ and that Gibrat's law and the extended-Gibrat's property are valid in two successive firm ages $(t, t+1)$. Confirming this assumption in the examined ORBIS database is difficult, because it does not include long-term data. For the same reason, the growth of firms cannot be observed directly by tracing the history of the sales of each firm. However, the database contains not only firm-size data but also the incorporation years of firms. Therefore, the growth of the average value of sales x_t ((10) and (12)) that are analytically derived can be approximately observed under the above assumption.

Third, using these data, we can observe the firm growth derived in the previous subsection. In Figure 4(a), by classifying the sales of Japanese firms in 2007 that are included in the database into age-rank bins with a 1-year width, the logarithmic averages of sales x_t in each bin $(t = 1, 2, \ldots, 50)$ are depicted by circles in a single logarithmic plot. Similarly, the logarithmical averages of sales in 2008 in Japan are also depicted by crosses. In this figure, to adjust the growth in 2007 to the growth in 2008, sales x_t in 2007 are divided by the total amount of sales in 2007 and multiplied by the total sales in 2008. In Figure 4(b), the logarithmic averages are plotted in a log-log scale. In Figure 4, the dashed line is an optimal power-law function, and the dotted line is an optimal exponential function. These figures show that, under $x_{\text{th}} \sim 10^3$, x_t is well fitted by the power-law function (10). At the same time, over $x_{\text{th}} \sim 10^3$, x_t is well fitted by an exponential function (12). Using the least square method, the parameters of exponential growth β and power-law growth γ are estimated by $\beta = 0.022 \pm 0.006$ and $\gamma = 0.89 \pm 0.05$, respectively. These parameters that are estimated by long-term data are not close to the parameters of the short-term data. This discrepancy probably comes from the breakdown of the assumption that the growth rate of sales with respect to firm age was always distributed in a similar shape between 2007 and 2008. At the same time, in Figure 4, the power-law growth (10) is not smoothly continuous to the exponential growth (12) because of the distortion of sales from ages 15 to 20. This corresponds to Japan's bubble economy from 1987 to 1992.

3. Discussion

In this study, we analytically showed that the logarithmic average sales of firms first follow a power-law growth and subsequently an exponential growth, if the growth-rate distributions of the sales obey the extended-Gibrat's property and Gibrat's law. Here, the extended-Gibrat's property and Gibrat's law are statistically observed in the short-term data for two successive years, which denote the dependence of the growth-rate distributions on the initial values. By employing the exhaustive sales data of Japanese firms in 2007 and 2008 that are contained in the ORBIS database, first, we confirmed the extended-Gibrat's property and Gibrat's law.

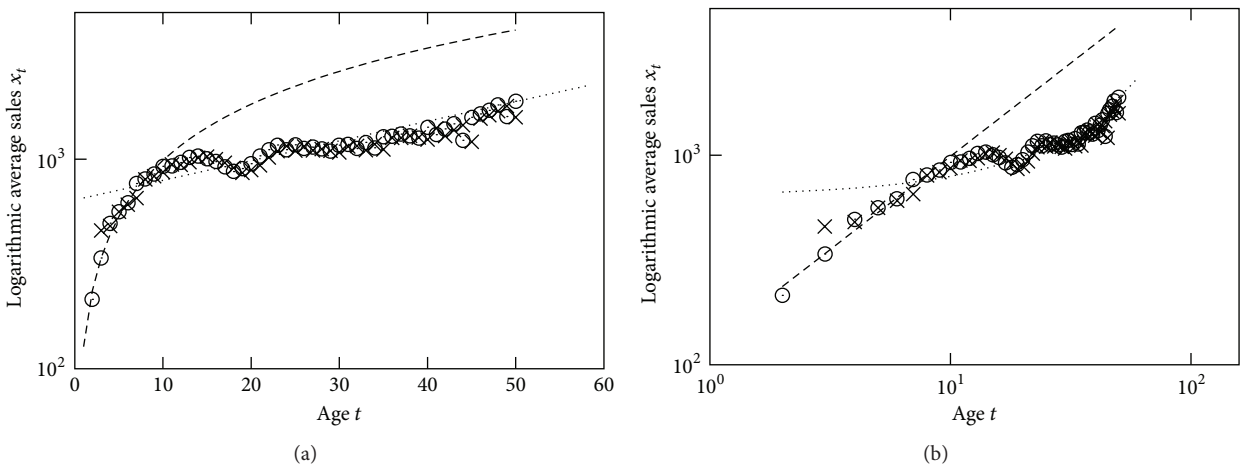

Figure 4: (a) By classifying sales of firms in 2007 in Japan included in ORBIS database into age-rank bins with 1-year width, logarithmic average of sales in each bin ($t = 1, 2, \ldots, 50$) is depicted by circles in single logarithmic plots. Similarly, logarithmic average of sales in 2008 in Japan is also depicted by crosses. (b) Logarithmic average of sales x_t classified into age-rank bins with a 1-year width is plotted in double logarithmic plots. In both figures, dashed and dotted lines represent optimal power-law and optimal exponential functions, respectively.

At the same time, we checked the consistency of the analytic derivation of firm growth by comparing the parameters of the extended-Gibrat's property, Gibrat's law, the power-law growth, and the exponential growth in short-term data. After that, we investigated the average firm growth of sales in 2007 and 2008 from Japan with respect to firm ages using the firm incorporation years in the database. We observed early power-law growth and subsequent exponential growth.

In the analytic derivation of firm growth, we did not assume inverse symmetry [24, 25] or non-Gibrat's law [28–31]. Instead, we proposed an extended-Gibrat's property under size threshold x_{th}, which is important to power-law growth. Gibrat's law over x_{th} is also important to exponential growth. Similar growths are known as the S-curve of a product. In S-curves, expansion periods correspond to power-law growth and later maturity periods correspond to exponential growth. The average growth rate of sales definitely exceeds 1 even in a Gibrat's law range where $x_t > x_{\text{th}}$. This feature is different from the growth of a product, which follows an S-curve.

It is fascinating that the extended-Gibrat's property and Gibrat's law, which are observed in short-term statistical data, can explain both power-law and exponential growth, which are long-term statistical laws. The boundary between Gibrat's and the extended-Gibrat's ranges presented in this study x_{th} is determined in all kinds of industries and firms in Japan. Perhaps it differs with respect to kinds of industries and other factors. In this case, mega firms might exist, which grow under a power law beyond average boundary x_{th}.

The discussions in this paper, constructed by analyzing sales data, are applicable to other firm-size variables, such as assets, number of employees, and positive profits. The results obtained here can also be applied to GDP growth analyses. The growth-rate distributions of GDP, which follow the extended-Gibrat's property, decide a GDP's power-law growth and exponential growth. This is not only an intriguing theme but also a significant one for such advanced countries that are suffering from sluggish GDP growth as Japan and the United States.

Competing Interests

The authors declare that they have no competing interests.

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