Hindawi
Advances in Mathematical Physics
Volume 2018, Article ID 2789412, 18 pages
https://doi.org/10.1155/2018/2789412

Research Article

Stability and Complexity Analysis of Temperature Index Model Considering Stochastic Perturbation

Jing Wang

Faculty of Science, Bengbu University, Bengbu 233030, China

Correspondence should be addressed to Jing Wang; crystalstella@126.com

Received 20 October 2017; Accepted 12 December 2017; Published 1 January 2018

Academic Editor: Giampaolo Cristadoro

A temperature index model with delay and stochastic perturbation is constructed in this paper. It explores the influence of parameters and stochastic factors on the stability and complexity of the model. Based on historical temperature data of four cities of Anhui Province in China, the temperature periodic variation trends of approximately sinusoidal curves of four cities are given, respectively. In addition, we analyze the existence conditions of the local stability of the temperature index model without stochastic term and estimate its parameters by using the same historical data of the four cities, respectively. The numerical simulation results of the four cities are basically consistent with the descriptions of their historical temperature data, which proves that the temperature index model constructed has good fitting degree. It also shows that unreasonable delay parameter can make the model lose stability and improve the complexity. Stochastic factors do not usually change the trend in temperature, but they can cause high frequency fluctuations in the process of temperature evolution. Stability control is successfully realized for unstable systems by the variable feedback control method. The trend of temperature changes in Anhui Province is deduced by analyzing four typical cities.

1. Introduction

Weather derivatives trading carried out in the world to actively respond to climate change and the development and prosperity of financial market, which has experienced great development since its inception, has made a positive contribution to the world economy to prevent the effect of climate change and to buffer weather disaster loss. Temperature is an important indicator of climate change but can also be used as the underlying asset of weather derivatives. In 2007-2008, 70% of all weather derivatives contracts traded were based on temperature as the underlying factor. According to Benth and Saltyte-Benth [1] and Fujita and Mori [2], 98-99% of weather derivatives traded are based on temperature. The majority of traded weather contracts are currently being written based on temperature. For this reason, the dynamic analysis of temperature processes is necessary. It helps to study climate change and the development of weather derivatives trading.

Previous studies focus on two different categories of fitting daily temperature variation processes correctly. Cao and Wei [3], Campbell and Diebold [4], and Jewson and Caballero [5] rely on a time series approach. Instead, a daily simulation approach, which we found in the scientific literature, is explored by Dischel [6, 7] and is refined by Dornier and Queruel [8]. Thereafter, Torró et al. [9], Alaton et al. [10], Brody et al. [11], and Benth et al. [1, 12, 13] extend the well-known financial diffusion processes in order to incorporate the basic statistical features of temperature models, combining both the time continuous approach and the econometric approach. In addition, more research has focused on derivatives pricing. Jewson and Brix [14] distinguish between three different approaches for the valuation of weather derivatives. Schiller et al. [15] propose a temperature model which uses splines to remove trend and seasonality effects from the temperature time series in a flexible way. Fortuna et al. [16] consider the solar radiation daily patterns, divided into clear sky, intermittent clear sky, completely cloud sky, and intermittent cloud sky, according to the original features-based classification strategy. The results show that features-based classification is superior to the traditional neural network classifier when operating on high-dimensional solar radiation patterns.

However, few papers focus on the complexity and stability of the temperature process or climate change. Balint et al. [17]

investigate the micro- and macroeconomics of climate change from the perspective of complexity science, and they discuss the challenges faced by this series of studies. Jacobson et al. [18] report that students use agent-based computer models to learn and understand the complex system of climate change. The experimental conditions and experimental procedures for obtaining the results are explained in detail. The paper points out that the research conclusion is consistent with theory and practice. Mihailović et al. [19] investigate the flux aggregation effect in the subgrid scale parameterization based on the dynamical system approach. The stability of turbulent heat exchange over the heterogeneous environmental interface in climate models is discussed with emphasis. Sun et al. [20] analyze the stability of annual temperature between winter and summer in the urban functional zone. The research conclusion of this article can be applied in urban planning to improve climate adaptability.

Different from previous studies, this paper discusses the stability and the complexity of temperature index model under intervene of delay and stochastic factors based on entropy theory and nonlinear dynamics theory. The work mainly includes the following: (1) the historical temperature changes trends of four cities of Anhui Province in China are given; (2) a temperature index model with delay and stochastic term is constructed; (3) the parameters of the model are estimated by using the historical temperature data of four cities; (4) the stability and the complexity of temperature index model in four cities are analyzed, respectively, under the influence of delay and stochastic term; (5) the fitting degree of the constructed temperature index model is explored by comparison with the temperature changes trend obtained from the historical temperature data; (6) the variable feedback control method is used to control the unstable system; (7) the overall temperature evolution trend of Anhui Province is discussed.

The present paper is organized as follows. We take a panoramic view of the situation to generate the temperature index model and simulate daily average temperature data from January 1, 2010, to May 31, 2016, in Anhui Province in Section 2. Based on the work of Alaton et al. [10], we give a general parameter estimation of the model in Section 3. In Section 4, the local stability condition of the delayed temperature index model without stochastic term is discussed. In Section 5, the temperature evolution trends of four cities with or without delay and stochastic perturbation are investigated through numerical simulation, respectively. In Section 6, the stability control is realized by applying the variable feedback control method. Brief summary and concluding remark are given in Section 7.

2. Temperature Index Model

Since we have decided to focus only on the temperature index as the underlying variable in the finance market, we will try to find a model that describes the temperature in this section. The goal is to find a dynamic process describing the movements of temperature. The latter part of this paper will analyze the influence of delay parameter and stochastic perturbation on the dynamic behavior of the model and evaluate the fitting degree of temperature index model.

Definition 1 (daily average temperature). Daily average temperature is defined as the average of the maximum and minimum temperature for that day [10]; that is, if the maximum temperature for any day is $T_{\max}$ and the minimum for the same day is $T_{\min}$, then daily average temperature T_{ave} is given by

$$T_{\text{ave}} = \frac{(T_{\max} + T_{\min})}{2}. \tag{1}$$

Historical temperature data usually exhibits a trend. The reason may be attributed not only to the effects of global warming, but also to urbanization effects that have led to local warming. We establish three assumptions:

(i) The interference of noise and other factors is taken into consideration, thus assuming the temperature model consists of deterministic part and stochastic part.

(ii) The period of oscillations is one year, but leap years have 366 days. In order to allow for assumption (iii) our model will neglect the effect of leap years.

(iii) The seasonality of the temperatures equals one year and the temperatures are modeled on a daily basis, thus setting $\omega = 2\pi/365$.

Modeling daily average temperature is a difficult task because of the existence of multiple variables that govern weather. Looking to the past, we will attempt to obtain precious information about the behavior of temperature.

2.1. Data Description. The dataset investigated includes temperature observations measured in centigrade degrees ($\pm^\circ$C) on four measurement stations of Anhui Province: Bozhou, Bengbu, Hefei, and Anqing. Data consist of observations on the daily maximum and minimum temperature from China's meteorological data network. The sample period extends from January 1, 2010, to May 31, 2016, with a total of 2349 observations per each measuring station.

To assist in finding a good model, we use a database with temperatures to analyze the trend of temperature change in the last several years from different cities in Anhui Province, China. The temperature data consists of daily average temperatures, computed according to (1). In Figure 1, we have plotted the daily average temperatures of the above-mentioned four cities in Anhui Province for six consecutive years.

From the temperature data in Figure 1, we clearly see that there is a strong seasonal variation in the temperature and fitting with the sine function. We will build a temperature model below and show the temperature trends of four cities during the same time. The temperature index model is evaluated by comparing it with Figure 1.

2.2. Modeling Temperature. The underlying weather indices of the derivatives traded at the CME for US cities are directly derived from the daily average temperature, which is defined as the average of the minimum and the maximum temperature at a weather station on a day t, $t \in N$. According to the mean reversion model by Alaton et al. [10]

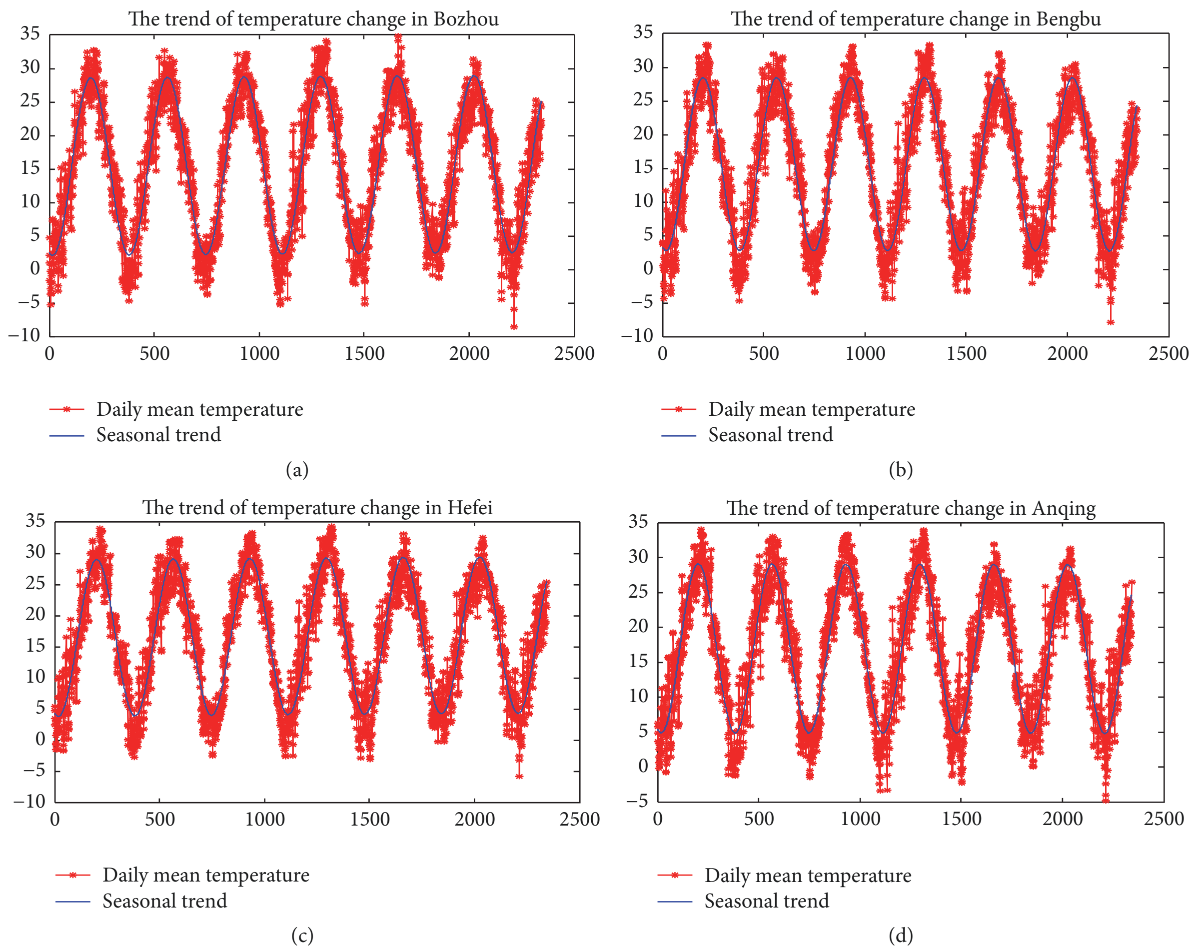

Figure 1: Daily average temperatures of four cities in Anhui Province during 2010–2016. (a) Bozhou; (b) Bengbu; (c) Hefei; (d) Anqing.

and the temperature long-term change trend, we assume the temperature index satisfies the following equation:

$$dX_t = d\theta(t) + k(\theta(t) - X_t)\,dt + \sigma_t dW_t, \qquad (2)$$

where k determines the speed of the mean reversion, σ_t is the daily volatility of temperature variation, and dW_t is a Brownian increment. The daily average temperature for day t is $\theta(t)$ $(t = 1, 2, \ldots, N)$, which represents the changes of seasonal.

As can be seen from Figure 1, the daily mean temperature changes process of four cities shows an obvious seasonal variation trend and exhibits periodicity. The seasonality of the temperature is modeled with a sine curve; it can be expressed as

$$\theta(t) = A + Bt + C\sin(\omega t + \varphi), \qquad (3)$$

where φ is the phase angle and the parameters A, B, C, and φ need to be estimated. In addition, the oscillation period of daily mean temperature is one year, so we ignore the effect of the leap year (excluding the temperature data on February 29 of the leap years) and assume $\omega = 2\pi/365$. The unknown parameters are estimated by using historical temperature data measured in four cities of Anhui Province in the last few years. The greenhouse effect brought by global warming or city urbanization causes the temperature to show a slightly rising trend, so Bt captures the trend of temperature change.

From (2) and (3) we can definitely say that the daily average temperature can be broken into the time term, seasonal term, and stochastic term. The next step will be to determine the unknown parameter of the so-specified model through estimation based on historical temperatures.

3. Parameter Estimation

For (3), we performed the following transformation:

$$\begin{aligned}\theta(t) &= A + Bt + C\sin(\omega t + \varphi) \\ &= A + Bt + C(\sin(\omega t)\cos\varphi + \cos(\omega t)\sin\varphi).\end{aligned} \qquad (4)$$

Let us consider $d = C \cdot \cos\varphi$, $e = C \cdot \sin\varphi$, $t_1 = \sin(\omega t)$, and $t_2 = \cos(\omega t)$. Therefore, (4) is equivalent to the linear regression equation:

$$\theta(t) = a + bt + dt_1 + et_2. \qquad (5)$$

Table 1: Parameters estimation.

City	Arguments							
	A	B	d	e	C	φ	k	σ_t
Bozhou	15.3292	$-1.8561e-004$	−3.629285	−12.75431	13.2606	−1.8480	0.109500	0.967525
	(0.129957)	(0.000096)	(0.091505)	(0.092323)			(0.0013)	
Bengbu	15.6110	$-1.9633e-005$	−3.893407	−12.25677	12.8603	−1.8784	0.125200	1.000233
	(0.128585)	(0.000095)	(0.090539)	(0.09138)			(0.0044)	
Hefei	16.3834	$2.7225e-004$	−3.908884	−11.96285	12.5853	−1.8866	0.086997	0.95595
	(0.13199)	(0.0000975)	(0.092937)	(0.093767)			(0.0057)	
Anqing	17.1427	$-2.2414e-004$	−0.443545	−11.91876	11.9270	−1.6080	0.079950	1.02330
	(0.156511)	(0.000124)	(0.110663)	(0.110126)			(0.0029)	

Data sources: China Meteorological Data Service Center (CMDC) http://data.cma.cn/site/index.html.

Then we have

$$A = a$$
$$B = b$$
$$C = \sqrt{d^2 + e^2} \tag{6}$$
$$\varphi = \arctan\left(\frac{e}{d}\right).$$

We made parameter estimation rely on the history data of the daily average temperature in four cities of Anhui Province with the least squares method by using Matlab software (Matrix Laboratory, MathWorks Company, America).

The second estimator is derived by (2) and considers the discretized equation as a regression equation, which can be seen as a regression of today's temperature on yesterday's temperature. It is natural to estimate the unknowns by minimizing the least squares criterion $\sum_{i=1}^{N}(X_i-\widehat{X}_i)^T(X_i-\widehat{X}_i)$, with $\widehat{X}_i$ being the estimate for X in the ith simulation runs with $i = 1, 2, \ldots, N$.

Based on above description and discussion, the parameters estimation results of four cities in Anhui Province are shown in Table 1.

Table 1 presents the estimates of the unknown parameters driving the daily average temperature of each city. The standard errors are reported in parentheses to measure the significance of parameters.

Thus far, substituting (3) into (2), the following equation of temperature index model is obtained:

$$dX_t = [B + C\omega\cos(\omega t + \varphi) + k(A + Bt + C\sin(\omega t + \varphi) - X_t)]\,dt + \sigma_t dW_t. \tag{7}$$

If stochastic factors are not taken into account, the deterministic part of (7) is as follows:

$$dX_t = [B + C\omega\cos(\omega t + \varphi) + k(A + Bt + C\sin(\omega t + \varphi) - X_t)]\,dt. \tag{8}$$

The daily average temperature in Anhui varies greatly in different periods, but the temperature in Anhui will have a short relatively stable state during the periodic change. We analyze the relative stability of the temperature change trend by observing the state of the temperature index model. We assume that the daily average temperature before τ ($\tau = 0, 1, 2, \ldots$) days can be used instead of the current daily average temperature. At this time, if the temperature index model is stable, the difference between the historical temperature and the current temperature is small, and the temperature change is relatively stable. Otherwise, the unstable model means that the temperature changes greatly fluctuate, and the relative steady change trend of temperature has changed. It is necessary to explain that we ignore the individual daily temperature mutation in the relative stable interval in the process of analysis. That is to say, it is a macro discussion and analysis. For this reason, on the basis of (7), we can get the following model with delay parameter:

$$dX_t = [B + C\omega\cos(\omega t + \varphi) + k(A + Bt + C\sin(\omega t + \varphi) - X(t-\tau))]\,dt + \sigma_t dW_t, \tag{9}$$

where τ represents the delay parameter, which means that the average temperature τ days before can be approximated as the current average temperature.

Now, the deterministic part of (9) is as follows:

$$dX_t = [B + C\omega\cos(\omega t + \varphi) + k(A + Bt + C\sin(\omega t + \varphi) - X(t-\tau))]\,dt. \tag{10}$$

In order to analyze the influence of delay parameter and stochastic factors on the stability and complexity of models, some numerical simulations are given in Section 5. In addition, Bozhou, Bengbu, Hefei, and Anqing are located in the northern, north central, south central, and southern part of Anhui Province, respectively, and they are four representative cities in the temperature changes. Thus, we adopt the parameter estimation of these four cities for numerical simulation, respectively. On the one hand, we find the trend of temperature changes in Anhui Province and observe the effects of delay parameter and stochastic factors on the trend of temperature changes. On the other hand, the fitting degree of the temperature index model is measured, that is, whether it reflects the changes trend of the real temperature.

This paper discusses four cases: the stochastic differential equation without delay (i.e., (7)), the deterministic differential equation without delay (i.e., (8)), the stochastic differential equation with delay (i.e., (9)), and the deterministic differential equation with delay (i.e., (10)). Next, the local stability conditions of (10) are analyzed.

4. Local Stability Analysis of (10)

Now we investigate the stability of (10). It can be rewritten as

$$\begin{aligned}\dot{X}_t &= B + C\omega\cos(\omega t+\varphi) \\ &\quad + k\left(A + Bt + C\sin(\omega t+\varphi) - X(t-\tau)\right).\end{aligned} \tag{11}$$

For convenience, we assume that the equilibrium point of (11) is $E(X^*)$. Next, (11) is linearized through Jacobian matrix as follows:

$$\dot{X}_t = -kX(t-\tau). \tag{12}$$

The characteristic equation of (12) is

$$\lambda + ke^{-\lambda\tau} = 0. \tag{13}$$

4.1. $\tau = 0$. For $\tau = 0$, (10) becomes (8). The characteristic equation (13) can be simplified to

$$\lambda + k = 0. \tag{14}$$

According to Routh-Hurwitz, we can draw the following conclusions.

Theorem 2. *If $k > 0$, the root of (14) will have negative real parts. Then, the equilibrium point $E(X^*)$ of (10) is locally stable without delay parameter.*

4.2. $\tau > 0$. In this case, the characteristic equation of (10) is still (13). Let $\lambda = i\omega_1$ $(\omega_1 > 0)$ be the root of (13). Then, we can obtain

$$i\omega_1 = -k\left(\cos\omega_1\tau - i\sin\omega_1\tau\right). \tag{15}$$

Separating the real and imaginary parts, we have

$$\begin{aligned} k\sin\omega_1\tau &= \omega_1 \\ k\cos\omega_1\tau &= 0.\end{aligned} \tag{16}$$

Further, we can obtain

$$\omega_1^2 = k^2. \tag{17}$$

Namely,

$$\omega_1 = \sqrt{k^2}. \tag{18}$$

From (16), if $k \neq 0$, we obtain

$$\tau^{(j)} = \frac{1}{\omega_1}\left(\arccos(0) + 2j\pi\right), \quad j = 0, 1, 2, \ldots, \tag{19}$$

and then $\pm i\omega_1$ is a pair of purely imaginary roots of (13) when $\tau = \tau^{(j)}$.

Defining

$$\begin{aligned}\omega_{10} &= \omega_1, \\ \tau_0 &= \min\left\{\tau^{(j)}\right\}, \quad j = 0, 1, 2, \ldots,\end{aligned} \tag{20}$$

let $\lambda(\tau) = \alpha(\tau) + i\omega_1(\tau)$ be the root of (13) near $\tau = \tau_0$ which satisfies $\alpha(\tau_0) = 0$, $\omega_1(\tau_0) = 0$.

Differentiating both sides of (13) with respect to τ, we can obtain

$$\frac{d\lambda}{d\tau} = \frac{k\lambda e^{-\lambda\tau}}{1 - k\tau e^{-\lambda\tau}}. \tag{21}$$

Accordingly,

$$\left[\frac{d\lambda}{d\tau}\right]^{-1} = \frac{1}{k\lambda e^{-\lambda\tau}} - \frac{\tau}{\lambda}. \tag{22}$$

Substituting $\lambda = i\omega_{10}$ $(\omega_{10} > 0)$ into (22), we have

$$\begin{aligned}\operatorname{Re}\left[\frac{d\lambda(\tau_0)}{d\tau}\right]^{-1} &= \operatorname{Re}\left[\frac{1}{k\lambda e^{-\lambda\tau_0}}\right]_{\lambda = i\omega_{10}} \\ &= \frac{k\omega_{10}\sin\omega_{10}\tau_0}{k^2\omega_{10}^2}.\end{aligned} \tag{23}$$

Since

$$\operatorname{sign}\operatorname{Re}\left[\frac{d\lambda(\tau_0)}{d\tau}\right] = \operatorname{sign}\left[\frac{d\left(\operatorname{Re}\lambda(\tau_0)\right)}{d\tau}\right]^{-1}, \tag{24}$$

if $\sin\omega_{10}\tau_0 \neq 0$, then $\operatorname{Re}[d\lambda(\tau_0)/d\tau]^{-1} \neq 0$. According to the corollary in [21–24], we have the following conclusions.

Theorem 3. *If $k \neq 0$ and $\sin\omega_{10}\tau_0 \neq 0$, then the equilibrium point $E(X^*)$ of (10) is asymptotically stable for $\tau \in [0, \tau_0)$, and it is unstable when $\tau > \tau_0$. In addition, (10) begins to lose stability near $\tau = \tau_0$.*

5. Simulate and Analyze

We consider the influence of delay parameter and stochastic factors on the temperature index model. The sequence of random numbers in (7) and (9) obeys a normal distribution with mean being 0 and variance being 1.

5.1. The Stability of (7) and (8) when $\tau = 0$

5.1.1. The Stability of (8). As shown in Table 1, the estimations of parameter k of Bozhou, Bengbu, Hefei, and Anqing are $k_{bz} = 0.109500$, $k_{bb} = 0.125200$, $k_{hf} = 0.086997$, and $k_{aq} = 0.079950$, respectively. The subscripts bz, bb, hf, and aq are the abbreviations of the above-mentioned four cities. Because $k_i > 0$, $i = bz, bb, hf, aq$, according to Theorem 2, the temperature index models (8) of four cities are stable. Taking the parameters estimation values of four cities into

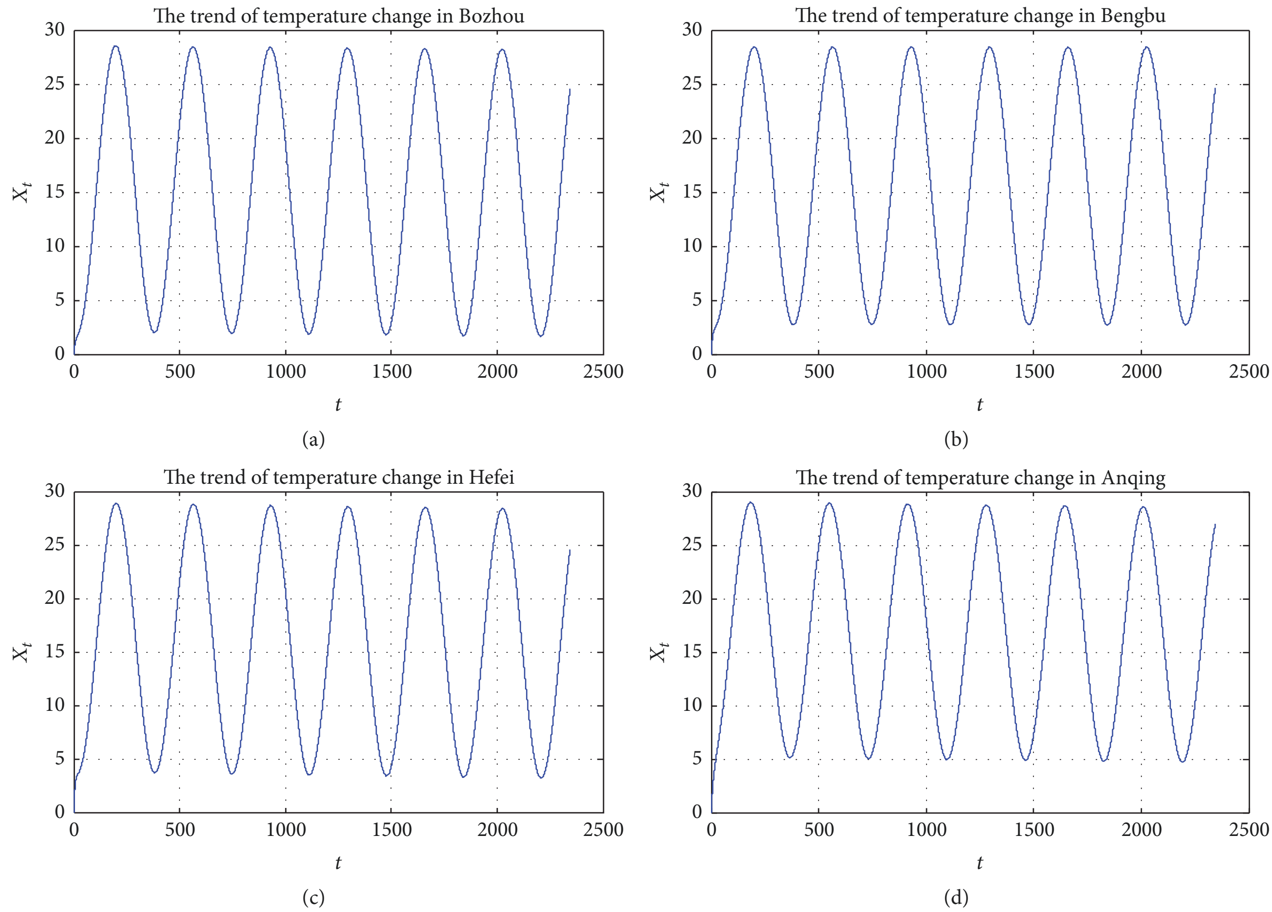

Figure 2: Daily average temperatures of four cities for (8) during 2010–2016. (a) Bozhou; (b) Bengbu; (c) Hefei; (d) Anqing.

(8), respectively, then their time series diagrams are shown in Figure 2.

We observed that the temperature changes in all four cities show stable periodic characteristics, similar to those of sinusoidal curves. The temperature changes trends of the four cities vary slightly. What is most obvious is that the minimum temperature in Anqing is greater than or equal to 5°C, which is higher than that of the other three cities. This is because Anqing is located in the south of Anhui; the daily average temperature is on the high side. Compared with Figure 1, the temperature changes of the four cities shown in Figure 2 are basically consistent with analysis results of historical temperature data. Therefore, (8) has good fitting degree without considering the delay parameter and stochastic factors, and it can better show the temperature changes trends of four cities in a longer period.

5.1.2. The Effect of A, B, C, k on the Stability of (8) in Anqing. Taking the parameter estimation of Anqing as an example, the influence of A, B, C, and k on the stability of (8) is analyzed. As can be seen from Figures 3(a)–3(c), no matter how A, B change, X_t remains at a low level when k is very small (the critical value is about 0.005). At this point, X_t will rise with an increase of C. However, in the remaining cases, k has little effect on X_t. Figure 3(a) shows that the increase of A and C will lead to the growth of X_t when $k \geq 0.005$; the effect on X_t by A is more significant. The growing process can be divided into four stages, which are represented by the light blue, green, yellow, and red, respectively. It is a slow process for C to cause a small increase in X_t. In Figure 3(b), the value of X_t is close to 20 when $k \geq 0.005$.

In Figure 3(c), X_t increases gradually with the increase of A when $k \geq 0.005$. The change process of X_t can be divided into five stages, with denoted dark blue, light blue, green, yellow, and red, respectively. Figure 3(d) shows the relationships among A, B, and C. We notice that B and C have little effect on X_t, and there is a gradual increase in X_t with the growth of A. What is interesting is that the trend is basically the same as Figure 3(c).

From the above analysis, we can draw the following conclusion: (1) the parameter A has a significant effect on X_t, whereas X_t is almost unaffected by B and C; (2) the effect of k on X_t is divided into two stages: X_t does not change with the increase of k when $k \geq 0.005$, and X_t decreases with the decrease of k when $k < 0.005$. Therefore, the reasonable range of parameters can maintain the stability of the system.

5.1.3. The Stability of (7). The parameters estimate values of four cities are taken into (7). Their corresponding time series diagrams are shown in Figure 4. Compared with Figure 2,

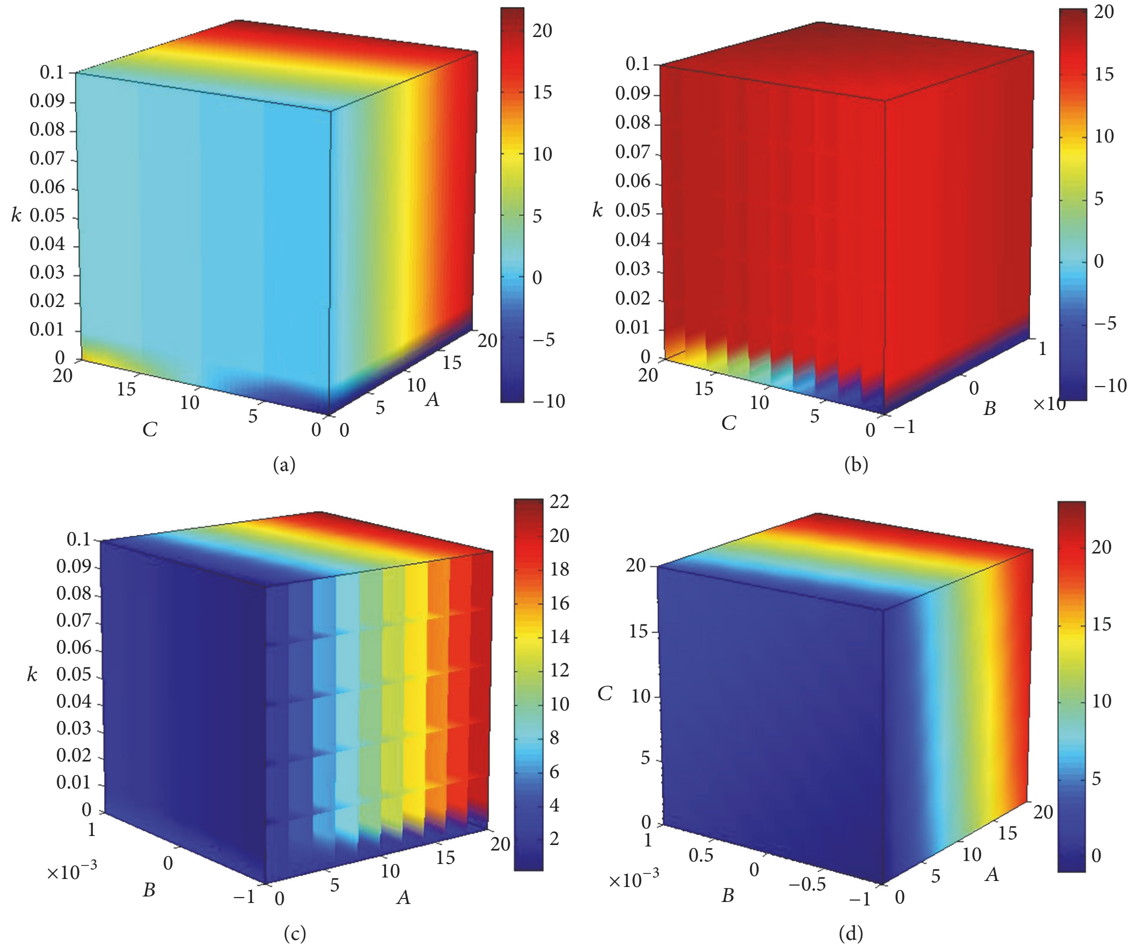

FIGURE 3: The effect of A, B, C, and k on (8) in Anqing; (a) $B = -0.00022414$; (b) $A = 17.1427$; (c) $C = 11.927$; (d) $k = 0.07995$.

the stochastic factors do not change the overall trends of temperature evolution in the four cities, but the regularity of periodic change is obviously affected. High frequency fluctuations accompany the whole process of temperature evolution, which seriously interfere with the accuracy of temperature changes. By comparing Figures 4 with 1, we can conclude that (7) is not ideal under stochastic factors interference, but it can reflect more realistic trends of historical temperature changes in four cities.

5.2. The Stability and Complexity of (9) and (10) when $\tau > 0$. According to (18), (19), and (20), we get E_m, ω_{m10}, and τ_{m0} ($m = bz, bb, hf, aq$) by using the parameters estimation values of four cities, respectively, in Table 1. The subscript 0 indicates the critical value of system stability. By combining Theorems 2 and 3, the stability of temperature index model of each city can be determined. The analysis results are shown in Table 2.

As a provincial capital city, Hefei is located in the central and southern part of Anhui Province, which is representative of the temperature change. Therefore, we take Hefei city as an example to explore the influence of delay parameter on the stability and complexity of temperature index model.

5.2.1. The Effect of τ_{hf} on (10) in Hefei. Figure 5 shows that the system moves from stable to unstable state as the τ_{hf} increases, which also improves the complexity of the system. As you can see from Figure 5(a), there is a large fluctuation when $\tau_{hf} > 17.44$; this unstable state is obviously not conducive to our analysis of temperature. We do a further analysis of (10) with the largest Lyapunov exponent (LLE). LLE means that the stability of system is judged according to the relation between the exponent value (LLE-ev) and zero. If LLE-ev is less than zero, the system is stable. It is unstable when LLE-ev is greater than zero. When LLE-ev is equal to zero, it represents the system losing stability. From Figure 5(b) we find that the critical value is $\tau_{hf0} = 17.44$; that is to say, the system is stable on the left of the critical value, but on the right it is unstable. Thus, Figures 5(a) and 5(b) describe the effect of τ_{hf} on the stability of the system from different angles, but they express the same meanings.

Complexity is another important factor to consider in system analysis. The more complex the system is, the harder it is to understand, and vice versa. Entropy indicates the degree of randomness of a system. The same applies to measuring the complexity of a system. Kolmogorov entropy (K entropy) is

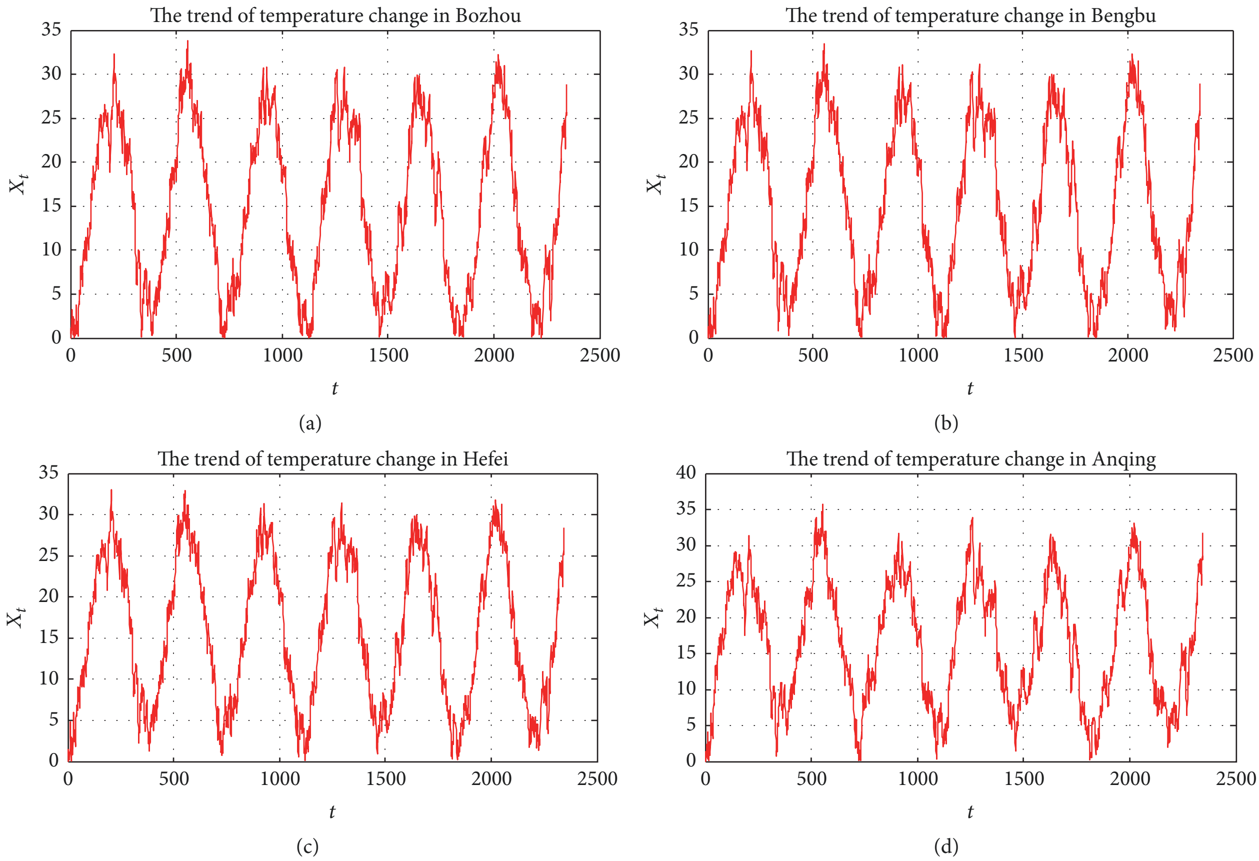

Figure 4: Daily average temperatures of four cities for (7) during 2010–2016. (a) Bozhou; (b) Bengbu; (c) Hefei; (d) Anqing.

Table 2: Calculation results and stability conditions of (10) for each city.

	Bozhou	Bengbu	Hefei	Anqing
k	0.109500	0.125200	0.086997	0.079950
$\sin \omega_{10}\tau_0$	1	1	1	1
$E_m(X^*)$	15.3244	15.6105	16.3769	17.1420
ω_{m10}	0.109500	0.125200	0.086997	0.079950
τ_{m0}	14.3452	12.5463	17.4400	19.6472
Stable	$\tau_{bz} \in [0, \tau_{bz0})$	$\tau_{bb} \in [0, \tau_{bb0})$	$\tau_{hf} \in [0, \tau_{hf0})$	$\tau_{aq} \in [0, \tau_{aq0})$
Losing stability	$\tau_{bz} = \tau_{bz0}$	$\tau_{bb} = \tau_{bb0}$	$\tau_{hf} = \tau_{hf0}$	$\tau_{aq} = \tau_{aq0}$
Unstable	$\tau_{bz} > \tau_{bz0}$	$\tau_{bb} > \tau_{bb0}$	$\tau_{hf} > \tau_{hf0}$	$\tau_{aq} > \tau_{aq0}$

one of the most commonly used entropy. Thus, this paper uses K entropy to measure the complexity of the system by using Matlab software. The rules of K entropy for judging system complexity are as follows: If the system is stable or regular movement, the entropy value equals zero. That is to say, the system retains the original complexity. On the other hand, if the system is unstable, the entropy value is greater than zero, which means the system is more complex than the original one. Therefore, it must be clear that the complexity of the system is positively related to the entropy value. Figure 5(c) provides an entropy diagram to describe the impact of τ_{hf} on the complexity of the system. We noticed that the entropy value is equal to zero when $\tau_{hf} < 17.44$, while entropy value is greater than zero as $\tau_{hf} > 17.44$. It can be observed intuitively that the entropy value increases continuously with the increase of τ_{hf}. In other words, the excessive increase of τ_{hf} will make the system more complex. From what is mentioned above, we may come to the conclusion that τ_{hf} must be controlled within a certain range in order to keep the system stable. Otherwise, the system will lose stability and increase the complexity.

From the above analysis, it is known that the system has different states on both sides of τ_{hf0}. Now, let $\tau_{hf} = 16 < \tau_{hf0}$ and $\tau_{hf} = 19 > \tau_{hf0}$, their corresponding time series diagrams are shown in Figures 6(a) and 6(b), respectively. Figure 6(a) shows the historical temperature changes trend of

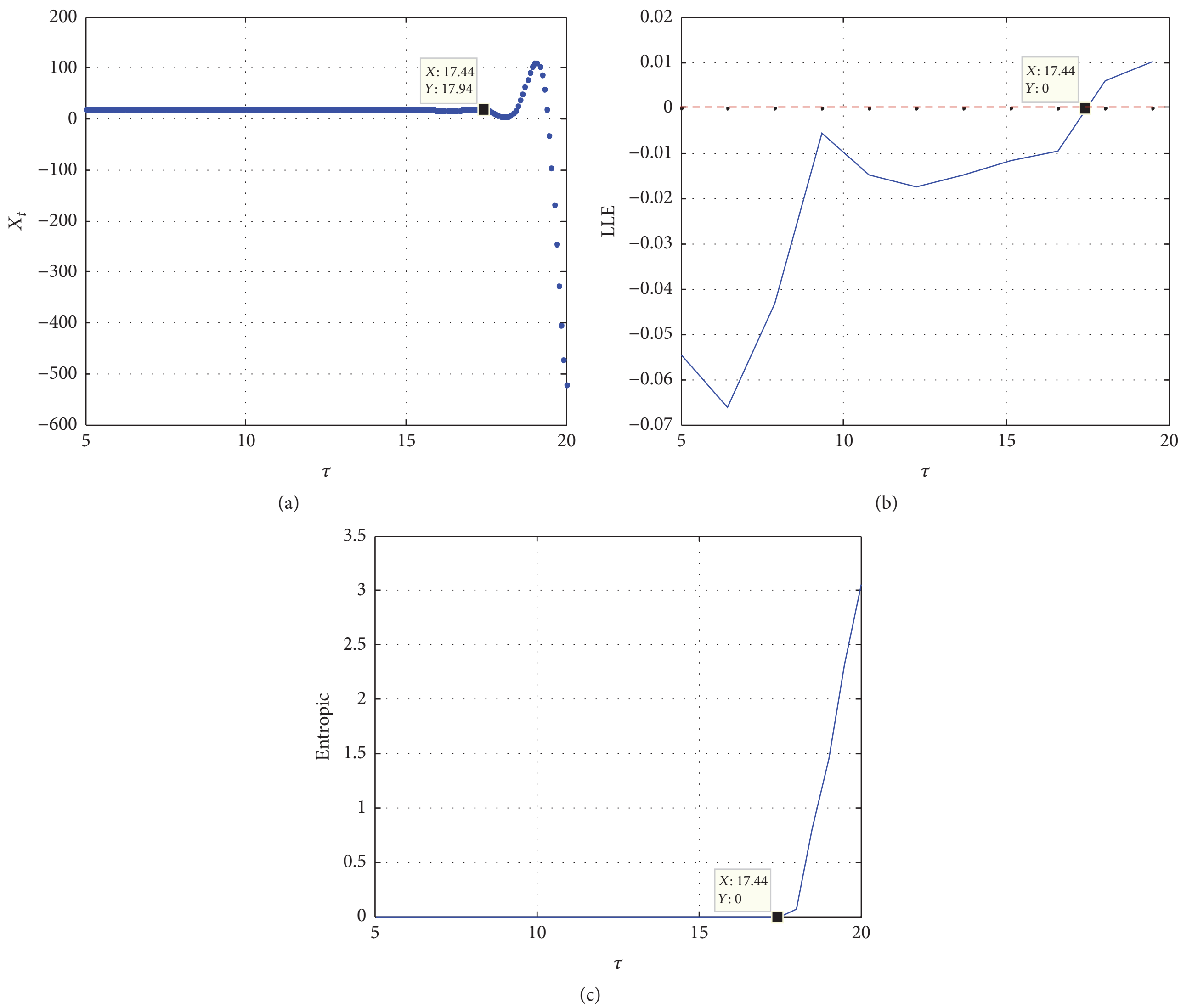

FIGURE 5: The effect of τ on the stability and complexity of (10) in Hefei. (a) Change trend diagram; (b) LLE diagram; (c) entropy diagram.

(10) for Hefei. The temperature exhibits a periodic variation when $\tau_{hf} = 16$. Figures 6(a) and 2(c) are basically the same, so we can infer that as long as the delay parameter τ_{hf} is within a reasonable range, the daily average temperature of history can replace the present daily average temperature, and it can also show the characteristics of periodic changes in temperature. By comparing Figures 6(a) and 1(c), we find that the results are basically the same based on the same historical temperature data. So (10) is an ideal model $\tau_{hf} = 16 < \tau_{hf0}$.

However, if the system is unstable, the temperature evolution is irregular and disorganized. The instability of the system is described in Figure 6(b). Thus, we should carefully select τ_{hf} to avoid adverse changes in system stability and complexity.

Through the simulation of (10) in Hefei, it is shown that the stability analysis is correct. In other words, when τ_{hf} increases to more than τ_{hf0}, (10) changes from a steady state to an unstable state, and the complexity of (10) will be improved. In addition, the smaller the τ_{hf} is, the better the fitting with the sine curve is, and vice versa. That is, the daily average temperature near the current date is a better choice.

5.2.2. The Effect of τ_{hf} on (9) in Hefei. The parameters estimation values of Hefei are taken into (9) and the delay parameters are the same as those shown in Figure 6. The dynamical characteristics are shown in Figure 7.

Comparing Figures 7(a) and 5(a), it can be found that the stochastic factors do not change the trend of the system evolution, but the system has smaller fluctuations in the process of change. However, there is a larger difference in the complexity of the system. From Figures 7(b) and 5(c) we find that stochastic disturbance leads to premature loss of stability and complexity of the system. Under the influence of stochastic factors, the complexity of the system has a reciprocating change in the process of increase, and the trend of overall increase has not changed.

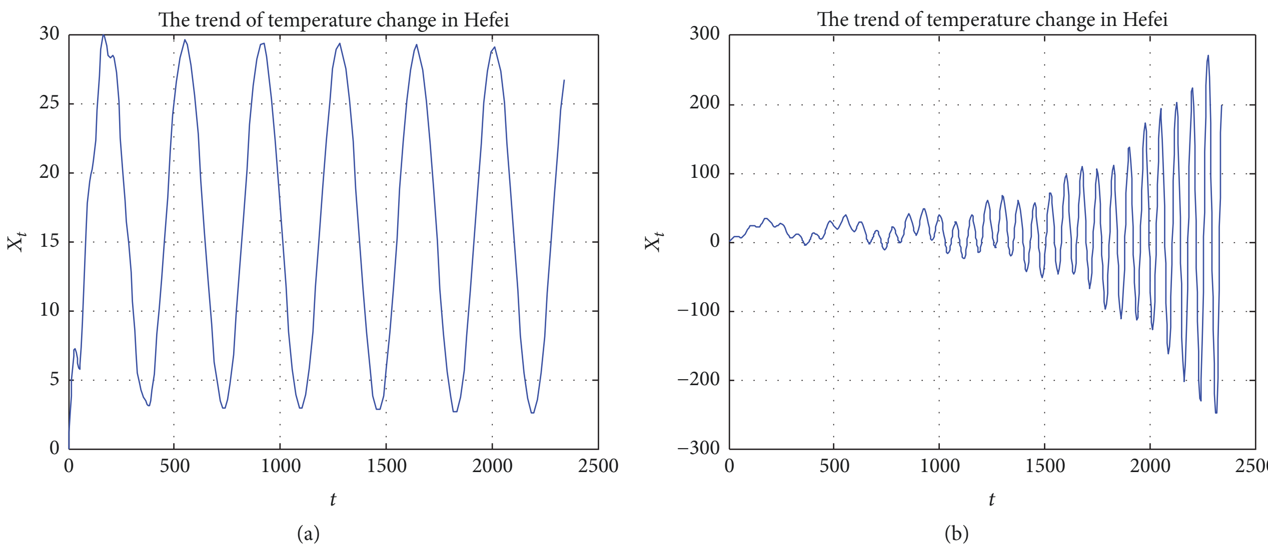

FIGURE 6: Time series diagram for (10) in Hefei. (a) $\tau_{hf} = 16$; (b) $\tau_{hf} = 19$.

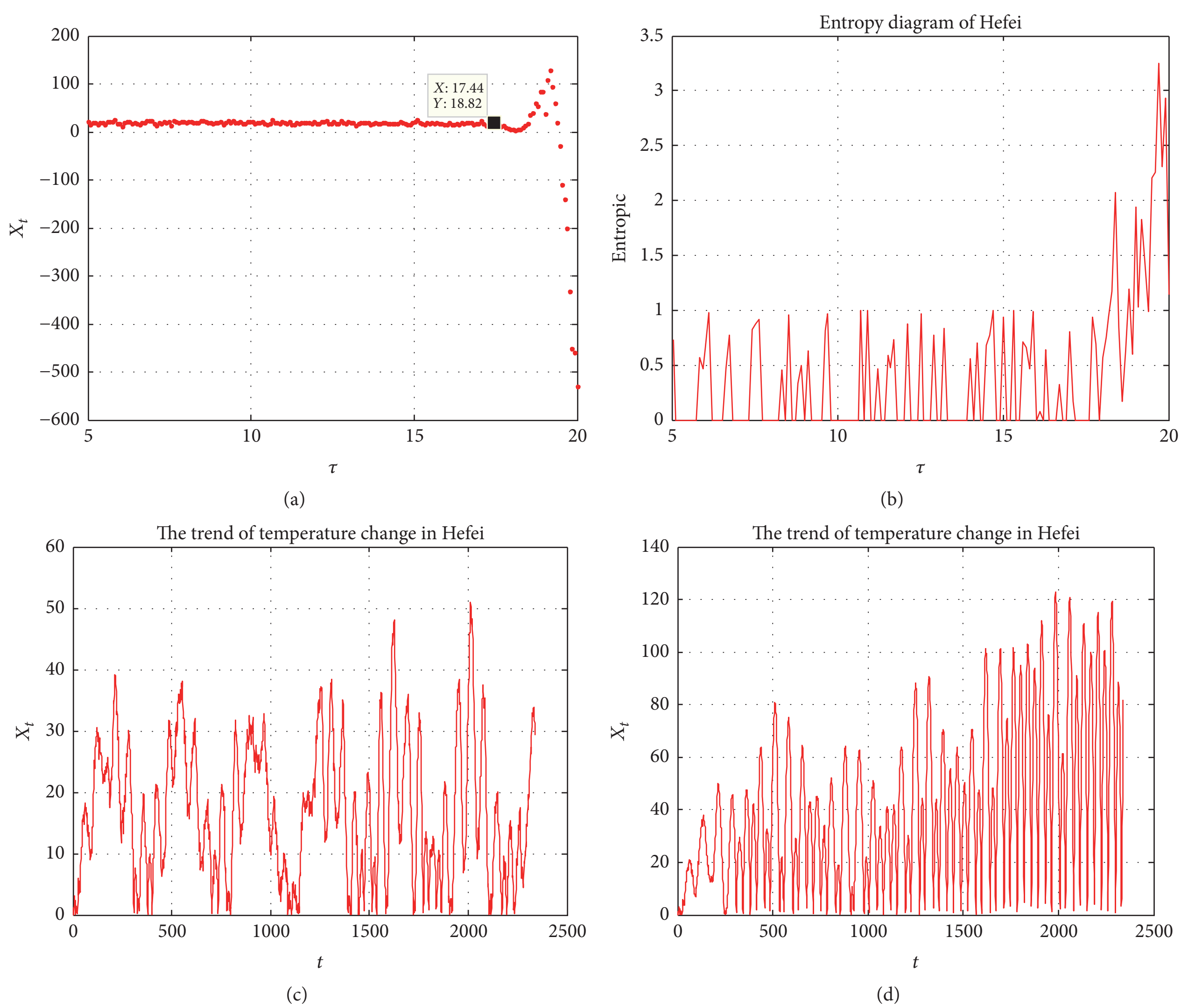

FIGURE 7: The effect of τ_{hf} on the stability and complexity of (9) in Hefei. (a) Change trend diagram; (b) entropy diagram; (c) $\tau_{hf} = 16$; (d) $\tau_{hf} = 19$.

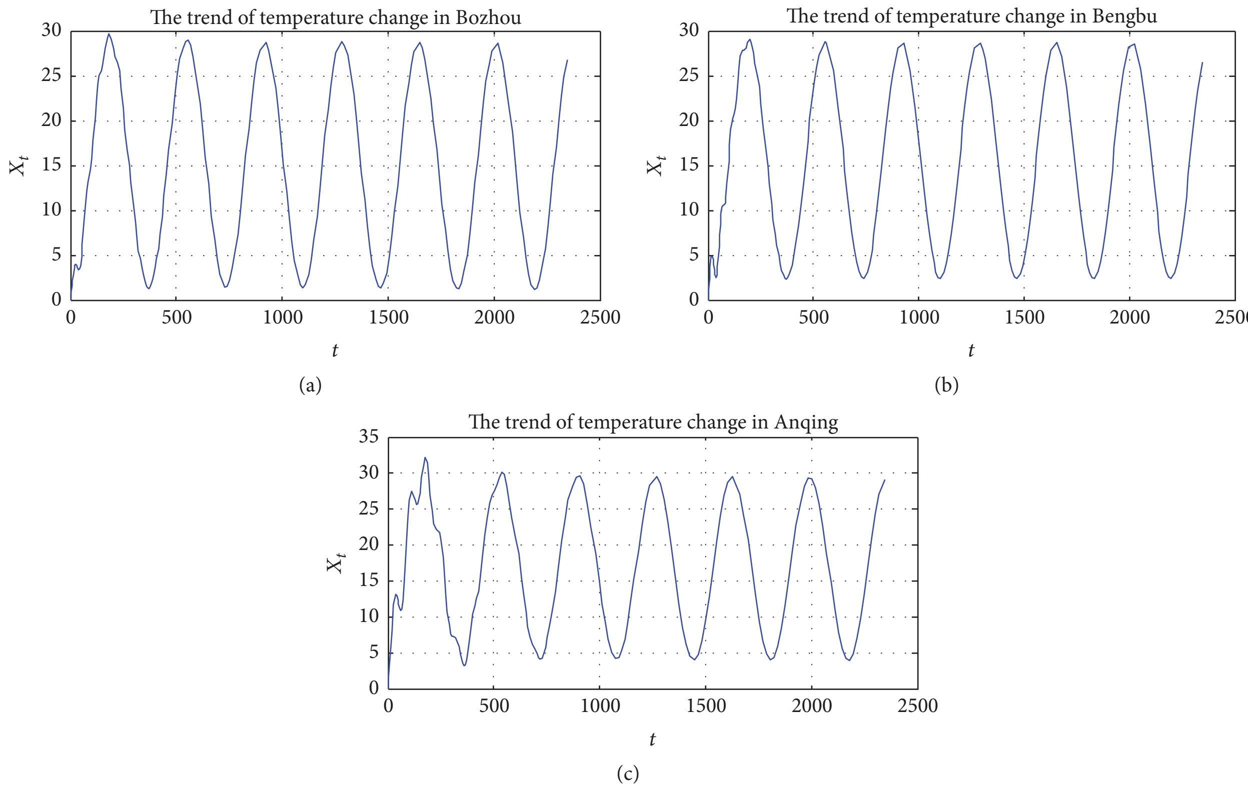

FIGURE 8: Daily average temperatures of (10) during 2010–2016. (a) Bozhou, $\tau_{bz} = 13 < \tau_{bz0}$; (b) Bengbu, $\tau_{bb} = 11 < \tau_{bb0}$; (c) Anqing, $\tau_{aq} = 18 < \tau_{aq0}$.

Figures 7(c) and 7(d) correspond to the two states. Relative to Figure 6, it is clear that the periodic variation of the temperature index model is not very regular and fluctuates greatly. It is worth noting that the random factors lead to temperatures greater than 0 except the initial temperature in Figure 7(d). In addition, we can deduce that the influence of stochastic factors on the stable system is more obvious than the unstable system. This is obviously not desirable, so we should minimize the impact of random factors on the system to ensure better system stability.

In a word, the effects of stochastic perturbations are more obvious, which accelerate the instability and complexity of the system.

In short, the delay differential equation (10) without stochastic term is stable when $\tau_{hf} \in [0, \tau_{hf0})$, and the complexity of the system does not change. The complexity increases and the system loses stability only when $\tau_{hf} > \tau_{hf0}$. However, the temperature variation shown by the delay differential equations (9) with stochastic term is seriously distorted with high frequency fluctuations. This corresponds to the increase in complexity to a certain extent, and the complexity of the system increases more clearly when $\tau_{hf} > \tau_{hf0}$. Therefore, the model with stochastic perturbation basically reflects the overall trend of temperature changes but has a negative impact on the stability and complexity of the system.

5.2.3. The Stability and Complexity of (10) in Bozhou, Bengbu, and Anqing. The analysis processes are similar to those in Sections 5.2.1 and 5.2.2, so the temperature evolution of the other three cities is comprehensively analyzed. In order to support the theoretical analysis of Table 2, we set the delay parameters values for three cities as shown in Table 3.

TABLE 3: Delay parameters selectionof (10) for three cities.

	Bozhou	Bengbu	Anqing
Stable	$\tau_{bz} = 13 < \tau_{bz0}$	$\tau_{bb} = 11 < \tau_{bb0}$	$\tau_{aq} = 18 < \tau_{aq0}$
Unstable	$\tau_{bz} = 15 > \tau_{bz0}$	$\tau_{bb} = 13 > \tau_{bb0}$	$\tau_{aq} = 20 > \tau_{aq0}$

The trends of temperature changes of three cities from January 1, 2010, to May 31, 2016, are shown in Figures 8 and 9. The entropy diagrams corresponding to them are shown in Figure 10. As you can see from Figure 8, the systems are stable when $\tau_m < \tau_{m0}$ ($m = bz, bb, aq$), and the temperature changes trends of the three cities are similar to that of the sine curve. Figure 9 shows that the systems are unstable as $\tau_m > \tau_{m0}$ ($m = bz, bb, aq$); at this time the temperature changes show chaotic and irregular trends. Those are consistent with the results of theoretical analysis. By comparing Figures 8 and 1, we find that (10) has a high fitting degree.

In addition, Figure 10 shows the state transition of the system from the point of view of system complexity. The critical points of system complexity and stability are the same. When the system is unstable, the complexity of the system rises with the increase of τ_m ($m = bz, bb, aq$).

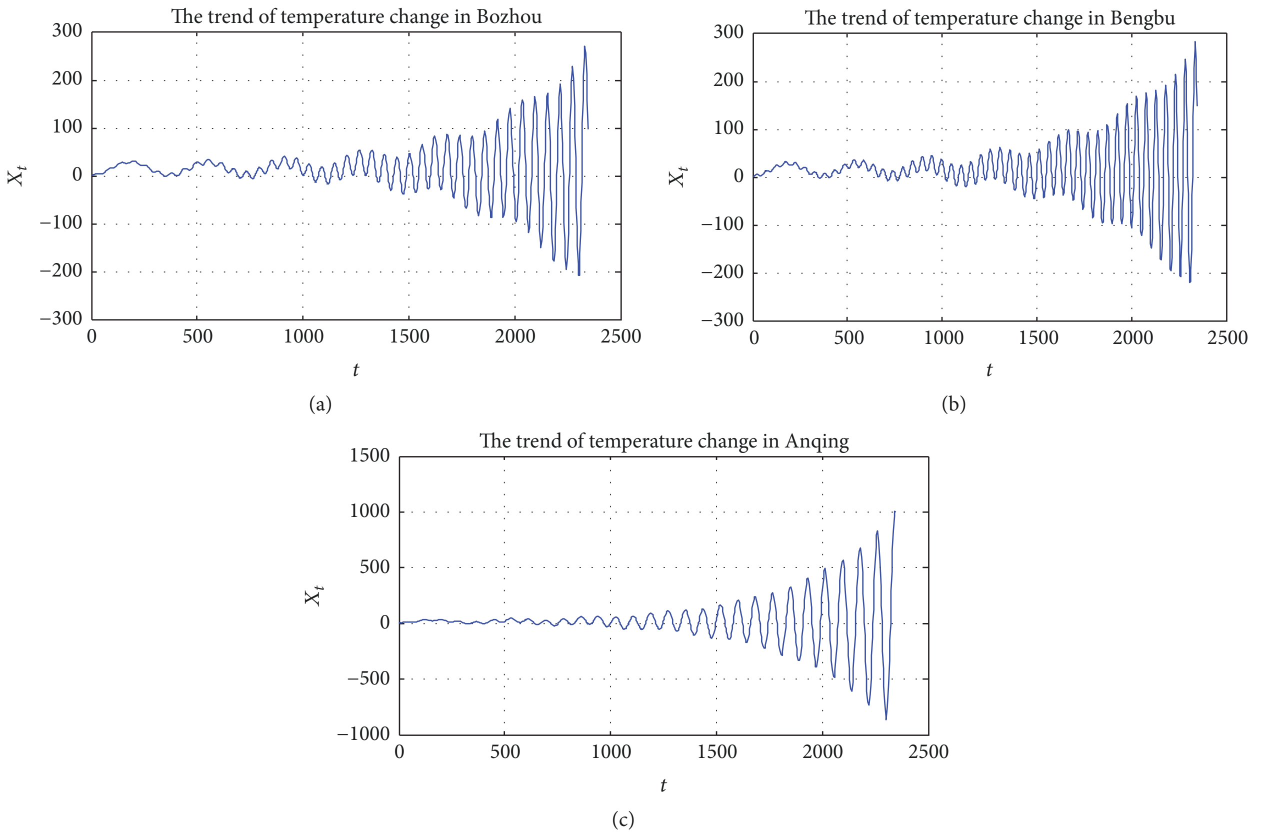

Figure 9: Daily average temperatures of (10) during 2010–2016. (a) Bozhou, $\tau_{bz} = 15 > \tau_{bz0}$; (b) Bengbu, $\tau_{bb} = 13 > \tau_{bb0}$; (c) Anqing, $\tau_{aq} = 20 > \tau_{aq0}$.

Comparison between Figures 10 and 5(c) shows that τ_{aq0} is the largest, τ_{bb0} is the smallest, τ_{hf0} and τ_{bz0} are in the middle. That is to say, the daily average temperature in Anqing is not very different over a period of time, and the average temperature of current day can be replaced by the daily average temperature of τ_{aq0} days ago at most. However, the daily average temperature varies greatly in Bengbu. The daily average temperature within τ_{bb0} days of the current date will be of reference value. The similar conclusions can be drawn from Hefei and Bozhou. The differences of τ_{m0} ($m = bz, bb, aq$) are closely related to the geographical position and surrounding environment of the four cities and conform to the actual situation.

5.2.4. The Stability and Complexity of (9) in Bozhou, Bengbu, and Anqing. The temperature evolution trends of three cities under stochastic perturbation are shown in Figures 11 and 12. It can be seen that the impacts of stochastic factors on the system are very obvious whether the systems are stable or not. The temperature changes are always accompanied by high frequency fluctuations. Comparing Figures 11 and 8, the stochastic factors seriously destroy the accuracy of temperature periodic variations. However, this can describe the periodic phenomenon of temperature changes macroscopically. The differences between Figures 11 and 1 are large. Equation (9) disturbed by stochastic factors cannot fit the trends of the historical temperature changes well.

Therefore, if stochastic factors play an important role, the trajectory of temperature changes will be unstable, which is not conducive to the study of the historical temperature and the future temperature. As compared with Figure 9, the unstable equation (9) shown in Figure 12 is even more chaotic. In other words, stochastic factors exacerbate the system's dramatic fluctuations and irregular motion.

In a word, the interference of stochastic factors enhances the instability and complexity of the system. Whether the system is stable or not, there is a lot of unstable factors in it, and the system becomes complex. The entropy diagrams describe the evolution process of the system complexity under stochastic perturbation in Figure 13. Compared with Figure 10, the interference of stochastic factors improves the complexity of the stable system and changes the trajectory of complexity in unstable system.

6. Stability Control

Based on above description and discussion, we know that an unstable system may not function properly and cannot achieve its intended goal. Therefore, we will take effective measures to ensure that the system is transformed from instability to stability. In other words, the unstable system

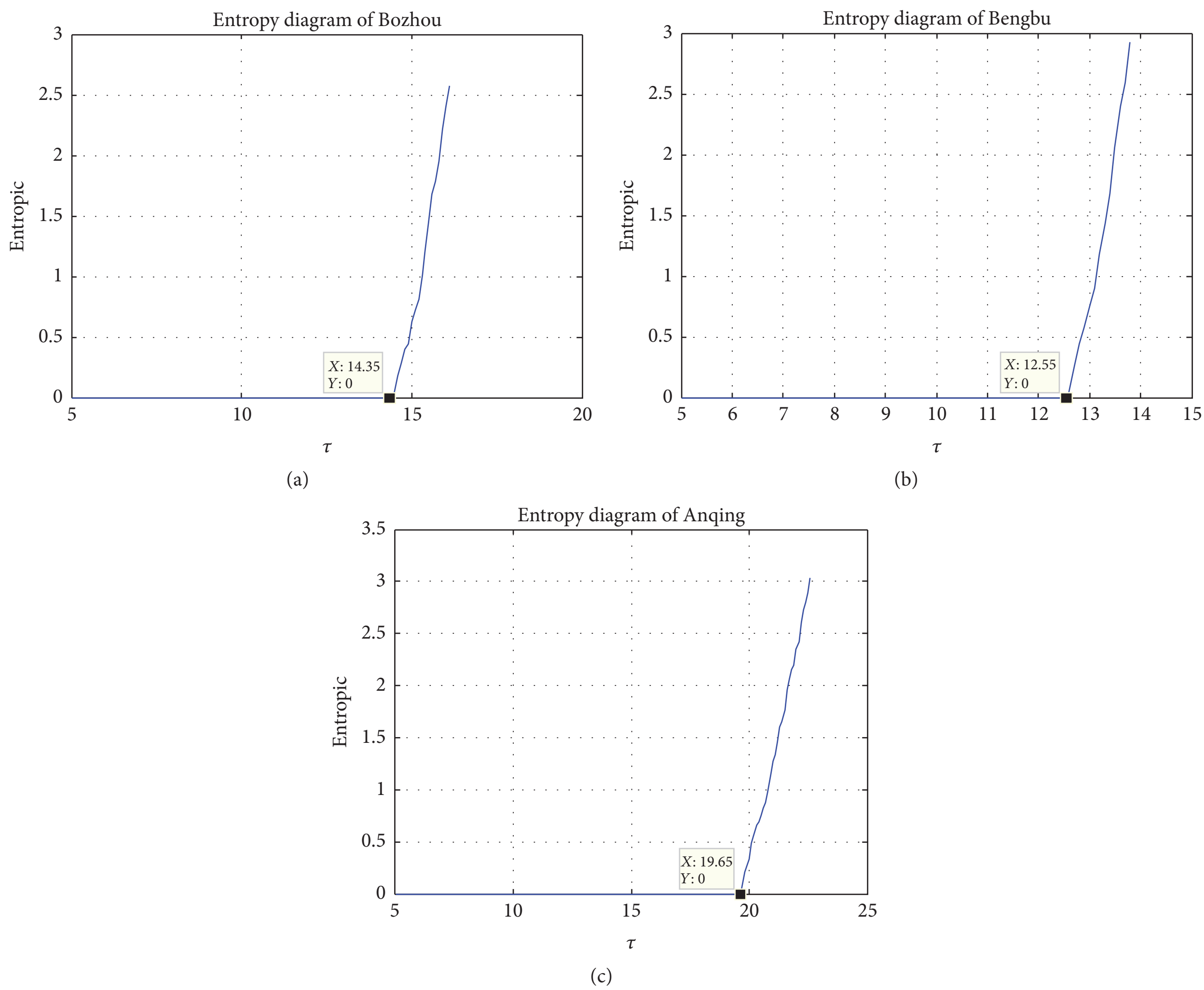

FIGURE 10: Entropy diagrams of (10). (a) Bozhou; (b) Bengbu; (c) Anqing.

is controlled by variable feedback method stabilized by adjusting the control parameters.

6.1. The Control of (10) in Hefei. Adding the control term in (10) based on parameter estimation of Hefei as follows:

$$\begin{aligned}\dot{X}_t &= 2.7225e-004+0.2169\cos(0.0172t-1.8866)\\&\quad+0.090069\,(16.3834+2.7225e-004t\\&\quad+12.5853\sin(0.0172t-1.8866)-X(t-\tau))\\&\quad-\mu X_t,\end{aligned} \tag{25}$$

where μ is a control parameters, the purpose of the following discussion is to achieve system control by adjusting μ.

According to the analysis of Section 5.2.1, the system is unstable when $\tau_{hf} = 19$, and the corresponding time series diagram is shown in Figure 6(b). Now, we study the fact that the effect of μ on the stability and complexity of (25) under the other parameters is the same. Figure 14(a) shows that (25) is unstable when $\mu < 0.01382$, and it is stable when $\mu > 0.01382$. The critical value is $\mu_0 = 0.01382$. Figure 14(b) describes the impact of μ on the system from a complexity perspective. Figures 14(a) and 14(b) express the same meaning. In addition, the smaller the μ is, the stronger the system instability is and the more complex it is, and vice versa. Therefore, the system returns to stable state only when $\mu > 0.01382$.

Next, let $\mu = 0.005 < \mu_0$, according to the previous analysis, (25) is unstable. Figure 15(a) confirms this conclusion. Then set $\mu = 0.02 > \mu_0$; as can be seen in Figure 15(b), (25) is stable and the temperature varies periodically. This is consistent with the above discussion. Therefore, it can be concluded that the unstable systems achieve stability control by adjusting control parameter μ.

6.2. The Control of (9) in Hefei. After adding the control item, (9) based on parameter estimation of Hefei can be changed to

$$\begin{aligned}dX_t &= [2.7225e-004+0.2169\cos(0.0172t-1.8866)\\&\quad+0.090069\,(16.3834+2.7225e-004t\\&\quad+12.5853\sin(0.0172t-1.8866)-X(t-\tau))]\,dt\\&\quad+0.95595dW_t-\beta X_t dt,\end{aligned} \tag{26}$$

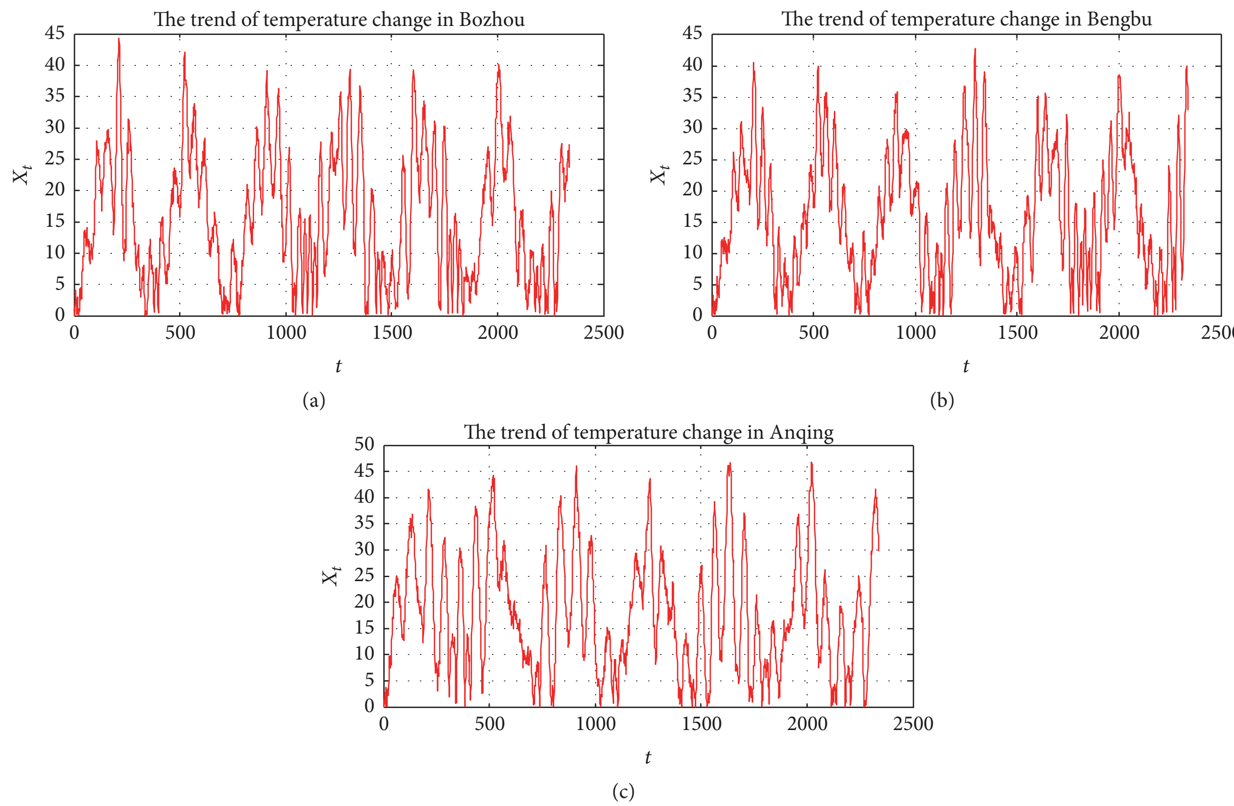

Figure 11: Daily average temperatures of (9) during 2010–2016. (a) Bozhou, $\tau_{bz} = 13 < \tau_{bz0}$; (b) Bengbu, $\tau_{bb} = 11 < \tau_{bb0}$; (c) Anqing, $\tau_{aq} = 18 < \tau_{aq0}$.

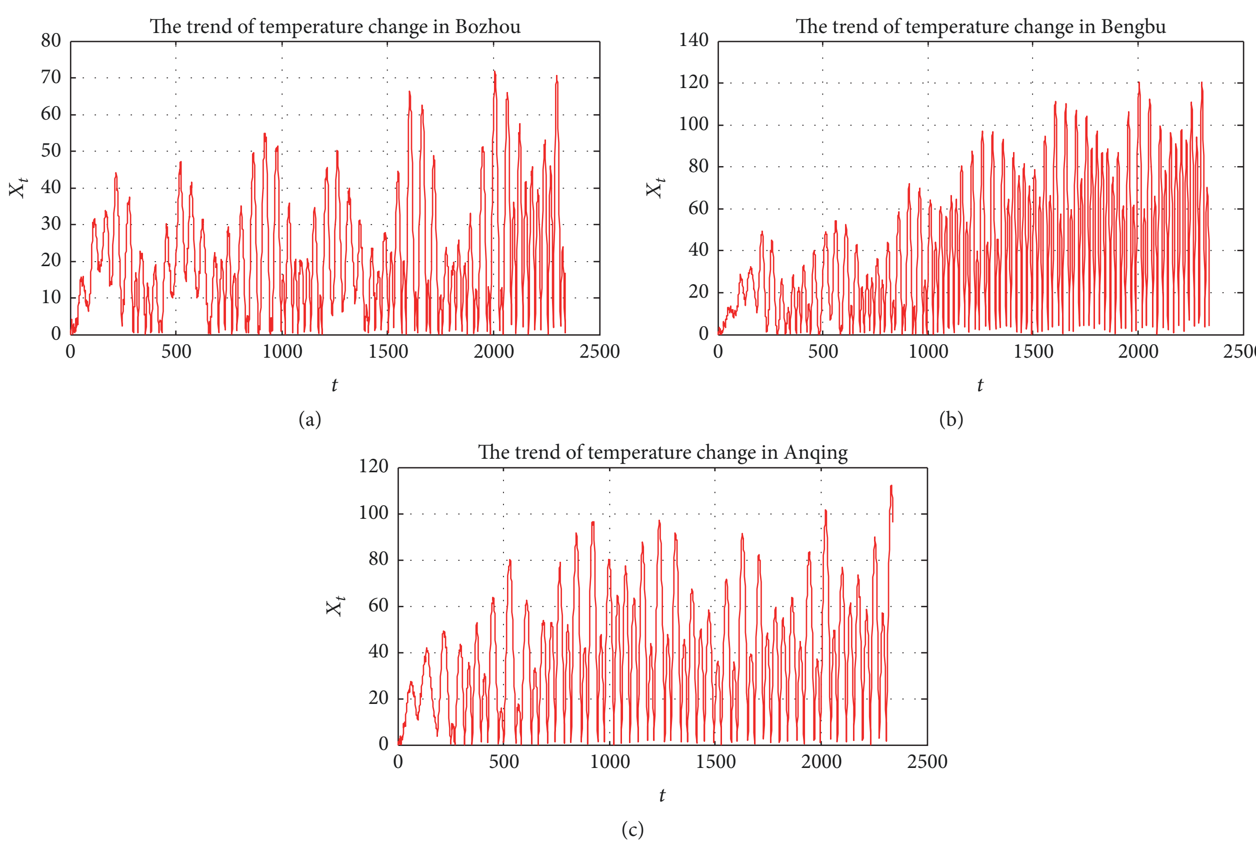

Figure 12: Daily average temperatures of (9) during 2010–2016. (a) Bozhou, $\tau_{bz} = 15 > \tau_{bz0}$; (b) Bengbu, $\tau_{bb} = 13 > \tau_{bb0}$; (c) Anqing, $\tau_{aq} = 20 > \tau_{aq0}$.

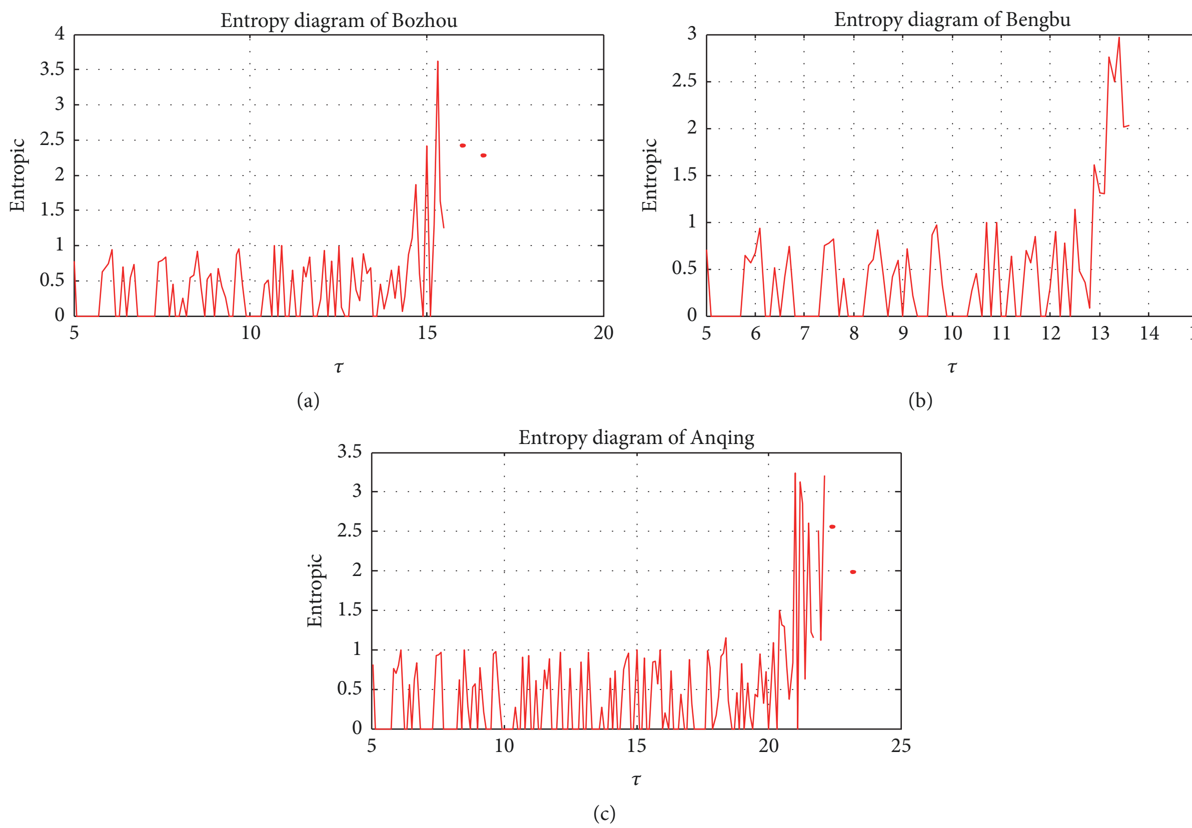

FIGURE 13: Entropy diagrams of (9) during 2010–2016. (a) Bozhou; (b) Bengbu; (c) Anqing.

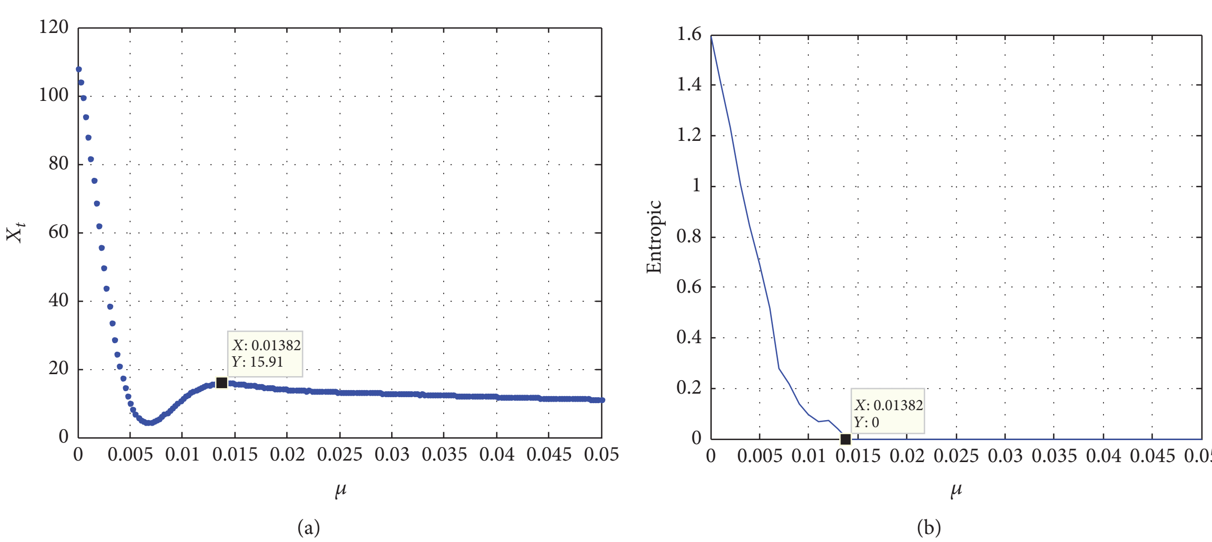

FIGURE 14: The effect of μ on the stability and complexity of (25) in Hefei when $\tau_{hf} = 19$. (a) Change trend diagram; (b) entropy diagram.

where β is a control parameter. The instability of (9) is described in Figure 7(d) when $\tau_{hf} = 19$. The analysis process is similar to that in Section 6.1. Next we analyze the effect of the control parameter β on (26) under other unchanged parameters.

Compared to Figures 14 and 16, the evolution trends of the control parameters μ and β are basically the same, but only a slight fluctuation occurs during the β change. That is to say, the influence of stochastic factors on the control parameter is limited. Further, the system is stable when $\beta > 0.01382$, and

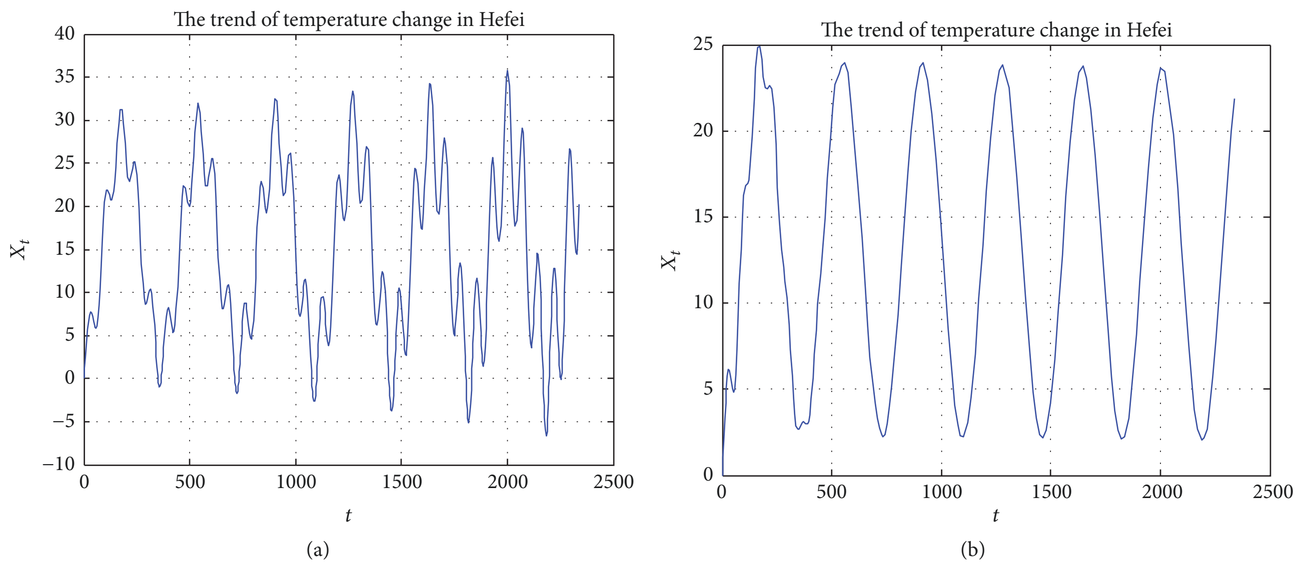

Figure 15: Time series diagrams of (25) in Hefei when $\tau_{hf} = 19$. (a) $\mu = 0.005$; (b) $\mu = 0.02$.

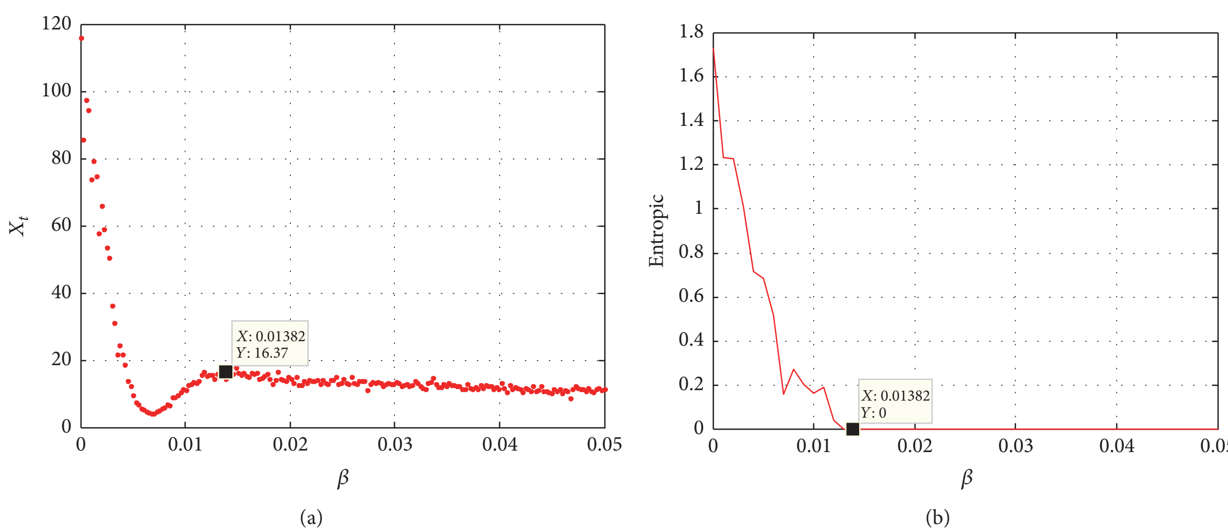

Figure 16: The effect of β on the stability and complexity of (26) in Hefei when $\tau_{hf} = 19$. (a) Change trend diagram; (b) entropy diagram.

it is unstable when $\beta < 0.01382$. The bigger the β is, the more stable the system is, so the unstable system can be returned to stable state by setting $\beta > 0.01382$.

Figures 17(a) and 17(b) show the control effect of (26) when $\beta = 0.005$ and $\beta = 0.03$, respectively. According to the analysis above, (26) is unstable when $\beta = 0.005$. In comparison with Figures 17(b) and 7(d), we find that the controlled system exhibits the trend of periodic variation when $\beta = 0.03 > 0.01382$. Because (26) is affected by random factors, compared with Figure 15(b), the periodic variation in Figure 17(b) is not very regular, but obvious control action has been shown with respect to Figure 7(d).

In brief, the unstable system is extremely likely to cause temperature oscillations and cannot obtain accurate results. However, adding control parameters in the original system can effectively avoid instability through the variable feedback control method. The adjustment range of the control parameters directly affects the stability and complexity of the system. Moreover, the variable feedback control method can control the deterministic differential equation and the stochastic differential equation well. However, the control effect of the deterministic differential equation is better than that of the stochastic differential equation.

7. Conclusion

In this paper, a temperature index model with delay and stochastic term is established. The temperature variation trends of four cities in Anhui Province are analyzed by using historical temperature data. The results show that the

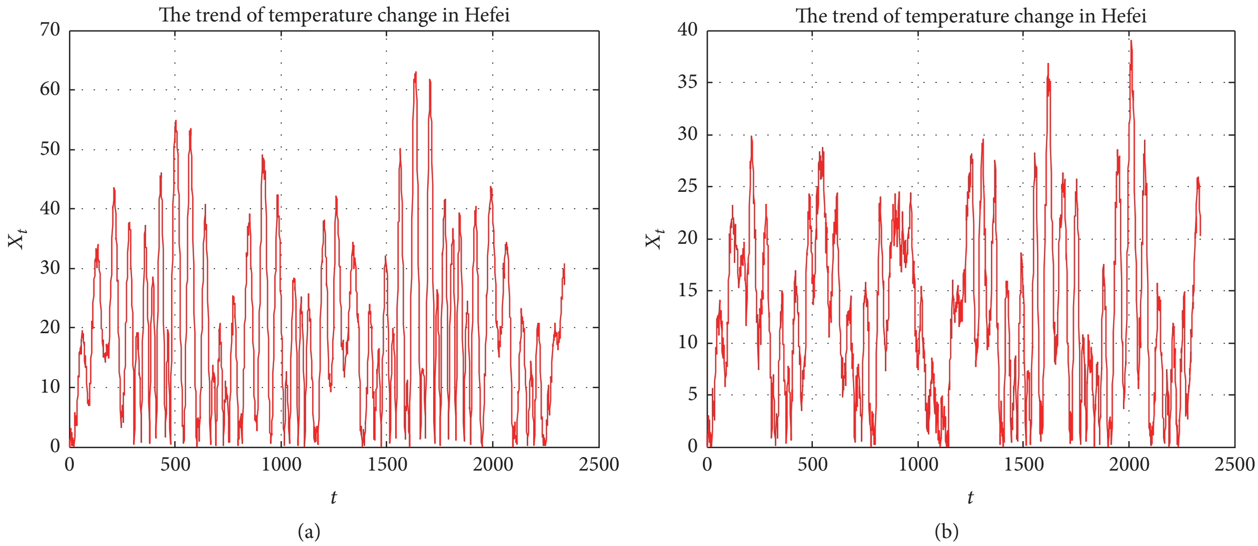

FIGURE 17: Time series diagram of (9) in Hefei when $\tau_{hf} = 19$. (a) $\beta = 0.005$; (b) $\beta = 0.03$.

historical temperatures of all the four cities exhibit periodic variation trends similar to that of sinusoidal curve. The local stability conditions of delay differential equation are explored. We analyze the effects of delay parameter and stochastic factors on the stability and complexity of the model through temperature change trend diagram, the largest Lyapunov exponent diagrams, time series diagrams, entropy diagrams, and 4D diagrams. We carried out numerical simulation on the following models: the temperature index model without delay and stochastic term, the temperature index model without delay under stochastic perturbation, delay temperature index model without stochastic term, and delay temperature index model with stochastic term. The simulation results are compared with historical temperature data to measure the fitting degree of the models. The variable feedback control method is adopted to return the unstable system to the stable state.

The conclusions of this paper are as follows: (1) the historical temperature evolution trends of four cities in Anhui Province periodically change; (2) the temperature index model without delay and stochastic term can better fit the temperature evolution process of four cities, similar to the sine curve; (3) the temperature index model without delay under stochastic perturbation can show the trend of temperature changes in four cities macroscopically, but the disturbance of stochastic factors makes the temperature changes always accompanied by the high frequency fluctuations; (4) delay temperature index model without stochastic term tells us that if the delay parameter is within a reasonable range, the model can reflect the trend of temperature changes better; otherwise, the system will lose stability and increase the complexity; (5) because of the influence of stochastic factors, even if the delay parameter is within a reasonable range, delay temperature index model with stochastic term cannot accurately describe the changes of temperature, and it shows the trend of periodic variation only macroscopically; (6) by comparing and analyzing simulation results, the model without stochastic term (delay parameter in reasonable range) can better fit the trend of actual historical temperature; (7) by analyzing the temperature evolution of four representative cities, it can be inferred that the overall temperature in Anhui is also periodically changed; (8) parameters k and A have more obvious effect on the stability and complexity of the system than parameters B and C; (9) the variable feedback control method successfully achieves the stability control of the deterministic differential equation and stochastic differential equation. Thus, the unstable systems can be stabilized by adjusting the control parameters. However, the control effect of deterministic differential equations is better than that of stochastic differential equations.

Conflicts of Interest

The author declares no conflicts of interest.

Acknowledgments

The author gratefully acknowledges financial support from the National Natural Science Foundation of China (11626033); the Philosophy and Social Science Planning Project of Anhui (AHSKQ2016D29); the University Outstanding Young Talent Support Project of Anhui (gxyq2017101); the Humanities and Social Science Research Project of Education Department of Anhui Province (SK2017A0434); the Social Science Planning Project of Bengbu (BB17C021); KYTD of BBC 2016-02.

References

[1] F. E. Benth and J. Saltyte-Benth, "Stochastic modelling of temperature variations with a view towards weather derivatives," *Applied Mathematical Finance*, vol. 12, no. 1, pp. 53–85, 2005.

[2] H. Fujita and H. Mori, "A hybrid intelligent system for designing a contract model for weather derivatives," in *Proceedings of the 2012 Complex Adaptive Systems Conference*, vol. 12, pp. 361–366, November 2012.

[3] M. Cao and J. Wei, "Weather derivatives valuation and market price of weather risk," *Journal of Futures Markets*, vol. 24, no. 11, pp. 1065–1089, 2004.

[4] S. D. Campbell and F. X. Diebold, "Weather forecasting for weather derivatives," *Journal of the American Statistical Association*, vol. 100, no. 469, pp. 6–16, 2005.

[5] S. Jewson and R. Caballero, "Seasonality in the statistics of surface air temperature and the pricing of weather derivatives," *Meteorological Applications*, vol. 10, no. 4, pp. 367–376, 2003.

[6] B. Dischel, "Black-Scholes Won't Do, Weather Risk Special Report," in *Energy and Power Risk Management*, pp. 8-9, 1998.

[7] B. Dischel, "At Least: a Model for Weather Risk, Weather Risk Special Report," in *Energy and Power Risk Management*, pp. 20-21, 1998.

[8] F. Dornier and M. Queruel, "Weather derivatives pricing: caution to the wind," *Energy & Power Risk Management*, vol. 8, pp. 30–32, 2000.

[9] H. Torró, V. Meneu, and E. Valor, "Single factor stochastic models with seasonality applied to underlying weather derivatives variables," *The Journal of Risk Finance*, vol. 4, no. 4, pp. 6–17, 2003.

[10] P. Alaton, B. Djehiche, and D. Stillberger, "On modelling and pricing weather derivatives," *Applied Mathematical Finance*, vol. 9, no. 1, pp. 1–20, 2002.

[11] D. C. Brody, J. Syroka, and M. Zervos, "Dynamical pricing of weather derivatives," *Quantitative Finance*, vol. 2, no. 3, pp. 189–198, 2002.

[12] F. E. Benth and J. Šaltytė-Benth, "The volatility of temperature and pricing of weather derivatives," *Quantitative Finance*, vol. 7, no. 5, pp. 553–561, 2007.

[13] F. E. Benth, J. Šaltyte Benth, and S. Koekebakker, "Putting a price on temperature," *Scandinavian Journal of Statistics*, vol. 34, no. 4, pp. 746–767, 2007.

[14] S. Jewson and A. Brix, *Weather Derivative Valuation: The Meteorological, Statistical, Financial and Mathematical Foundations*, Cambridge University Press, Cambridge, UK, 2005.

[15] F. Schiller, G. Seidler, and M. Wimmer, "Temperature models for pricing weather derivatives," *Quantitative Finance*, vol. 12, no. 3, pp. 489–500, 2012.

[16] L. Fortuna, G. Nunnari, and S. Nunnari, "A new fine-grained classification strategy for solar daily radiation patterns," *Pattern Recognition Letters*, vol. 81, pp. 110–117, 2016.

[17] T. Balint, F. Lamperti, A. Mandel, M. Napoletano, A. Roventini, and A. Sapio, "Complexity and the Economics of Climate Change: A Survey and a Look Forward," *Ecological Economics*, vol. 138, pp. 252–265, 2017.

[18] M. J. Jacobson, L. Markauskaite, A. Portolese, M. Kapur, P. K. Lai, and G. Roberts, "Designs for learning about climate change as a complex system," *Learning and Instruction*, vol. 52, pp. 1–14, 2017.

[19] D. Mihailović, V. Kostić, G. Mimić, and L. Cvetković, "Stability analysis of turbulent heat exchange over the heterogeneous environmental interface in climate models," *Applied Mathematics and Computation*, vol. 265, pp. 79–90, 2015.

[20] R. Sun, Y. Lü, L. Chen, L. Yang, and A. Chen, "Assessing the stability of annual temperatures for different urban functional zones," *Building and Environment*, vol. 65, pp. 90–98, 2013.

[21] Y. A. Kuznetsov, *Elements of Applied Bifurcation Theory*, Springer, New York, NY, USA, 2004.

[22] S. G. Ruan and J. J. Wei, "On the zeros of transcendental functions with applications to stability of delay differential equations with two delays," *Dynamics of Continuous, Discrete & Impulsive Systems A: Mathematical Analysis*, vol. 10, no. 6, pp. 863–874, 2003.

[23] J. Wang, F. Si, Y. Wang, and S. Duan, "Entropy and stability analysis of delayed energy supply-demand model," *Entropy*, vol. 18, no. 12, article no. 434, 2016.

[24] Y. Wang, S. Guo, and S. Duan, "Computation of the stability and complexity about triopoly price game model with delay decision," *Advances in Mathematical Physics*, vol. 2017, Article ID 2717352, 10 pages, 2017.