Research Article

Ground State Solutions to a Critical Nonlocal Integrodifferential System

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Received 30 April 2018; Accepted 18 July 2018; Published 8 August 2018

Academic Editor: Emmanuel Lorin

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Consider the following nonlocal integrodifferential system: $\mathcal{L}_K u + \lambda_1 u + \mu_1 |u|^{2_*-2} u + G_u(x, u, v) = 0$ in Ω , $\mathcal{L}_K v + \lambda_2 v + \mu_2 |v|^{2_*-2} v + G_v(x, u, v) = 0$ in Ω , $u = 0$, $v = 0$ in $\mathbb{R}^N \setminus \Omega$, where $\mathcal{L}_K$ is a general nonlocal integrodifferential operator, $\lambda_1, \lambda_2, \mu_1, \mu_2 > 0$, $2_* := 2N/(N-2s)$ is a fractional Sobolev critical exponent, $0 < s < 1$, $N > 2s$, $G(x, u, v)$ is a lower order perturbation of the critical coupling term, and Ω is an open bounded domain in $\mathbb{R}^N$ with Lipschitz boundary. Under proper conditions, we establish an existence result of the ground state solution to the nonlocal integrodifferential system.

1. Introduction

Recently, fractional Sobolev spaces and the corresponding nonlocal equations are applied to many subjects, such as, anomalous diffusion, elliptic problems with measure data, gradient potential theory, minimal surfaces, nonuniformly elliptic problems, optimization, phase transitions, quasi-geostrophic flows, singular set of minima of variational functionals, and water waves (see [1] and the references therein). For more details, please refer the book [2]. In [3], the authors considered the following equation:

$$\begin{aligned} \mathcal{L}_K u + \lambda u + |u|^{2_*-2} u + f(x, u) &= 0 \quad \text{in } \Omega, \\ u &= 0 \quad \text{in } \mathbb{R}^N \setminus \Omega, \end{aligned} \quad (1)$$

where $\mathcal{L}_K$ is a general nonlocal integrodifferential operator of order s which will be defined later and f is a lower order perturbation of the critical power $|u|^{2_*-2} u$, and, under proper conditions, an existence result of its solutions was obtained.

In present paper, we use some ideas and techniques in [3–5] to extend the results in [3] to the system case.

Consider the following nonlocal integrodifferential system:

$$\mathcal{L}_K u + \lambda_1 u + \mu_1 |u|^{2_*-2} u + G_u(x, u, v) = 0 \quad \text{in } \Omega,$$

$$\mathcal{L}_K v + \lambda_2 v + \mu_2 |v|^{2_*-2} v + G_v(x, u, v) = 0 \quad \text{in } \Omega,$$

$$u = 0, \quad v = 0$$

$$\text{in } \mathbb{R}^N \setminus \Omega, \quad (2)$$

where $\lambda_1, \lambda_2, \mu_1, \mu_2 > 0$, $2_* := 2N/(N-2s)$ is a fractional Sobolev critical exponent, $0 < s < 1$, $N > 2s$, $G(x, u, v)$ is a lower order perturbation of the critical coupling term, Ω is an open bounded domain in $\mathbb{R}^N$ with Lipschitz boundary, and $\mathcal{L}_K$ is a general nonlocal integrodifferential operator defined as

$$\begin{aligned} \mathcal{L}_K u(x) &= \int_{\mathbb{R}^N} (u(x+y) + u(x-y) - 2u(x)) K(y) dy, \\ x &\in \mathbb{R}^N. \end{aligned} \quad (3)$$

Here, $K : \mathbb{R}^N \setminus \{0\} \rightarrow (0, +\infty)$ is a function satisfying

$$mK \in L^1(\mathbb{R}^N), \quad \text{where } m(x) = \min\{1, |x|^2\}, \quad (4)$$

$$\exists \theta > 0 \text{ such that } K(x) \geq \frac{\theta}{|x|^{N+2s}}, \quad (5)$$

$$\forall x \in \mathbb{R}^N \setminus \{0\},$$

$$K(-x) = K(x) \quad \text{for any } x \in \mathbb{R}^N \setminus \{0\}. \quad (6)$$

Define the Hilbert space $D_K(\Omega)$ as the completion of $C_c^\infty(\Omega)$ with respect to the norm $\|\cdot\|_{D_K}$ induced by the scalar product $\langle \cdot, \cdot \rangle_{D_K}$ given by

$$\langle u, v \rangle_{D_K} := \int_{\mathbb{R}^{2N}} (u(x) - u(y))(v(x) - v(y)) \cdot K(x - y) dx dy. \quad (7)$$

If Ω is an open bounded Lipschitz domain, then $D_K(\Omega)$ coincides with the Sobolev space

$$X_0 := \{f \in X : f = 0 \text{ a.e. in } \Omega^c\}, \quad (8)$$

where X is a linear space of Lebesgue measurable functions from $\mathbb{R}^N$ to $\mathbb{R}$ such that the restriction to Ω of any function f in X belongs to $L^2(\Omega)$ and the map $(x, y) \mapsto (f(x) - f(y))\sqrt{K(x - y)}$ is in $L^2(\mathbb{R}^{2N} \setminus (\Omega^c \times \Omega^c), dx dy)$, and Ω^c is the complement of Ω in $\mathbb{R}^N$.

The solutions of (2) coincide with the critical points of the following energy functional $E_K : \mathcal{D}_K \rightarrow \mathbb{R}$:

$$\begin{aligned} E_K(u, v) &= \frac{1}{2} \|(u, v)\|_{\mathcal{D}_K}^2 - \frac{1}{2} (\lambda_1 |u|_{2, \Omega}^2 + \lambda_2 |v|_{2, \Omega}^2) \\ &\quad - \frac{1}{2_*} (\mu_1 |u|_{2_*, \Omega}^{2_*} + \mu_2 |v|_{2_*, \Omega}^{2_*}) \\ &\quad - \int_{\Omega} G(x, u(x), v(x)) dx, \end{aligned} \quad (9)$$

where $|u|_{p, \Omega} = (\int_{\Omega} |u|^p dx)^{1/p}$, $1 \leq p < +\infty$, $\mathcal{D}_K(\Omega) := D_K(\Omega) \times D_K(\Omega)$, endowed with norm $\|(u, v)\|_{\mathcal{D}_K}^2 := \|u\|_{D_K}^2 + \|v\|_{D_K}^2$, and $G(x, s, t)$ is a nonnegative Carathéodory function from $\Omega \times \mathbb{R} \times \mathbb{R}$ to $\mathbb{R}$; namely,

$$\begin{aligned} \forall \delta > 0, \sup \left\{ \frac{|G(x, s, t)|}{|s| + |t|} : \text{a.e. } x \in \Omega, |(s, t)| \right. \\ \left. := \sqrt{s^2 + t^2} \leq \delta \right\} < +\infty, \end{aligned} \quad (10)$$

$$\lim_{(s, t) \rightarrow (0, 0)} \frac{G(x, s, t)}{|s|^2 + |t|^2} = 0 \quad \text{uniformly in } x \in \Omega, \quad (11)$$

$$\lim_{|(s, t)| \rightarrow +\infty} \frac{G(x, s, t)}{|s|^{2_*} + |t|^{2_*}} = 0 \quad \text{uniformly in } x \in \Omega. \quad (12)$$

Consider the following Nehari manifold:

$$\begin{aligned} \mathcal{M}_K &:= \left\{ (u, v) \in \mathcal{D}_K(\Omega) \setminus \{(0, 0)\} : \|(u, v)\|_{\mathcal{D}_K}^2 \right. \\ &= \lambda_1 |u|_{2, \Omega}^2 + \lambda_2 |v|_{2, \Omega}^2 + \mu_1 |u|_{2_*, \Omega}^{2_*} + \mu_2 |v|_{2_*, \Omega}^{2_*} \\ &\quad + \int_{\Omega} (G_u(x, u(x), v(x)) u(x) \\ &\quad \left. + G_v(x, u(x), v(x)) v(x)) dx \right\}, \end{aligned} \quad (13)$$

define the ground state energy of (2) by

$$B_K := \inf_{(u, v) \in \mathcal{M}_K} E_K(u, v), \quad (14)$$

and call a solution (u, v) by a ground state solution if $E_K(u, v) = B_K$. Define

$$\begin{aligned} \bar{S}_K &:= \inf_{\substack{(u, v) \in \mathcal{D}_K(\mathbb{R}^N) \\ (u, v) \neq (0, 0)}} \frac{\|(u, v)\|_{\mathcal{D}_K}^2}{\left(\mu_1 |u|_{2_*, \mathbb{R}^N}^{2_*} + \mu_2 |v|_{2_*, \mathbb{R}^N}^{2_*} \right)^{2/2_*}}, \\ \bar{S}_{K, \lambda_1, \lambda_2} & \end{aligned} \quad (15)$$

$$:= \inf_{\substack{(u, v) \in \mathcal{D}_K(\Omega) \\ (u, v) \neq (0, 0)}} \frac{\|(u, v)\|_{\mathcal{D}_K}^2 - \lambda_1 |u|_{2, \Omega}^2 - \lambda_2 |v|_{2, \Omega}^2}{\left(\mu_1 |u|_{2_*, \Omega}^{2_*} + \mu_2 |v|_{2_*, \Omega}^{2_*} \right)^{2/2_*}}.$$

Then $\bar{S}_{K, \lambda_1, \lambda_2} \leq \min\{\bar{S}_K, S_{K, \lambda_1}, S_{K, \lambda_2}\}$, where

$$S_{K, \lambda_1} := \inf_{\substack{u \in D_K(\Omega) \\ u \neq 0}} \frac{\|u\|_{D_K}^2 - \lambda_1 |u|_{2, \Omega}^2}{\left(\mu_1 |u|_{2_*, \Omega}^{2_*} \right)^{2/2_*}}, \quad (16)$$

$$S_{K, \lambda_2} := \inf_{\substack{v \in D_K(\Omega) \\ v \neq 0}} \frac{\|v\|_{D_K}^2 - \lambda_2 |v|_{2, \Omega}^2}{\left(\mu_2 |v|_{2_*, \Omega}^{2_*} \right)^{2/2_*}}.$$

Theorem 1. Assume that $\lambda_1, \lambda_2 \in (0, \lambda_K(\Omega))$, where $\lambda_K(\Omega)$ is the first eigenvalue of the nonlocal operator $-\mathcal{L}_K$ with homogeneous Dirichlet boundary conditions. If there exists $(u_0, v_0) \in \mathcal{D}_K(\Omega) \setminus \{(0, 0)\}$ with $u_0, v_0 \geq 0$ a.e. in $\mathbb{R}^N$, such that

$$\sup_{t > 0} E_K(tu_0, tv_0) < \frac{s}{N} \bar{S}_{K, \lambda_1, \lambda_2}^{N/2s}, \quad (17)$$

then (2) has a nontrivial ground state solution.

2. Preliminaries

We need the following results, which have been proved in [6].

Lemma 2. There exists a positive constant C_s such that, for any $u \in D_K(\Omega)$,

$$C_s \int_{\mathbb{R}^{2N}} \frac{|u(x) - u(y)|^2}{|x - y|^{N+2s}} dx dy \geq |u|_{2_*, \mathbb{R}^N}^2 = |u|_{2_*, \Omega}^2, \quad (18)$$

where C_s is only depending on N and s .

3. Proof of Theorem 1

Lemma 3. Assume that $G : \Omega \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is a Carathéodory function; that is, (10)–(12) hold. Then, for any $\varepsilon > 0$, there exist $a(\varepsilon) > 0$ and $b(\varepsilon) > 0$ such that for a.e. $x \in \Omega$ and, for any $s, t \in \mathbb{R}$,

$$|G(x, s, t)| \leq \varepsilon (|s|^{2_*} + |t|^{2_*}) + a(\varepsilon) (|s| + |t|), \quad (19)$$

$$|G(x, s, t)| \leq \varepsilon (|s|^2 + |t|^2) + b(\varepsilon) (|s|^{2_*} + |t|^{2_*}). \quad (20)$$

Proof. For any $\varepsilon > 0$, by (12), there exists $\delta = \delta(\varepsilon) > 0$ such that, for a.e. $x \in \Omega$ and for any $(s, t) \in \mathbb{R}^2$ with $|s| + |t| > \delta$,

$$|G(x, s, t)| \leq \varepsilon (|s|^{2_*} + |t|^{2_*}). \quad (21)$$

It follows from (10) that there exists $a(\varepsilon) = a(\delta(\varepsilon)) > 0$ satisfying, for a.e. $x \in \Omega$ and for any $(s, t) \in \mathbb{R} \times \mathbb{R}$ with $|(s, t)| \leq \delta$,

$$|G(x, s, t)| \leq a(\varepsilon) (|s| + |t|). \quad (22)$$

Combining (21) and (22), we see that (19) holds.

By (11), for any $\varepsilon > 0$, there exists $\sigma = \sigma(\varepsilon) > 0$ such that, for a.e. $x \in \Omega$ and for any $(s, t) \in \mathbb{R}^2$ with $|(s, t)| < \sigma$,

$$|G(x, s, t)| \leq \varepsilon (|s|^2 + |t|^2). \quad (23)$$

It is easy to see from (19) that

$$\begin{aligned} |G(x, s, t)| &\leq |s|^{2^*} + |t|^{2^*} + a(1) (|s| + |t|) \\ &= (|s|^{2^*} + |t|^{2^*}) \left(1 + \frac{a(1) (|s| + |t|)}{|s|^{2^*} + |t|^{2^*}} \right). \end{aligned} \quad (24)$$

Therefore, there exists $b(\varepsilon) = b(\sigma(\varepsilon)) > 0$ satisfying, for a.e. $x \in \Omega$ and for any $(s, t) \in \mathbb{R}^2$ with $|(s, t)| \geq \sigma$,

$$|G(x, s, t)| \leq b(\varepsilon) (|s|^{2^*} + |t|^{2^*}). \quad (25)$$

Then, (20) follows from (23) and (25). $\square$

Lemma 4. Assume that $\lambda_1, \lambda_2 \in (0, \lambda_K(\Omega))$. Then, there exist $\rho, \tau > 0$ and $e = (e_1, e_2) \in \mathcal{D}_K(\Omega)$ such that

- (i) $E_K(u, v) \geq \tau$ for any $(u, v) \in \mathcal{D}_K(\Omega)$ with $\|(u, v)\|_{\mathcal{D}_K} = \rho$;
- (ii) $e_1, e_2 \geq 0$ a.e. in $\mathbb{R}^N$, $\|(e_1, e_2)\|_{\mathcal{D}_K} > \rho$, and $E_K(e_1, e_2) < \tau$.

In particular, if (17) holds, then we can construct e by $e = t_0(u_0, v_0)$, where (u_0, v_0) is given by (17) and $t_0 > 0$ is large enough.

Proof. For $(u, v) \in \mathcal{D}_K(\Omega)$ and any $\varepsilon > 0$, by (20), Lemma 2, $H^s(\mathbb{R}^N) \hookrightarrow L^{2^*}(\mathbb{R}^N)$, and (5), we deduce that

$$\begin{aligned} E_K(u, v) &\geq \frac{1}{2} \|u, v\|_{\mathcal{D}_K}^2 - \frac{1}{2} (\lambda_1 |u|_{2, \Omega}^2 + \lambda_2 |v|_{2, \Omega}^2) \\ &\quad - \frac{1}{2_*} (\mu_1 |u|_{2^*, \Omega}^{2^*} + \mu_2 |v|_{2^*, \Omega}^{2^*}) \\ &\quad - \int_{\Omega} \varepsilon [(u^2 + v^2) + b(\varepsilon) (|u|^{2^*} + |v|^{2^*})] dx \\ &\geq \frac{1}{2} \left(1 - \frac{\lambda_1}{\lambda_K(\Omega)} \right) \|u\|_{D_K}^2 + \frac{1}{2} \left(1 - \frac{\lambda_2}{\lambda_K(\Omega)} \right) \\ &\quad \cdot \|v\|_{D_K}^2 - \varepsilon |\Omega|^{(2^*-2)/2_*} (|u|_{2^*, \Omega}^{2^*} + |v|_{2^*, \Omega}^{2^*}) - \left(\frac{\mu_1}{2_*} \right. \\ &\quad \left. + b(\varepsilon) \right) |u|_{2^*, \Omega}^{2^*} - \left(\frac{\mu_2}{2_*} + b(\varepsilon) \right) |v|_{2^*, \Omega}^{2^*} \geq \frac{1}{2} \left(1 \right. \\ &\quad \left. - \frac{\lambda_1}{\lambda_K(\Omega)} \right) \|u\|_{D_K}^2 + \frac{1}{2} \left(1 - \frac{\lambda_2}{\lambda_K(\Omega)} \right) \|v\|_{D_K}^2 \\ &\quad - \varepsilon |\Omega|^{(2^*-2)/2_*} C_s \left(\int_{\mathbb{R}^{2N}} \frac{|u(x) - u(y)|^2}{|x - y|^{N+2s}} dx dy \right. \\ &\quad \left. + \int_{\mathbb{R}^{2N}} \frac{|v(x) - v(y)|^2}{|x - y|^{N+2s}} dx dy \right) - \left(\frac{\mu_1}{2_*} + b(\varepsilon) \right) \\ &\quad \cdot C_s^{2_*/2} \left(\int_{\mathbb{R}^{2N}} \frac{|u(x) - u(y)|^2}{|x - y|^{N+2s}} dx dy \right)^{2_*/2} - \left(\frac{\mu_2}{2_*} \right. \\ &\quad \left. + b(\varepsilon) \right) C_s^{2_*/2} \left(\int_{\mathbb{R}^{2N}} \frac{|v(x) - v(y)|^2}{|x - y|^{N+2s}} dx dy \right)^{2_*/2} \\ &\geq \left[\frac{1}{2} \left(1 - \frac{\lambda_1}{\lambda_K(\Omega)} \right) - \frac{\varepsilon C_s |\Omega|^{(2^*-2)/2_*}}{\theta} \right] \|u\|_{D_K}^2 \\ &\quad + \left[\frac{1}{2} \left(1 - \frac{\lambda_2}{\lambda_K(\Omega)} \right) - \frac{\varepsilon C_s |\Omega|^{(2^*-2)/2_*}}{\theta} \right] \|v\|_{D_K}^2 \\ &\quad - \left(\frac{\mu_1}{2_*} + b(\varepsilon) \right) \left(\frac{C_s}{\theta} \right)^{2_*/2} \|u\|_{D_K}^{2_*} - \left(\frac{\mu_2}{2_*} + b(\varepsilon) \right) \\ &\quad \cdot \left(\frac{C_s}{\theta} \right)^{2_*/2} \|v\|_{D_K}^{2_*}. \end{aligned} \quad (26)$$

For $\varepsilon < \theta(\lambda_K(\Omega) - \max\{\lambda_1, \lambda_2\})/2C_s\lambda_K(\Omega)|\Omega|^{(2^*-2)/2_*}$, we have

$$E_K(u, v) \geq a \|(u, v)\|_{\mathcal{D}_K}^2 (1 - b \|(u, v)\|_{\mathcal{D}_K}^{2_*-2}), \quad (27)$$

where a and b are suitable positive constants. Let $(u, v) \in \mathcal{D}_K$ satisfy $\|(u, v)\|_{\mathcal{D}_K} = \rho > 0$, where ρ is small enough such that $1 - b\rho^{2_*-2} > 0$. Therefore,

$$\inf_{\substack{(u,v) \in \mathcal{D}_K(\Omega) \\ \|(u,v)\|_{\mathcal{D}_K} = \rho}} E_K(u, v) \geq a\rho^2 (1 - b\rho^{2_*-2}) := \tau > 0, \quad (28)$$

which proves (i).

Fix $(u, v) \in \mathcal{D}_K(\Omega)$ with $\|(u, v)\|_{\mathcal{D}_K} = 1$ and $u \geq 0, v \geq 0$ a.e. in $\mathbb{R}^N$. Choosing $\varepsilon = (\min\{\mu_1, \mu_2\})/(2 \cdot 2_*)$ in (19), we get that

$$\begin{aligned} |G(x, u, v)| &\leq \frac{\min\{\mu_1, \mu_2\}}{2 \cdot 2_*} (|u|^{2^*} + |v|^{2^*}) \\ &\quad + a \left(\frac{\min\{\mu_1, \mu_2\}}{2 \cdot 2_*} \right) (|u| + |v|). \end{aligned} \quad (29)$$

For $t > 0$, by (29), we have

$$\begin{aligned} E_K(tu, tv) &= \frac{t^2}{2} \|u, v\|_{\mathcal{D}_K}^2 - \frac{t^2}{2} (\lambda_1 |u|_{2, \Omega}^2 + \lambda_2 |v|_{2, \Omega}^2) \\ &\quad - \frac{t^{2_*}}{2_*} (\mu_1 |u|_{2^*, \Omega}^{2^*} + \mu_2 |v|_{2^*, \Omega}^{2^*}) \\ &\quad - \int_{\Omega} G(x, tu, tv) dx \\ &\leq \frac{t^2}{2} - \frac{t^{2_*} \min\{\mu_1, \mu_2\}}{2_*} (|u|_{2^*, \Omega}^{2^*} + |v|_{2^*, \Omega}^{2^*}) \end{aligned}$$

$$\begin{aligned}
& + \frac{t^{2*} \min \{\mu_1, \mu_2\}}{2 \cdot 2_*} \int_{\Omega} (|u|^{2*} + |v|^{2*}) dx \\
& + ta \left(\frac{\min \{\mu_1, \mu_2\}}{2 \cdot 2_*} \right) \int_{\Omega} (|u| + |v|) dx \\
& \leq \frac{t^2}{2} - \frac{\min \{\mu_1, \mu_2\}}{2 \cdot 2_*} (|u|_{2^*, \Omega}^{2*} + |v|_{2^*, \Omega}^{2*}) t^{2*} \\
& + a \left(\frac{\min \{\mu_1, \mu_2\}}{2 \cdot 2_*} \right) (|u|_{2, \Omega}^2 + |v|_{2, \Omega}^2) t,
\end{aligned} \tag{30}$$

which implies that $E_K(tu, tv) \rightarrow -\infty$ as $t \rightarrow +\infty$. Then, (ii) follows by taking $e = (e_1, e_2) = (tu, tv)$ with t sufficiently large.

In particular, if (17) holds, then we may fix (u, v) by $(u, v) = (u_0, v_0) / \|(u_0, v_0)\|_{\mathcal{D}_K}$. Thus, we can construct e by $e = (tu_0, tv_0) / \|u_0, v_0\|_{\mathcal{D}_K} = t_0(u_0, v_0)$, where t_0 is large enough. $\square$

Define

$$\mathcal{B}_K := \inf_{h \in \Gamma} \sup_{0 < t < 1} E_K(h(t)), \tag{31}$$

where

$$\begin{aligned}
\Gamma &:= \{h \in C([0, 1], \mathcal{D}_K(\Omega)) : h(0) = 0, h(1) = e \\
&= (e_1, e_2)\}
\end{aligned} \tag{32}$$

with $e = t_0(u_0, v_0)$ given in Lemma 4. It is standard to see that

$$\mathcal{B}_K = \mathbb{B}_K = B_K, \tag{33}$$

where $\mathbb{B}_K = \inf_{(u, v) \in \mathcal{D}_K(\Omega) \setminus \{(0, 0)\}} \sup_{t > 0} E_K(tu, tv)$ and B_K is given in (14).

Lemma 5. $\tau \leq B_K < (s/N) \bar{S}_{K, \lambda_1, \lambda_2}^{N/2s}$, where τ is given in Lemma 4 and $\bar{S}_{K, \lambda_1, \lambda_2}$ is defined in (15).

Proof. Noting that, for any $h \in \Gamma$, the function $t \mapsto \|h(t)\|_{\mathcal{D}_K}$ is continuous in $[0, 1]$; we get that $\|h(0)\|_{\mathcal{D}_K} = \|(0, 0)\|_{\mathcal{D}_K} = 0 < \rho$ and $\|h(1)\|_{\mathcal{D}_K} = \|e\|_{\mathcal{D}_K} > \rho$, where ρ is given by Lemma 4. Thus, we may find $t_h \in (0, 1)$ such that $\|h(t_h)\|_{\mathcal{D}_K} = \rho$. Therefore,

$$\sup_{0 < t < 1} E_K(h(t)) \geq E_K(h(t_h)) \geq \inf_{\substack{(u, v) \in \mathcal{D}_K(\Omega) \\ \|(u, v)\|_{\mathcal{D}_K} = \rho}} E_K(u, v) \tag{34}$$

$$\geq \tau,$$

which implies that $B_K \geq \tau$. Recalling that $e = t_0(u_0, v_0)$ and the special pass $[0, 1] \ni t \mapsto tt_0(u_0, v_0)$ belongs to Γ , by (17), we deduce that

$$\begin{aligned}
B_K &= \inf_{h \in \Gamma} \sup_{0 < t < 1} E_K(h(t)) \leq \sup_{t > 0} E_K(tt_0(u_0, v_0)) \\
&< \frac{s}{N} \bar{S}_{K, \lambda_1, \lambda_2}^{N/2s}.
\end{aligned} \tag{35}$$

$\square$

Lemma 6. Every sequence $\{(u_n, v_n)\}$ with $E_K(u_n, v_n) \rightarrow c$ and $E'_K(u_n, v_n) \rightarrow 0$ in $\mathcal{D}'_K(\Omega)$ as $n \rightarrow \infty$ ((PS) $_c$ sequence of E_K) is bounded in $\mathcal{D}_K(\Omega)$.

Proof. From

$$\begin{aligned}
c + o(1) &= E_K(u_n, v_n) \\
&= \frac{1}{2} \|(u_n, v_n)\|_{\mathcal{D}_K}^2 \\
&\quad - \frac{1}{2} (\lambda_1 |u_n|_{2, \Omega}^2 + \lambda_2 |v_n|_{2, \Omega}^2) \\
&\quad - \frac{1}{2_*} (\mu_1 |u_n|_{2^*, \Omega}^{2*} + \mu_2 |v_n|_{2^*, \Omega}^{2*}) \\
&\quad - \int_{\Omega} G(x, u_n, v_n) dx
\end{aligned} \tag{36}$$

and

$$\begin{aligned}
o(1) &= E'_K(u_n, v_n)(u_n, v_n) \\
&= \|(u_n, v_n)\|_{\mathcal{D}_K}^2 - (\lambda_1 |u_n|_{2, \Omega}^2 + \lambda_2 |v_n|_{2, \Omega}^2) \\
&\quad - (\mu_1 |u_n|_{2^*, \Omega}^{2*} + \mu_2 |v_n|_{2^*, \Omega}^{2*}) \\
&\quad - \int_{\Omega} (G_u(x, u_n, v_n) u_n + G_v(x, u_n, v_n) v_n) dx,
\end{aligned} \tag{37}$$

it can be seen that

$$\begin{aligned}
c + o(1) &= E_K(u_n, v_n) - \frac{1}{2} E'_K(u_n, v_n)(u_n, v_n) \\
&= \left(\frac{1}{2} - \frac{1}{2_*} \right) (\mu_1 |u_n|_{2^*, \Omega}^{2*} + \mu_2 |v_n|_{2^*, \Omega}^{2*}) \\
&\quad - \int_{\Omega} G(x, u_n, v_n) dx \\
&\quad + \frac{1}{2} \int_{\Omega} (G_u(x, u_n, v_n) u_n + G_v(x, u_n, v_n) v_n) dx \\
&\geq \frac{s}{N} (\mu_1 |u_n|_{2^*, \Omega}^{2*} + \mu_2 |v_n|_{2^*, \Omega}^{2*}) \\
&\quad - k \int_{\Omega} |G(x, u_n, v_n)| dx
\end{aligned} \tag{38}$$

for some suitable positive constant k . For $u \in D_K(u)$, it follows from Lemma 2 and (5) that

$$\begin{aligned}
\int_{\Omega} |u| dx &\leq |\Omega|^{(2^*-1)/2_*} |u|_{2^*, \Omega} \\
&\leq |\Omega|^{(2^*-1)/2_*} \left(C_s \int_{\mathbb{R}^{2N}} \frac{|u(x) - u(y)|^2}{|x - y|^{N+2s}} dx dy \right)^{1/2} \\
&\leq \Theta \|u\|_{D_K}
\end{aligned} \tag{39}$$

for some positive constant Θ . By (19) and (39), we have

$$\begin{aligned} \int_{\Omega} |G(x, u_n, v_n)| dx &\leq \varepsilon (|u_n|_{2^*, \Omega}^{2^*} + |v_n|_{2^*, \Omega}^{2^*}) \\ &\quad + a(\varepsilon) \Theta (\|u_n\|_{D_K} + \|v_n\|_{D_K}) \\ &\leq \varepsilon (|u_n|_{2^*, \Omega}^{2^*} + |v_n|_{2^*, \Omega}^{2^*}) \\ &\quad + C(\varepsilon) \|(u_n, v_n)\|_{\mathcal{D}_K}, \end{aligned} \quad (40)$$

where $C(\varepsilon)$ is a positive constant relevant to ε and Ω . Choosing ε small enough such that $(s/N) \min\{\mu_1, \mu_2\} - k\varepsilon > 0$, then (38) and (40) yield that

$$|u_n|_{2^*, \Omega}^{2^*} + |v_n|_{2^*, \Omega}^{2^*} \leq C (1 + \|(u_n, v_n)\|_{\mathcal{D}_K}). \quad (41)$$

Letting $\varepsilon = 1$ in (40) and noting $0 < \lambda_1, \lambda_2 < \lambda_K(\Omega)$, by (41), we infer that

$$\begin{aligned} c + o(1) &= E_K(u_n, v_n) \\ &\geq \frac{1}{2} \left(1 - \frac{\lambda_1}{\lambda_K(\Omega)}\right) \|u_n\|_{D_K}^2 \\ &\quad + \frac{1}{2} \left(1 - \frac{\lambda_2}{\lambda_K(\Omega)}\right) \|v_n\|_{D_K}^2 \\ &\quad - \frac{1}{2_*} (\mu_1 |u_n|_{2^*, \Omega}^{2^*} + \mu_2 |v_n|_{2^*, \Omega}^{2^*}) \\ &\quad - \int_{\Omega} |G(x, u_n, v_n)| dx \\ &\geq \frac{1}{2} \left(1 - \frac{\max\{\lambda_1, \lambda_2\}}{\lambda_K(\Omega)}\right) \|(u_n, v_n)\|_{\mathcal{D}_K}^2 \\ &\quad - \left(\frac{\max\{\mu_1, \mu_2\}}{2_*} + 1\right) (|u_n|_{2^*, \Omega}^{2^*} + |v_n|_{2^*, \Omega}^{2^*}) \\ &\quad - C(1) \|(u_n, v_n)\|_{\mathcal{D}_K} \\ &\geq C_1 \|(u_n, v_n)\|_{\mathcal{D}_K}^2 - C_2 (1 + \|(u_n, v_n)\|_{\mathcal{D}_K}) \end{aligned} \quad (42)$$

for suitable constants C_1 and C_2 , which implies that $\{(u_n, v_n)\}$ is bounded in $\mathcal{D}_K(\Omega)$. $\square$

Proof of Theorem 1. It follows from $G(x, 0, 0) = 0$ that $E_K(0, 0) = 0 < \tau$, where τ is given in Lemma 4. By Lemma 4, we see that E_K has a Mountain Pass Structure (e.g., [7]). By (33) as well as Lemma 5 and Theorem 2.2 in [8], there is $(PS)_{B_K}$ sequence $\{(u_n, v_n)\}$ for E_K . By Lemma 6, we get that $\{(u_n, v_n)\}$ is bounded in $\mathcal{D}_K(\Omega)$, and then, going if necessary to a subsequence, still denoted by $\{(u_n, v_n)\}$, there exists $(u_{\infty}, v_{\infty}) \in \mathcal{D}_K(\Omega)$ such that $(u_n, v_n) \rightharpoonup (u_{\infty}, v_{\infty})$ weakly in $\mathcal{D}_K(\Omega)$; i.e.,

$$\begin{aligned} \int_{\mathbb{R}^{2N}} (u_n(x) - u_n(y)) (\phi(x) - \phi(y)) \\ \cdot K(x - y) dx dy + \int_{\mathbb{R}^{2N}} (v_n(x) - v_n(y)) \end{aligned}$$

$$\cdot (\psi(x) - \psi(y)) K(x - y) dx dy \longrightarrow$$

$$\begin{aligned} \int_{\mathbb{R}^{2N}} (u_{\infty}(x) - u_{\infty}(y)) (\phi(x) - \phi(y)) \\ \cdot K(x - y) dx dy + \int_{\mathbb{R}^{2N}} (v_{\infty}(x) - v_{\infty}(y)) \\ \cdot (\psi(x) - \psi(y)) K(x - y) dx dy \end{aligned} \quad (43)$$

for any $(\phi, \psi) \in \mathcal{D}_K(\Omega)$. It follows from (41) and the boundedness of $\{(u_n, v_n)\}$ in $\mathcal{D}_K(\Omega)$ that

$$\{u_n\} \text{ and } \{v_n\} \text{ are bounded in } L^{2^*}(\Omega). \quad (44)$$

Then, it is standard to see that

$$u_n \rightharpoonup u_{\infty},$$

$$v_n \rightharpoonup v_{\infty}$$

weakly in $L^{2^*}(\Omega)$,

$$u_n \longrightarrow u_{\infty},$$

$$v_n \longrightarrow v_{\infty}$$

strongly in $L^p(\Omega)$,

$$u_n \longrightarrow u_{\infty},$$

$$v_n \longrightarrow v_{\infty}$$

a.e. in Ω ,

$$|u_n|^{2^*-2} u_n \rightharpoonup |u_{\infty}|^{2^*-2} u_{\infty},$$

$$|v_n|^{2^*-2} v_n \rightharpoonup |v_{\infty}|^{2^*-2} v_{\infty}$$

weakly in $L^{2^*/(2^*-1)}(\Omega)$,

up to a subsequence, where $p \in [1, 2_*)$. By (19), (44), and the boundedness of Ω , we see that

$$G_u(\cdot, u_n(\cdot), v_n(\cdot)),$$

$$G_v(\cdot, u_n(\cdot), v_n(\cdot)) \quad (46)$$

are bounded in $L^{2^*/(2^*-1)}(\Omega)$.

Since $G(x, u, v)$ is superlinear with respect to u and v , we get that the mappings $t \mapsto G_u(x, t, v)$ and $t \mapsto G_v(x, u, t)$ are continuous in $\mathbb{R}$. Therefore,

$$G_u(\cdot, u_n(\cdot), v_n(\cdot)) \longrightarrow G_u(\cdot, u_{\infty}(\cdot), v_{\infty}(\cdot)),$$

$$G_v(\cdot, u_n(\cdot), v_n(\cdot)) \longrightarrow G_v(\cdot, u_{\infty}(\cdot), v_{\infty}(\cdot)) \quad (47)$$

a.e. in Ω .

Then, by (46) and (47), we have

$$G_u(\cdot, u_n(\cdot), v_n(\cdot)) \rightharpoonup G_u(\cdot, u_{\infty}(\cdot), v_{\infty}(\cdot)),$$

$$G_v(\cdot, u_n(\cdot), v_n(\cdot)) \rightharpoonup G_v(\cdot, u_{\infty}(\cdot), v_{\infty}(\cdot)) \quad (48)$$

weakly in $L^{2^*/(2^*-1)}(\Omega)$.

Noting that $L^{2^*}(\Omega)$ is the dual space of $L^{2^*/(2^*-1)}(\Omega)$, we deduce that

$$\begin{aligned} \int_{\Omega} G_u(x, u_n, v_n) \phi \, dx &\longrightarrow \int_{\Omega} G_u(x, u_{\infty}, v_{\infty}) \phi \, dx, \\ \int_{\Omega} G_v(x, u_n, v_n) \psi \, dx &\longrightarrow \int_{\Omega} G_v(x, u_{\infty}, v_{\infty}) \psi \, dx \quad (49) \\ &\text{for any } \phi, \psi \in L^{2^*}(\Omega), \end{aligned}$$

and then, in particular,

$$\begin{aligned} \int_{\Omega} G_u(x, u_n, v_n) \phi \, dx &\longrightarrow \int_{\Omega} G_u(x, u_{\infty}, v_{\infty}) \phi \, dx, \\ \int_{\Omega} G_v(x, u_n, v_n) \psi \, dx &\longrightarrow \int_{\Omega} G_v(x, u_{\infty}, v_{\infty}) \psi \, dx \quad (50) \\ &\text{for any } (\phi, \psi) \in \mathcal{D}_K(\Omega). \end{aligned}$$

For any $(\phi, \psi) \in \mathcal{D}_K(\Omega)$, we have

$$\begin{aligned} 0 &\leftarrow E'_K(u_n, v_n)(\phi, \psi) = \int_{\mathbb{R}^{2N}} (u_n(x) - u_n(y)) \\ &\quad \cdot (\phi(x) - \phi(y)) K(x - y) \, dx \, dy \\ &\quad + \int_{\mathbb{R}^{2N}} (v_n(x) - v_n(y)) (\psi(x) - \psi(y)) \\ &\quad \cdot K(x - y) \, dx \, dy - \int_{\Omega} (\lambda_1 u_n \phi + \lambda_2 v_n \psi) \, dx \\ &\quad - \int_{\Omega} (\mu_1 |u_n|^{2^*-2} u_n \phi + \mu_2 |v_n|^{2^*-2} v_n \psi) \, dx \\ &\quad - \int_{\Omega} (G_u(x, u_n, v_n) \phi + G_v(x, u_n, v_n) \psi) \, dx. \end{aligned} \quad (51)$$

Then, it follows from (43), (45), and (50) that

$$\begin{aligned} &\int_{\mathbb{R}^{2N}} (u_{\infty}(x) - u_{\infty}(y)) (\phi(x) - \phi(y)) \\ &\quad \cdot K(x - y) \, dx \, dy + \int_{\mathbb{R}^{2N}} (v_{\infty}(x) - v_{\infty}(y)) \\ &\quad \cdot (\psi(x) - \psi(y)) K(x - y) \, dx \, dy \\ &\quad - \int_{\Omega} (\lambda_1 u_{\infty} \phi + \lambda_2 v_{\infty} \psi) \, dx \\ &\quad - \int_{\Omega} (\mu_1 |u_{\infty}|^{2^*-2} u_{\infty} \phi + \mu_2 |v_{\infty}|^{2^*-2} v_{\infty} \psi) \, dx \\ &\quad - \int_{\Omega} (G_u(x, u_{\infty}, v_{\infty}) \phi + G_v(x, u_{\infty}, v_{\infty}) \psi) \, dx \\ &= 0, \end{aligned} \quad (52)$$

which means that (u_{∞}, v_{∞}) is solution of (2). It remains to show that (u_{∞}, v_{∞}) is nontrivial. Suppose by contradiction

that $(u_{\infty}, v_{\infty}) = (0, 0)$ in Ω . Then, by (19) and boundedness of $\{u_n\}$ and $\{v_n\}$ in $L^{2^*}(\Omega)$, we have

$$\begin{aligned} \left| \int_{\Omega} G(x, u_n, v_n) \, dx \right| &\leq \varepsilon \int_{\Omega} (|u_n|^{2^*} + |v_n|^{2^*}) \, dx \\ &\quad + a(\varepsilon) \int_{\Omega} (|u_n| + |v_n|) \, dx \quad (53) \\ &\leq C\varepsilon + a(\varepsilon) (|u_n|_{1,\Omega} + |v_n|_{1,\Omega}), \end{aligned}$$

and similarly

$$\begin{aligned} \left| \int_{\Omega} G_u(x, u_n, v_n) u_n \, dx \right| &\leq C\varepsilon + a(\varepsilon) (|u_n|_{1,\Omega} + |v_n|_{1,\Omega}), \\ \left| \int_{\Omega} G_v(x, u_n, v_n) v_n \, dx \right| &\leq C\varepsilon + a(\varepsilon) (|u_n|_{1,\Omega} + |v_n|_{1,\Omega}). \end{aligned} \quad (54)$$

Noting that $u_n \rightarrow 0$, $v_n \rightarrow 0$ in $L^p(\Omega)$ for any $p \in [1, 2_*)$ and letting $n \rightarrow \infty$, we get that

$$\begin{aligned} \limsup_{n \rightarrow \infty} \left| \int_{\Omega} G(x, u_n, v_n) \, dx \right| &\leq C\varepsilon, \\ \limsup_{n \rightarrow \infty} \left| \int_{\Omega} G_u(x, u_n, v_n) u_n \, dx \right| &\leq C\varepsilon, \\ \limsup_{n \rightarrow \infty} \left| \int_{\Omega} G_v(x, u_n, v_n) v_n \, dx \right| &\leq C\varepsilon. \end{aligned} \quad (55)$$

Taking $\varepsilon \rightarrow 0^+$, we have

$$\begin{aligned} &\int_{\Omega} G(x, u_n, v_n) \, dx, \\ &\int_{\Omega} G_u(x, u_n, v_n) u_n \, dx, \\ &\int_{\Omega} G_v(x, u_n, v_n) v_n \, dx \\ &\quad \downarrow \\ &0. \end{aligned} \quad (56)$$

Therefore,

$$\begin{aligned} &\|(u_n, v_n)\|_{\mathcal{D}_K}^2 - \mu_1 |u_n|_{2^*,\Omega}^{2^*} - \mu_2 |v_n|_{2^*,\Omega}^{2^*} \\ &= E'_K(u_n, v_n)(u_n, v_n) + \lambda_1 |u_n|_{2,\Omega}^2 + \lambda_2 |v_n|_{2,\Omega}^2 \\ &\quad + \int_{\Omega} G_u(x, u_n, v_n) u_n \, dx \\ &\quad + \int_{\Omega} G_v(x, u_n, v_n) v_n \, dx \longrightarrow 0. \end{aligned} \quad (57)$$

Assume, up to a subsequence, that $\|(u_n, v_n)\|_{\mathcal{D}_K}^2 \rightarrow b$. Then, $\mu_1 |u_n|_{2^*,\Omega}^{2^*} + \mu_2 |v_n|_{2^*,\Omega}^{2^*} \rightarrow b$, and

$$B_K + o(1) = E_K(u_n, v_n) = \frac{b}{2} - \frac{b}{2_*} + o(1), \quad (58)$$

which means that $B_K = (s/N)b$. By Lemma 5, we see that $b > 0$. Noting (15), we have

$$\|(u_n, v_n)\|_{\mathcal{D}_K}^2 \geq \bar{S}_{K,\lambda_1,\lambda_2} (\mu_1 |u_n|_{2^*,\Omega}^{2^*} + \mu_2 |v_n|_{2^*,\Omega}^{2^*})^{2/2^*}, \quad (59)$$

which implies that $b \geq S_{K,\lambda_1,\lambda_2} b^{2/2^*}$. Therefore, $B_K \geq (s/N)S_{K,\lambda_1,\lambda_2}^{2^*/(2^*-2)} = (s/N)S_{K,\lambda_1,\lambda_2}^{N/2s}$, contradicting with Lemma 5. Hence, (u_∞, v_∞) can not be $(0, 0)$. Besides (45), up to a subsequence, we may assume that

$$\begin{aligned} G(x, u_n, v_n) &\longrightarrow G(x, u_\infty, v_\infty) \\ &\text{strongly in } L^1(\Omega), \\ G_u(x, u_n, v_n) &\longrightarrow G_u(x, u_\infty, v_\infty), \\ G_v(x, u_n, v_n) &\longrightarrow G_v(x, u_\infty, v_\infty) \\ &\text{strongly in } L^{q/(q-1)}(\Omega), \end{aligned} \quad (60)$$

where $q \in (2, 2_*)$ is the power sum of G with respect to u and v . From

$$\begin{aligned} \langle u_n, \phi \rangle_{D_K} + \langle v_n, \psi \rangle_{D_K} - \lambda_1 \int_{\Omega} u_n \phi \, dx \\ - \lambda_2 \int_{\Omega} v_n \psi \, dx - \mu_1 \int_{\Omega} |u_n|^{2^*-2} u_n \phi \, dx \\ - \mu_2 \int_{\Omega} |v_n|^{2^*-2} v_n \psi \, dx \\ - \int_{\Omega} (G_u(x, u_n, v_n) \phi + G_v(x, u_n, v_n) \psi) \, dx \\ = o(1), \end{aligned} \quad (61)$$

we get that

$$\begin{aligned} \langle u_\infty, \phi \rangle_{D_K} + \langle v_\infty, \psi \rangle_{D_K} - \lambda_1 \int_{\Omega} u_\infty \phi \, dx \\ - \lambda_2 \int_{\Omega} v_\infty \psi \, dx \\ - \mu_1 \int_{\Omega} |u_\infty|^{2^*-2} u_\infty \phi \, dx - \mu_2 \int_{\Omega} |v_\infty|^{2^*-2} v_\infty \psi \, dx \\ - \int_{\Omega} (G_u(x, u_\infty, v_\infty) \phi + G_v(x, u_\infty, v_\infty) \psi) \, dx \\ = 0, \end{aligned} \quad (62)$$

where $(\phi, \psi) \in \mathcal{D}_K(\Omega)$; i.e., $E'_K(u, v) = 0$ in $\mathcal{D}'_K(\Omega)$. Setting $\omega_n = u_n - u$ and $\sigma_n = v_n - v$, then $\omega_n \rightharpoonup 0, \sigma_n \rightharpoonup 0$ weakly in $D_K(\Omega)$. Thus, $\omega_n \rightarrow 0, \sigma_n \rightarrow 0$ strongly in $L^2(\Omega)$, $\omega_n \rightarrow 0, \sigma_n \rightarrow 0$ a.e. in Ω , $G_u(x, \omega_n, \sigma_n) \omega_n \rightarrow 0$, $G_v(x, \omega_n, \sigma_n) \sigma_n \rightarrow 0$, $G(x, \omega_n, \sigma_n) \rightarrow 0$ in $L^1(\Omega)$, up to a subsequence. Brezis-Lieb Lemma guarantees that

$$\begin{aligned} |u_n|_{2^*,\Omega}^{2^*} &= |u_\infty|_{2^*,\Omega}^{2^*} + |\omega_n|_{2^*,\Omega}^{2^*} + o(1), \\ |v_n|_{2^*,\Omega}^{2^*} &= |v_\infty|_{2^*,\Omega}^{2^*} + |\sigma_n|_{2^*,\Omega}^{2^*} + o(1), \end{aligned}$$

$$\begin{aligned} \int_{\Omega} G(x, u_n, v_n) \, dx &= \int_{\Omega} G(x, u_\infty, v_\infty) \, dx \\ &+ \int_{\Omega} G(x, \omega_n, \sigma_n) \, dx \\ &+ o(1). \end{aligned} \quad (63)$$

Since $E'_K(u_n, v_n)(u_n, 0) = o(1)$, $E'_K(u_n, v_n)(0, v_n) = o(1)$ and $E'_K(u_\infty, v_\infty)(u_\infty, v_\infty) = 0$, by (63), we deduce that

$$\begin{aligned} \|\omega_n\|_{D_K}^2 - \mu_1 |\omega_n|_{2^*,\Omega}^{2^*} &= o(1), \\ \|\sigma_n\|_{D_K}^2 - \mu_2 |\sigma_n|_{2^*,\Omega}^{2^*} &= o(1), \end{aligned} \quad (64)$$

and

$$I_K(\omega_n, \sigma_n) = E_K(u_n, v_n) - E_K(u_\infty, v_\infty) + o(1), \quad (65)$$

where

$$\begin{aligned} I_K(u, v) &= \frac{1}{2} \|(u, v)\|_{\mathcal{D}_K}^2 \\ &- \frac{1}{2_*} (\mu_1 |u|_{2^*,\mathbb{R}^N}^{2^*} + \mu_2 |v|_{2^*,\mathbb{R}^N}^{2^*}) \end{aligned} \quad (66)$$

Assume that

$$\begin{aligned} \lim_{n \rightarrow \infty} \mu_1 |\omega_n|_{2^*,\Omega}^{2^*} &= a_1, \\ \lim_{n \rightarrow \infty} \mu_2 |\sigma_n|_{2^*,\Omega}^{2^*} &= a_2, \end{aligned} \quad (67)$$

up to a subsequence. By (64), we see that

$$I_K(\omega_n, \sigma_n) = \left(\frac{1}{2} - \frac{1}{2_*} \right) (a_1 + a_2) + o(1). \quad (68)$$

Letting $n \rightarrow \infty$, we have

$$\begin{aligned} B_K &= \lim_{n \rightarrow \infty} E_K(u_n, v_n) \\ &= \left(\frac{1}{2} - \frac{1}{2_*} \right) (a_1 + a_2) + E_K(u_\infty, v_\infty) \geq 0. \end{aligned} \quad (69)$$

Suppose by contradiction that $(u_\infty, v_\infty) = (u_\infty, 0)$, where $u_\infty \not\equiv 0$. Then, u_∞ is a nontrivial solution of $\mathcal{L}_K u + \lambda_1 u + \mu_1 |u|^{2^*-2} u = 0$. Hence, by (69), we deduce that $B_K \geq E_K(u_\infty, 0) \geq (s/N)S_{K,\lambda_1}^{N/2s} \geq (s/N)\bar{S}_{K,\lambda_1,\lambda_2}^{N/2s}$, contradiction with Lemma 5. Therefore, (u_∞, v_∞) can not be $(u_\infty, 0)$, where $u_\infty \not\equiv 0$. Similarly, (u_∞, v_∞) can not be $(0, v_\infty)$, where $v_\infty \not\equiv 0$. Thus, (2) has a nontrivial ground state solution, which completes the proof. $\square$

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work is supported by NSFC (11701248).

References

- [1] E. Di Nezza, G. Palatucci, and E. Valdinoci, “Hitchhiker’s guide to the fractional Sobolev spaces,” *Bulletin des Sciences mathématiques*, vol. 136, no. 5, pp. 521–573, 2012.
- [2] G. M. Bisci, V. D. Radulescu, and R. Servadei, *Variational Methods for Nonlocal Fractional Problems*, vol. 162, Cambridge University Press, 2016.
- [3] R. Servadei and E. Valdinoci, “The Brezis-Nirenberg result for the fractional Laplacian,” *Transactions of the American Mathematical Society*, vol. 367, no. 1, pp. 67–102, 2015.
- [4] Z. Chen and W. Zou, “Positive least energy solutions and phase separation for coupled Schrödinger equations with critical exponent,” *Archive for Rational Mechanics and Analysis*, vol. 205, no. 2, pp. 515–551, 2012.
- [5] Z. Chen and W. Zou, “Positive least energy solutions and phase separation for coupled Schrödinger equations with critical exponent: higher dimensional case,” *Calculus of Variations and Partial Differential Equations*, vol. 52, no. 1-2, pp. 423–467, 2015.
- [6] R. Servadei and E. Valdinoci, “Mountain pass solutions for non-local elliptic operators,” *Journal of Mathematical Analysis and Applications*, vol. 389, no. 2, pp. 887–898, 2012.
- [7] A. Ambrosetti and P. H. Rabinowitz, “Dual variational methods in critical point theory and applications,” *Journal of Functional Analysis*, vol. 14, pp. 349–381, 1973.
- [8] H. Brezis and L. Nirenberg, “Positive solutions of nonlinear elliptic equations involving critical Sobolev exponents,” *Communications on Pure and Applied Mathematics*, vol. 36, no. 4, pp. 437–477, 1983.