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Research Article

Quantum Controlled Teleportation of Arbitrary Two-Qubit State via Entangled States

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We put forward an efficient quantum controlled teleportation scheme, in which arbitrary two-qubit state is transmitted from the sender to the remote receiver via two entangled states under the control of the supervisor. In this paper, we use the combination of one two-qubit entangled state and one three-qubit entangled state as quantum channel for achieving the transmission of unknown quantum states. We present the concrete implementation processes of this scheme. Furthermore, we calculate the successful probability and the amount of classical information of our protocol.

1. Introduction

Quantum teleportation theoretical scheme was first proposed by Bennett et al. in 1993 [1], where one unknown quantum state could be transmitted via Einstein-Podolsky-Rosen (EPR) pair with the help of classical information. In fact, what is transferred is the information contained in quantum state, not the particle itself. And by teleportation we will achieve transfer of information from one particle to another particle, which is not material as in fiction. In addition, due to its high confidentiality and reliability, it is superior to conventional electrical communication. In the teleportation, both of the sender and the receiver do not know the transmitted quantum state in advance.

The quantum controlled teleportation was originally presented by Karlsson et al. [2]. Its main difference from the initial quantum teleportation [3–8] is that this scheme introduces the third party to supervise channel, so that the information of unknown state can not be transmitted without the cooperation of all third parties. Due to its potential applications, it has been further studied and many theoretical protocols have been proposed. Yan and Wang proposed the controlled teleportation of both one-qubit and two-qubit unknown quantum states [9]. Yang et al. put forward controlled teleportation protocol of teleporting multiqubit states [10]. Gao et al. presented a protocol for controlled and secure direct communication using GHZ state [11]. Xiu et al. proposed a protocol for teleporting a one-qubit state via a three-particle entangled state [12].

In many researches on quantum controlled teleportation, it has been realized that one-qubit unknown state can be transmitted by using of the three-particle entangled states as quantum channel. In this paper, based on quantum controlled teleportation protocol we propose a method for transmitting arbitrary two-qubit state via one two-qubit entangled state and one three-qubit entangled state as quantum channel, where the Greenberger-Horne-Zeilinger (GHZ) state and Bell state are utilized. After that, we calculate the successful probability and the amount of classical information of our protocol.

2. Controlled Teleportation of Arbitrary Two-Qubit State

We assume that the sender Alice, the receiver Bob, and the supervisor Charlie are spatially separated from each other. For achieving the transmission of two-qubit state, we use the combination of one GHZ state and one Bell state as quantum channel. The arbitrary two-qubit state transported from the sender Alice to the receiver Bob can be expressed as

$$|\psi\rangle_{i_1 i_2} = \left(a_0\,|00\rangle + a_1\,|01\rangle + a_2\,|10\rangle + a_3\,|11\rangle\right)_{i_1 i_2} \quad (1)$$

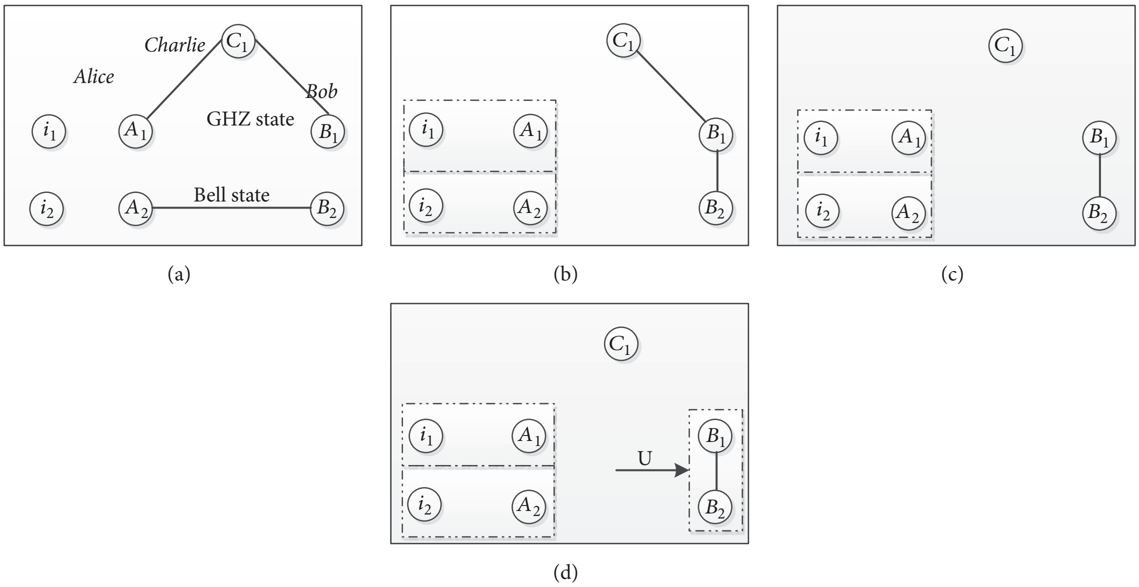

FIGURE 1: (a) shows the preparation and distribution of entangled states; (b) expresses the measurement of particles i_1 and A_1, i_2 and A_2; (c) shows the measurement of particle C_1; (d) shows the unitary operation for states of particles B_1 and B_2.

where i_1 and i_2 are used to represent particles of two unknown states to be transmitted; factors a_i ($i = 0,1,2,3$) satisfy the normalization condition $\sum_{i=0}^{3}|a_i|^2 = 1$. Quantum channel consists of two entangled states below:

$$|\psi\rangle_{A_1B_1C_1} = \frac{1}{\sqrt{2}}|000\rangle_{A_1B_1C_1} + \frac{1}{\sqrt{2}}|111\rangle_{A_1B_1C_1} \tag{2}$$

$$|\psi\rangle_{A_2B_2} = \frac{1}{\sqrt{2}}|00\rangle_{A_2B_2} + \frac{1}{\sqrt{2}}|11\rangle_{A_2B_2} \tag{3}$$

where three-qubit entangled states are shared by Alice, Bob, and Charlie. For convenience, we use A_1 and A_2, B_1 and B_2 to represent the two particles held by Alice and Bob and use C_1 to represent the particle held by Charlie. The whole system can be expressed as

$$|\psi\rangle_{total} = |\psi\rangle_{i_1i_2} \otimes |\psi\rangle_{A_1B_1C_1} \otimes |\psi\rangle_{A_2B_2} \tag{4}$$

In our scheme, the detailed realization procedures for the quantum controlled teleportation of arbitrary two-qubit state are as follows. Also, the corresponding flow chart is shown in Figure 1.

Step 1. Alice performs a Bell-state measurement of the particles i_1 and A_1, i_2 and A_2 it possesses. There would be four kinds of measurement results of particles i_1 and A_1, particles i_2 and A_2, respectively. So we can obtain sixteen outcomes. Based on the measurement results, the total state of system can be expressed as

$$\begin{aligned}
|\psi\rangle_{total} &= |\psi\rangle_{i_1i_2} \otimes |\psi\rangle_{A_1B_1C_1} \otimes |\psi\rangle_{A_2B_2} = |\phi^+\rangle_{i_1A_1} \\
&\otimes |\phi^+\rangle_{i_2A_2} \otimes \frac{1}{4}(a_0|000\rangle + a_1|001\rangle + a_2|110\rangle \\
&+ a_3|111\rangle)_{B_1C_1B_2} + |\phi^+\rangle_{i_1A_1} \otimes |\phi^-\rangle_{i_2A_2} \\
&\otimes \frac{1}{4}(a_0|000\rangle - a_1|001\rangle + a_2|110\rangle \\
&- a_3|111\rangle)_{B_1C_1B_2} + |\phi^+\rangle_{i_1A_1} \otimes |\psi^+\rangle_{i_2A_2} \\
&\otimes \frac{1}{4}(a_0|001\rangle + a_1|000\rangle + a_2|111\rangle \\
&+ a_3|110\rangle)_{B_1C_1B_2} + |\phi^+\rangle_{i_1A_1} \otimes |\psi^-\rangle_{i_2A_2} \\
&\otimes \frac{1}{4}(-a_0|001\rangle + a_1|000\rangle - a_2|111\rangle \\
&+ a_3|110\rangle)_{B_1C_1B_2} + |\phi^-\rangle_{i_1A_1} \otimes |\phi^+\rangle_{i_2A_2} \\
&\otimes \frac{1}{4}(a_0|000\rangle + a_1|001\rangle - a_2|110\rangle \\
&- a_3|111\rangle)_{B_1C_1B_2} + |\phi^-\rangle_{i_1A_1} \otimes |\phi^-\rangle_{i_2A_2} \\
&\otimes \frac{1}{4}(a_0|000\rangle - a_1|001\rangle - a_2|110\rangle \\
&+ a_3|111\rangle)_{B_1C_1B_2} + |\phi^-\rangle_{i_1A_1} \otimes |\psi^+\rangle_{i_2A_2} \\
&\otimes \frac{1}{4}(a_0|001\rangle + a_1|000\rangle - a_2|111\rangle \\
&- a_3|110\rangle)_{B_1C_1B_2} + |\phi^-\rangle_{i_1A_1} \otimes |\psi^-\rangle_{i_2A_2} \\
&\otimes \frac{1}{4}(-a_0|001\rangle + a_1|000\rangle + a_2|111\rangle \\
&- a_3|110\rangle)_{B_1C_1B_2} + |\psi^+\rangle_{i_1A_1} \otimes |\phi^+\rangle_{i_2A_2} \\
&\otimes \frac{1}{4}(a_0|100\rangle + a_1|101\rangle + a_2|010\rangle \\
&+ a_3|011\rangle)_{B_1C_1B_2} + |\psi^+\rangle_{i_1A_1} \otimes |\phi^-\rangle_{i_2A_2}
\end{aligned}$$

$$\begin{aligned}
&\otimes \frac{1}{4}\left(a_0 |100\rangle - a_1 |101\rangle + a_2 |010\rangle\right.\\
&\left.- a_3 |011\rangle\right)_{B_1C_1B_2} + |\psi^+\rangle_{i_1A_1} \otimes |\psi^+\rangle_{i_2A_2}\\
&\otimes \frac{1}{4}\left(a_0 |101\rangle + a_1 |100\rangle + a_2 |011\rangle\right.\\
&\left.+ a_3 |010\rangle\right)_{B_1C_1B_2} + |\psi^+\rangle_{i_1A_1} \otimes |\psi^-\rangle_{i_2A_2}\\
&\otimes \frac{1}{4}\left(-a_0 |101\rangle + a_1 |100\rangle - a_2 |011\rangle\right.\\
&\left.+ a_3 |010\rangle\right)_{B_1C_1B_2} + |\psi^-\rangle_{i_1A_1} \otimes |\phi^+\rangle_{i_2A_2}\\
&\otimes \frac{1}{4}\left(-a_0 |100\rangle - a_1 |101\rangle + a_2 |010\rangle\right.\\
&\left.+ a_3 |011\rangle\right)_{B_1C_1B_2} + |\psi^-\rangle_{i_1A_1} \otimes |\phi^-\rangle_{i_2A_2}\\
&\otimes \frac{1}{4}\left(-a_0 |100\rangle + a_1 |101\rangle + a_2 |010\rangle\right.\\
&\left.- a_3 |011\rangle\right)_{B_1C_1B_2} + |\psi^-\rangle_{i_1A_1} \otimes |\psi^+\rangle_{i_2A_2}\\
&\otimes \frac{1}{4}\left(-a_0 |101\rangle - a_1 |100\rangle + a_2 |011\rangle\right.\\
&\left.+ a_3 |010\rangle\right)_{B_1C_1B_2} + |\psi^-\rangle_{i_1A_1} \otimes |\psi^-\rangle_{i_2A_2}\\
&\otimes \frac{1}{4}\left(a_0 |101\rangle - a_1 |100\rangle - a_2 |011\rangle\right.\\
&\left.+ a_3 |010\rangle\right)_{B_1C_1B_2}
\end{aligned} \tag{5}$$

Here

$$\begin{aligned}
|\phi^\pm\rangle &= \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right),\\
|\psi^\pm\rangle &= \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)
\end{aligned} \tag{6}$$

Step 2. Alice needs to inform Bob and Charlie of the measurement results via classic channel. Based on measurement results of the particles i_1 and A_1, i_2 and A_2, the quantum states of particles B_1, B_2, and C_1 will also collapse to corresponding states. For example, if the measurement results of i_1 and A_1, i_2 and A_2 are $|\phi^+\rangle_{i_1A_1}$ and $|\phi^+\rangle_{i_2A_2}$, respectively, Bob and Charlie can judge that the quantum state of system has converted into

$$\begin{aligned}
&|\psi\rangle_{B_1C_1B_2}\\
&\quad = \left(a_0 |000\rangle + a_1 |001\rangle + a_2 |110\rangle + a_3 |111\rangle\right)_{B_1C_1B_2}
\end{aligned} \tag{7}$$

Step 3. If the supervisor Charlie consents to assist the communication between Alice and Bob, Charlie needs to perform projection measurement of the particle C_1. There may be two results for the measurement of C_1. Based on measurement results, the quantum states of particles can also be expressed as

$$\begin{aligned}
&|\psi\rangle_{B_1C_1B_2}\\
&\quad = \left(a_0 |000\rangle + a_1 |001\rangle + a_2 |110\rangle + a_3 |111\rangle\right)_{B_1C_1B_2}\\
&\quad = |+\rangle_{C_1}\\
&\qquad \otimes \frac{1}{4\sqrt{2}}\left(a_0 |00\rangle + a_1 |01\rangle + a_2 |10\rangle + a_3 |11\rangle\right)_{B_1B_2}\\
&\qquad + |-\rangle_{C_1}\\
&\qquad \otimes \frac{1}{4\sqrt{2}}\left(a_0 |00\rangle + a_1 |01\rangle - a_2 |10\rangle - a_3 |11\rangle\right)_{B_1B_2}
\end{aligned} \tag{8}$$

where $|\pm\rangle_{C_1} = (1/\sqrt{2})(|0\rangle \pm |1\rangle)_{C_1}$.

Step 4. Charlie informs Bob of the measurement results of particle C_1 via classic channel. Based on measurement results, the quantum states of particles B_1 and B_2 will also collapse to corresponding states. If the measurement results of C_1 are $|+\rangle_{C_1}$, Bob can judge that the quantum states of system have collapsed to

$$|\psi\rangle_{B_1B_2} = \left(a_0 |00\rangle + a_1 |01\rangle + a_2 |10\rangle + a_3 |11\rangle\right)_{B_1B_2} \tag{9}$$

Similarly, if the results are $|-\rangle_{C_1}$, the quantum states of system have collapsed to

$$|\psi\rangle_{B_1B_2} = \left(a_0 |00\rangle + a_1 |01\rangle - a_2 |10\rangle - a_3 |11\rangle\right)_{B_1B_2} \tag{10}$$

Step 5. According to the classic information from Alice and Charlie, Bob needs to perform the Pauli transformation to recover the two-qubit state. The Pauli transformations σ_x, σ_y, σ_z are as follows:

$$\begin{aligned}
\sigma_x &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\\
\sigma_y &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\\
\sigma_z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\end{aligned} \tag{11}$$

When (9) is obtained, the quantum teleportation is achieved, in which the two-qubit state is transferred to the two particles owned by Bob. In this case, Bob does not need to take any action to get the two-qubit state. When (10) is obtained, Bob needs to perform σ_z transformation on particle B_1 and no transformation is performed on the particle B_2. And, for other possible measurement results, we need to perform appropriate transformation for achieving successfully quantum controlled teleportation.

3. The Successful Probability and Classical Information

There are thirty-two measurement results. Based on the measurement results of particles i_1, A_1, i_2, A_2 and C_1, the unknown two-qubit state can be constructed by Bob via the corresponding Pauli transformation on particle B_1. The probability of each result is

$$P_i = \left(\frac{1}{4\sqrt{2}}\right)^2 = \frac{1}{32}, \quad (i = 1, 2, \ldots, 32) \tag{12}$$

The total successful probability can be expressed as

$$P_{total} = \sum_{i=1}^{32} P_i = 100\% \tag{13}$$

The amount of classic information S can be calculated as

$$S = -\sum_{i=1}^{32} P_i \log P_i = 5 \tag{14}$$

4. Conclusions

In summary, we propose an efficient quantum controlled teleportation scheme, in which arbitrary two-qubit state is transmitted from the sender Alice to the remote receiver Bob via two entangled states under the control of the supervisor Charlie. In this paper, we use one GHZ entangled state and one Bell state as quantum channel for achieving the transfer of unknown quantum state. In contrast to the previous schemes [3–5, 5, 9, 11], our scheme is easier to implement. First of all, we use the combination of one GHZ state and one Bell state as quantum channel for achieving the transmission of two-qubit state. Second, Charlie only needs to operate one particle C_1 for supervising the process of the whole quantum controlled teleportation. Third, according to the method mentioned in this paper, the number of unknown qubit states to be transmitted can be expanded from two particles to n particles, and we only need to increase the number of Bell states to achieve quantum controlled teleportation of n-qubit state. The scheme can not only be available in the large capacity information transmission field based on the optical fiber or free space quantum communication, but also play an important role in quantum computation field. Quantum teleportation is one of the typical methods to realize future quantum communication. With the further improvement of this scheme, it contributes to future development in computation and communication. On the one hand, it enables large capacity information transmission, and on the other hand it can achieve long-distance communication between satellites and satellites and satellites and ground. Finally, we hope that the scheme proposed will be experimentally verified in the near future.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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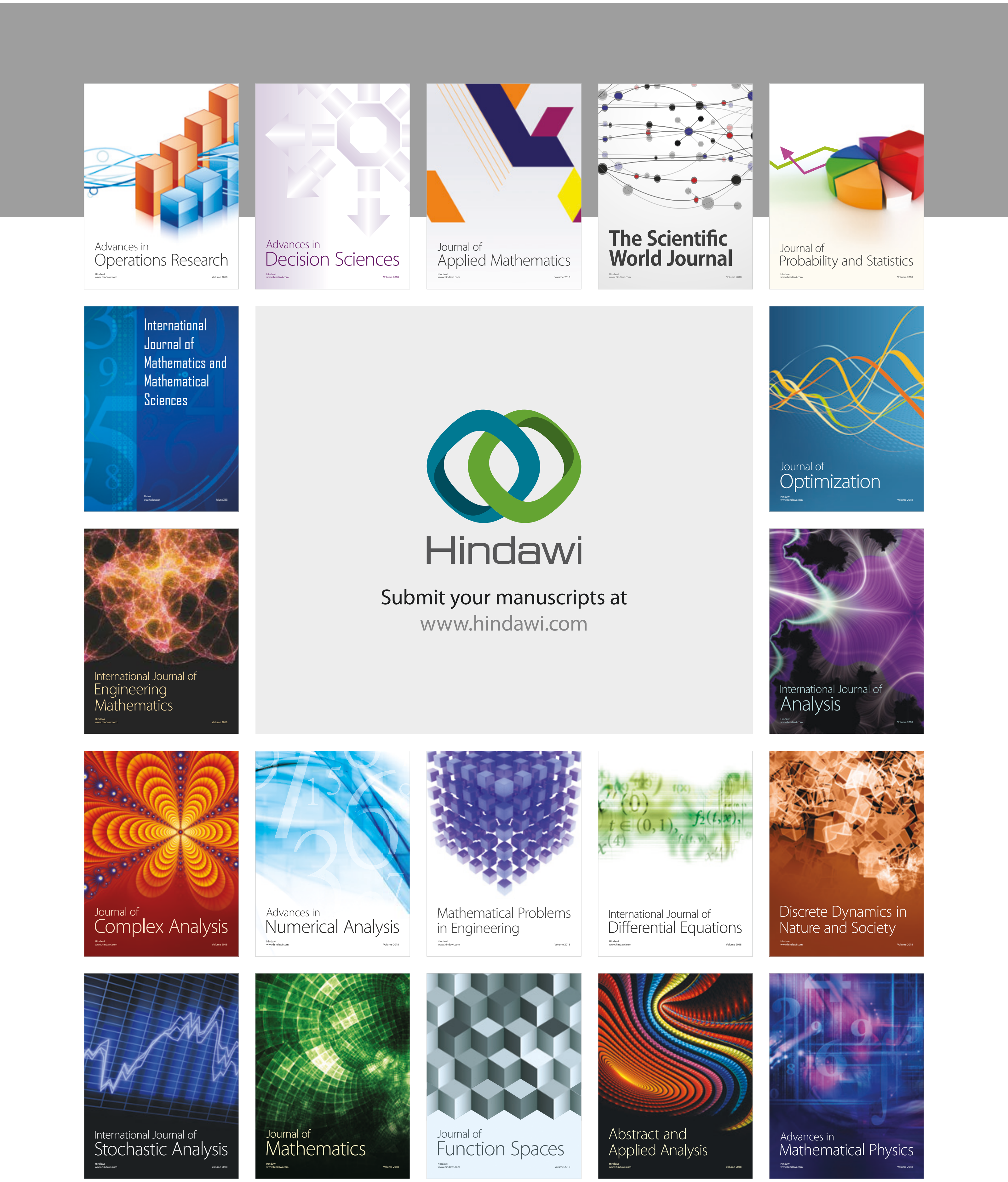
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