

Research Article

Identification of a Time-Dependent Coefficient in Heat Conduction Problem by New Iteration Method

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This paper investigates the problem of identifying unknown coefficient of time dependent in heat conduction equation by new iteration method. In order to use new iteration method, we should convert the parabolic heat conductive equation into an integral equation by integral calculus and initial condition. This method constructs a convergent sequence of function, which approximates the exact solution with a few iterations and does not need complex calculation. Illustrative examples are given to demonstrate the efficiency and validity.

1. Introduction

In this paper, we will consider an inverse problem of determining a control function $p(t)$ in the following parabolic partial differential equation:

$$u_t(x, t) = u_{xx} + qu_x + p(t)u + F(x, t), \quad x \in (0, 1), \quad t \in (0, T], \quad (1)$$

$$u(x, 0) = f(x), \quad x \in (0, 1), \quad (2)$$

$$u(0, t) = g_1(t), \quad t \in (0, T], \quad (3)$$

$$u(1, t) = g_2(t), \quad t \in (0, T], \quad (4)$$

and the overspecified condition

$$u(x^*, t) = E(t), \quad t \in (0, T], \quad (5)$$

or

$$\int_0^{s(t)} u dx = E(t), \quad t \in (0, T], \quad (6)$$

where $F(x, t)$, $f(x)$, $g_1(t)$, $g_2(t)$, $E(t) \neq 0$, and $0 < s(t) \leq 1$ are known functions, q is a known constant, and $x^* \in (0, 1)$

is a fixed observation point; the condition (6) represents the specification of a relative heat content of a portion of the conductor. The unknown function pair (u, p) will be determined.

Nonlocal boundary specifications like (6) arise from many important applications in heat transfer, thermoelasticity, control theory, life science, etc. For example, in a heat transfer process, if we let u represent the temperature distribution, then (1)–(4) and (6) can be regarded as a control problem with source control. A source control parameter $p(t)$ needs to be determined so that a desired thermal energy can be obtained for a portion of the spatial domain.

Determination of unknown coefficients in inverse heat conduction problems is well known as inverse coefficient problems (ICPs). Identification of physical properties such as conductivity using measured temperature or heat flux values at some sites is an important inverse problem, and these problems have been studied by many authors and some types of ICPs have been solved by some numerical or analysis methods; for instance, Cannon JR and Lin YP researched the parameter $p(t)$ in some quasilinear parabolic differential equations in [1], Kerimov NB and Ismailov MI studied the existence, uniqueness and continuous dependence upon the data of the solution of the ICPs using the generalized Fourier

method in [2], Pourgholi R and Rostamian M solved an ICPs using Tikhonov regularization method in [3], Hussein MS and Lesnic D studied the problem of time and space-dependent coefficients in a parabolic equation using the finite difference method in [4], in [5] Kamynin VL researched the existence and uniqueness theorems of the solution of the inverse problem of simultaneous determination of the right-hand side and the lowest coefficient in multidimensional parabolic equations with integral observation, Fatullayev AG and Cula S determined an unknown space-dependent coefficient in a parabolic equation using the finite difference method in [6], Ozbilge E and Demir A researched the inverse problem for a time-fractional parabolic equation using semigroup method in [7], variational iteration method was applied to solve an inverse parabolic equation in [8], and other methods can be referred to literature [9–12].

Daftardar-Gejji and Jafari [13] proposed the new iteration method (NIM) which is based upon the Adomian decomposition method. This method has been applied to various equations such as algebraic equations, integral equations, and ordinary and partial differential equations of integer and fractional order. This method was compared with other numerical methods by some authors; for instance, in [14] Bhalekar and Daftardar-Gejji applied the NIM to fractional-order logistic equation and compared with the Adomian decomposition and homotopy perturbation method, and the results showed that the NIM converges faster to the approximate solutions. Srivastava and Rai [15] used the NIM and modified Adomian decomposition method to solve the multiterm fractional diffusion equation for different conditions, the results also showed that the NIM is direct and straightforward, and it avoids the volume of calculations resulting from computing the Adomian polynomials.

In this paper, we will use the NIM to solve inverse heat conduction problems (1)–(6).

This paper has been organized as follows. In Section 2 the basis idea of the NIM is simply described, and in Section 3 determination of unknown coefficient in inverse heat conduction problem has been verified using the NIM. Illustrative examples have been presented in Section 4 and the conclusion is given in Section 5.

2. The NIM

Consider the general functional equation

$$u = N(u) + f, \quad (7)$$

where N is a continuous nonlinear operator from $U \rightarrow V$ ($U, V \subseteq B$, B is a Banach space), f is a known function, and $\|f(x)\| \leq M$, where M is a positive constant. Assume that (7) has the following series solution:

$$u = \sum_{i=0}^{\infty} u_i. \quad (8)$$

The nonlinear operator N can be decomposed as [13]

$$\begin{aligned} N(u) &= N\left(\sum_{i=0}^{\infty} u_i\right) \\ &= N(u_0) + \sum_{i=1}^{\infty} \left\{ N\left(\sum_{j=0}^i u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right) \right\}. \end{aligned} \quad (9)$$

In view of (8) and (9), (7) can be rewritten as

$$\begin{aligned} u &= N(u) + f \\ &= f + N(u_0) + \sum_{i=1}^{\infty} \left\{ N\left(\sum_{j=0}^i u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right) \right\}. \end{aligned} \quad (10)$$

Define the recursion relation as follows:

$$\begin{aligned} u_0 &= f, \\ u_1 &= N(u_0), \\ u_{m+1} &= N(u_0 + u_1 + \cdots + u_m) \\ &\quad - N(u_0 + u_1 + \cdots + u_{m-1}), \end{aligned} \quad (11)$$

$m = 1, 2, \dots$

From (11), we have

$$\begin{aligned} u_1 + u_2 + \cdots + u_{m+1} &= N(u_0 + u_1 + \cdots + u_m), \\ m &= 1, 2, \dots, \end{aligned} \quad (12)$$

and

$$u = f + \sum_{i=1}^{\infty} u_i. \quad (13)$$

If N is a contraction operator, that is, $\|N(u) - N(v)\| \leq K\|u - v\|$, $0 < K < 1$, then

$$\begin{aligned} \|u_{m+1}\| &= \|N(u_0 + u_1 + \cdots + u_m) \\ &\quad - N(u_0 + u_1 + \cdots + u_{m-1})\| \leq K\|u_{m-1}\| \\ &\leq K^2\|u_{m-2}\| \leq \cdots \leq K^m\|u_0\|, \quad m = 1, 2, \dots \end{aligned} \quad (14)$$

Since the numerical series $\sum_{i=0}^{\infty} K^i\|u_0\|$ is convergent, therefore, the series $\sum_{i=0}^{\infty} u_i$ uniformly converges to a solution of (7) [16], which is unique in view of the Banach fixed point theorem [17].

If N is a linear operator, from (11) we obtain

$$u_m = N(u_{m-1}), \quad m = 1, 2, \dots, \quad (15)$$

and

$$u = f + \sum_{i=1}^{\infty} N(u_{i-1}). \quad (16)$$

Apply the NIM to Volterra integral equation of the second kind as follows:

$$u(x) = f(x) + \int_a^x \Gamma(x, t, u(t)) dt, \quad x \in \Omega \subset R^1, \quad (17)$$

where Ω is a closed subset, $|x-a| \leq h$, $|t-a| \leq h$, h is a positive constant, and $f(x)$ is analytical function. Γ is a continuous function of its arguments and satisfies Lipschitz Condition; that is, $|\Gamma(x, t, \Phi) - \Gamma(x, t, \Psi)| < L|\Phi - \Psi|$, where L is a positive constant independent of x, t, Φ , and Ψ .

Let $|\Gamma(x, t, u)| \leq M$, where M is a positive constant; from (11) we define

$$\begin{aligned} u_0(x) &= f(x), \\ u_1(x) &= \int_a^x \Gamma(x, t, u_0(t)) dt, \\ u_{m+1}(x) &= \int_a^x (\Gamma(x, t, u_0 + \dots + u_m) \\ &\quad - \Gamma(x, t, u_0 + \dots + u_{m-1})) dt, \quad m = 1, 2, \dots \end{aligned} \quad (18)$$

Then the series $\sum_{i=0}^{\infty} u_i$ is uniformly convergent.

In fact,

$$\begin{aligned} |u_1(x)| &\leq \int_a^x |\Gamma(x, t, u_0(t))| dt \leq M(x-a), \\ |u_2(x)| &\leq \int_a^x |\Gamma(x, t, u_0(t) + u_1(t)) \\ &\quad - \Gamma(x, t, u_0(t))| dt \leq L \int_a^x |u_1(t)| dt \leq ML \\ &\quad \cdot \frac{(x-a)^2}{2!} \leq \frac{M}{L} \frac{(Lh)^2}{2!}, \\ |u_{m+1}(x)| &\leq \int_a^x |\Gamma(x, t, u_0(t) + \dots + u_m(t)) \\ &\quad - \Gamma(x, t, u_0(t) + \dots + u_{m-1}(t))| dt \\ &\leq L \int_a^x |u_m(t)| dt \leq ML^m \frac{(x-a)^{m+1}}{(m+1)!} \leq \frac{M}{L} \\ &\quad \cdot \frac{(Lh)^{m+1}}{(m+1)!}, \quad m = 1, 2, \dots \end{aligned} \quad (19)$$

Since the numerical series $\sum_{i=1}^{\infty} (M/L)((Lh)^m/m!)$ is convergent, so the series $\sum_{i=0}^{\infty} u_i(x)$ uniformly converges to the solution of (17).

3. Apply NIM to the Inverse Problem

We begin our investigation with a pair of invertible transformations for (1)-(6):

$$r(t) = \exp\left(-\int_0^t \left(p(s) - \frac{q^2}{4}\right) ds\right), \quad (20)$$

$$v(x, t) = u(x, t) r(t) \exp\left(\frac{q}{2}x\right). \quad (21)$$

We can rewrite (1)-(4) and (5) or (6) as follows:

$$\begin{aligned} v_t(x, t) &= v_{xx} + r(t) \exp\left(\frac{q}{2}x\right) F(x, t), \\ x &\in (0, 1), \quad t \in (0, T], \end{aligned} \quad (22)$$

$$v(x, 0) = f(x) \exp\left(\frac{q}{2}x\right), \quad x \in (0, 1), \quad (23)$$

$$v(0, t) = r(t) g_1(t), \quad t \in (0, T], \quad (24)$$

$$v(1, t) = r(t) g_2(t) \exp\left(\frac{q}{2}\right), \quad t \in (0, T], \quad (25)$$

such that

$$r(t) = \frac{v(x^*, t)}{E(t)} \exp\left(-\frac{q}{2}x^*\right), \quad (26)$$

or

$$r(t) = \frac{\int_0^{s(t)} v(x, t) \exp(-(q/2)x) dx}{E(t)}. \quad (27)$$

It is clear that the original inverse problem (1)-(6) is equivalent to the auxiliary problem (22)-(27). If $r(t)$ can be made known, then we can use proper method to solve (22)-(25) which seemed as a direct problem. So the key to the solution of the inverse problem lies in getting $r(t)$ from overspecified data (26) or (27), where $r(t)$ is related to unknown function $v(x, t)$. Cannon JR and Lin YP [1, 18] have showed the existence and uniqueness of a smooth global solution pair (v, r) which depends continuously upon the data under certain assumptions on the data of the auxiliary problem (22)-(27). Thus a unique solution pair (u, p) can be obtained through the following inverse transformations to (20) and (21):

$$\begin{aligned} u(x, t) &= \frac{v(x, t)}{r(t)} \exp\left(-\frac{q}{2}x\right), \\ p(t) &= -\frac{r'(t)}{r(t)} + \frac{q^2}{4}. \end{aligned} \quad (28)$$

In this paper, we will focus on the numerical approach.

Now we proceed to approximate solution pair (v, r) by the NIM. By integrating both sides of (22) with respect to t from 0 to t and using (23), we obtain

$$\begin{aligned} v(x, t) \\ &= v(x, 0) \end{aligned} \quad (29)$$

$$+ \int_0^t \left(v_{xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right) ds,$$

which of the same form as (17) can be solved by the NIM, where

$$\int_0^t \left(v_{xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right) ds \quad (30)$$

is a linear operator with respect to $v(x, s)$. We obtain the recursion relation from (15) and (18) as follows:

$$\begin{aligned}
 v_0(x, t) &= v(x, 0), \\
 v_1(x, t) &= \int_0^t \left(v_{0xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right) ds, \\
 v_{m+1}(x, t) &= \int_0^t \left(v_{mxx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right) ds, \\
 m &= 1, 2, \dots
 \end{aligned} \tag{31}$$

Here, the notation $v_{mxx}(x, s) = \partial^2 v_m(x, s) / \partial x^2$. If (29) satisfies the conditions of (17), then the series $v = \sum_{i=0}^{\infty} v_i$ is convergent and the pair (v, r) is found; we can obtain the solution of the inverse problem from (28).

We can summarize the procedure from the analysis above. Firstly, we change the inverse problem (1)–(6) into the equivalent problem (22)–(27) from a pair of invertible transformation (21) and (20); secondly, the problem (22)–(25) can be seemed as a direct problem when $r(t)$ can be made known from the overspecified data (26) or (27); thirdly, by integrating both sides of the (22) with respect to t from 0 to t and using (23), (22) has been changed into (29) which of the same form as (17) can be solved by the NIM; finally, using $v(x, t)$ which has been obtained from the third step and (26) or (27), $r(t)$ can be determined; then we can obtain the solution of the inverse problem (1)–(6) from (28).

4. Illustrative Examples

In this section, several examples of inverse heat conduction problems are given to illustrate the efficiency and validity of the NIM.

4.1. Example 1. Consider (1)–(5) with the given data

$$\begin{aligned}
 u(x, 0) &= x, \\
 u(0, t) &= 0, \\
 u(1, t) &= \exp(t), \\
 F(x, t) &= -(2 + xt^2) \exp(t), \\
 E(t) &= \frac{1}{2} \exp(t),
 \end{aligned} \tag{32}$$

with $q = 2$, $x^* = 1/2$.

Using the (31) and letting $v_0(x, t) = v(x, 0) = x \exp(x)$, we obtain

$$\begin{aligned}
 v_1(x, t) &= \int_0^t \left\{ v_{0xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right\} ds \\
 &= \left(t - \frac{t^3}{3}\right) x \exp(x), \\
 v_2(x, t) &= \int_0^t \left\{ v_{1xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right\} ds \\
 &= \frac{(t - t^3/3)^2}{2!} x \exp(x), \\
 v_3(x, t) &= \int_0^t \left\{ v_{2xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right\} ds \\
 &= \frac{(t - t^3/3)^3}{3!} x \exp(x),
 \end{aligned} \tag{33}$$

and so on. Generally we obtain

$$v_n(x, t) = \frac{(t - t^3/3)^n}{n!} x \exp(x), \quad n = 0, 1, \dots, \tag{34}$$

the solution in a closed form is

$$v(x, t) = \sum_{i=0}^{\infty} v_i = \exp\left(t - \frac{t^3}{3}\right) x \exp(x), \tag{35}$$

and using (26)

$$r(t) = \exp\left(-\frac{t^3}{3}\right). \tag{36}$$

Applying (28), the exact solution of this inverse problem is

$$\begin{aligned}
 u(x, t) &= x \exp(t), \\
 p(t) &= 1 + t^2.
 \end{aligned} \tag{37}$$

It can be seen that the same results are obtained using the variational iteration method [8]. At the same time, it is worth pointing out that the NIM does not need to approximately identify the general Lagrange multipliers via the variational theory. The overall results show the computation efficiency of the NIM for the studied model.

4.2. Example 2. Consider (1)–(5) with the given data

$$\begin{aligned}
u(x, 0) &= \sin\left(\frac{\pi}{2}x\right), \\
u(0, t) &= 0, \\
u(1, t) &= \exp(t), \\
F(x, t) &= \left[\left(\frac{\pi^2}{4} - t \right) \sin\left(\frac{\pi}{2}x\right) - \pi \cos\left(\frac{\pi}{2}x\right) \right] \exp(t), \\
E(t) &= \frac{\sqrt{2}}{2} \exp(t),
\end{aligned} \tag{38}$$

with $q = 2, x^* = 1/2$.

Using (31) and letting $v_0(x, t) = v(x, 0) = \sin((\pi/2)x) \exp(x)$, we have

$$\begin{aligned}
v_1(x, t) &= \int_0^t \left\{ v_{0xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right\} ds \\
&= \left(t - \frac{t^2}{2} \right) \sin\left(\frac{\pi}{2}x\right) \exp(x), \\
v_2(x, t) &= \int_0^t \left\{ v_{1xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right\} ds \\
&= \frac{(t - t^2/2)^2}{2!} \sin\left(\frac{\pi}{2}x\right) \exp(x), \\
v_3(x, t) &= \int_0^t \left\{ v_{2xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right\} ds \\
&= \frac{(t - t^2/2)^3}{3!} \sin\left(\frac{\pi}{2}x\right) \exp(x),
\end{aligned} \tag{39}$$

and so on. Generally we obtain

$$v_n(x, t) = \frac{(t - t^2/2)^n}{n!} \sin\left(\frac{\pi}{2}x\right) \exp(x), \tag{40}$$

$n = 0, 1, \dots,$

the solution in a closed form is

$$v(x, t) = \sum_{i=0}^{\infty} v_i = \exp\left(t - \frac{t^2}{2}\right) \sin\left(\frac{\pi}{2}x\right) \exp(x), \tag{41}$$

and using (26)

$$r(t) = \exp\left(-\frac{t^2}{2}\right). \tag{42}$$

Applying the (28), the exact solution of this inverse problem is

$$\begin{aligned}
u(x, t) &= \sin\left(\frac{\pi}{2}x\right) \exp(t), \\
p(t) &= 1 + t.
\end{aligned} \tag{43}$$

4.3. Example 3. Consider (1)–(5) with the given data

$$\begin{aligned}
u(x, 0) &= \cos(\pi x) + x, \\
u(0, t) &= \exp(t), \\
u(1, t) &= 0, \\
F(x, t) &= \pi^2 \exp(t) \cos(\pi x) \\
&\quad - t^2 \exp(t) (\cos(\pi x) + x), \\
E(t) &= \left(\frac{\sqrt{2}}{2} + \frac{1}{4} \right) \exp(t),
\end{aligned} \tag{44}$$

with $q = 0, x^* = 1/4$.

Using (31) and letting $v_0(x, t) = v(x, 0) = \cos(\pi x) + x$, we obtain

$$\begin{aligned}
v_1(x, t) &= \int_0^t \left\{ v_{0xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right\} ds \\
&= -\frac{t^3}{3} (\cos(\pi x) + x), \\
v_2(x, t) &= \int_0^t \left\{ v_{1xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right\} ds \\
&= \frac{t^6}{18} (\cos(\pi x) + x), \\
v_3(x, t) &= \int_0^t \left\{ v_{2xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right\} ds \\
&= -\frac{t^9}{162} (\cos(\pi x) + x),
\end{aligned} \tag{45}$$

and so on. Generally we obtain

$$v_n(x, t) = \frac{(-t^3/3)^n}{n!} (\cos(\pi x) + x), \quad n = 0, 1, \dots, \tag{46}$$

the solution in a closed form is

$$v(x, t) = \sum_{i=0}^{\infty} v_i = \exp\left(-\frac{t^3}{3}\right) (\cos(\pi x) + x), \tag{47}$$

and using (26)

$$r(t) = \exp\left(-t - \frac{t^3}{3}\right). \tag{48}$$

Applying (28), the exact solution of this inverse problem is

$$\begin{aligned} u(x, t) &= (\cos(\pi x) + x) \exp(t), \\ p(t) &= 1 + t^2. \end{aligned} \quad (49)$$

4.4. *Example 4.* Consider (1)–(4) and (6) with the given data

$$\begin{aligned} u(x, 0) &= \cos(\pi x) + x, \\ u(0, t) &= \exp(t), \\ u(1, t) &= 0, \\ F(x, t) &= (\pi^2 + 2t) \exp(t) \cos(\pi x) + 2xt \exp(t), \\ E(t) & \end{aligned} \quad (50)$$

$$= \left\{ \frac{\sin\left(\frac{\pi}{2}(1 + \sqrt{t})\right)}{\pi} + \frac{(1 + \sqrt{t})^2}{8} \right\} \exp(t),$$

with $q = 0$, $s(t) = (1/2)(1 + \sqrt{t})$.

Using (31) and letting $v_0(x, t) = v(x, 0) = \cos(\pi x) + x$, we have

$$\begin{aligned} v_1(x, t) &= \int_0^t \left\{ v_{0xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right\} ds \\ &= t^2 (\cos(\pi x) + x), \\ v_2(x, t) &= \int_0^t \left\{ v_{1xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right\} ds \\ &= \frac{t^4}{2!} (\cos(\pi x) + x), \\ v_3(x, t) &= \int_0^t \left\{ v_{2xx}(x, s) + r(s) \exp\left(\frac{q}{2}x\right) F(x, s) \right\} ds \\ &= \frac{t^6}{3!} (\cos(\pi x) + x), \end{aligned} \quad (51)$$

and so on. Generally we obtain

$$v_n(x, t) = \frac{(t^2)^n}{n!} (\cos(\pi x) + x), \quad n = 0, 1, \dots, \quad (52)$$

the solution in a closed form is

$$v(x, t) = \sum_{i=0}^{\infty} v_i = \exp(t^2) (\cos(\pi x) + x), \quad (53)$$

and using (27)

$$r(t) = \exp(t^2 - t). \quad (54)$$

Applying (28), the exact solution of this inverse problem is

$$\begin{aligned} u(x, t) &= (\cos(\pi x) + x) \exp(t), \\ p(t) &= 1 - 2t. \end{aligned} \quad (55)$$

We obtain the following conclusion from the examples above. Comparing the NIM with the variational iteration method, it does not need to approximately identify the general Lagrange multipliers via the variational theory and reduces the computational difficulties; comparing this method with other numerical method, for instance, the finite difference method [4], it does not require discretization of the variables, then it is not effected by computation round-off errors, and one is not faced with necessity of large computer memory and time. It provides the solution with high accuracy and minimal calculation in a rapidly convergent series which lead to the solution in a closed form by using the initial condition only. The solutions obtained are highly in agreement with the exact solutions.

5. Conclusion

In this work, we have successfully utilized the NIM to an inverse heat conduction problem. It is observed that the present method reduces the computational difficulties of variational iteration method, it does not need to approximately identify the general Lagrange multipliers via complex calculation, and all the calculation can be made in simple manipulations. It does not require discretization of the variables; it is not effected by computation round-off errors and not faced with necessity of large computer memory and time. It provides the solution with high accuracy and minimal calculation in a rapidly convergent series where the series may lead to the solution in a closed form by using the initial condition only. The solutions obtained are highly in agreement with the exact solutions; thus we can say the NIM is very simple and straightforward for the studied model.

Data Availability

(1) The new iteration method in this paper was used to support this study and is available at doi:10.1016/j.jmaa.2005.05.009 and doi:10.1016/j.jmaa.2005.05.009. These prior studies are cited at relevant places within the text as [13, 14]. (2) The theory of the existence and uniqueness of solution under studied problem was used to support this study and is available at doi:10.1088/0266-5611/4/1/006; these studies are cited at relevant places within the text as [1, 18]. (3) Examples 1 and 2 in this study are available at doi:10.1016/j.physleta.2008.02.042 and are cited at relevant places within the text as [8].

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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