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Research Article

The Perturbed Riemann Problem for the Aw-Rascle Model with Modified Chaplygin Gas Pressure

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This paper is concerned with the perturbed Riemann problem for the Aw-Rascle model with the modified Chaplygin gas pressure. We obtain constructively the solutions when the initial values are three piecewise constant states. The global structure and the large-time asymptotic behaviors of the solutions are discussed case by case. Further, we obtain the stability of the corresponding Riemann solutions as the initial perturbed parameter tends to zero.

1. Introduction

In the present paper we study the Aw-Rascle (AR) macroscopic model of traffic flow which is expressed by

$$\begin{aligned} \rho_t + (\rho u)_x &= 0, \\ (\rho(u+p))_t + (\rho u(u+p))_x &= 0, \end{aligned} \tag{1}$$

where $\rho \geq 0$, $u \geq 0$ are the density and the velocity, respectively, and p is the velocity offset which is called the "pressure" inspired from gas dynamics. The AR model was established to remedy the deficiencies of second order models of car traffic presented by Daganzo [1] and was independently obtained by Zhang [2]. For the related work, we refer to [3–10] and the references cited therein.

In [10], Wang obtained the Riemann solutions to (1) with

$$p = -\frac{B_1}{\rho^\alpha}, \quad 0 < \alpha \leq 1, \ B_1 > 0, \tag{2}$$

which is called the generalized Chaplygin gas. Wang observed that the delta shock wave occurs in the solutions. Let $B_1 = 1$ and $\alpha = 1$, we call it the Chaplygin gas which was introduced by Chaplygin [11]. Chaplygin gas (see [12–15]) is a good candidate for dark energy.

In [16], we constructed the solutions of the elementary wave interactions for the generalized AR model (1) with (2) by resolving two initial value problems case by case and found that the Riemann solutions are globally stable.

In [17], Wang, Liu, and Yang constructed the Riemann solutions for the AR model (1) with

$$p = B_2\rho - \frac{B_1}{\rho^\alpha}, \quad B_2 > 0, \tag{3}$$

which is called the modified Chaplygin gas. The authors analyzed the limit behavior when the pressure tends to zero.

In [18], Cheng and Yang studied the Riemann problem for (1) with

$$p = B_2\rho - \frac{B_1}{\rho}, \tag{4}$$

and they investigated the limits of the solutions when the pressure tends to the Chaplygin gas pressure.

In our present paper, we study the wave interactions for (1) with the state equation (3) which can be regarded as a combination of the perfect fluid and Chaplygin gas. In order to construct the solutions of the all possible wave interactions, we consider (1) and (3) with the following three piecewise constants:

$$(u,\rho)(x,0) = \begin{cases} (u_-,\rho_-), & x < -\zeta, \\ (u_m,\rho_m), & -\zeta < x < \zeta, \\ (u_+,\rho_+), & x > \zeta, \end{cases} \tag{5}$$

where the perturbation parameter $\zeta > 0$ is sufficiently small. The initial data (5) is a small perturbation on the corresponding Riemann initial values

$$(u,\rho)(x,0) = (u_{\pm},\rho_{\pm}), \quad \pm x > 0, \tag{6}$$

where $u_{\pm}, \rho_{\pm} > 0$. We will face the interesting question that whether the solutions of the perturbed Riemann problem (1), (3), and (5) converge to the corresponding Riemann solutions of (1), (3), and (6) as ζ tends to zero.

The paper is organized as follows. In Section 2, for readers' convenience we restate the results of the Riemann problem (1), (3), and (6). In Section 3, the elementary wave interactions are investigated case by case. And we obtain that the perturbed Riemann solutions of the modified AR model (1) and (3) converge to the corresponding Riemann solutions when ζ tends to zero which shows that the corresponding Riemann solutions of (1), (3), and (6) are globally stable.

2. Preliminaries

In this section we first sketch some results on the Riemann solutions to the modified AR model (1) and (3) and the detailed study can be found in [17].

The AR model (1) has two eigenvalues: $\lambda_1 = u - B_2\rho - B_1\alpha/\rho^{\alpha}$, $\lambda_2 = u$, and the corresponding right eigenvectors are given by

$$\begin{gathered}\vec{r}_1 = \left(-B_2 - \frac{B_1\alpha}{\rho^{1+\alpha}}, 1\right)^T, \\ \vec{r}_2 = (0,1)^T.\end{gathered} \tag{7}$$

By a direct calculation, we have

$$\begin{gathered}\nabla\lambda_1 \cdot \vec{r}_1 = -2B_2 - \frac{(1-\alpha)B_1\alpha}{\rho^{1+\alpha}} < 0, \\ \nabla\lambda_2 \cdot \vec{r}_2 \equiv 0.\end{gathered} \tag{8}$$

Therefore, λ_1 is genuinely nonlinear, and λ_2 is always linearly degenerate. And we know that the Riemann solutions of (1), (3), and (6) can be constructed by shock wave, rarefaction wave, or the contact discontinuity connecting the two constant states (u_-,ρ_-) and (u_+,ρ_+).

For the reason that system (1) and the initial values (6) are invariant under stretching of coordinates, i.e., $(x,t) \to (\beta x, \beta t)$ (β is constant), we look for the self-similar solution $(u,\rho)(x,t) = (u,\rho)(\eta)$, $\eta = x/t$. Thus we get the following boundary value problem of the ordinary differential equations:

$$\begin{gathered}-\eta\rho_{\eta} + (\rho u)_{\eta} = 0, \\ -\eta\left(\rho\left(u + B_2\rho - \frac{B_1}{\rho^{\alpha}}\right)\right)_{\eta} + \left(\rho u\left(u + B_2\rho - \frac{B_1}{\rho^{\alpha}}\right)\right)_{\eta} \\ = 0,\end{gathered} \tag{9}$$

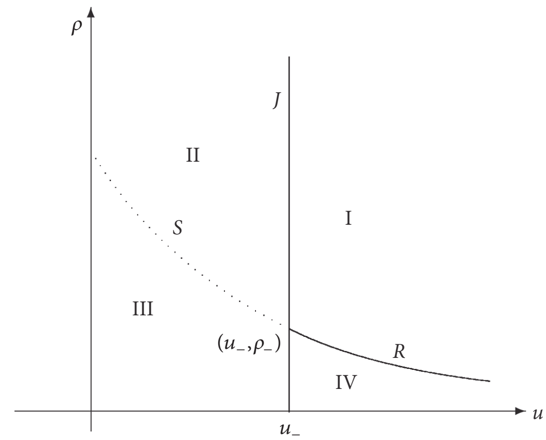


Figure 1: Wave curves in the phase plane (u,ρ).

and $(u,\rho)(\pm\infty) = (u_{\pm},\rho_{\pm})$. By solving the above problem, we obtain the constant state solution $(u,\rho) = constant$, and the rarefaction wave solution

$$R: \begin{cases}\eta = \lambda_1 = u - B_2\rho - \dfrac{B_1\alpha}{\rho^{\alpha}}, \\ u = u_- - B_2\rho + \dfrac{B_1}{\rho^{\alpha}} + B_2\rho_- - \dfrac{B_1}{\rho_-^{\alpha}}, \\ \rho < \rho_-,\ u > u_-.\end{cases} \tag{10}$$

For a bounded discontinuity at $\eta = \sigma$, the Rankine-Hugoniot conditions are as follows:

$$\begin{gathered}-\sigma[\rho] + [\rho u] = 0, \\ -\sigma\left[\rho\left(u + B_2\rho - \frac{B_1}{\rho^{\alpha}}\right)\right] + \left[\rho u\left(u + B_2\rho - \frac{B_1}{\rho^{\alpha}}\right)\right] \\ = 0,\end{gathered} \tag{11}$$

where $[\rho] = \rho_r - \rho_l$, $\rho_l = \rho(\sigma - 0)$, $\rho_r = \rho(\sigma + 0)$, etc.

By solving (11) and using the Lax entropy inequalities, we get the shock wave solution

$$S: \begin{cases}\sigma = u - \dfrac{B_1}{\rho^{\alpha}} - B_2\rho_- - B_1\dfrac{\rho_-^{1-\alpha} - \rho^{1-\alpha}}{\rho - \rho_-}, \\ u = u_- - B_2\rho + \dfrac{B_1}{\rho^{\alpha}} + B_2\rho_- - \dfrac{B_1}{\rho_-^{\alpha}}, \\ \rho > \rho_-,\ u < u_-.\end{cases} \tag{12}$$

Because λ_2 is always linearly degenerate, from (11) we get the contact discontinuity

$$J: \eta = u = u_-, \quad \rho < \rho_- \text{ or } \rho > \rho_-. \tag{13}$$

R, S, and J are called the elementary waves of (1). Notice that system (1) can be written by $Y_t + (uY)_x = 0$ [19]; i.e., it is the Temple type; we can see that the shock curves coincide with the rarefaction curves in the phase plane (u,ρ) (Figure 1). This property simplifies the discussion of the wave interactions.

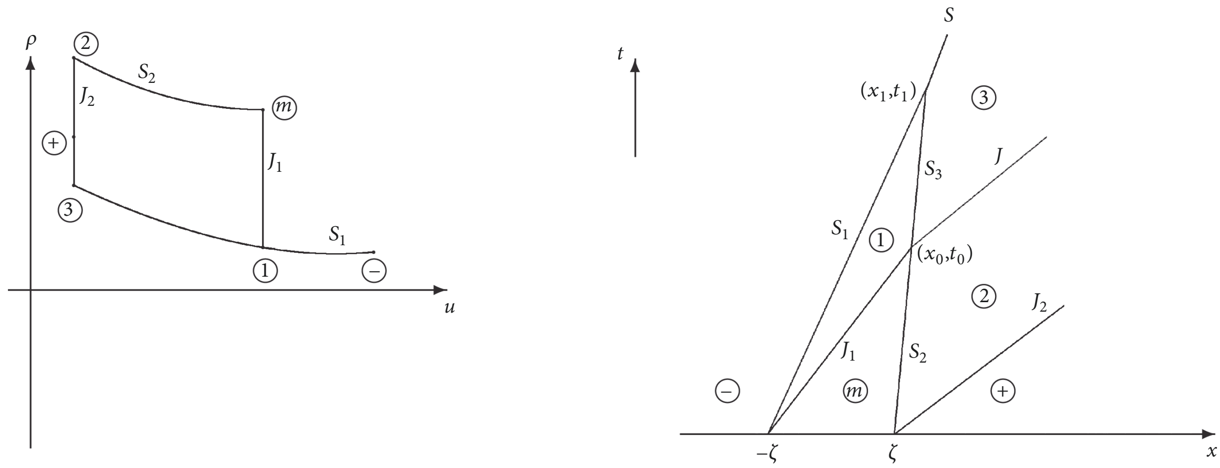


FIGURE 2: Wave interactions as $u_+ < u_m < u_-$.

Since $u_\rho = -B_2 - B_1\alpha/\rho^{\alpha+1} < 0$ and $u_{\rho\rho} = \alpha(\alpha+1)B_1/\rho^{\alpha+2} > 0$, the above two wave curves are monotonic decreasing and convex. And we know that the shock wave curve interacts with ρ-axis, and the rarefaction wave curve has the u-axis as its asymptote.

Based on the above analysis, we have the following result.

Theorem 1. *There exists a unique Riemann solution for (1) and (4) with the initial data (6). When $(u_+, \rho_+) \in I$ or IV, the unique Riemann solution is $R + J$; when $(u_+, \rho_+) \in II$ or III, the unique Riemann solution is $S + J$.*

3. Wave Interactions and Large-Time Behaviors

In this section, we consider the perturbed problem (1) and (3) with initial data (5). The data (5) is a small perturbation to the corresponding Riemann initial values (6). We want to determine whether the solutions of the perturbed Riemann problem (1), (3), and (5) converge to the corresponding Riemann solutions of (1), (3), and (6) as ζ tends to zero, where $(u_\zeta, \rho_\zeta)(x,t)$ are the solutions of (1), (3), and (5).

In order to cover all the cases completely, we divide our discussions into the following four cases: $u_+ < u_m < u_-$, $u_+ > u_m > u_-$, $u_m > u_\pm$, and $u_m < u_\pm$.

Case 1 ($u_+ < u_m < u_-$). In this case, we consider the interaction of $S + J$ emitted from $(-\zeta, 0)$ and $R + J$ emitted from $(\zeta, 0)$ (Figure 2), where "+" means "followed by". The occurrence of this case depends on the condition $u_+ < u_m < u_-$.

The propagation speed of J_1 and S_2 is $\tau_1 = u_m = u_1$ and $\sigma_2 = u_2 - B_1/\rho_2^\alpha - -B_2\rho_m - B_1((\rho_m^{1-\alpha} - \rho_2^{1-\alpha})/(\rho_2 - \rho_m))$, respectively. Since $\sigma_2 < u_2 = u_+ < u_m = \tau_1$, we know that J_1 will overtake S_2 at a point (x_0, t_0) which is determined by

$$\begin{aligned} x_0 + \zeta &= u_m t_0, \\ x_0 - \zeta &= \sigma_2 t_0. \end{aligned} \tag{14}$$

It follows that

$$(x_0, t_0) = \left(\frac{(u_m + \sigma_2)\zeta}{u_m - \sigma_2}, \frac{2\zeta}{u_m - \sigma_2} \right). \tag{15}$$

At the interaction point (x_0, t_0), the new Riemann problem with the initial data (u_1, ρ_1) and (u_2, ρ_2) is formed, which is resolved by a new S_3 and a new J. For the reason that S_3 propagates with the same speed as that of S_2, we obtain that the propagation direction of S_2 is unchanged. From $\tau = \tau_2$ we get that J has the same propagation speed as that of J_2 which implies that J is parallel to J_2.

When $t > t_0$, due to

$$\begin{aligned} \sigma_1 &= u_1 - \frac{B_1}{\rho_1^\alpha} - B_2\rho_- - B_1\frac{\rho_-^{1-\alpha} - \rho_1^{1-\alpha}}{\rho_1 - \rho_-}, \\ \sigma_3 &= u_3 - \frac{B_1}{\rho_3^\alpha} - B_2\rho_1 - B_1\frac{\rho_1^{1-\alpha} - \rho_3^{1-\alpha}}{\rho_3 - \rho_1}, \end{aligned} \tag{16}$$

and $\rho_- < \rho_1 < \rho_3$, by a direct computation it follows that $\sigma_1 - \sigma_3 > 0$. Then we know S_1 will overtake S_3 at (x_1, t_1) which satisfies

$$\begin{aligned} x_1 + \zeta &= \sigma_1 t_1, \\ x_1 - x_0 &= \sigma_3 (t_1 - t_0). \end{aligned} \tag{17}$$

From (17) we get $(x_1, t_1) = ((\sigma_1 x_0 - \sigma_1\sigma_3 t_0 + \sigma_3\zeta)/(\sigma_1 - \sigma_3), (x_0 + \zeta - \sigma_3 t_0)/(\sigma_1 - \sigma_3))$. When $t > t_1$, a new shock wave S will occur and its propagating speed is $u_3 - B_1/\rho_3^\alpha - B_2\rho_- - B_1((\rho_-^{1-\alpha} - \rho^{1-\alpha})/(\rho_3 - \rho_-)) < u_3$, which yields that S cannot overtake J_3 forever.

When ζ tends to zero, we get that $(x_0, t_0) \to (0, 0)$ and $(x_1, t_1) \to (0, 0)$; J and J_2 will coincide with each other. Thus, as the time t is large enough, the solution of the perturbed Riemann problem is $S + J$. For this case, the limit of the perturbed Riemann solution to (1), (3), and (5) is no other than the corresponding Riemann solution to the initial problem (1), (3), and (6) which implies the global stability of the Riemann solution to (1), (3), and (6).

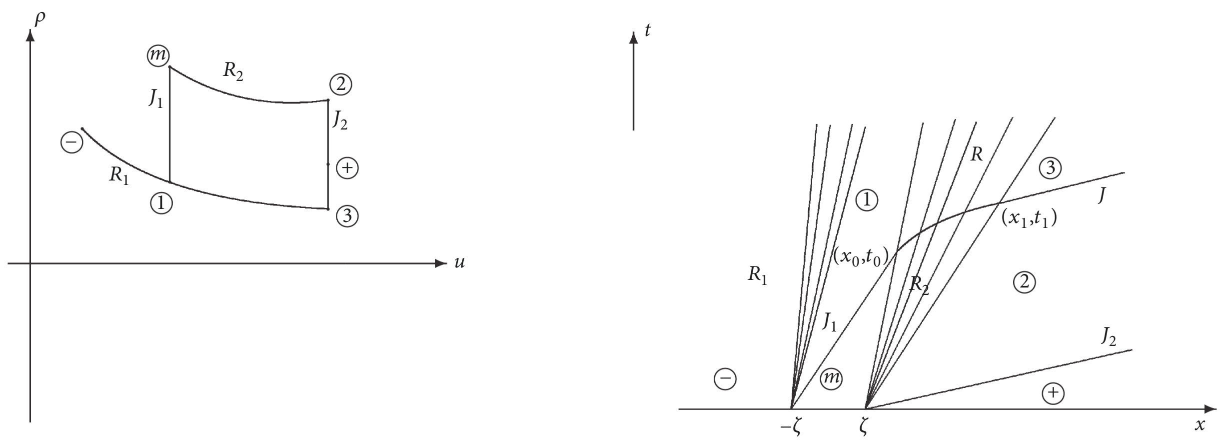


FIGURE 3: Wave interactions as $u_+ > u_m > u_-$.

Case 2 ($u_+ > u_m > u_-$). In this case, we investigate the interaction of $R + J$ emitted from $(-\zeta, 0)$ and $R + J$ emitted from $(\zeta, 0)$ (Figure 3).

Due to $\tau_1 = u_m > \lambda_2 = u_m - B_2\rho_m - B_1\alpha/\rho_m^\alpha$, we know that J_1 will overtake R_2 at the point (x_0, t_0) which satisfies

$$\begin{aligned} x_0 + \zeta &= u_m t_0, \\ x_0 - \zeta &= \lambda_2 t_0. \end{aligned} \tag{18}$$

A direct computation yields that

$$\begin{aligned} x_0 &= \frac{u_m + \lambda_2}{u_m - \lambda_2}\zeta, \\ t_0 &= \frac{2\zeta}{u_m - \lambda_2}. \end{aligned} \tag{19}$$

When $t > t_0$, J_1 begins to penetrate R_2, and during the penetration J_1 satisfies

$$\begin{aligned} &\frac{dx}{dt} = u, \\ &x - \zeta = \left(u - B_2\rho - \frac{B_1\alpha}{\rho^\alpha}\right)t, \\ &u - u_m = -B_2\rho + \frac{B_1}{\rho^\alpha} + B_2\rho_m - \frac{B_1}{\rho_m^\alpha}, \\ &x(t_0) = x_0, \\ &\qquad\qquad \rho_2 \le \rho \le \rho_m. \end{aligned} \tag{20}$$

Differentiating with respect to t in the second equation of (20), we know that

$$\begin{aligned} \frac{dx}{dt} = {} & \left(u - B_2\rho - \frac{B_1\alpha}{\rho^\alpha}\right) \\ & + \left(\frac{du}{dt} - B_2\frac{d\rho}{dt} + B_1\alpha^2\rho^{-\alpha-1}\frac{d\rho}{dt}\right)t. \end{aligned} \tag{21}$$

Differentiating with respect to t in the third equation of (20), we obtain that

$$\frac{du}{dt} = \left(-B_2 - B_1\alpha\rho^{-\alpha-1}\right)\frac{d\rho}{dt}. \tag{22}$$

Substituting the first equation of (20) into the above representations yields that

$$\frac{d\rho}{dt} = \frac{B_2\rho + B_1\alpha}{\left(-2B_2 - B_1\alpha\rho^{-\alpha-1} + B_1\alpha^2\rho^{-\alpha-1}\right)t} < 0, \tag{23}$$

and

$$\begin{aligned} \frac{d^2x}{dt^2} &= \frac{du}{dt} \\ &= \left(B_2\rho + \frac{B_1\alpha}{\rho^\alpha}\right)\frac{1}{t}\frac{-B_2 - B_1\alpha\rho^{-\alpha-1}}{-2B_2 - B_1\alpha\rho^{-\alpha-1} + B_1\alpha^2\rho^{-\alpha-1}} \\ &> 0. \end{aligned} \tag{24}$$

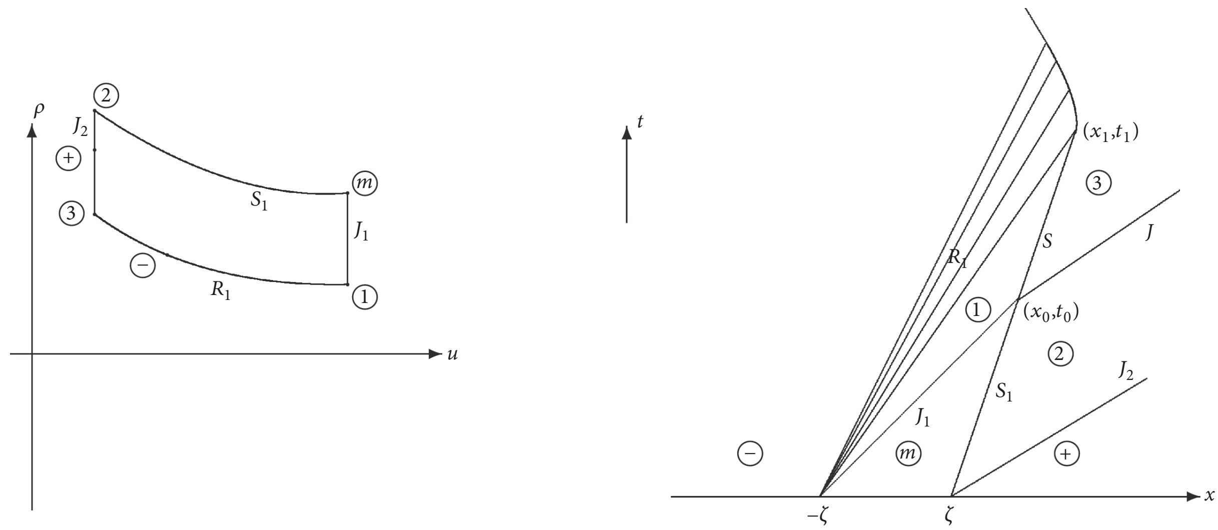
From the above discussions, we conclude that J_1 accelerates during the process of penetration.

From (23) and the condition $\rho_2 < \rho < \rho_m$, we conclude that the contact discontinuity J_1 can penetrate completely the rarefaction wave R_2 in a finite time.

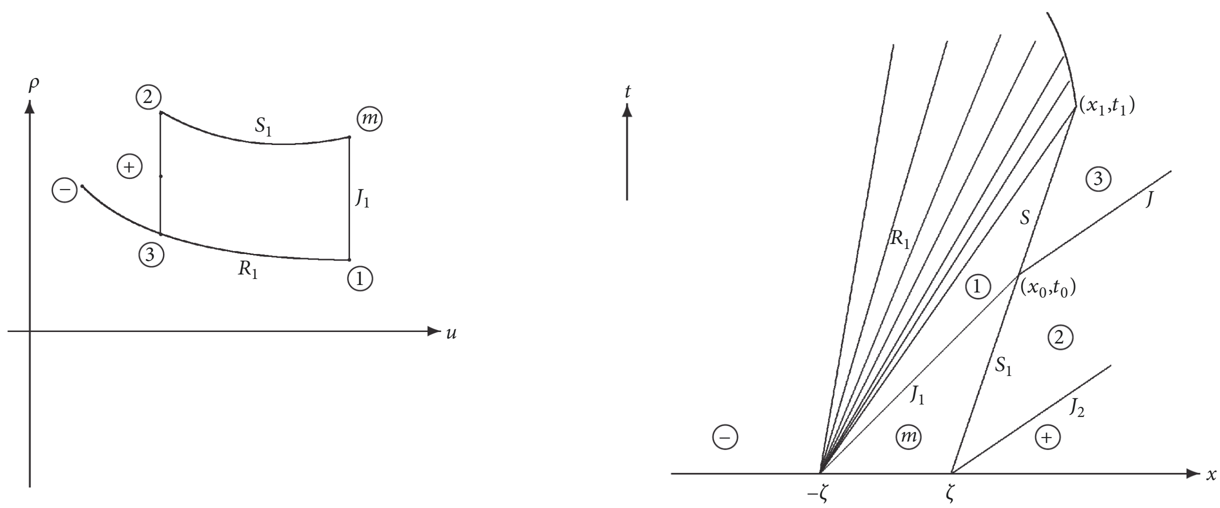
After the penetration, a new R and a new J will occur. Due to $u_1 - B_2\rho_1 - B_1/\rho_1 = u_m - B_2\rho_m - B_1/\rho_m$ and $u_3 - B_2\rho_3 - B_1/\rho_3 = u_2 - B_2\rho_2 - B_1/\rho_2$, the propagation direction of R keeps unchanged during the whole penetration. Notice that J is parallel to J_2 since $\tau = \tau_2$.

When ζ tends to zero, the points (x_0, t_0) and (x_1, t_1) tend to $(0, 0)$ and J coincides with J_2. Since the propagation speed of the wave back in R equals that of the wave front in R_1, it follows that R coincides with R_1 when ζ tends to zero. Thus, we know for this case the result is $R + J$ for large enough time. And the perturbed Riemann solution converges to the corresponding Riemann solution.

Case 3 ($u_m > u_\pm$). In this case, we discuss the interaction of $R + J$ emitted from $(-\zeta, 0)$ and $S + J$ emitted from $(\zeta, 0)$ (Figure 4).



(a) Wave interactions as $u_m > u_- > u_+$



(b) Wave interactions as $u_- \leq u_+ < u_m$

FIGURE 4

By similar discussions to that in Case 1, a new shock S and a new J will occur after the interaction of J_1 with S_1. Notice that S propagates with S_1 at the same speed and J is parallel to J_2 due to $\tau = \tau_2$. J_1 intersects with S_1 at (x_0, t_0) which is determined by (14).

Because the propagation speed of the wave front in R_1 is larger than the propagation speed in S, it follows that S will penetrate R_1 at (x_1, t_1) which satisfies

$$
\begin{aligned}
x_1 + \zeta &= \lambda_1 t_1, \\
x_1 - x_0 &= \sigma\left(t_1 - t_0\right),
\end{aligned}
\tag{25}
$$

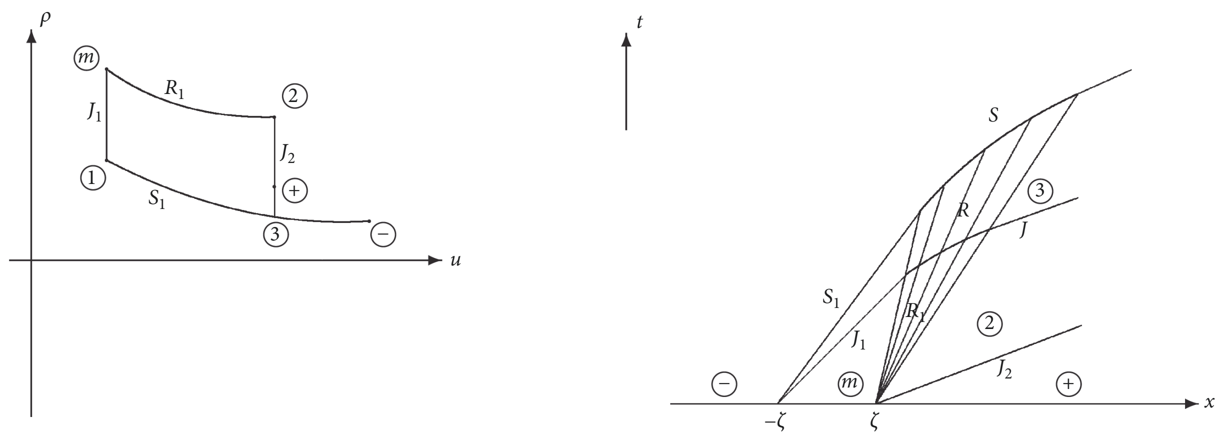
where $\lambda_1 = u_1 - B_2\rho_1 - B_1\alpha/\rho_1^{\alpha}$ and $\sigma = u_3 - B_1/\rho_3^{\alpha} - -B_2\rho_1 - B_1((\rho_1^{1-\alpha} - \rho_3^{1-\alpha})/(\rho_3 - \rho_1))$ are, respectively, the propagation speed of the wave front in R_1 and the propagation speed of S.

By a direct computation, we get $(x_1, t_1) = ((\lambda_1 x_0 - \lambda_1\sigma t_0 + \sigma\zeta)/(\lambda_1 - \sigma), (x_0 + \zeta - \sigma t_0)/(\sigma_1 - \sigma))$. When $t > t_1$, S begins to penetrate R_1 and we have

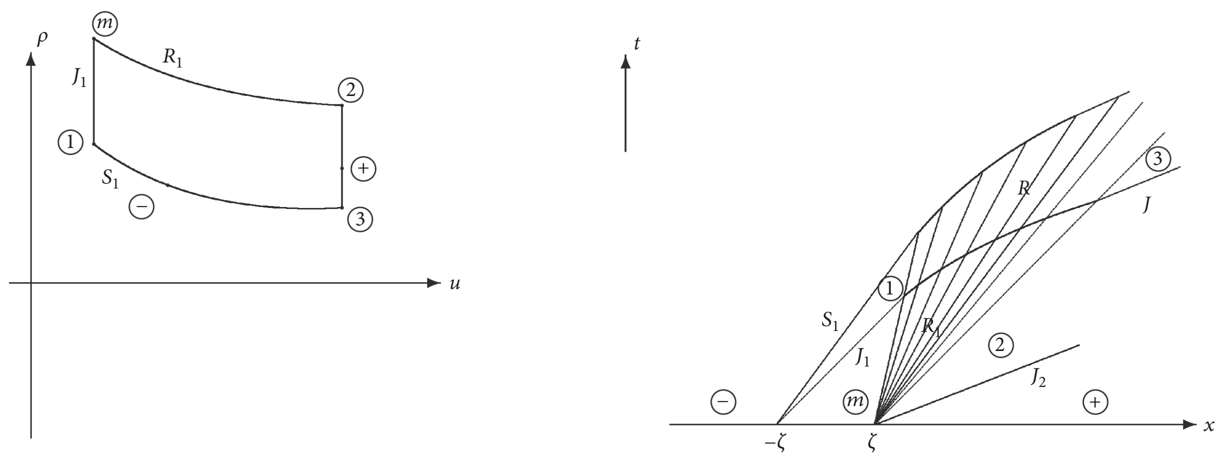
$$
\begin{aligned}
&\frac{dx}{dt} = u - \frac{B_1}{\rho^{\alpha}} - B_2\rho_- - B_1\frac{\rho_-^{1-\alpha} - \rho^{1-\alpha}}{\rho - \rho_-}, \\
&x + \zeta = \left(u - B_2\rho - \frac{B_1\alpha}{\rho^{\alpha}}\right)t, \\
&u - u_- = -B_2\rho + \frac{B_1}{\rho^{\alpha}} + B_2\rho_- - \frac{B_1}{\rho_-^{\alpha}}, \\
&x\left(t_1\right) = x_1, \\
&\qquad u_- \leq u \leq u_1, \ \ \rho_1 \leq \rho \leq \rho_-.
\end{aligned}
\tag{26}
$$

From the first and the second expression in (26) we know that

$$
\begin{aligned}
&\frac{du}{dt} + \left(-B_2 + B_1\alpha^2\rho^{-\alpha-1}\right)\frac{d\rho}{dt} \\
&\quad = \frac{B_2\left(\rho - \rho_-\right) + B_1\left(\alpha - 1\right)/\rho^{\alpha}}{t},
\end{aligned}
\tag{27}
$$



(a) Wave interactions as $u_m < u_+ < u_-$



(b) Wave interactions as $u_+ \geq u_- > u_m$

Figure 5

and from the third expression we obtain that

$$\frac{du}{dt} = \left(-B_2 - B_1 \alpha \rho^{-\alpha-1}\right) \frac{d\rho}{dt}. \tag{28}$$

Substituting (28) into (27), due to $\rho < \rho_-$ we have

$$\frac{d\rho}{dt} = \frac{B_2 (\rho - \rho_-) + B_1 (\alpha - 1) / \rho^{\alpha}}{\left(-2B_2 - B_1 \alpha \rho^{-\alpha-1} + B_1 \alpha^2 \rho^{-\alpha-1}\right) t} > 0. \tag{29}$$

From (28)

$$\frac{du}{dt} = \left(-B_2 - B_1 \alpha \rho^{-\alpha-1}\right) \frac{d\rho}{dt} < 0. \tag{30}$$

From (26), (28), and (29), we have

$$\begin{aligned} \frac{d^2 x}{dt^2} = \Bigg[-B_2 - \frac{B_1 (1 - \alpha) \rho^{-\alpha} (\rho_- - \rho)}{(\rho - \rho_-)^2} \\ - \frac{B_1 \left(\rho^{1-\alpha} - \rho_-^{1-\alpha}\right)}{(\rho - \rho_-)^2} \Bigg] \frac{d\rho}{dt} < 0, \end{aligned} \tag{31}$$

telling us that S decelerates during the above penetration. From (29) we conclude that for the case $u_- > u_+$, S will penetrate R_1 completely due to $\rho_3 > \rho_-$ at the finite time. While for the other case $u_- \leq u_+$, S cannot penetrate R_1 forever because $t \to \infty$ when $\rho \to \rho_3$ and S has $x + \zeta = (u_3 - B_2 \rho_3 - B_1 \alpha / \rho_3^{\alpha}) t$ as its asymptote.

For the large time, the solution is $S + J$ (Figure 4(a)) as $u_+ < u_-$, and the solution is $R + J$ (Figure 4(b)) as $u_+ \geq u_-$. For this case, we obtain the stability of the Riemann solution to the initial problem (1), (3), and (6).

Case 4 ($u_m < u_\pm$). In this case, we study the interaction of $S + J$ emitted from $(-\zeta, 0)$ and $R + J$ emitted from $(\zeta, 0)$ (Figure 5).

By similar discussions to that of Case 2, a new R and a new J will occur after the intersection of J_1 and R_1. Notice that R keeps the same propagation speed and J is parallel to J_2 because $\tau = \tau_2$.

Similarly to Case 3, as $u_+ < u_-$, we obtain that S_1 will penetrate R completely in a finite time and a new shock will occur whose propagation speed is less than that of J. It implies that for the large time the solution is $S + J$ (Figure 5(a)). As $u_+ \geq u_-$, S_1 cannot penetrate R forever and it has $x - \zeta = (u_- - B_2 \rho_- - B_1 \alpha / \rho_-^{\alpha}) t$ as its asymptote, which shows that for the large time the solution is $R + J$ (Figure 5(b)).

For this case, we get the global stability of the corresponding Riemann solution.

Now we have discussed all possible wave interactions in the phase plane (u, ρ). Based on the above discussions, we obtain our main result.

Theorem 2. *The limits of the perturbed Riemann solutions of (1), (3), and (5) are exactly the corresponding Riemann solutions of (1), (3), and (6). The asymptotic behavior of the perturbed Riemann solutions is governed completely by the states $(u_\pm, \rho_\pm)$. The Riemann solutions of the initial value problem (1), (3), and (6) are stable under such small perturbation to the initial values.*

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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