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Research Article

A New Integrable Variable-Coefficient 2 + 1-Dimensional Long Wave-Short Wave Equation and the Generalized Dressing Method

Ting Su [1] **and Hui Hui Dai** [2]

[1] *Department of Mathematical and Physical Science, Henan Institute of Engineering, Zhengzhou, China*
[2] *Department of Mathematics, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon, Hong Kong*

Correspondence should be addressed to Ting Su; suting1976@163.com

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Based on the generalized dressing method, we propose a new integrable variable-coefficient 2 + 1-dimensional long wave-short wave equation and derive its Lax pair. Using separation of variables, we have derived the explicit solutions of the equation. With the aid of Matlab, the curves of the solutions are drawn.

1. Introduction

It is well known that the interactions of long wave and short wave play an important role in fluid dynamics. The model is described by the following equation:

$$\begin{gathered} i\partial_t S + ic_g\partial_x S - \beta LS - \gamma\partial_{xx}^2 S - \delta\left|S\right|^2 S = 0, \\ \partial_t L + c_l\partial_y L + \alpha\partial_x\left|S\right|^2 = 0. \end{gathered} \tag{1}$$

The inverse scattering technique proposed in [1] plays an important role in constructing the complete solution of the long wave and short wave resonance equations. The N soliton solution of long wave and short wave has been obtained in [2]. Radha et al. in [3] derived periodic solutions and localized solutions of (1). Lai and Chow in [4] studied positon and dromion solutions of 2 + 1-dimensional long wave and short wave resonance interaction equations. Researchers have focused on the long wave and short wave equation [5–10] by using different methods. Serkin et al. [11] discussed integrable variable-coefficient nonlinear evolution equations. By utilizing the exp-function method, the generalized solitary solution and periodic solution of soliton equations can be given in [12]. Authors in [13] discussed the interactions of dark soliton and bright soliton in a double-mode optical fiber. In [14, 15], authors (Dai and Jeffrey [14], Jeffrey and Dai [15]) extended the dressing method [16, 17] to a generalized version for solving soliton equations associated with matrix nonspectral problems and variable-coefficient cases. The generalized dressing method was based on the problems of factorization of an integral operator F on the line into the product of two Volterra type integral operators $K_{\pm}$, from which the Gel'fand-Levitan-Marchenko (GLM) equation is obtained. These Volterra operators are then used to construct dressed operators (N_1, N_2) starting from a pair of initial operators (M_1, M_2). Integrable variable-coefficient nonlinear equations are obtained from the compatibility of the dressed operators. There are some differences between the original dressing method and the generalized dressing method. In the original dressing method, the constant coefficient operators have transformed into different constant coefficient operators. The generalized dressing method transforms the variable-coefficient operators into different variable-coefficient ones. The advantages in the generalized dressing method lie in deriving integrable variable-coefficient nonlinear evolution equation and corresponding Lax pairs. However, the original dressing method is a system way to study constant coefficient nonlinear evolution equation [18–20]. Authors (Dai and Jeffrey [14]) presented the generalized dressing method; we also discussed some integrable variable-coefficient evolution equations [21–25]. In fact, the dressing method can

be thought as a rather general formulation of the inverse scattering method, which has the advantage of bypassing the scattering problem. The common point between the two methods is that two methods can deal with the initial boundary value problem.

In the paper, we applied the generalized dressing method to derive a new integrable variable-coefficient 2 + 1-dimensional long wave-short wave equation:

$$\begin{gathered} i\alpha_2 w_{xy} + \beta_2 w_{xx} + \beta_2 (uv)_x = 0, \\ -i\alpha_1 u_t + \beta_1 \left(v_{xx} - u_{xx} - u_{yy}\right) + 2\beta_1 \left(u^2 v + u w_x\right) = 0, \\ i\alpha_1 v_t - \beta_1 v_{yy} + 2\beta_1 \left(u v^2 + v w_x\right) = 0, \end{gathered} \tag{2}$$

where α_2 and β_2 are functions of t and y. α_1 and β_1 are functions of t. Particularly, the above equation is reduced to a new 2 + 1-dimensional integrable variable-coefficient equation:

$$\begin{aligned} & i\alpha_1 (uv)_t + \beta_1 \left[(uv)_{xx} + (uv)_{yy}\right] \\ & \quad - \beta_1 \left[(u - v) v_{xx} + 2uv_{yy} + 2\left(u_x v_x + u_y v_y\right)\right] \\ & = 0, \end{aligned} \tag{3}$$

in view of $\alpha_2 = \beta_2$.

Furthermore, under the transformations $\alpha_1 = 1, \beta_1 = 1/t$, (3) can be read as the cylindrical equation:

$$\begin{aligned} & i (uv)_t + \frac{1}{t} \left[(uv)_{xx} + (uv)_{yy}\right] \\ & \quad - \frac{1}{t} \left[(u - v) v_{xx} + 2uv_{yy} + 2\left(u_x v_x + u_y v_y\right)\right] \\ & = 0. \end{aligned} \tag{4}$$

Moreover, (2) are written as a 2 + 1-dimensional integrable modified long wave-short wave equation for $\alpha_2 = \beta_2$ and $\alpha_1 = \beta_1$:

$$\begin{gathered} i w_{xy} + w_{xx} + (uv)_x = 0, \\ -i u_t + v_{xx} - u_{xx} - u_{yy} + 2\left(u^2 v + u w_x\right) = 0, \\ i v_t - v_{yy} + 2\left(u v^2 + v w_x\right) = 0. \end{gathered} \tag{5}$$

The outline of the paper is as follows. In Section 2, we briefly describe the generalized dressing method and its properties. Moreover, we introduce two dressing operators. In Section 3, new integrable variable-coefficient 2 + 1-dimensional long wave-short wave equations and their Lax pairs are derived with the aid of the generalized dressing method. In Section 4, as an application, we obtain explicit solutions of these equations and draw the curves of the solutions.

2. The Generalized Dressing Method and Dressing Operators

First, we consider three integral operators $\mathbf{F}(x, z, y)$, $\mathbf{K}_+(x, z, y)$, and $\mathbf{K}_-(x, z, y)$ defined by [16]

$$\begin{gathered} \mathbf{K}_+ \psi(x) \equiv \int_x^\infty K_+(x, z, y)\, \psi(z)\, dz, \\ \mathbf{K}_- \psi(x) \equiv \int_{-\infty}^x K_-(x, z, y)\, \psi(z)\, dz, \\ \mathbf{F} \psi(x) \equiv \int_{-\infty}^\infty F(x, z, y)\, \psi(z)\, dz. \end{gathered} \tag{6}$$

We assume that $(\mathbf{I} + \mathbf{K}_+)^{-1}$ exists and $\mathbf{F}$ admits the triangular factorization

$$\mathbf{I} + \mathbf{F} = (\mathbf{I} + \mathbf{K}_+)^{-1} (\mathbf{I} + \mathbf{K}_-), \tag{7}$$

where $\mathbf{I}$ is the identity operator. From (7), a direct calculation shows that $\mathbf{F}$ and $\mathbf{K}_+$ satisfy the Gel'fand-Levitan-Marchenko (GLM) equation [16]:

$$\begin{aligned} & K_+(x, z, y) + F(x, z, y) \\ & \quad + \int_x^\infty K_+(x, s, y)\, F(s, z, y)\, ds = 0, \quad z > x, \end{aligned} \tag{8}$$

here it is supposed that $K_\pm(x, z, y)$ and $F(x, z, y)$ satisfy the condition

$$\begin{gathered} \sup \int_{x_0}^{+\infty} \left|K_\pm(x, z, y)\right| \psi(z)\, dz < +\infty, \\ \sup \int_{x_0}^{+\infty} \left|F(x, z, y)\right| \psi(z)\, dz < +\infty, \quad x_0 > -\infty. \end{gathered} \tag{9}$$

We now introduce two differential operators $\mathbf{M_1}$ and $\mathbf{M_2}$ defined by

$$\begin{gathered} \mathbf{M}_1 = \alpha_1 \partial_t + i\beta_1 \Theta \partial_{xx}^2, \\ \mathbf{M}_2 = \alpha_2 \partial_y - i\beta_2 \Theta \partial_x, \end{gathered} \tag{10}$$

with α_1 and β_1 being matrix functions of t. α_2 and β_2 are matrix functions of t and y:

$$\Theta = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{11}$$

Suppose that the operator $\mathbf{F}$ commutes with $\mathbf{M_1}$ and $\mathbf{M_2}$; that is,

$$\begin{gathered} [\mathbf{M_1}, \mathbf{F}] = \mathbf{M_1 F} - \mathbf{F M_1} = 0, \\ [\mathbf{M_2}, \mathbf{F}] = \mathbf{M_2 F} - \mathbf{F M_2} = 0, \end{gathered} \tag{12}$$

which together with (10) implies the following equations:

$$\alpha_1 F_t + i\beta_1 \Theta F_{xx} + i F_{zz} \beta_1 \Theta = 0, \tag{13}$$

$$\alpha_2 F_y - i\beta_2 \Theta F_x - i F_z \beta_2 \Theta = 0. \tag{14}$$

In what follows, we obtain the dressing operators $\mathbf{N_1}$ and $\mathbf{N_2}$ with the aid of operators $\mathbf{M_1}$ and $\mathbf{M_2}$. The dressing procedure is accomplished through the relations [14, 15]

$$\mathbf{N_1}\left(\mathbf{I}+\mathbf{K}_+\right)-\left(\mathbf{I}+\mathbf{K}_+\right)\mathbf{M_1}=0, \tag{15a}$$

$$\mathbf{N_2}\left(\mathbf{I}+\mathbf{K}_+\right)-\left(\mathbf{I}+\mathbf{K}_+\right)\mathbf{M_2}=0. \tag{15b}$$

The difference between the original dressing method and the generalized dressing method lies in the differential operators $\mathbf{M_1}$ and $\mathbf{M_2}$ which satisfied the relation

$$\left[\mathbf{M_1},\mathbf{M_2}\right]=\phi_1\mathbf{M_1}+\phi_2\mathbf{M_2}, \tag{16}$$

with ϕ_1 and ϕ_2 being arbitrary functions of their arguments. In view of [14, 15], the corresponding dressed operators obey the equation

$$\left[\mathbf{N_1},\mathbf{N_2}\right]=\phi_1\mathbf{N_1}+\phi_2\mathbf{N_2}. \tag{17}$$

Letting $\mathbf{N_1}=\mathbf{M_1}+u_1^{(1)}\partial_x+u_0^{(1)}$, $\mathbf{N_2}=\mathbf{M_2}+u_0^{(2)}$, from (15a) and (15b), we have

$$\begin{aligned}
u_0^{(1)}&=u_1^{(1)}\widehat{K}+i\beta_1\Theta\left(\widehat{K}_x+K_x\big|_{z=x}\right)+i\widehat{K}_z\big|_{z=x}\beta_1\Theta,\\
u_1^{(1)}&=i\beta_1\left(\Theta\widehat{K}-\widehat{K}\Theta\right),\\
u_0^{(2)}&=i\beta_2\left(\widehat{K}\Theta-\Theta\widehat{K}\right).
\end{aligned} \tag{18}$$

In view of (16), it is easy to obtain

$$\begin{aligned}
\alpha_1\alpha_{2t}&=\phi_2\alpha_2,\\
\alpha_1\beta_{2t}&=\phi_2\beta_2,\\
\phi_1&=0;
\end{aligned} \tag{19}$$

thus, we have $\alpha_2=e^{\int(\phi_2/\alpha_1)dt+c_1(y)}$, $\beta_2=e^{\int(\phi_2/\alpha_1)dt+c_2(y)}$. Here, ϕ is an arbitrary function of t and y. $c_1(y)$ and $c_2(y)$ are arbitrary functions of y.

3. A New Integrable Variable-Coefficient 2+1-Dimensional Long Wave-Short Wave Equation and Its Lax Pair

In this section, based on the generalized dressing method, we derive a new integrable variable-coefficient 2+1-dimensional long wave-short wave equation. From (17), we have

$$\begin{aligned}
&\alpha_1u_{0t}^{(2)}+i\beta_1\Theta u_{0xx}^{(2)}-\alpha_2u_{0y}^{(1)}+i\beta_2\Theta u_{0x}^{(1)}+u_0^{(1)}u_0^{(2)}\\
&\quad-u_0^{(2)}u_0^{(1)}+u_1^{(1)}u_{0x}^{(2)}=\phi_2u_0^{(2)},
\end{aligned} \tag{20}$$

$$\begin{aligned}
&2i\beta_1\Theta u_{0x}^{(2)}-\alpha_2u_{1y}^{(1)}+i\beta_2\Theta u_{1x}^{(1)}-i\beta_2u_0^{(1)}\Theta+i\beta_2\Theta u_0^{(1)}\\
&\quad+u_1^{(1)}u_0^{(2)}-u_0^{(2)}u_1^{(1)}=0.
\end{aligned} \tag{21}$$

We denote

$$\begin{aligned}
\widehat{K}&=K\left(x,z,y,t\right)\big|_{x=z}=\begin{pmatrix}\widehat{k}_{11}&\widehat{k}_{12}\\\widehat{k}_{21}&\widehat{k}_{22}\end{pmatrix},\\
&\widehat{k}_{11}=w,\ \widehat{k}_{12}=u,\ \widehat{k}_{21}=v.
\end{aligned} \tag{22}$$

In view of (16), we have

$$\begin{aligned}
i\alpha_2\widehat{k}_{12y}+\beta_2\widehat{k}_{12}\widehat{k}_{22}+\beta_2k_{12x}\big|_{z=x}&=0,\\
i\alpha_2\widehat{k}_{21y}-\beta_2\widehat{k}_{21}\widehat{k}_{11}+\beta_2k_{21z}\big|_{z=x}&=0.
\end{aligned} \tag{23}$$

Based on (18) and (22)-(23), we obtain

$$u_1^{(1)}=i\beta_1\begin{pmatrix}0&\widehat{k}_{12}\\-\widehat{k}_{21}&0\end{pmatrix}=i\beta_1\begin{pmatrix}0&u\\-v&0\end{pmatrix}, \tag{24}$$

$$u_0^{(2)}=i\beta_2\begin{pmatrix}0&-\widehat{k}_{12}\\\widehat{k}_{21}&0\end{pmatrix}=i\beta_2\begin{pmatrix}0&-u\\v&0\end{pmatrix}, \tag{25}$$

$$\begin{aligned}
&u_0^{(1)}\\
&=i\beta_1\begin{pmatrix}\widehat{k}_{12}\widehat{k}_{21}+2\widehat{k}_{11x}&\widehat{k}_{12x}+k_{12x}\big|_{z=x}+\widehat{k}_{12}\widehat{k}_{22}\\-\widehat{k}_{11}\widehat{k}_{21}+k_{12z}\big|_{z=x}&-\widehat{k}_{12}\widehat{k}_{21}\end{pmatrix}\\
&=i\beta_1\begin{pmatrix}uv+2w_x&u_x-i\dfrac{\alpha_2}{\beta_2}u_y\\-i\dfrac{\alpha_2}{\beta_2}v_y&-uv\end{pmatrix}.
\end{aligned} \tag{26}$$

From (20), we derive new integrable variable-coefficient 2+1-dimensional long wave-short wave equation with the aid of (22)–(26):

$$\begin{aligned}
&i\alpha_2w_{xy}+\beta_2\left(uv\right)_x+\beta_2w_{xx}=0,\\
&-i\alpha_1u_t+\beta_1\left(v_{xx}-u_{xx}-u_{yy}\right)+2\beta_1\left(u^2v+uw_x\right)\\
&\quad=0,\\
&-i\alpha_1\beta_2v_t-\alpha_2\beta_1v_{yy}+2\beta_2\beta_1\left(uv^2+vw_x\right)=0.
\end{aligned} \tag{27}$$

Particularly, the above equations are reduced to the cylindrical form:

$$\begin{aligned}
iw_y+w_x+uv&=0,\\
-iu_t+\frac{1}{t}\left(v_{xx}-u_{xx}-u_{yy}\right)+\frac{2}{t}\left(u^2v+uw_x\right)&=0,\\
iv_t-\frac{1}{t}v_{yy}+\frac{2}{t}\left(uv^2+vw_x\right)&=0,
\end{aligned} \tag{28}$$

where $\alpha_2=\beta_2$, $\alpha_1=1$, $\beta_1=1/t$, and the integration constant is zero.

The Lax pairs of (27) are $\mathbf{N_1}$ and $\mathbf{N_2}$. $u_1^{(1)}$, $u_0^{(1)}$, and $u_0^{(2)}$ are presented in (24)–(26).

Particularly, we consider the case for $y=x$; then (28) is reduced to a new coupled equation:

$$\begin{aligned}
(i+1)\,w_x+uv&=0,\\
-iu_t+\frac{1}{t}v_{xx}-\frac{2}{t}u_{xx}+\frac{2}{t}\left(u^2v+uw_x\right)&=0,\\
iv_t-\frac{1}{t}v_{xx}+\frac{2}{t}\left(uv^2+vw_x\right)&=0.
\end{aligned} \tag{29}$$

4. Explicit Solutions and the Curves of Solutions

In this section, we shall apply the generalized dressing method to construct explicit solutions of these obtained 2 + 1-dimensional long wave-short wave equation and its reductions. We assume that F and K have solutions in the form of separation of variables:

$$F(x,z,y,t)=\sum_{j=1}^{N} f_j(x,y,t)\, g_j(z,y,t), \tag{30}$$

$$K(x,z,y,t)=\sum_{j=1}^{N} k_j(x,y,t)\, g_j(z,y,t), \tag{31}$$

where $f_j(x,y,t)$, $g_j(z,y,t)$ are some 2×2 matrices.

Substituting (30) and (31) into the GLM equation (8) yields the following:

$$\begin{aligned} K(x,x,y,t) &= \sum_{j=1}^{N} k_j(x,y,t)\, g_j(x,y,t) \\ &= -(f_1,f_2,\ldots,f_N)\, L^{-1}(g_1,g_2,\ldots,g_N)^T, \end{aligned} \tag{32}$$

where $L=(L_{jl})_{2N\times 2N}$ is defined by

$$L_{jl}=\delta_{jl}+\int_x^{\infty} g_j(s,y,t)\, f_l(s,y,t)\, ds, \qquad 1\le j,\ l\le N, \tag{33}$$

and δ_{jl} is the Kronecker delta.

We denote $F=(F_{ij})_{2\times 2}$, and, in view of (13) and (14), we obtain

$$\begin{aligned} \alpha_1 F_{11t}+i\beta_1(F_{11xx}+F_{11zz}) &= 0,\\ \alpha_2 F_{11y}-i\beta_2(F_{11x}+F_{11z}) &= 0,\\ \alpha_1 F_{12t}+i\beta_1 F_{12xx} &= 0,\\ \alpha_1 F_{21t}+i\beta_1 F_{21zz} &= 0,\\ \alpha_2 F_{12y}-i\beta_2 F_{12x} &= 0,\\ \alpha_2 F_{21y}-i\beta_2 F_{21z} &= 0,\\ \alpha_1 F_{22t} &= 0,\\ \alpha_2 F_{22y} &= 0. \end{aligned} \tag{34}$$

In the what follows, we obtain one soliton solution for the case of $N=1$ of (30). Let

$$F(x,z,y,t)=f(x,y,t)\,g(z,y,t)=\begin{pmatrix} e^{-(i\beta_1/\alpha_1)(l_1^2+m_1^2)t+l_1x+i(l_1+m_1)y} & e^{-i(\beta_1/\alpha_1)l_2^2t+l_2x+i(\beta_2/\alpha_2)l_2y} \\ e^{-i(\beta_1/\alpha_1)m_1^2t+l_3x+i(\beta_2/\alpha_2)m_1y} & 0 \end{pmatrix}\begin{pmatrix} e^{m_1z} & 0 \\ 0 & e^{m_2z} \end{pmatrix}. \tag{35}$$

Then, from (32) we obtain

$$K(x,x,y,t)=\frac{1}{\Delta}\begin{pmatrix} \hat{k}_{11} & \hat{k}_{12} \\ \hat{k}_{21} & \hat{k}_{22} \end{pmatrix}, \tag{36}$$

with

$$\begin{aligned} \Delta &= 1-\frac{1}{m_1+l_1}e^{-i(\beta_1/\alpha_1(l_1^2+m_1^2))t+(m_1+l_1)x+i(\beta_2/\alpha_2)(m_1+l_1)y}-\frac{1}{(m_1+l_2)(m_2+l_3)}e^{-i(\beta_1/\alpha_1)(l_2^2+l_3^2)t+(m_1+m_2+l_2+l_3)x+i(\beta_2/\alpha_2)(l_2+m_1)y},\\ \hat{k}_{11} &= \frac{1}{\Delta}e^{-i(\beta_1/\alpha_1)(l_1^2+m_1^2)t+(l_1+m_1)(x+i(\beta_2/\alpha_2)y)+(1/(m_1+l_2))e^{-2i(\beta_1/\alpha_1)l_2^2t+2(m_1+l_2)x+2i(\beta_2/\alpha_2)y}},\\ \hat{k}_{12} &= \frac{1}{\Delta}e^{-i(\beta_1/\alpha_1)l_2^2t+l_2x+i(\beta_2/\alpha_2)l_2y}+\frac{1}{m_2+l_3}e^{-i(\beta_1/\alpha_1)(l_1^2+m_1^2+l_3^2)t+(2m_2+l_1+l_3)x+i(\beta_2/\alpha_2)(l_1+2m_1)y}\\ &\quad -\frac{1}{m_1+l_1}e^{-i(\beta_1/\alpha_1)(l_1^2+l_2^2+m_1^2)t+(l_1+l_2+m_1+m_2)x+i(\beta_2/\alpha_2)(l_1+l_2+m_1)y},\\ \hat{k}_{21} &= \frac{1}{\Delta}e^{-i(\beta_1/\alpha_1)l_3^2t+(m_1+l_3)x+i(\beta_2/\alpha_2)m_1y},\\ \hat{k}_{22} &= \frac{1}{\Delta}\frac{1}{m_2+l_3}e^{-2i(\beta_1/\alpha_1)l_3^2t+2(m_2+l_3)x+2i(\beta_2/\alpha_2)m_1y}. \end{aligned} \tag{37}$$

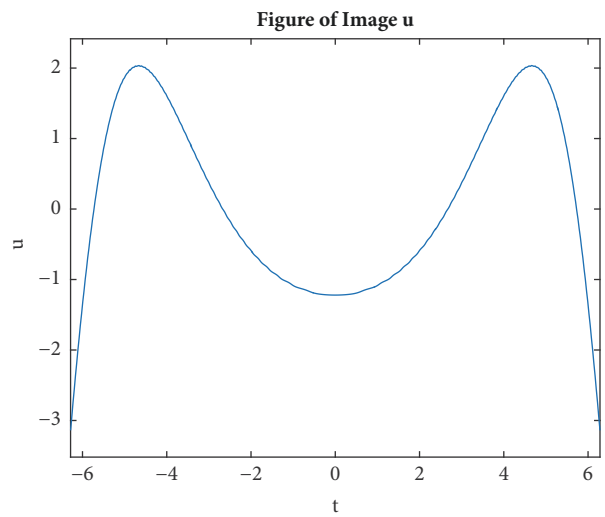

FIGURE 1: $x = 0.9$, $l_1 = 0.3$, $l_2 = 0.5$, $l_3 = 0.8$, $m_1 = 1$, $m_2 = 2$, $y = 3$, and $t \in [-6, 6]$.

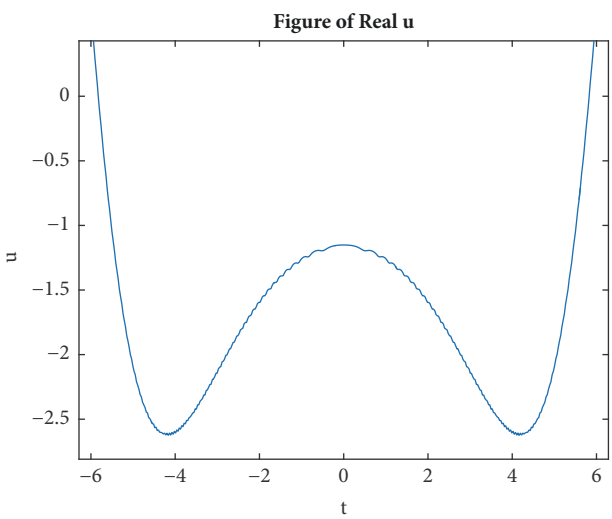

FIGURE 2: $x = 0.9$, $l_1 = 0.3$, $l_2 = 0.5$, $l_3 = 0.8$, $m_1 = 1$, $m_2 = 2$, $y = 3$, and $t \in [-6, 6]$.

Using (22), we obtain the solutions of (27). Particularly, for $\alpha_2 = \beta_2$, $\alpha_1 = 1$, and $\beta_1 = 1/t$, we derive the solutions of (28). In what follows, we draw the curves of the solutions for $\alpha_2 = \beta_2$, $\alpha_1 = 1$, and $\beta_1 = t$. Figures 1 and 2 describe the imaginary of u and real of u, respectively. From the curves, we can see that the forms are similar. The imaginary of v and real of v are shown by Figures 3 and 4, respectively. From the curves, we can see that the forms are different and with diminishing energy. Figures 5 and 6 construct the imaginary of w and real of w, respectively. In view of the solution curves, we can read the difference between the imaginary of w and real of w. Furthermore, we find that imaginary of v and that of w are similar. At the same time, we find that real of v and that of w are similar. Similarly, in later paper, we will discuss two soliton solutions and $N-$soliton solutions.

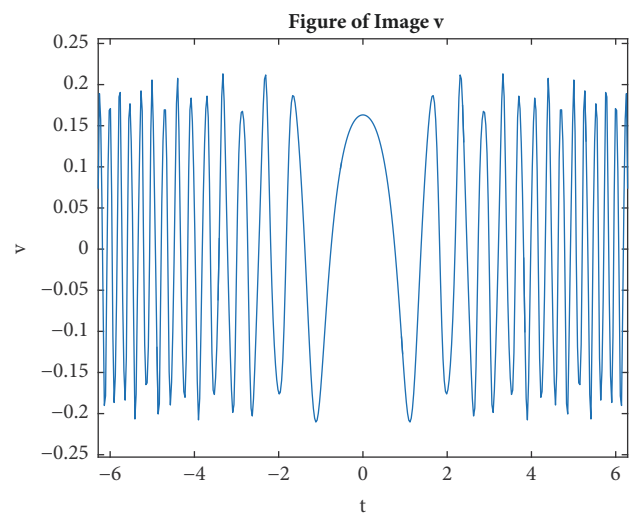

FIGURE 3: $x = 0.9$, $l_1 = 0.3$, $l_2 = 0.5$, $l_3 = 0.8$, $m_1 = 1$, $m_2 = 2$, $y = 3$, and $t \in [-6, 6]$.

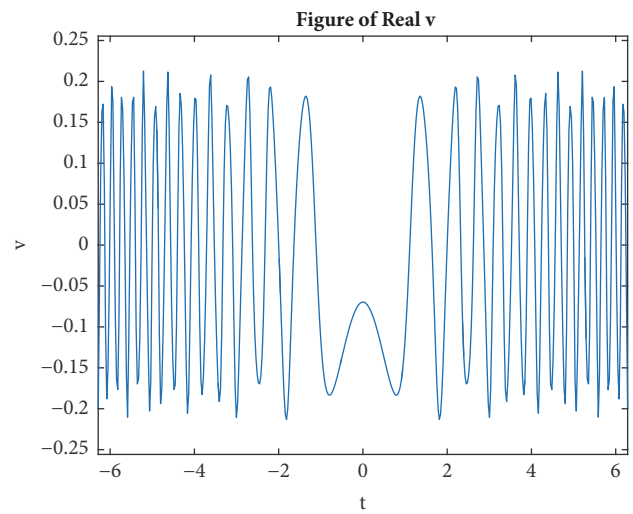

FIGURE 4: $x = 0.9$, $l_1 = 0.3$, $l_2 = 0.5$, $l_3 = 0.8$, $m_1 = 1$, $m_2 = 2$, $y = 3$, and $t \in [-6, 6]$.

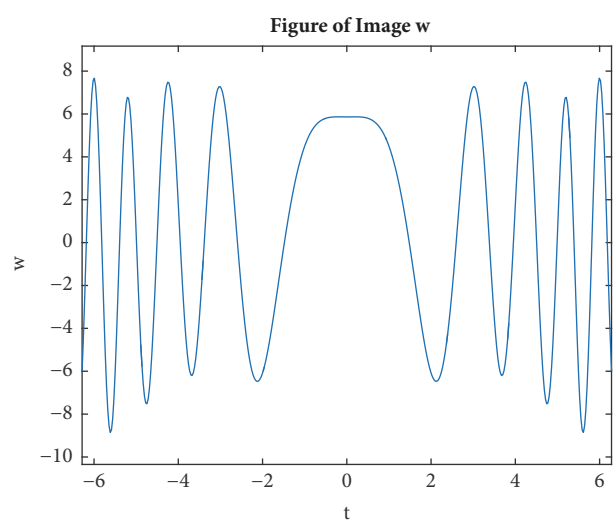

FIGURE 5: $x = 0.9$, $l_1 = 0.3$, $l_2 = 0.5$, $l_3 = 0.8$, $m_1 = 1$, $m_2 = 2$, $y = 3$, and $t \in [-6, 6]$.

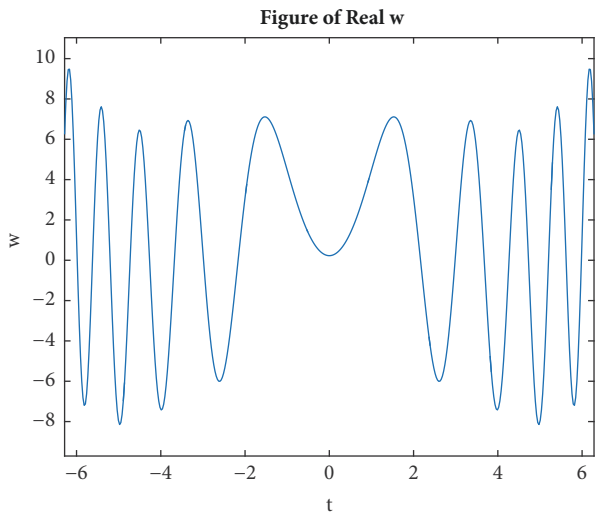

Figure 6: $x = 0.9$, $l_1 = 0.3$, $l_2 = 0.5$, $l_3 = 0.8$, $m_1 = 1$, $m_2 = 2$, $y = 3$, and $t \in [-6, 6]$.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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