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*Research Article*

# Adaptive Fuzzy Synchronization of Fractional-Order Chaotic Neural Networks with Backlash-Like Hysteresis

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An adaptive fuzzy synchronization controller is designed for a class of fractional-order neural networks (FONNs) subject to backlash-like hysteresis input. Fuzzy logic systems are used to approximate the system uncertainties as well as the unknown terms of the backlash-like hysteresis. An adaptive fuzzy controller, which can guarantee the synchronization errors tend to an arbitrary small region, is given. The stability of the closed-loop system is rigorously analyzed based on fractional Lyapunov stability criterion. Fractional adaptation laws are established to update the fuzzy parameters. Finally, some simulation examples are provided to indicate the effectiveness and the robust of the proposed control method.

## 1. Introduction

In the past two decades, study results of fractional calculus have received more and more attention because, compared with the classical integer-order calculus, the fractional-order one has many interesting and special properties. It has also been proven that a lot kinds of actual systems, ranging from life science and engineering to secret communication and system control, can be better modeled by using fractional-order differential equations (FDE) [1–10]. The nonlinear system, which is described by FDE, has memory. This advantage makes it possible to describe the hereditary as well as memory characters of many systems and processes. On this account, a lot of scholars employed the fractional-order derivative to replace the integer-order one in neural networks to get the FONNs [11–18]. It is known that the fractional model equips the neurons with more powerful computation ability, and these abilities could be used in information processing, frequency-independent phase shifts of oscillatory neuronal firing, and stimulus anticipation [13, 19]. By far, lots of methods have been given to synchronize FONNs [5, 12, 13, 20–22]. It should be mentioned that, in above works, the model of the master FONN should be known in advance. How to design synchronization controller when the master system's model is unknown is a challenging but interesting work.

It is well known that hysteresis can be found in a great mount of physical systems or devices, for instance, biology optics, mechanical actuators, electromagnetism, and electronic circuits [6, 23–26]. Hysteresis can damage the control performance or even lead to the instability of the controlled system. How to construct proper controller for these kinds of systems is an interesting work. With respect to integer-order systems subject to hysteresis, a lot of results have been given. In [27], a feedback controller was introduced to control nonlinear systems with hysteresis. The control of systems subject to Prandtl-Ishlinskii hysteresis was studied in [28]. To see more results on the control of integer-order systems with hysteresis please refer to [29–33]. However, with respect to fractional-order nonlinear systems with hysteresis, the related literatures are very few.

Up to now, fuzzy control methods have been studied extensively [34–42]. Specially, this approach has been particularly used to synchronize or control integer-order neural networks (IONNs) [43–47]. In above literature, fuzzy logic systems were employed to approximate the uncertain functions. To enhance the approximation ability of the fuzzy system, some robust terms, for example, sliding mode control, $H_\infty$ control should be used together with the main fuzzy adaptive control term. It should be pointed out that the above results are limited to uncertain IONNs. It is advisable to discuss the synchronization problem for uncertain FONNs.

In our paper, an adaptive fuzzy control approach is introduced for synchronizing two uncertain FONNs. Based on some fractional Lyapunov stability theorems, the stability analysis and the controller implement are given. To show the effectiveness of the proposed synchronization method, some illustrative examples are presented. Bearing the results of aforementioned works in mind, the main contributions of our study consist of the following: (1) by designing an adaptive fuzzy controller, a practical synchronization is proposed for a class of uncertain FONNs. To the best of our knowledge, how to construct fuzzy adaptive control for FONNs has not been previously investigated up to now, except some preliminaries works in [8, 46]. It should be pointed out that, in these works, the integer-order stability analysis method is used. However, in this paper, we will use the fractional stability analysis approach, and the stability of the closed-loop system is proved rigorously. (2) The models of the FONNs are assumed to be fully unknown (i.e., the controller designed is free of the models of both master and slave systems). (3) The control of fractional-order nonlinear systems with backlash-like hysteresis input is studied.

## 2. Preliminaries

*2.1. Some Basic Results of Fractional Calculus.* The $q$th fractional integral is defined by

$$\mathbb{I}^{-q} f(t) = \frac{1}{\Gamma(q)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-q}} d\tau, \tag{1}$$

where $\Gamma(q) = \int_0^{+\infty} \tau^{q-1} e^{-\tau} d\tau$. The $q$th fractional-order derivative is given as

$$\mathbb{D}^{q} f(t) = \frac{1}{\Gamma(n-q)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{q+1-n}} d\tau, \tag{2}$$

where $n-1 \le q < n$ $(n \in \mathbb{N})$. The Laplace transform of the Caputo fractional derivative is

$$\int_0^\infty e^{-st} \mathbb{D}^q f(t)\, dt = s^q F(s) - \sum_{k=0}^{n-1} s^{q-k-1} f^{(k)}(0), \tag{3}$$

where $F(s) = \mathscr{L}\{f(t)\}$. For convenience, we always assume that $0 < q \le 1$ in the rest of this paper.

The following results on fractional calculus will help us to facilitate the synchronization controller design as well as the stability analysis.

*Definition 1* (see [1]). The Mittag-Leffler function is defined by

$$E_{q_1,q_2}(\zeta) = \sum_{k=0}^{\infty} \frac{\zeta^k}{\Gamma(q_1 k + q_2)}, \tag{4}$$

where $q_1, q_2 > 0$, and $\zeta \in \mathbb{C}$.

The Laplace transform of (4) is as follows [1]:

$$\mathscr{L}\left\{t^{\beta-1} E_{q_1,q_2}\left(-a t^{q_1}\right)\right\} = \frac{s^{q_1-q_2}}{s^{q_1}+a}. \tag{5}$$

**Lemma 2** (see [1]). *Let $x(t) \in C^1[0,T]$ with $T > 0$; then one has*

$$\begin{aligned} {}_0\mathbb{I}_t^{-q} \mathbb{D}^q x(t) &= x(t) - x(0), \\ \mathbb{D}^q\, {}_0\mathbb{I}_t^{-q} x(t) &= x(t). \end{aligned} \tag{6}$$

**Lemma 3** (see [1]). *Let $\beta \in \mathbb{C}$ and to constants $q, \mu$ satisfying $0 < q < 2$ and*

$$\frac{\pi q}{2} < \mu < \min\{\pi, \pi q\}, \tag{7}$$

*and then the following equality holds:*

$$\begin{gathered} E_{q,\beta}(z) = -\sum_{j=1}^{n} \frac{1}{\Gamma(\beta - qj) z^j} + o\left(\frac{1}{|z|^{n+1}}\right), \\ |z| \longrightarrow \infty, \ \mu \le \left|\arg(z)\right| \le \pi. \end{gathered} \tag{8}$$

**Lemma 4** (see [1]). *Let $0 < q < 2$ and $\beta \in \mathbb{R}$. If there exists a positive constant $\mu$ such that $\pi q/2 < \mu \le \min\{\pi, \pi q\}$, then one has*

$$\left|E_{q,\beta}(z)\right| \le \frac{b_0}{1+|z|}, \tag{9}$$

*where $b_0$ is a positive real constant, $\mu \le |\arg(z)| \le \pi$ and $|z| \ge 0$.*

**Lemma 5** (see [2]). *Let $x(t) = 0$ be an equilibrium point of the following fractional-order nonlinear system:*

$$\mathbb{D}^q x(t) = f(t, x(t)). \tag{10}$$

*If one can find a Lyapunov function $V(t, x(t))$ as well as three class-K functions $g_i$, $i = 1, 2, 3$ such that*

$$\begin{aligned} g_1(\|x(t)\|) \le V(t, x(t)) &\le g_2(\|x(t)\|), \\ \mathbb{D}^q V(t, x(t)) &\le -g_3(\|x(t)\|), \end{aligned} \tag{11}$$

*then system (10) will be asymptotically stable.*

**Lemma 6** (see [4]). *Let $x(t) \in \mathscr{R}^n$ be a continuous and derivable function. Then, for any $t > 0$,*

$$\frac{1}{2} \mathbb{D}^q x^T(t) x(t) \le x^T(t) \mathbb{D}^q x(t). \tag{12}$$

**Lemma 7** (see [3]). *Let $x(t) \in \mathscr{R}^n$ be a continuous and derivable function. Then, for any $t > 0$,*

$$\frac{1}{2} \mathbb{D}^q x^T(t) B x(t) \le x^T(t) B \mathbb{D}^q x(t), \tag{13}$$

*where $B \in \mathscr{R}^{n\times n}$ is a positive definite constant matrix.*

*2.2. Description of a Fuzzy System.* A fuzzy logic system consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on the fuzzy rules, and the defuzzifier [34–40, 48]. Usually, a fuzzy logic system is modeled by

$$\hat{f}(x(t)) = \frac{\sum_{j\in J} \vartheta_j(t)\mu_j(x(t))}{\sum_{j\in J}\mu_j(x(t))}, \tag{14}$$

where $\hat{f}$ (a Lipschitz-continuous mapping from a compact subset $\Omega \subseteq \mathscr{R}^n$ to the real line $\mathscr{R}$) is called the output of the fuzzy logic system, $x = [x_1, \dots, x_n]^T \in C^1[\mathscr{I}, \Omega]$ (the set of all continuous mappings from $\mathscr{I} = [0, +\infty) \subseteq \mathscr{R}$ to $\Omega$ which have continuous derivatives) is called the input vector, $J = \prod_{i=1}^n \mathscr{F}_i$, $\mathscr{F}_i$ consists of $N_i$ fuzzy sets ($1 \le i \le n$), $\mu_j$ (a mapping from $\mathscr{R}^n$ to the closed unit interval $[0,1] \subseteq \mathscr{R}$) is called the membership function of rule $j$ ($j \in J$), and $\vartheta_j$ (a mapping from $\mathscr{I}$ to $\mathscr{R}$) is called the centroid of the $j$th consequent set ($j \in J$); we may identify $J$ with $\{1, 2, \dots, N\}$ for the sake of convenience. Write $\vartheta(t) = [\vartheta_1(t), \dots, \vartheta_N(t)]^T$ and $\varphi(x(t)) = [q_1(x(t)), q_2(x(t)), \dots, q_N(x(t))]^T$, where $q_j$ (called the $j$th fuzzy basis function, $j \in J$) is a continuous mapping (and thus $\varphi : \Omega \to \mathscr{R}^N$ is continuous) defined by

$$q_j(x(t)) = \frac{\vartheta_j(t)}{\sum_{s\in J}\mu_s(x(t))}. \tag{15}$$

Then system (14) can be rewritten as

$$\hat{f}(x(t)) = \vartheta^T(t)\varphi(x(t)). \tag{16}$$

## 3. Main Results

*3.1. Problem Description.* Consider a class of FONNs described as

$$\mathbb{D}^q x_i(t) = -a_i x_i(t) + \sum_{k=1}^n b_{ik} f_k(x_k(t)) + I_i, \tag{17}$$

where $i = 1, \dots, n$, $x_i(t)$ is the state variable, $a_i > 0$ and $b_{ik}$, $k = 1, 2, \dots, m$ are constants, $I_i$ represents the external input, and $f_k(\cdot)$ is a smooth nonlinear function.

Write $x(t) = [x_1(t), \dots, x_n(t)]^T \in \mathbb{R}^n$, $f(\cdot) = [f_1(\cdot), \dots, f_n(\cdot)]^T \in \mathbb{R}^n$, $I = [I_1, \dots, I_n]^T \in \mathbb{R}^n$, $A = -\operatorname{diag}(a_1, \dots, a_n) \in \mathbb{R}^{n\times n}$, $u(t) = [u_1(t), \dots, u_m(t)] \in \mathbb{R}^m$,

$$B = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{1n} & \cdots & b_{nn} \end{bmatrix} \in \mathbb{R}^{n\times n}, \tag{18}$$

then (17) can be written into the following compact form:

$$\mathbb{D}^q x(t) = Ax(t) + Bf(x(t)) + I. \tag{19}$$

To guarantee the existence and uniqueness of the solutions of the fractional-order neural network (19) (see, [3]), we assume that the functions $f$ are Lipschitz-continuous, i.e., for all $x(t), y(t) \in \mathscr{R}^n$,

$$\|f(x(t)) - f(y(t))\| \le \sigma \|x(t) - y(t)\|, \tag{20}$$

where $\sigma$ is a positive constant.

The slave system is expressed by

$$\mathbb{D}^\alpha y(t) = Ay(t) + Bf(y(t)) + I + G\nu(t) + d(t), \tag{21}$$

where $y(t) \in \mathscr{R}^n$ is the state vector of the slave system, $G \in \mathscr{R}^{n\times n}$ is a positive definite control gain matrix, $d(t) = [d_1(t), d_2(t), \dots, d_n(t)]^T \in \mathscr{R}^n$ is an unknown external disturbance, and $\nu(t) \in \mathbb{R}^n$ represents hysteresis type of nonlinear control input which is described by

$$\frac{d\nu(t)}{dt} = \gamma_1 \left|\frac{du(t)}{dt}\right| (\gamma_3 u(t) - \nu(t)) + \gamma_2 \frac{du(t)}{dt}, \tag{22}$$

where $u(t) = [u_1(t), \dots, u_n(t)]^T \in \mathbb{R}^n$ is the control input, $\gamma_1$, $\gamma_2$ and $\gamma_3$ are three constants satisfying $\gamma_3 > 0$ and $\gamma_3 > \gamma_2$. One can rewrite (22) as

$$\begin{aligned} \nu(t) &= \gamma_3 u(t) + \hbar(t) \\ \hbar(t) &= [\omega(0) - cu(0)]\, e^{-a(u(t)-u(0))\operatorname{sign}(\dot{u}(t))} \\ &\quad + e^{-au(t)\operatorname{sign}(\dot{u}(t))} \int_{u(0)}^{u(t)} (b-c)\, e^{a\tau \operatorname{sign}(\dot{u}(t))} d\tau. \end{aligned} \tag{23}$$

When $\gamma_1 = 1.1$, $\gamma_3 = 3.22$, $\gamma_2 = 0.44$, $u(t) = 5.1\sin(2t)$ and $\nu(0) = 0$, the behavior of the backlash-like hysteresis is depicted in Figure 1.

The objective of this paper is to construct an adaptive fuzzy controller such that the slave system (21) synchronizes the master system (19). To proceed, the following assumption is needed.

*Assumption 8.* The external disturbance is bounded, i.e., $|d_i(t)| \le \overline{d}_i$, $i = 1, 2, \dots, n$ where $\overline{d}_i$ is an unknown positive constant.

*3.2. Synchronization Controller Implement.* In this part, we will give the detailed procedure of the adaptive fuzzy controller design as well as the stability analysis. Let the synchronization error be $e(t) = x(t) - y(t)$. It follows from (19), (21), and (23) that

$$\begin{aligned} \mathbb{D}^q e(t) &= Ae(t) + A(f(x(t)) - f(y(t))) - \gamma_3 Gu(t) \\ &\quad - G\hbar(t) - d(t). \end{aligned} \tag{24}$$

Denote $\Xi = (1/\gamma_3)G^{-1}$; (24) implies that

$$\Xi\mathbb{D}^q e(t) = \varpi(e(t)) - u(t), \tag{25}$$

where $\varpi(e(t)) = \Xi(Ae(t) + A(f(x(t)) - f(y(t))) - G\hbar(t) - d(t))$ is an unknown nonlinear function.

*Remark 9.* In fact, it is easy to know that $\varpi$ is a function of $x(t)$, $y(t)$, $e(t)$, and $t$. In this paper, the synchronization

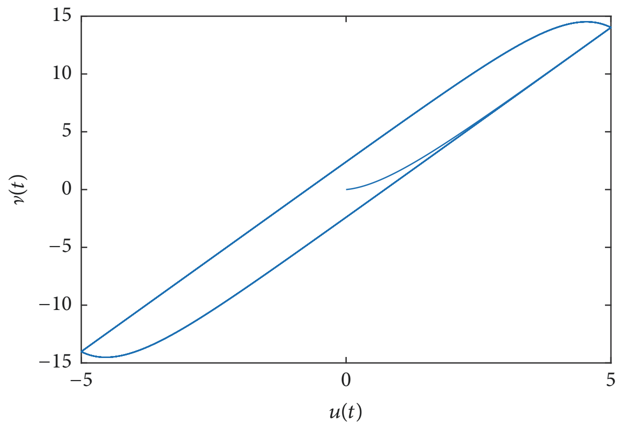


Figure 1: Hysteresis curve.

error will be used as the input of the fuzzy logic systems. In addition, the synchronization error $e(t)$ can be seen as a bridge between the signals $x(t)$ and $y(t)$. Consequently, we will use the denotation $\varpi(e(t))$ for convenience.

Firstly, let us consider an ideal condition. Suppose that $\varpi(e(t))$ is known (i.e., $d(t)$, $f(x(t))$, and $f(y(t))$ are all known in advance). Thus, the ideal controller can be given by

$$u_{eq} = \varpi(e(t)) + Ke(t), \tag{26}$$

where $K = \mathrm{diag}(k_1, \ldots, k_n)$ with $k_i > 0$, $i = 1, 2, \ldots, n$. Then, we can give the following theorem.

**Theorem 10.** *If the models of the master and slave systems are known, and the synchronization controller is given by (26), then we have that the synchronization error converges to zero asymptotically.*

*Proof.* Substituting (26) into (25), one has

$$\Xi \mathbb{D}^q e(t) = -Ke(t). \tag{27}$$

Define the following Lyapunov function:

$$\mathbb{V}(t) = \frac{1}{2} e^T(t)\, \Xi e(t). \tag{28}$$

According to Lemma 7, the $q$th derivative of $\mathbb{V}(t)$ is bounded by

$$\begin{aligned}\mathbb{D}^\alpha \mathbb{V}(t) &= \frac{1}{2}\mathbb{D}^q e^T(t)\, \Xi e(t) \le e^T(t)\, \Xi \mathbb{D}^q e(t) \\ &\le -\underline{k}(t),\end{aligned} \tag{29}$$

where $\underline{k} = \min\{k_1, k_2, \ldots, k_n\}$. Thus, according to Lemma 5 and (29) that the synchronization error will converge to zero asymptotically. □

Since $\varpi(e(t))$ is an unknown function, the above controller (26) may be inapplicable. In such a case, by using the fuzzy logic system, let us approximate $\varpi(e(t))$ (16) as

$$\widehat{\varpi}_i(\vartheta_i(t), e(t)) = \vartheta_i^T(t)\, \varphi_i(e(t)), \tag{30}$$

where $\varphi_i(x(t))$ is a fuzzy basis function, and $\vartheta_i(t)$ is an adjustable parameter of the fuzzy system which drive $\sup\{|\varpi_i(e(t)) - \widehat{\varpi}_i(\vartheta_i^*, e(t))|$ small enough. Let the parameter estimation error with respect to $\vartheta_i$ be

$$\widetilde{\vartheta}_i(t) = \vartheta_i(t) - \vartheta_i^*, \tag{31}$$

and the fuzzy approximation error with respect to $\vartheta_i^*$ be

$$\varepsilon_i(e(t)) = \varpi_i(e(t)) - \widehat{\varpi}_i(\vartheta_i^*, e(t)). \tag{32}$$

According to the universal approximation theorem, one can know that the fuzzy systems do not violate the universal approximator property. Consequently, one can make the following assumption.

*Assumption 11.* There exists an unknown constant $\varepsilon^* > 0$ such that

$$\sup\{|\varepsilon_i(e(t))|\} \le \varepsilon^*. \tag{33}$$

Based on the above analysis, one has

$$\begin{aligned}&\widehat{\varpi}_i(\vartheta_i(t), e(t)) - \varpi_i(e(t)) \\ &\quad = \widehat{\varpi}_i(\vartheta_i(t), e(t)) - \widehat{\varpi}_i(\vartheta_i^*, e(t)) + \widehat{\varpi}_i(\vartheta_i^*, e(t)) \\ &\qquad - \varpi_i(e(t)) \\ &\quad = \widehat{\varpi}_i(\vartheta_i(t), e(t)) - \widehat{\varpi}_i(\vartheta_i^*, e(t)) - \varepsilon_i(e(t)) \\ &\quad = \widetilde{\vartheta}_i^T(t)\, \varphi_i(e(t)) - \varepsilon_i(e(t)).\end{aligned} \tag{34}$$

Denote $\vartheta(t)\varphi(e(t)) = [\vartheta_1^T(t)\varphi_1(e(t)), \vartheta_2^T(t)\varphi_2(e(t)), \ldots, \vartheta_n^T(t)\varphi_n(e(t))]^T$ and $\widetilde{\vartheta}(t)\varphi(e(t)) = [\widetilde{\vartheta}_1^T(t)\varphi_1(e(t)), \widetilde{\vartheta}_2^T(t)\varphi_2(e(t)), \ldots, \widetilde{\vartheta}_n^T(t)\varphi_n(e(t))]^T$, then one can rewrite (34) as

$$\widehat{\varpi}(\vartheta(t), e(t)) - \varpi(e(t)) = \widetilde{\vartheta}^T(t)\, \varphi(e(t)) - \varepsilon(e(t)), \tag{35}$$

where $\vartheta(t) = [\vartheta_1^T(t), \vartheta_2^T(t), \ldots, \vartheta_n^T(t)]^T \in \mathbb{R}^{N \times n}$ ($N$ represents the amount of the fuzzy rules).

To simplify the stability analysis, we give the following results first.

**Theorem 12.** *Suppose that $h(t) \in \mathbb{R}$ is a positive definite smooth function. $\delta_1 > 0$, $\delta_2 \in \mathbb{R}$ are two adjustable parameters. If it holds that*

$$\mathbb{D}^q h(t) \le -\delta_1 h(t) + \delta_2, \tag{36}$$

*then $h(t)$ will be small enough eventually if proper parameters are chosen.*

*Proof.* According to (36), one can find a function $\zeta(t)$ satisfying $\zeta(t) \ge 0$ and

$$\mathbb{D}^q h(t) + \zeta(t) = -\delta_1 h(t) + \delta_2. \tag{37}$$

It follows from (37) that

$$H(s) = \frac{s^{q-1}}{s^q + \delta_1} h(0) + \frac{s^{q-(1+q)}\delta_2}{(s^q + \delta_1)} - \frac{Z(s)}{s^q + \delta_1}, \tag{38}$$

where $H(s) = \mathscr{L}\{h(t)\}$ and $Z(s) = \mathscr{L}\{\zeta(t)\}$. One solves (38) according to (5) as

$$\begin{aligned} h(t) &= h(0) E_{q,1}(-\delta_1 t^q) + \delta_2 t^q E_{q,1+q}(-\delta_1 t^q) - \zeta(t) \\ &\quad * t^{-1} E_{q,0}(-\delta_1 t^q), \end{aligned} \tag{39}$$

where $*$ represents the convolution operator. Since that $t^{-1}E_{q,0}(-\delta_1 t^q)$ and $\zeta(t)$ are all nonnegative, one has $\zeta(t) * t^{-1}E_{q,0}(-\delta_1 t^q) \ge 0$. Consequently, one has

$$h(t) \le h(0) E_{q,1}(-\delta_1 t^q) + \delta_2 t^q E_{q,1+q}(-\delta_1 t^q). \tag{40}$$

It is easy to know that $\arg(-\delta_1 t^q) = -\pi$, $|-\delta_1 t^q| \ge 0$ and $q \in (0,2)$; thus, by using Lemma 4, one obtains that

$$\left|E_{q,1}(-\delta_1 t^q)\right| \le \frac{\kappa}{1 + \delta_1 t^q}, \tag{41}$$

where $\kappa > 0$, i.e.,

$$\lim_{t\to\infty} h(0) E_{q,1}(-\delta_1 t^q) = 0. \tag{42}$$

Thus, by using Lemma 3, one has for every $\varepsilon > 0$ and large enough time $t$ that

$$t^\alpha E_{\alpha,\alpha+1}(-At^\alpha) \le \frac{B}{A} + \frac{\varepsilon}{3}. \tag{43}$$

That is to say, if the design parameters are chosen as $\delta_2/\delta_1 \le \varepsilon$, then according to (40) and (43) one has

$$h(t) < \varepsilon. \tag{44}$$

This completes the proof of Theorem 12. □

Then, one can obtain the following theorem.

**Theorem 13.** *Under Assumptions 8 and 11 and proper control parameters, if $\hat{\varepsilon}^*(t)$ is the estimation of $\varepsilon^*$, $u(t)$ is implemented as*

$$u(t) = Ke(t) + \vartheta^T(t)\varphi(e(t)) + \hat{\varepsilon}^*(t)\operatorname{sign}(e(t)), \tag{45}$$

*and $\vartheta_i(t)$ and $\hat{\varepsilon}^*(t)$ are, respectively, updated by*

$$\mathbb{D}^q \vartheta_i(t) = \varsigma_{1i} e_i(t)\varphi_i(e(t)) - \varsigma_{1i}\varsigma_{2i}\vartheta_i(t), \tag{46}$$

$$\mathbb{D}^q \hat{\varepsilon}^*(t) = \varsigma_3 \sum_{i=1}^n |e_i(t)| - \varsigma_3\varsigma_4\hat{\varepsilon}^*(t), \tag{47}$$

*with $\varsigma_{1i}$, $\varsigma_{2i}$, $\varsigma_3$, $\varsigma_4$ being positive design parameters, then one has that the synchronization error eventually converges to an arbitrary small region of zero.*

*Proof.* It follows from (25), (35), and (45) that

$$\begin{aligned} \Xi\mathbb{D}^q e(t) &= \varpi(e(t)) - u(t) \\ &= \varpi(e(t)) - Ke(t) - \vartheta^T(t)\varphi(e(t)) \\ &\quad - \hat{\varepsilon}^*(t)\operatorname{sign}(e(t)) \\ &= -Ke(t) - \tilde{\vartheta}^T(t)\varphi(e(t)) \\ &\quad - \hat{\varepsilon}^*(t)\operatorname{sign}(e(t)) + \varepsilon(e(t)). \end{aligned} \tag{48}$$

Multiplying $e^T(t)$ to both sides of (48) gives

$$\begin{aligned} e^T(t)\Xi\mathbb{D}^q e(t) &= -e^T(t)Ke(t) - e^T(t)\tilde{\vartheta}^T(t)\varphi(e(t)) \\ &\quad + e^T(t)\varepsilon(e(t)) \\ &\quad - e^T(t)\hat{\varepsilon}^*(t)\operatorname{sign}(e(t)) \\ &\le -\underline{k}e^T(t)e(t) \\ &\quad - \sum_{i=1}^n e_i(t)\tilde{\vartheta}_i^T(t)\varphi_i(e(t)) \\ &\quad + \varepsilon^*\sum_{i=1}^n |e_i(t)| - \hat{\varepsilon}^*(t)\sum_{i=1}^n |e_i(t)| \\ &= -\underline{k}e^T(t)e(t) \\ &\quad - \sum_{i=1}^n e_i(t)\tilde{\vartheta}_i^T(t)\varphi_i(e(t)) \\ &\quad - \tilde{\varepsilon}^*(t)\sum_{i=1}^n |e_i(t)|, \end{aligned} \tag{49}$$

where

$$\tilde{\varepsilon}^*(t) = \hat{\varepsilon}^*(t) - \varepsilon^* \tag{50}$$

is the estimation error of $\varepsilon^*$.

It is known that the $\mathbb{D}^q C = 0$ where $C$ is an arbitrary constant. Thus, it follows from (31) and (50) that $\mathbb{D}^q\tilde{\vartheta}_i(t) = \mathbb{D}^q\vartheta_i(t)$ and $\mathbb{D}^q\tilde{\varepsilon}^*(t) = \mathbb{D}^q\hat{\varepsilon}^*(t)$.

Define the Lyapunov function as

$$\begin{aligned} \mathscr{V}(t) &= \frac{1}{2}e^T(t)\Xi e(t) + \sum_{i=1}^n \frac{1}{2\varsigma_{1i}}\tilde{\vartheta}_i^T(t)\tilde{\vartheta}_i(t) \\ &\quad + \frac{1}{2\varsigma_3}\tilde{\varepsilon}^{*2}(t). \end{aligned} \tag{51}$$

Using (46), (47), (49), and Lemma 6 one has

$$
\begin{aligned}
\mathbb{D}^q \mathscr{V}(t) &\le e^T(t)\,\Xi \mathbb{D}^q e(t) + \sum_{i=1}^{n} \frac{1}{\varsigma_{1i}} \tilde{\vartheta}_i^T(t)\, \mathbb{D}^q \tilde{\vartheta}_i(t) \\
&\quad + \frac{1}{\varsigma_3} \tilde{\varepsilon}^*(t)\, \mathbb{D}^q \tilde{\varepsilon}^*(t) \\
&= -\underline{k} e^T(t)\, e(t) - \sum_{i=1}^{n} e_i(t)\, \tilde{\vartheta}_i^T(t)\, \varphi_i(e(t)) \\
&\quad - \tilde{\varepsilon}^*(t) \sum_{i=1}^{n} |e_i(t)| + \sum_{i=1}^{n} \frac{1}{\varsigma_{1i}} \tilde{\vartheta}_i^T(t)\, \mathbb{D}^q \vartheta_i(t) \\
&\quad + \frac{1}{\varsigma_3} \tilde{\varepsilon}^*(t)\, \mathbb{D}^q \tilde{\varepsilon}^*(t) \\
&= -\underline{k} e^T(t)\, e(t) - \sum_{i=1}^{n} e_i(t)\, \tilde{\vartheta}_i^T(t)\, \varphi_i(e(t)) \\
&\quad + \frac{1}{\varsigma_3} \tilde{\varepsilon}^*(t) \left( \varsigma_3 \sum_{i=1}^{n} |e_i(t)| - \varsigma_3 \varsigma_4 \hat{\varepsilon}^*(t) \right) \\
&\quad - \tilde{\varepsilon}^*(t) \sum_{i=1}^{n} |e_i(t)| \\
&\quad + \sum_{i=1}^{n} \frac{1}{\varsigma_{1i}} \tilde{\vartheta}_i^T(t) \left( \varsigma_{1i} e_i(t)\, \varphi_i(e(t)) - \varsigma_{1i} \varsigma_{2i} \vartheta_i(t) \right) \\
&= -\underline{k} e^T(t)\, e(t) - \sum_{i=1}^{n} \varsigma_{2i} \tilde{\vartheta}_i^T(t)\, \vartheta_i(t) - \varsigma_4 \tilde{\varepsilon}^*(t)\, \hat{\varepsilon}^*(t) \\
&= -\underline{k} e^T(t)\, e(t) - \sum_{i=1}^{n} \varsigma_{2i} \tilde{\vartheta}_i^T(t)\, \tilde{\vartheta}_i(t) - \varsigma_4 \tilde{\varepsilon}^*(t)\, \tilde{\varepsilon}^*(t) \\
&\quad - \varsigma_4 \tilde{\varepsilon}^*(t)\, \varepsilon^*(t) - \sum_{i=1}^{n} \varsigma_{2i} \tilde{\vartheta}_i^T(t)\, \vartheta_i^* \\
&\le -\underline{k} e^T(t)\, e(t) - \sum_{i=1}^{n} \frac{\varsigma_{2i}}{2} \tilde{\vartheta}_i^T(t)\, \tilde{\vartheta}_i(t) - \frac{\varsigma_4}{2} \tilde{\varepsilon}^{*2}(t) \\
&\quad + \sum_{i=1}^{n} \frac{\varsigma_{2i}}{2} \vartheta_i^{*T} \vartheta_i^* + \frac{\varsigma_4}{2} \varepsilon^{*2} \le -\delta_1 \mathscr{V}(t) + \delta_2
\end{aligned} \tag{52}
$$

with $\delta_1 = \min\{2\underline{k}, \varsigma_{1i}\varsigma_{2i}, \varsigma_2\varsigma_4\}$ and $\delta_2 = \sum_{i=1}^{n}(\varsigma_{2i}/2)\vartheta_i^{*T}\vartheta_i^* + (\varsigma_4/2)\varepsilon^{*2}$ being two positive constants.

Thus, based on (52) and Theorem 12, one knows that the synchronization error eventually converges to an arbitrary small region of zero if proper control parameters are chosen. This completes the proof of Theorem 13. □

*Remark 14.* It should be pointed out that the fractional-order adaptation law was also introduced in [3, 5, 49, 50]. However, the above adaptation laws only contain a positive term (i.e., the adaptation law is designed as $\mathbb{D}^q\vartheta(t) = \sigma e_i(t)\varphi_i(t)$). Despite using this kind of adaptation law, the asymptotical stability of the system can be guaranteed. Yet, the boundedness of the control parameters cannot be ensured. The proposed adaptation law contains a negative term (for example, $-\varsigma_{1i}\varsigma_{2i}\vartheta_i(t)$ in (46)) which will drive the updated parameter tends to a small neighborhood of the origin eventually (see the proof of Theorem 13).

*Remark 15.* To enforce the synchronization error tending to a region as small as possible, one must make $\delta_2/\delta_1$ small enough. To meet this objective, one should choose large $k_i$, $\varsigma_{1i}$, $\varsigma_3$ and choose small $\varsigma_{2i}$, $\varsigma_4$.

*Remark 16.* It is worth mentioning that, in [51], to discuss the stability of the fractional-order nonlinear systems, a very complicated boundary condition

$$
\left\| \sum_{k=1}^{\infty} \frac{\Gamma(1+\alpha)}{\Gamma(1+k)\,\Gamma(1-k+\alpha)} \mathbb{D}^k x(t)\, \mathbb{D}^{\alpha-k} x(t) \right\| \le a \|x\| \tag{53}
$$

is assumed to be known. The above condition was proven in [51]. Yet, how to get the exact value of $a$ is a challenging work. But in our paper, by using the quadratic Lyapunov functions, the aforementioned problem is solved.

*Remark 17.* It should be pointed out that the proposed control method does not need the prior knowledge of systems models. Therefore, the control method can be easily extended to the following domains: control of fractional-order nonlinear systems, synchronization of fractional-order chaotic system, and secret communication, and so on. And relating out control method to a potential application is one of our future research directions.

## 4. Simulation Studies

In (19), letting $x(t) \in \mathbb{R}^3$, $x(0) = [-0.301, 0.400, 0.299]^T$, $q = 0.96$, $f_i(x_i(t)) = \tanh(x_i(t))$, $a_i = 1$, $I_i = 0$, $u(t) \equiv 0$, and

$$
B = \begin{bmatrix} 2.001 & -1.201 & 0 \\ 2.000 & 1.713 & 1.154 \\ -4.751 & 0 & 1.101 \end{bmatrix}, \tag{54}
$$

the FONN (19) shows chaotic behavior, which is depicted in Figure 2.

### *4.1. Synchronization of FONN with Constant System Parameters.*

Let the initial condition of the slave FONN (21) be $y(0) = [4.2, -3.1, -1.9]$, and $G = I_{3\times3}$. The external disturbance is defined as $d(t) = [\sin t, \cos t, \sin t + \cos t]^T$. The control design parameters are given as $k_1 = k_2 = k_3 = 0.5$. $\varsigma_{11} = \varsigma_{12} = \varsigma_{13} = 1.1$, $\varsigma_{21} = \varsigma_{22} = \varsigma_{23} = 0.05$, $\varsigma_3 = 1.55$, $\varsigma_4 = 0.02$. In the simulation, the noncontinuous sign function in (45) is replaced with arctan $(10\cdot)$.

With respect to the fuzzy logic systems, its input variable is chosen as the synchronization error $e_i(t)$. For each input, we give seven Gaussian membership functions on $[-5\ \ 5]$. The parameters of the proposed membership functions, which are

Table 1: Fuzzy parameters.

| Membership functions | Parameter $b$ | Parameter $a$ |
|---|---|---|
| Membership function 1 | 1 | −5 |
| Membership function 2 | 0.8 | −2 |
| Membership function 3 | 0.2 | −0.3 |
| Membership function 4 | 0.09 | 0 |
| Membership function 5 | 0.2 | 0.3 |
| Membership function 6 | 0.8 | 2 |
| Membership function 7 | 1 | 5 |

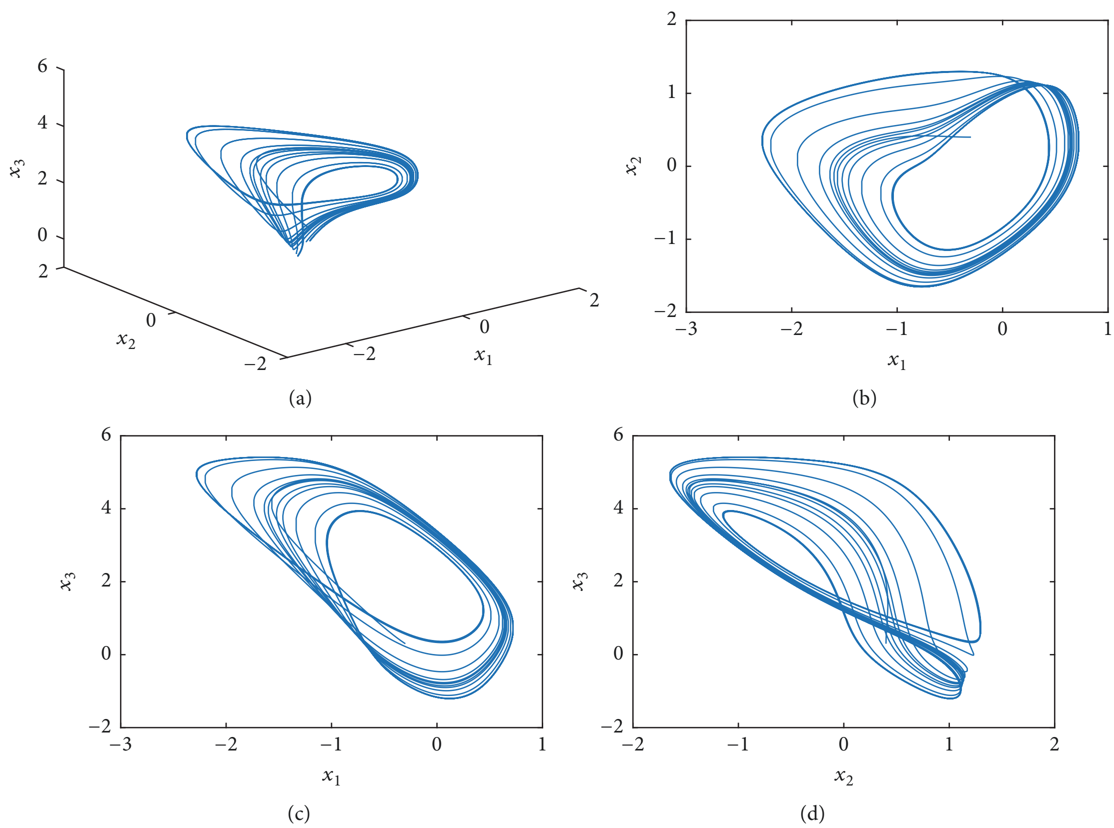


Figure 2: Chaotic dynamic of the FONN (19) in (a) 3D space; (b) $x_1$-$x_2$ plane; (c) $x_1$-$x_3$ plane; (d) $x_2$-$x_3$ plane.

defined as $e^{-(e_i(t)-a)^2/2b^2}$, are given in Table 1. These functions are depicted in Figure 3. The initial conditions of the fuzzy parameters are given as $\vartheta_1(0) = \vartheta_2(0) = \vartheta_3(0) = \underline{0} \in \mathscr{R}^{343}$.

The simulation results are depicted in Figures 4–6. The results that the signals $x_1(t)$, $x_2(t)$, and $x_3(t)$, respectively, track $y_1(t)$, $y_2(t)$, and $y_3(t)$, as well as the time response of the synchronization errors are presented in Figure 4. The smoothness and the boundedness of the control inputs are given in Figure 5. The fuzzy parameters, which can be concluded to be bounded according to the proposed adaptation law (46), are shown in Figure 6. From these figures one knows that the proposed controller works well and has good synchronization performance.

It is well known that the conventional systems usually suffer from discontented performance resulting from modeled errors, parametric uncertainties, input nonlinearities, and external disturbances, because it is impossible to provide accurate mathematical models of practical systems. These system uncertainties can damage the control performance or even lead to unstable of the controlled system if they are not well handled. To show the robust of the proposed method, let us consider the condition that the master system (19) suffers from time-varying system parameters and uncertainties. Suppose that the model of the master FONN (19) is replaced with

$$\mathbb{D}^q x(t) = \Delta A x(t) + B f(x(t)) + \Delta C \xi(t) + I, \tag{55}$$

where $\Delta A = A + \overline{A} \in \mathbb{R}n \times n$, $\Delta C \in \mathbb{R}^{n\times m}$, and $\xi \in \mathbb{R}^{m\times n}$ is an unknown external input signal. $\overline{A}$ and $\overline{C}$ are two

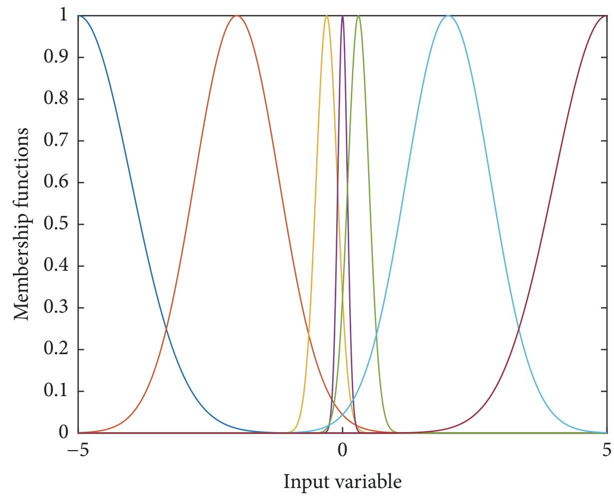


Figure 3: Membership functions.

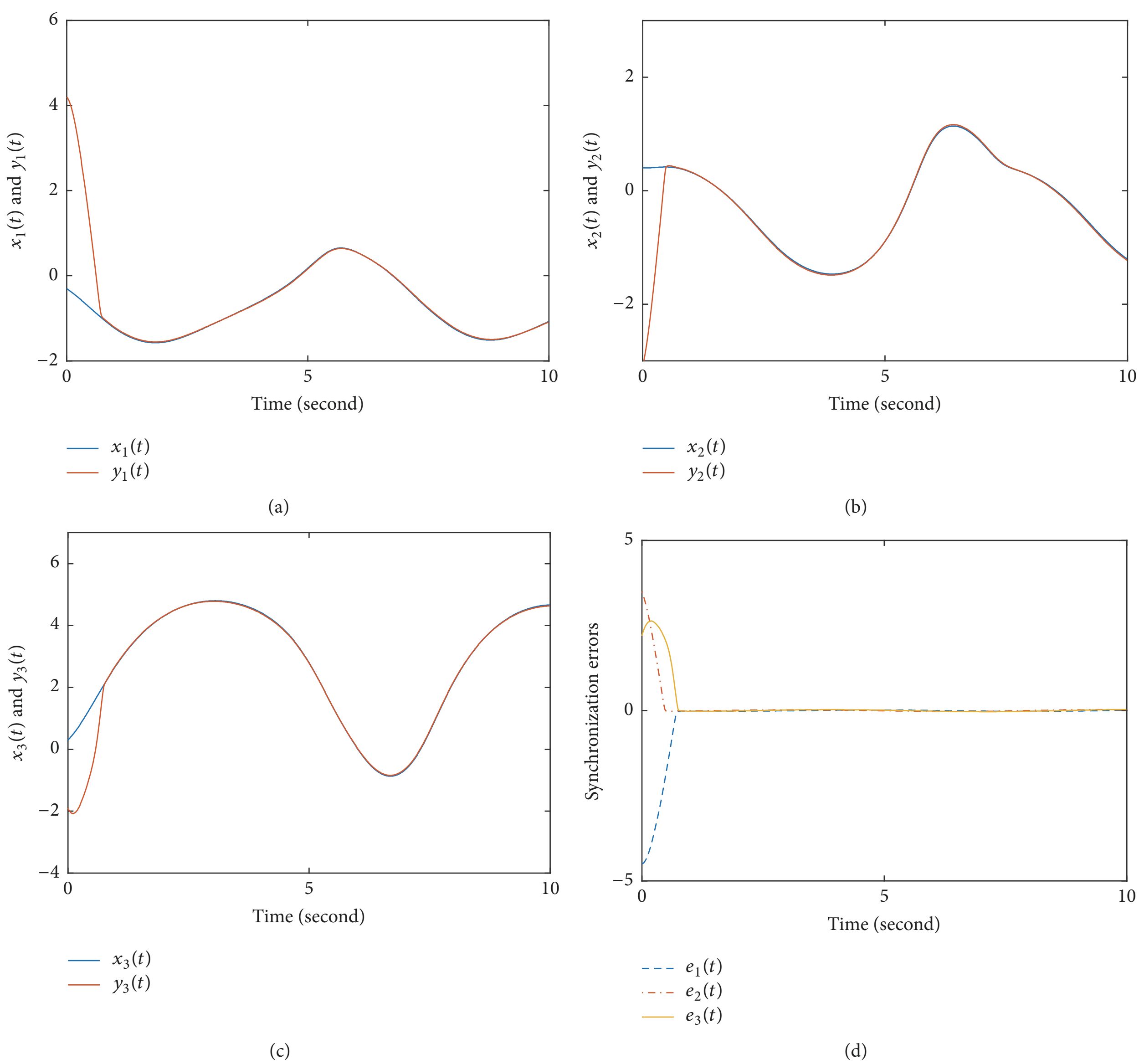


Figure 4: Simulation results in (a) $x_1(t)$ and $y_1(t)$; (b) $x_2(t)$ and $y_2(t)$; (c) $x_3(t)$ and $y_3(t)$; (d) synchronization errors.

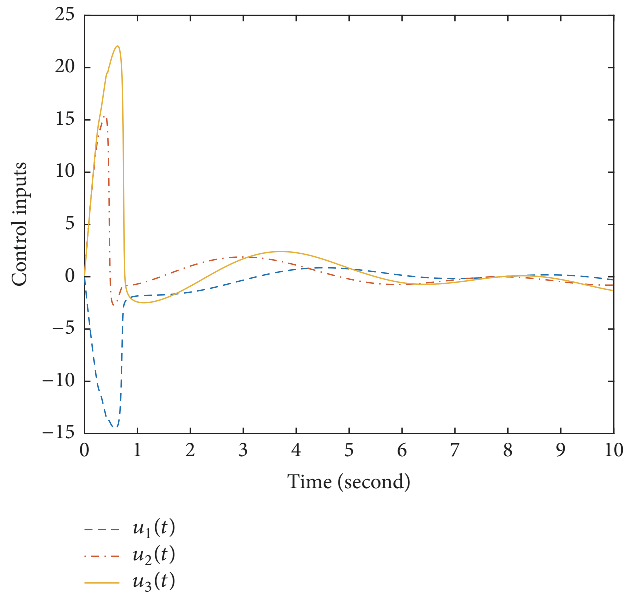


Figure 5: Control inputs.

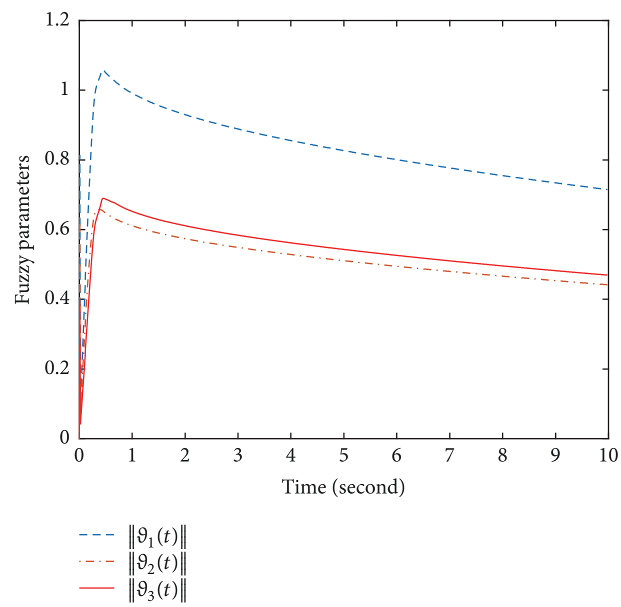


Figure 6: Fuzzy parameters.

unknown matrices. That is, $\overline{A}$ represents unknown system parameter perturbations, and $\Delta C\xi(t)$ denotes unknown input nonlinearities.

For convenience, let us suppose that $[\overline{A}, \overline{C}] = N_1 D(t) [N_2, N_3]$, where $N_1 \in \mathbb{R}^{n\times g}$, $N_2 \in \mathbb{R}^{g\times n}$ and $N_3 \in \mathbb{R}^{g\times m}$ are known matrices, and $D(t) \in \mathbb{R}^{g\times g}$ is an unknown matrix.

In the simulation, let $N_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$, $N_2 = \begin{bmatrix} 0.1 & 0.1 & 0 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}$, $N_3 = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0.1 & 0.1 & 0 \end{bmatrix}$, and $D(t) = \begin{bmatrix} 0.2\sin t & 0 \\ 0 & 0.2\cos t \end{bmatrix}$. It is easy to know that $\overline{A}$ and $\Delta C\xi(t)$ are time-varying matrices.

The simulations are presented in Figure 7. It should be pointed out that, in the simulation, the slave system and the controller are chosen to be the same as those in Section 4.1. From the simulation results we can see that good synchronization performance has been achieved even the master system suffers from time-varying parameters and input nonlinearities. That is, the proposed method has good robustness.

To indicate the effectiveness of our methods, the simulation results when $\xi(t) = -[2\sin(10t) + 15\mathrm{rand}(t), 2\cos(10t) +$

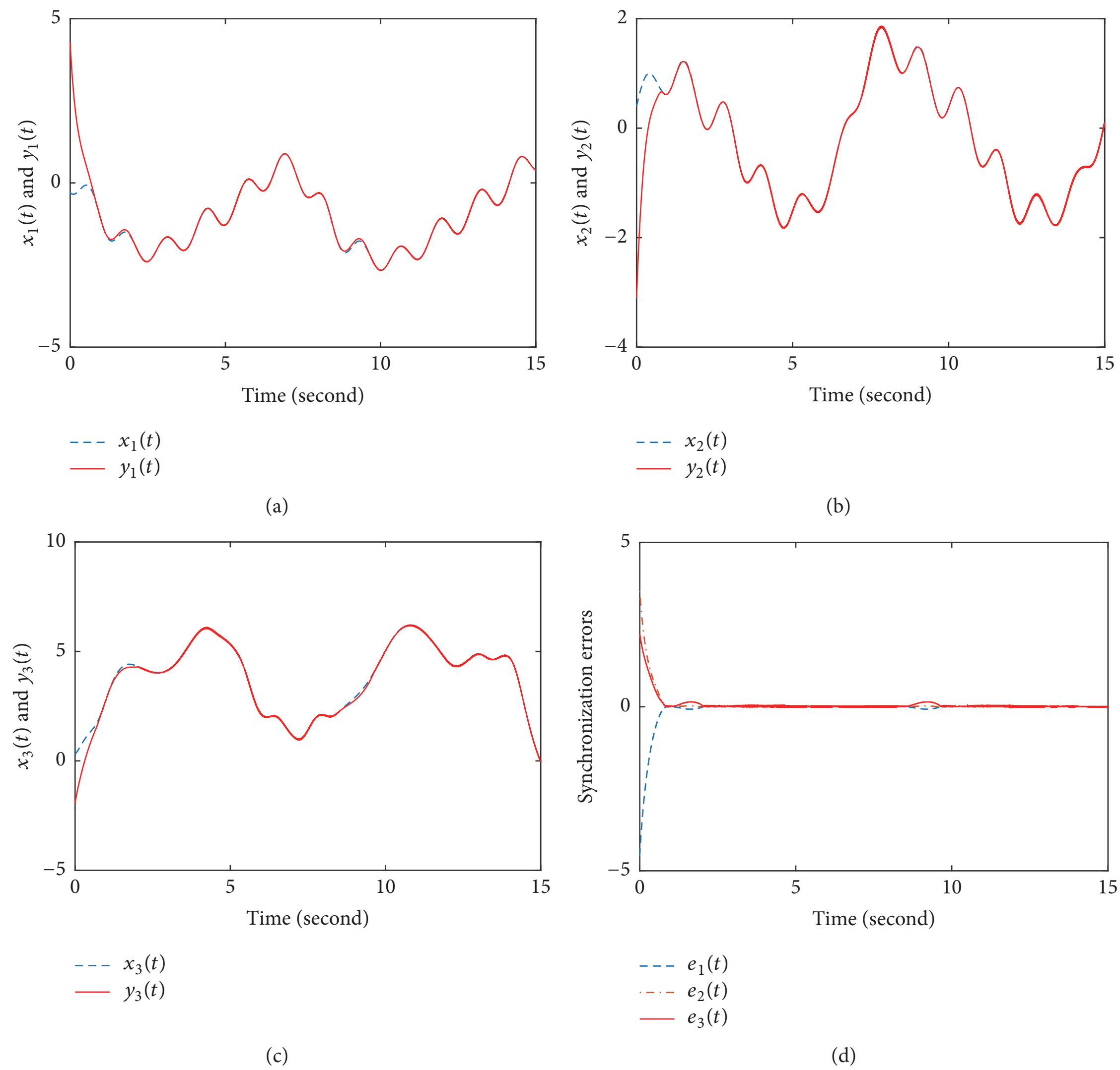


Figure 7: Simulation results when $u(t) = [5\sin(20t), 3\cos(20t), 4\sin(10t)]^T$ in (a) $x_1(t)$ and $\hat{x}_1(t)$; (b) $x_2(t)$ and $\hat{x}_2(t)$; (c) $x_3(t)$ and $\hat{x}_3(t)$; (d) synchronization errors.

$20\text{rand}(t), 2\sin(5t)+18\text{rand}(t)]^T$ where rand$(\cdot)$ represents the random function produced in MATLAB software are shown in Figure 8.

## 5. Conclusions

In this paper, an synchronization method was proposed for a class of FONNs subject to backlash-like hysteresis by means of adaptive fuzzy control. We showed that fuzzy logic systems can be employed to estimate nonlinear functions in fractional-order nonlinear systems. Based on the fractional stability theorems, an adaptive fuzzy synchronization controller, which can guarantee the synchronization error tends to an arbitrary small region of zero, was constructed. How to combine the proposed method with other control method, such as fractional-order sliding mode control, is one of our future research directions.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors do not have a direct financial relation with any commercial identity mentioned in their paper that might lead to conflicts of interest for any of the authors.


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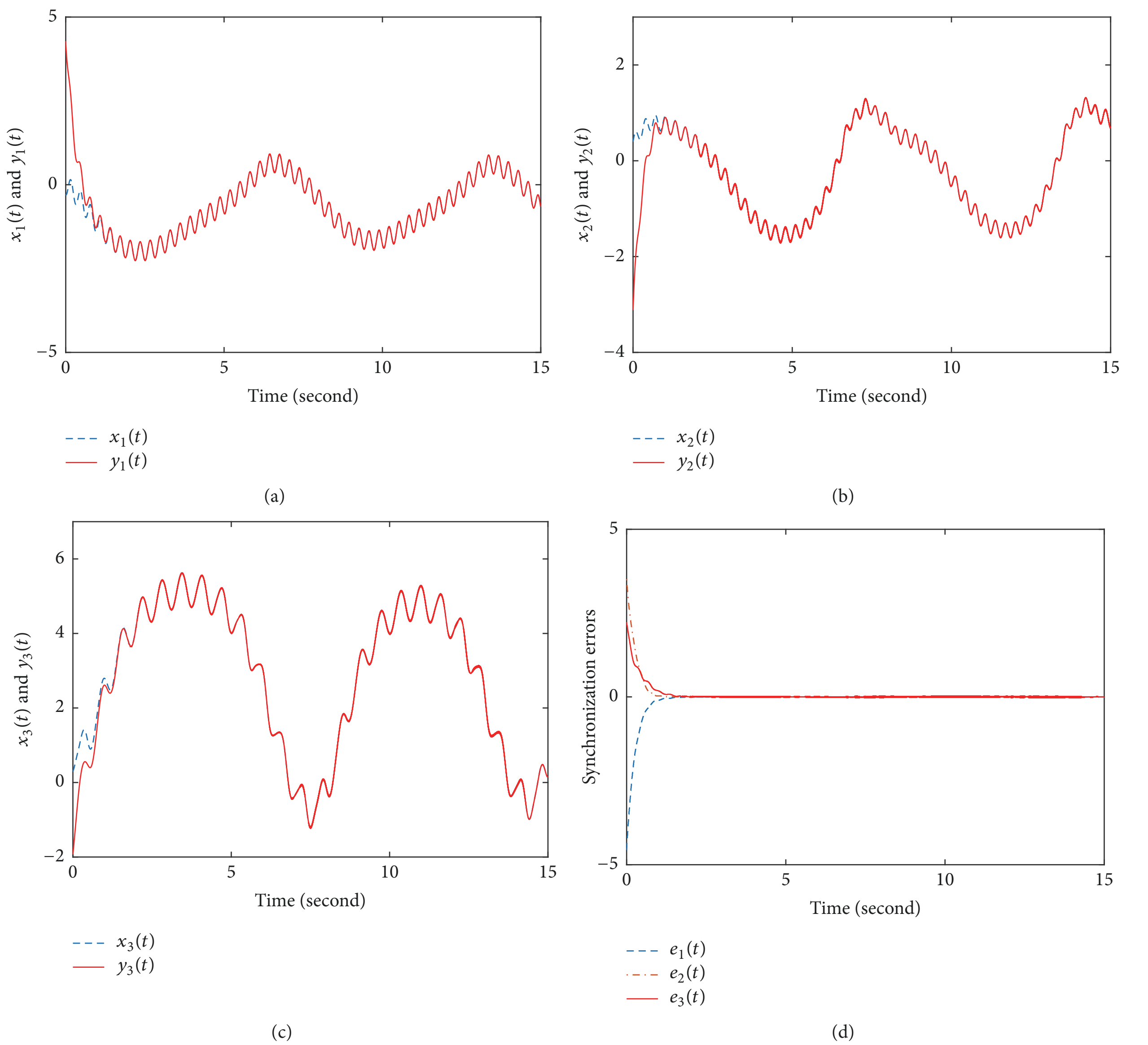


Figure 8: Simulation results when $\xi(t) = -[2\sin(10t) + 15\mathrm{rand}(t); 2\cos(10t) + 20\mathrm{rand}(t); 2\sin(5t) + 18\mathrm{rand}(t)]^T$ in (a) $x_1(t)$ and $y_1(t)$; (b) $x_2(t)$ and $y_2(t)$; (c) $x_3(t)$ and $y_3(t)$; (d) synchronization errors.

## References


[1] I. Podlubny, *Fractional Differential Equations*, vol. 198 of *Mathematics in Science and Engineering*, Academic Press, San Diego, Calif, USA, 1999.

[2] Y. Li, Y. Chen, and I. Podlubny, "Mittag-Leffler stability of fractional order nonlinear dynamic systems," *Automatica*, vol. 45, no. 8, pp. 1965–1969, 2009.

[3] H. Liu, S. Li, Y. Sun, and H. Wang, "Adaptive fuzzy synchronization for uncertain fractional-order chaotic systems with unknown non-symmetrical control gain," *Acta Physica Sinaca*, vol. 64, no. 7, Article ID 070503, 2015.

[4] N. Aguila-Camacho, M. A. Duarte-Mermoud, and J. A. Gallegos, "Lyapunov functions for fractional order systems," *Communications in Nonlinear Science and Numerical Simulation*, vol. 19, no. 9, pp. 2951–2957, 2014.

[5] H. Liu, S. Li, H. Wang, Y. Huo, and J. Luo, "Adaptive synchronization for a class of uncertain fractional-order neural networks," *Entropy. An International and Interdisciplinary Journal of Entropy and Information Studies*, vol. 17, no. 10, pp. 7185–7200, 2015.

[6] Y. Wu and H. Lv, "Adaptive neural network backstepping control for a class of uncertain fractional-order chaotic systems with unknown backlash-like hysteresis," *AIP Advances*, vol. 6, no. 8, Article ID 085121, 2016.

[7] K. Nakkeeran, "Mathematical description of differential equation solving electrical circuits," *Journal of Circuits, Systems and Computers*, vol. 18, no. 5, pp. 985–991, 2009.

[8] H. Liu, S. Li, J. D. Cao, A. G. Alsaedi, and F. E. Alsaadi, "Adaptive fuzzy prescribed performance controller design for a class of uncertain fractional-order nonlinear systems with external disturbances," *Neurocomputing*, vol. 219, pp. 422–430, 2017.

[9] C. Volos, V.-T. Pham, E. Zambrano-Serrano, J. M. Munoz-Pacheco, S. Vaidyanathan, and E. Tlelo-Cuautle, "Analysis of a 4-D hyperchaotic fractional-order memristive system with hidden attractors," in *Advances in Memristors, Memristive Devices and Systems*, vol. 701, pp. 207–235, Springer, 2017.

[10] Z.-Y. Sun, P. G. Kevrekidis, X. Yu, and K. Nakkeeran, "Theoretical and computational advances in nonlinear dynamical systems," *Advances in Mathematical Physics*, Article ID 3925964, Art. ID 3925964, 3 pages, 2017.

[11] I. Petráš, "A note on the fractional-order cellular neural networks," in *Proceedings of the International Joint Conference on Neural Networks (IJCNN '06)*, pp. 1021–1024, can, July 2006.

[12] I. Stamova, "Global Mittag-Leffler stability and synchronization of impulsive fractional-order neural networks with time-varying delays," *Nonlinear Dynamics*, vol. 77, no. 4, pp. 1251–1260, 2014.

[13] H.-B. Bao and J.-D. Cao, "Projective synchronization of fractional-order memristor-based neural networks," *Neural Networks*, vol. 63, pp. 1–9, 2015.

[14] L. P. Chen, R. C. Wu, J. Cao, and J.-B. Liu, "Stability and synchronization of memristor-based fractional-order delayed neural networks," *Neural Networks*, vol. 71, pp. 37–44, 2015.

[15] R. Rakkiyappan, R. Sivaranjani, G. Velmurugan, and J. Cao, "Analysis of global O(t-$\alpha$) stability and global asymptotical periodicity for a class of fractional-order complex-valued neural networks with time varying delays," *Neural Networks*, vol. 77, pp. 51–69, 2016.

[16] G. Velmurugan and R. Rakkiyappan, "Hybrid projective synchronization of fractional-order memristor-based neural networks with time delays," *Nonlinear Dynamics*, vol. 83, no. 1-2, pp. 419–432, 2016.

[17] H. Liu, Y. Pan, S. Li, and Y. Chen, "Synchronization for fractional-order neural networks with full/under-actuation using fractional-order sliding mode control," *International Journal of Machine Learning & Cybernetics*, pp. 1–14, 2017.

[18] H. Liu, S. Li, H. Wang, and Y. Sun, "Adaptive fuzzy control for a class of unknown fractional-order neural networks subject to input nonlinearities and dead-zones," *Information Sciences*, vol. 454-455, pp. 30–45, 2018.

[19] R. Wu, Y. Lu, and L. Chen, "Finite-time stability of fractional delayed neural networks," *Neurocomputing*, vol. 149, pp. 700–707, 2015.

[20] G. Velmurugan, R. Rakkiyappan, V. Vembarasan, J. Cao, and A. Alsaedi, "Dissipativity and stability analysis of fractional-order complex-valued neural networks with time delay," *Neural Networks*, vol. 86, pp. 42–53, 2017.

[21] A. Wu, L. Liu, T. Huang, and Z. Zeng, "Mittag-Leffler stability of fractional-order neural networks in the presence of generalized piecewise constant arguments," *Neural Networks*, vol. 85, pp. 118–127, 2017.

[22] E. Kaslik and I. R. Rădulescu, "Dynamics of complex-valued fractional-order neural networks," *Neural Networks*, vol. 89, pp. 39–49, 2017.

[23] X. Zhang and Y. Lin, "An adaptive output feedback dynamic surface control for a class of nonlinear systems with unknown backlash-like hysteresis," *Asian Journal of Control*, vol. 15, no. 2, pp. 489–500, 2013.

[24] H. Wang, B. Chen, K. Liu, X. Liu, and C. Lin, "Adaptive neural tracking control for a class of nonstrict-feedback stochastic nonlinear systems with unknown backlash-like hysteresis," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 25, no. 5, pp. 947–958, 2014.

[25] K. Chen, J. Wang, Y. Zhang, and Z. Liu, "Adaptive consensus of nonlinear multi-agent systems with unknown backlash-like hysteresis," *Neurocomputing*, vol. 175, pp. 698–703, 2015.

[26] L. Liu, Z. Wang, and H. Zhang, "Adaptive dynamic surface error constrained control for MIMO systems with backlash-like hysteresis via prediction error technique," *Nonlinear Dynamics*, vol. 84, no. 4, pp. 1989–2002, 2016.

[27] J. Zhou, C. Wen, and T. Li, "Adaptive output feedback control of uncertain nonlinear systems with hysteresis nonlinearity," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 57, no. 10, pp. 2627–2633, 2012.

[28] S. Liu, C.-Y. Su, and Z. Li, "Robust adaptive inverse control of a class of nonlinear systems with Prandtl-Ishlinskii hysteresis model," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 59, no. 8, pp. 2170–2175, 2014.

[29] R. Dong, Y. Tan, and K. Janschek, "Nonsmooth Predictive Control for Wiener Systems with Backlash-Like Hysteresis," *IEEE/ASME Transactions on Mechatronics*, vol. 21, no. 1, pp. 17–28, 2016.

[30] L. Bai, Q. Zhou, L. Wang, Z. Yu, and H. Li, "Observer-based adaptive control for stochastic nonstrict-feedback systems with unknown backlash-like hysteresis," *International Journal of Adaptive Control and Signal Processing*, vol. 31, no. 10, pp. 1481–1490, 2017.

[31] G. Cui, S. Xu, B. Zhang, J. Lu, Z. Li, and Z. Zhang, "Adaptive tracking control for uncertain switched stochastic nonlinear pure-feedback systems with unknown backlash-like hysteresis," *Journal of The Franklin Institute*, vol. 354, no. 4, pp. 1801–1818, 2017.

[32] H. A. Yousef, M. Hamdy, and K. Nashed, "Adaptive fuzzy controller for a class of nonlinear systems with unknown backlash-like hysteresis," *International Journal of Systems Science*, pp. 1–12, 2017.

[33] H. Wang, P. X. Liu, and S. Liu, "Adaptive neural synchronization control for bilateral teleoperation systems with time delay and backlash-like hysteresis," *IEEE Transactions on Cybernetics*, 2017.

[34] A. Boulkroune, A. Bouzeriba, T. Bouden, and A. T. Azar, "Fuzzy adaptive synchronization of uncertain fractional-order chaotic systems," in *Advances in chaos theory and intelligent control*, vol. 337, pp. 681–697, Springer, 2016.

[35] Y. Li, S. Sui, and S. Tong, "Adaptive fuzzy control design for stochastic nonlinear switched systems with arbitrary switchings and unmodeled dynamics," *IEEE Transactions on Cybernetics*, vol. 47, no. 2, pp. 403–414, 2017.

[36] S. Yin, P. Shi, and H. Yang, "Adaptive fuzzy control of strict-feedback nonlinear time-delay systems with unmodeled dynamics," *IEEE Transactions on Cybernetics*, vol. 46, no. 8, pp. 1926–1938, 2016.

[37] H. Li, L. Wang, and H. Du, "Adaptive fuzzy backstepping tracking control for strict-feedback systems with input delay," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 3, pp. 642–652, 2017.

[38] H. Li, J. Wang, and P. Shi, "Output-feedback based sliding mode control for fuzzy systems with actuator saturation," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 6, pp. 1282–1293, 2016.

[39] X. Ma, J. Zhan, M. I. Ali, and N. Mehmood, "A survey of decision making methods based on two classes of hybrid soft set models," *Artificial Intelligence Review*, vol. 49, no. 4, pp. 511–529, 2018.

[40] Y.-J. Liu and S. Tong, "Barrier Lyapunov functions-based adaptive control for a class of nonlinear pure-feedback systems with full state constraints," *Automatica*, vol. 64, pp. 70–75, 2016.

[41] Y. Pan and H. Yu, "Composite learning from adaptive dynamic surface control," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 61, no. 9, pp. 2603–2609, 2016.

[42] Y. Pan, Y. Liu, B. Xu, and H. Yu, "Hybrid feedback feedforward: An efficient design of adaptive neural network control," *Neural Networks*, vol. 76, pp. 122–134, 2016.

[43] Y. Liu, C. Li, T. Huang, and X. Wang, "Robust adaptive lag synchronization of uncertain fuzzy memristive neural networks with time-varying delays," *Neurocomputing*, vol. 190, pp. 188–196, 2016.

[44] S. Wen, Z. Zeng, T. Huang, and Y. Zhang, "Exponential adaptive lag synchronization of memristive neural networks via fuzzy method and applications in pseudo random number generators," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 6, pp. 1704–1713, 2014.

[45] J. Zhan and J. C. R. Alcantud, "A novel type of soft rough covering and its application to multicriteria group decision making," *Artificial Intelligence Review*, pp. 1–30, 2018.

[46] H. Liu, Y. Pan, S. Li, and Y. Chen, "Adaptive Fuzzy Backstepping Control of Fractional-Order Nonlinear Systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 8, pp. 2209–2217, 2017.

[47] Y. Pan and H. Yu, "Biomimetic hybrid feedback feedforward neural-network learning control," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 6, pp. 1481–1487, 2017.

[48] Y. Pan, M. J. Er, D. Huang, and Q. Wang, "Adaptive fuzzy control with guaranteed convergence of optimal approximation error," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 5, pp. 807–818, 2011.

[49] H. Liu, S. Li, G. Li, and H. Wang, "Adaptive controller design for a class of uncertain fractional-order nonlinear systems: an adaptive fuzzy approach," *International Journal of Fuzzy Systems*, vol. 20, no. 2, pp. 366–379, 2018.

[50] H. Liu, Y. Chen, G. Li, W. Xiang, and G. Xu, "Adaptive fuzzy synchronization of fractional-order chaotic (hyperchaotic) systems with input saturation and unknown parameters," *Complexity*, vol. 2017, 16 pages, 2017.

[51] F. Zhong, H. Li, and S. Zhong, "State estimation based on fractional order sliding mode observer method for a class of uncertain fractional-order nonlinear systems," *Signal Processing*, vol. 127, pp. 168–184, 2016.