Hindawi
Advances in Mathematical Physics
Volume 2019, Article ID 2085318, 10 pages
https://doi.org/10.1155/2019/2085318

Research Article

Combined Synchronization among Three Inconsistent Networks

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Received 14 September 2018; Revised 18 December 2018; Accepted 24 December 2018; Published 13 February 2019

Academic Editor: Zengtao Chen

A new type of network-combination synchronization is proposed between two drive networks, small-world and scale-free, and one response community network based on the model of multiple system combination. According to Lyapunov Stability Theorem, combined controllers are designed to control three disparate networks with different topological structures. A realistic combination model of multiple inhomogeneous networks and a universal controller are devised in the combination design. The paper is characterized by two parts of one response community network loaded from the combined signals of two drive networks. Numerical simulations are presented to show the full and partial control of the proposed method.

1. Introduction

The phenomenon of synchronization is commonplace in the present world. Since the creation of chaotic synchronization method [1], the wide use of various synchronization designs in every area has been proposed to better explain the phenomena of nature [2–6], such as complete synchronization [7], generalized synchronization [8], and adaptive synchronization [9], which, however, are drive-response system types and only remain within one-to-one synchronization schemes in the first research phase. Besides, these methods, the one-to-one synchronization in particular, are incapable of meeting more complex real requirements and vulnerable to information transfer due to the neglect of more chaotic systems by most researchers. Faced with this, it is imperative to develop some new synchronization methods of extending the synchronization of one-to-one system to that among many systems from which combination between many drive systems and a response system is drawn in the second research phase. However, despite their contributions to diversified synchronization [10–17], more exploration is needed and research on the synchronization problems about combination synchronization is still ongoing and extensible since the existing methods fail to meet the synchronization of multiple networks in view of the complexity of natural and realistic synchronous types and their tendency to combine network synchronization. There are many different forms of interacting networks, for example, a brain network, or more probably, a brain community network, constituted by two community structures of cerebrum and cerebellum [18]. The way in which the brain network receives signals from external multinetworks of the cosmos is very similar to the combination synchronization between the combined drive signals of audio-visual networks and the signals transmitted to the brain. Analogously, the signals of the recipient brain network synchronize with multisignals from outside combined networks. These studies have contributed to diversified synchronization, which should be further explored.

Inspired by the above discussions, a novel model of network combination with two drive networks, small-world and scale-free, and one response community network is proposed in this paper. Combined state variables of corresponding nodes in the two classic complex networks of small-world and scale-free are chosen as combined drive signals, which drive the two different types of response signals from state variables of corresponding nodes in one response community network. This kind of synchronization is defined as network-combination synchronization, with combination synchronization among multicorresponding nodes and selective control as its highlights compared with traditional methods of combination synchronization.

Compared with prior work, an innovative theoretical model of network-combination synchronization with multisignals from three inhomogeneous networks, i.e., two different drive networks and one response network with two community structures, is built in this paper. In contrast to

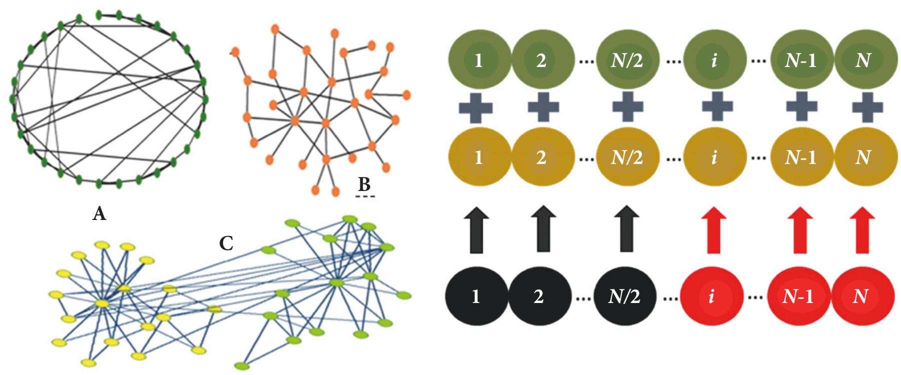

FIGURE 1: (Color online) Schematic of different topological structures of three networks.

the compound synchronization in [19–23], the combined synchronization of identical and different chaotic systems and of integer- and fractional-order chaotic systems are investigated within the category of full control without considering partial synchronization. Compared with achievements in [24, 25], these researches study the efficiency gains of the theory of finite-time synchronization, which realizes the synchronization between two complex-variable chaotic systems and the real combination synchronization of three complex-variable chaotic systems in a given finite time with identifying the unknown parameters. Especially in [25], the research of combined synchronization strategy has shifted from real-valued chaotic systems to complex-variable chaotic systems. However, there is not a combined control theory applicable to multiple networks. Motivated and compared by those results, this paper extends the combined theory of complex systems to that of multiple networks. The innovation of the paper lies in the combined synchronization among the corresponding nodes from three inconsistent networks under the full and partial control law by adjusting combined projective matrix, which increases the operating controllability of compound network synchronization and is different from general chaotic synchronization in earlier studies. The type of network-combination synchronization is a new theoretical framework requiring an even more elaborate work-around.

The organization of this paper is as follows. Besides introduction and conclusion, the basic concept and special design of network-combination synchronization is introduced in the second section. While in section three, the network-combination synchronization among Rössler small-world network, Lü scale-free network, and Lorenz Chen community network is realized by Lyapunov Stability Theorem, and two types of numerical simulations are presented to show the comprehensiveness and diversity of the combined design.

2. Scheme of Network-Combination Synchronization

Network-combination synchronization among multiple inhomogeneous networks, two drive networks, and one response network will be introduced in this part.

The first drive network is presented as

$$\dot{\boldsymbol{x}}_i(t) = \boldsymbol{F}_1(\boldsymbol{x}_i(t)) + k_1 \sum_{i=1}^{N} d_{ij} \boldsymbol{x}_i(t) \quad (i = 1, 2, \ldots, N), \tag{1}$$

and the second drive network is given by

$$\dot{\boldsymbol{y}}_i(t) = \boldsymbol{F}_2(\boldsymbol{y}_i(t)) + k_2 \sum_{i=1}^{N} g_{ij} \boldsymbol{y}_i(t) \quad (i = 1, 2, \ldots, N), \tag{2}$$

where $\boldsymbol{x}_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T \in \boldsymbol{R}^n$ and $\boldsymbol{y}_i(t) = [y_{i1}(t), y_{i2}(t), \ldots, y_{in}(t)]^T \in \boldsymbol{R}^n$ are the state variables of drive networks, $k_i > 0$ being the coupling strength.

The response community network is represented by

$$\dot{\boldsymbol{z}}_i(t) = \boldsymbol{F}_3^r(\boldsymbol{z}_i(t)) + k_3 \sum_{i=1}^{N} h_{ij} \boldsymbol{z}_i(t) + \boldsymbol{U}_i(x_i, y_i, z_i) \qquad (i = 1, 2, \ldots, N) \;\; i \in G_r, \tag{3}$$

where $\boldsymbol{z}_i(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T \in \boldsymbol{R}^n$ is the system's n-dimensional state variable in the response system. $\boldsymbol{F}_1 : \boldsymbol{R}^n \longrightarrow \boldsymbol{R}^n$, $\boldsymbol{F}_2 : \boldsymbol{R}^n \longrightarrow \boldsymbol{R}^n$, and $\boldsymbol{F}_3^r : \boldsymbol{R}^n \longrightarrow \boldsymbol{R}^n$ are the nonlinear parts of (1)–(3). In $i \in \{1, 2, \ldots, N\}$, $r \in \{1, 2, \ldots, v\}$, $G_1 = \{1, 2, \ldots, P_1\}$, $G_2 = \{P_1 + 1, P_1 + 2, \ldots, P_1 + P_2\}, \ldots,$ $G_v = \{P_1 + P_2 + \ldots + P_{v-1} + 1, \ldots, P_1 + P_2 + \ldots + P_v\}$, $1 \leqslant v \leqslant N$ and $1 \leqslant P_r \leqslant N$, satisfying $\sum_{r=1}^{v} P_r = N$ for $r \in \{1, 2, \ldots, v\}$. $\boldsymbol{U}_i(x_i, y_i, z_i) \in \boldsymbol{R}^n$ is a designed controller for network-combination synchronization.

Definition 1. Network-combination synchronization among the drive small-world network (1), drive scale-free network (2), and the community network (3) can be realized, given that the following condition is satisfied:

$$\lim_{t \longrightarrow \infty} \left\| \boldsymbol{C}_i \boldsymbol{z}_i - \boldsymbol{A} \boldsymbol{x}_i - \boldsymbol{B} \boldsymbol{y}_i \right\| = 0 \quad (i = 1, 2, \ldots, N), \tag{4}$$

where $\|.\|$ denotes Euclidean norm and constant matrices satisfy $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}_i \in \boldsymbol{R}^n$ and $\boldsymbol{C}_i \neq 0$.

As shown in the left panel in Figure 1, the three classic networks of small-world network (A), scale-free network (B), and community network (C) are selected as research subjects. The right panel displays network-combination synchronization with N green balls representing small-world network nodes at the top layer, N amber balls indicating scale-free network nodes at the middle layer, and $N/2$ black balls and red balls showing the nodes of two communities in a response network at the bottom layer. The first two networks can be combined as drive combination networks, the N nodes of which can output N signals as combined drive signals. Correspondingly, one response network at the bottom layer has N receiving nodes and each community has $N/2$ different receiving nodes.

3. Numerical Results

With two types of combination control, full and partial, network-combination synchronization is designed to upgrade one-on-one situation to synchronization among multinetworks in order to be more consistent with the ever-developing trend of real-world combined synchronization. In this section, the full and partial control of network-combination synchronization among three topologically inequivalent networks will be analyzed. Firstly, the double drive networks are described as [26–29]

$$\begin{aligned}
\dot{x}_{i1} &= -x_{i2} - x_{i3} + k_1 \sum_{i=1}^{N} g_{ij} x_{i1}, \\
\dot{x}_{i2} &= x_{i1} + a_1 x_{i2} + k_1 \sum_{i=1}^{N} g_{ij} x_{i2}, \\
\dot{x}_{i3} &= b_1 + x_{i3}\left(x_{i2} - c_1\right) + k_1 \sum_{i=1}^{N} g_{ij} x_{i3}, \\
& \qquad i = (1, 2, \dots, N), \\
\dot{y}_{i1} &= a_2\left(y_{i2} - y_{i1}\right) + k_2 \sum_{i=1}^{N} d_{ij} y_{i1}, \\
\dot{y}_{i2} &= c_2 y_{i1} - y_{i1} y_{i3} + k_2 \sum_{i=1}^{N} d_{ij} y_{i2}, \\
\dot{y}_{i3} &= y_{i1} y_{i2} - b_2 y_{i3} + k_2 \sum_{i=1}^{N} d_{ij} y_{i3}, \\
& \qquad i = (1, 2, \dots, N),
\end{aligned} \tag{5}$$

where the system parameters are $a_1 = 0.2$, $b_1 = 0.2$, $c_1 = 5.7$ and $a_2 = 36$, $b_2 = 3$, $c_2 = 20$, respectively, and the response community networks built by Lorenz system and Chen system are depicted as

$$\begin{aligned}
\dot{z}_{i1} &= a_3\left(z_{i2} - z_{i1}\right) + k_3 \sum_{i=1}^{N} h_{ij} z_{i1} + u_{i1}, \\
\dot{z}_{i2} &= -b_3 z_{i1} - z_{i2} - z_{i1} z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i2} + u_{i2}, \\
\dot{z}_{i3} &= z_{i1} z_{i2} - c_3 z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i3} + u_{i3}, \\
& \qquad i = \left(1, 2, \dots, \frac{N}{2}\right) \ i \in G_1, \\
\dot{z}_{i1} &= a_4\left(z_{i2} - z_{i1}\right) + k_3 \sum_{i=1}^{N} h_{ij} z_{i1} + u_{i1}, \\
\dot{z}_{i2} &= \left(c_4 - a_4\right) z_{i1} + c_4 z_{i2} - z_{i1} z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i2} + u_{i2}, \\
\dot{z}_{i3} &= z_{i1} z_{i2} - b_4 z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i3} + u_{i3}, \\
& \qquad i = \left(\frac{N}{2} + 1, \frac{N}{2} + 2, \dots, N\right) \ i \in G_2,
\end{aligned} \tag{6}$$

where the system parameters are $a_3 = 10$, $b_3 = 28$, $c_3 = 2.667$ and $a_4 = 35$, $b_4 = 3$, $c_4 = 28$, respectively, u_{i1}, u_{i2}, u_{i3} being community controllers.

For the convenience of discussion, $\boldsymbol{A} = \operatorname{diag}(\alpha_1, \alpha_2, \alpha_3)$, $\boldsymbol{B} = \operatorname{diag}(\beta_1, \beta_2, \beta_3)$, and $\mathbf{C}_{\boldsymbol{i}} = \operatorname{diag}(\gamma_{i1}, \gamma_{i2}, \gamma_{i3})$ are assumed in the synchronization scheme, with N error systems shown as

$$\begin{aligned}
e_{i1} &= \gamma_{i1} z_{i1} - \alpha_1 x_{i1} - \beta_1 y_{i1} \\
e_{i2} &= \gamma_{i2} z_{i2} - \alpha_2 x_{i2} - \beta_2 y_{i2} \\
e_{i3} &= \gamma_{i3} z_{i3} - \alpha_3 x_{i3} - \beta_3 y_{i3} \\
& \qquad i = (1, 2, \dots, N),
\end{aligned} \tag{7}$$

which denote $U_{i1} = \gamma_{i1} u_{i1}$, $U_{i2} = \gamma_{i2} u_{i2}$, $U_{i3} = \gamma_{i3} u_{i3}$.

Theorem 2. *If the community controllers are designed as follows:*

$$\begin{aligned}
U_{i1} &= -\left(\gamma_{i1} z_{i1} - \alpha_1 x_{i1} - \beta_1 y_{i1}\right) \\
&\quad - a_2\left(\gamma_{i2} z_{i2} - \alpha_2 x_{i2} - \beta_2 y_{i2}\right) \\
&\quad - \gamma_{i1}\left[a_3\left(z_{i2} - z_{i1}\right) + k_3 \sum_{i=1}^{N} h_{ij} z_{i1}\right] \\
&\quad + \alpha_1\left(-x_{i2} - x_{i3} + k_1 \sum_{i=1}^{N} g_{ij} x_{i1}\right) \\
&\quad + \beta_1\left[a_2\left(y_{i2} - y_{i1}\right) + k_2 \sum_{i=1}^{N} d_{ij} y_{i1}\right] \\
U_{i2} &= -\left(\gamma_{i2} z_{i2} - \alpha_2 x_{i2} - \beta_2 y_{i2}\right) \\
&\quad + a_2\left(\gamma_{i1} z_{i1} - \alpha_1 x_{i1} - \beta_1 y_{i1}\right) \\
&\quad + a_3\left(\gamma_{i3} z_{i3} - \alpha_3 x_{i3} - \beta_3 y_{i3}\right)
\end{aligned}$$

$$
\begin{aligned}
&- \gamma_{i2}\left(-b_3 z_{i1} - z_{i2} - z_{i1} z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i2}\right) \\
&+ \alpha_2 \left(x_{i1} + a_1 x_{i2} + k_1 \sum_{i=1}^{N} g_{ij} x_{i2}\right) \\
&+ \beta_2 \left(c_2 y_{i1} - y_{i1} y_{i3} + k_2 \sum_{i=1}^{N} d_{ij} y_{i2}\right)
\end{aligned}
$$

$$
\begin{aligned}
U_{i3} &= -\left(\gamma_{i3} z_{i3} - \alpha_3 x_{i3} - \beta_3 y_{i3}\right) \\
&- a_3 \left(\gamma_{i2} z_{i2} - \alpha_2 x_{i2} - \beta_2 y_{i2}\right) \\
&- \gamma_{i3}\left[z_{i1} z_{i2} - c_3 z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i3}\right] \\
&+ \alpha_3 \left[b_1 + x_{i3}\left(x_{i2} - c_2\right) + k_1 \sum_{i=1}^{N} g_{ij} x_{i3}\right] \\
&+ \beta_3 \left(y_{i1} y_{i2} - b_2 y_{i3} + k_2 \sum_{i=1}^{N} d_{ij} y_{i3}\right) \\
&\qquad i = \left(1, 2, \ldots, \frac{N}{2}\right),
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
U_{i1} &= -\left(\gamma_{i1} z_{i1} - \alpha_1 x_{i1} - \beta_1 y_{i1}\right) \\
&- a_2 \left(\gamma_{i2} z_{i2} - \alpha_2 x_{i2} - \beta_2 y_{i2}\right) \\
&- \gamma_{i1}\left[a_4\left(z_{i2} - z_{i1}\right) + k_3 \sum_{i=1}^{N} h_{ij} z_{i1}\right] \\
&+ \alpha_1 \left(-x_{i2} - x_{i3} + k_1 \sum_{i=1}^{N} g_{ij} x_{i1}\right) \\
&+ \beta_1 \left[a_2\left(y_{i2} - y_{i1}\right) + k_2 \sum_{i=1}^{N} d_{ij} y_{i1}\right]
\end{aligned}
$$

$$
\begin{aligned}
U_{i2} &= -\left(\gamma_{i2} z_{i2} - \alpha_2 x_{i2} - \beta_2 y_{i2}\right) \\
&+ a_2 \left(\gamma_{i1} z_{i1} - \alpha_1 x_{i1} - \beta_1 y_{i1}\right) \\
&+ a_3 \left(\gamma_{i3} z_{i3} - \alpha_3 x_{i3} - \beta_3 y_{i3}\right) \\
&- \gamma_{i2}\left(\left(c_4 - a_4\right) z_{i1} + c_4 z_{i2} - z_{i1} z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i2}\right) \\
&+ \alpha_2 \left(x_{i1} + a_1 x_{i2} + k_1 \sum_{i=1}^{N} g_{ij} x_{i2}\right) \\
&+ \beta_2 \left(c_2 y_{i1} - y_{i1} y_{i3} + k_2 \sum_{i=1}^{N} d_{ij} y_{i2}\right)
\end{aligned}
$$

$$
\begin{aligned}
U_{i3} &= -\left(\gamma_{i3} z_{i3} - \alpha_3 x_{i3} - \beta_3 y_{i3}\right) \\
&- a_3 \left(\gamma_{i2} z_{i2} - \alpha_2 x_{i2} - \beta_2 y_{i2}\right) \\
&- \gamma_{i3}\left[z_{i1} z_{i2} - b_4 z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i3}\right] \\
&+ \alpha_3 \left[b_1 + x_{i3}\left(x_{i2} - c_2\right) + k_1 \sum_{i=1}^{N} g_{ij} x_{i3}\right] \\
&+ \beta_3 \left(y_{i1} y_{i2} - b_2 y_{i3} + k_2 \sum_{i=1}^{N} d_{ij} y_{i3}\right) \\
&\qquad i = \left(\frac{N}{2} + 1, \frac{N}{2} + 2, \ldots, N\right),
\end{aligned} \tag{9}
$$

then the network-combination synchronization between the drive systems (1) and (2) and the response system (3) will be achieved.

Proof. A constructor Lyapunov function is given as the following:

$$
V\left(e_{i1}, e_{i2}, e_{i3}\right) = \frac{1}{2}\left(\sum_{i=1}^{N} e_{i1}^2 + \sum_{i=1}^{N} e_{i2}^2 + \sum_{i=1}^{N} e_{i3}^2\right), \tag{10}
$$

$$
\begin{aligned}
\dot{V} &= \sum_{i=1}^{N} e_{i1}\dot{e}_{i1} + \sum_{i=1}^{N} e_{i2}\dot{e}_{i2} + \sum_{i=1}^{N} e_{i3}\dot{e}_{i3} \quad i = \left(1, 2, \ldots, \frac{N}{2}\right) \\
&= \sum_{i=1}^{N} e_{i1}\left\{\gamma_{i1}\left[a_3\left(z_{i2} - z_{i1}\right) + k_3 \sum_{i=1}^{N} h_{ij} z_{i1}\right] - \alpha_1\left(-x_{i2}\right.\right. \\
&\quad \left. - x_{i3} + k_1 \sum_{i=1}^{N} g_{ij} x_{i1}\right) - \beta_1\left[a_2\left(y_{i2} - y_{i1}\right)\right. \\
&\quad \left.\left. + k_2 \sum_{i=1}^{N} d_{ij} y_{i1}\right]\right\} + \sum_{i=1}^{N} e_{i1}\gamma_{i1}u_{i1} + \sum_{i=1}^{N} e_{i2}\left\{\gamma_{i2}\left(-b_3 z_{i1}\right.\right. \\
&\quad \left. - z_{i2} - z_{i1} z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i2}\right) - \alpha_2\left(x_{i1} + a_1 x_{i2}\right. \\
&\quad \left. + k_1 \sum_{i=1}^{N} g_{ij} x_{i2}\right) - \beta_2\left(c_2 y_{i1} - y_{i1} y_{i3}\right. \\
&\quad \left.\left. + k_2 \sum_{i=1}^{N} d_{ij} y_{i2}\right)\right\} + \sum_{i=1}^{N} e_{i2}\gamma_{i2}u_{i2} + \sum_{i=1}^{N} e_{i3}\left\{\gamma_{i3}\left[z_{i1} z_{i2}\right.\right. \\
&\quad \left. - c_3 z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i3}\right] - \alpha_3\left[b_1 + x_{i3}\left(x_{i2} - c_1\right)\right. \\
&\quad \left. + k_1 \sum_{i=1}^{N} g_{ij} x_{i3}\right] - \beta_3\left(y_{i1} y_{i2} - b_2 y_{i3}\right. \\
&\quad \left.\left. + k_2 \sum_{i=1}^{N} d_{ij} y_{i3}\right)\right\} + \sum_{i=1}^{N} e_{i3}\gamma_{i3}u_{i3},
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
\dot{V} ={}& \sum_{i=1}^{N} e_{i1} \left\{ \gamma_{i1} \left[a_3 \left(z_{i2} - z_{i1} \right) + k_3 \sum_{i=1}^{N} h_{ij} z_{i1} \right] \right. \\
& - \alpha_1 \left(-x_{i2} - x_{i3} + k_1 \sum_{i=1}^{N} g_{ij} x_{i1} \right) - \beta_1 \left[a_2 \left(y_{i2} \right. \right. \\
& \left. \left. \left. - y_{i1} \right) + k_2 \sum_{i=1}^{N} d_{ij} y_{i1} \right] \right\} + \sum_{i=1}^{N} e_{i1} \left\{ - \left(\gamma_{i1} z_{i1} - \alpha_1 x_{i1} \right. \right. \\
& \left. - \beta_1 y_{i1} \right) - a_2 \left(\gamma_{i2} z_{i2} - \alpha_2 x_{i2} - \beta_2 y_{i2} \right) \\
& - \gamma_{i1} \left[a_3 \left(z_{i2} - z_{i1} \right) + k_3 \sum_{i=1}^{N} h_{ij} z_{i1} \right] + \alpha_1 \left(-x_{i2} \right. \\
& \left. - x_{i3} + k_1 \sum_{i=1}^{N} g_{ij} x_{i1} \right) + \beta_1 \left[a_2 \left(y_{i2} - y_{i1} \right) \right. \\
& \left. \left. + k_2 \sum_{i=1}^{N} d_{ij} y_{i1} \right] \right\} \\
& + \sum_{i=1}^{N} e_{i2} \left\{ \gamma_{i2} \left(-b_3 z_{i1} - z_{i2} - z_{i1} z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i2} \right) \right. \\
& - \alpha_2 \left(x_{i1} + a_1 x_{i2} + k_1 \sum_{i=1}^{N} g_{ij} x_{i2} \right) - \beta_2 \left(c_2 y_{i1} \right. \\
& \left. \left. - y_{i1} y_{i3} + k_2 \sum_{i=1}^{N} d_{ij} y_{i2} \right) \right\} + \sum_{i=1}^{N} e_{i2} \left\{ - \left(\gamma_{i2} z_{i2} - \alpha_2 x_{i2} \right. \right. \\
& \left. - \beta_2 y_{i2} \right) + a_2 \left(\gamma_{i1} z_{i1} - \alpha_1 x_{i1} - \beta_1 y_{i1} \right) + a_3 \left(\gamma_{i3} z_{i3} \right. \\
& \left. - \alpha_3 x_{i3} - \beta_3 y_{i3} \right) - \gamma_{i2} \left(-b_3 z_{i1} - z_{i2} - z_{i1} z_{i3} \right. \\
& \left. + k_3 \sum_{i=1}^{N} h_{ij} z_{i2} \right) + \alpha_2 \left(x_{i1} + a_1 x_{i2} + k_1 \sum_{i=1}^{N} g_{ij} x_{i2} \right) \\
& \left. + \beta_2 \left(c_2 y_{i1} - y_{i1} y_{i3} + k_2 \sum_{i=1}^{N} d_{ij} y_{i2} \right) \right\} \\
& + \sum_{i=1}^{N} e_{i3} \left\{ \gamma_{i3} \left[z_{i1} z_{i2} - c_3 z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i3} \right] - \alpha_3 \left[b_1 \right. \right. \\
& \left. + x_{i3} \left(x_{i2} - c_1 \right) + k_1 \sum_{i=1}^{N} g_{ij} x_{i3} \right] - \beta_3 \left(y_{i1} y_{i2} - b_2 y_{i3} \right. \\
& \left. \left. + k_2 \sum_{i=1}^{N} d_{ij} y_{i3} \right) \right\} + \sum_{i=1}^{N} e_{i3} \left\{ - \left(\gamma_{i3} z_{i3} - \alpha_3 x_{i3} \right. \right. \\
& \left. - \beta_3 y_{i3} \right) - a_3 \left(\gamma_{i2} z_{i2} - \alpha_2 x_{i2} - \beta_2 y_{i2} \right) - \gamma_{i3} \left[z_{i1} z_{i2} \right. \\
& \left. - c_3 z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i3} \right] + \alpha_3 \left[b_1 + x_{i3} \left(x_{i2} - c_1 \right) \right. \\
& \left. + k_1 \sum_{i=1}^{N} g_{ij} x_{i3} \right] + \beta_3 \left(y_{i1} y_{i2} - b_2 y_{i3} \right. \\
& \left. \left. + k_2 \sum_{i=1}^{N} d_{ij} y_{i3} \right) \right\} \\
={}& \sum_{i=1}^{N} e_{i1} \left(-e_{i1} - a_2 e_{i2} \right) + \sum_{i=1}^{N} e_{i2} \left(-e_{i2} + a_2 e_{i1} + a_3 e_{i3} \right) \\
& + \sum_{i=1}^{N} e_{i3} \left(-e_{i3} - a_3 e_{i2} \right) = - \sum_{i=1}^{N} e_{i1}^2 - \sum_{i=1}^{N} e_{i2}^2 - \sum_{i=1}^{N} e_{i3}^2,
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
\dot{V} ={}& \sum_{i=1}^{N} e_{i1} \dot{e}_{i1} + \sum_{i=1}^{N} e_{i2} \dot{e}_{i2} + \sum_{i=1}^{N} e_{i3} \dot{e}_{i3} \\
& \qquad i = \left(\frac{N}{2} + 1, \frac{N}{2} + 2, \ldots, N \right) \\
={}& \sum_{i=1}^{N} e_{i1} \left\{ \gamma_{i1} \left[a_4 \left(z_{i2} - z_{i1} \right) + k_3 \sum_{i=1}^{N} h_{ij} z_{i1} \right] - \alpha_1 \left(-x_{i2} \right. \right. \\
& \left. - x_{i3} + k_1 \sum_{i=1}^{N} g_{ij} x_{i1} \right) - \beta_1 \left[a_2 \left(y_{i2} - y_{i1} \right) \right. \\
& \left. \left. + k_2 \sum_{i=1}^{N} d_{ij} y_{i1} \right] \right\} + \sum_{i=1}^{N} e_{i1} \gamma_{i1} u_{i1} \\
& + \sum_{i=1}^{N} e_{i2} \left\{ \gamma_{i2} \left(\left(c_4 - a_4 \right) z_{i1} + c_4 z_{i2} - z_{i1} z_{i3} \right. \right. \\
& \left. + k_3 \sum_{i=1}^{N} h_{ij} z_{i2} \right) - \alpha_2 \left(x_{i1} + a_1 x_{i2} + k_1 \sum_{i=1}^{N} g_{ij} x_{i2} \right) \\
& \left. - \beta_2 \left(c_2 y_{i1} - y_{i1} y_{i3} + k_2 \sum_{i=1}^{N} d_{ij} y_{i2} \right) \right\} + \sum_{i=1}^{N} e_{i2} \gamma_{i2} u_{i2} \\
& + \sum_{i=1}^{N} e_{i3} \left\{ \gamma_{i3} \left[z_{i1} z_{i2} - b_4 z_{i3} + k_3 \sum_{i=1}^{N} h_{ij} z_{i3} \right] - \alpha_3 \left[b_1 \right. \right. \\
& \left. + x_{i3} \left(x_{i2} - c_1 \right) + k_1 \sum_{i=1}^{N} g_{ij} x_{i3} \right] - \beta_3 \left(y_{i1} y_{i2} - b_2 y_{i3} \right. \\
& \left. \left. + k_2 \sum_{i=1}^{N} d_{ij} y_{i3} \right) \right\} + \sum_{i=1}^{N} e_{i3} \gamma_{i3} u_{i3},
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
\dot{V} ={}& \sum_{i=1}^{N} e_{i1} \left\{ \gamma_{i1} \left[a_4 \left(z_{i2} - z_{i1} \right) + k_3 \sum_{i=1}^{N} h_{ij} z_{i1} \right] \right. \\
& - \alpha_1 \left(-x_{i2} - x_{i3} + k_1 \sum_{i=1}^{N} g_{ij} x_{i1} \right) \\
& \left. - \beta_1 \left[a_2 \left(y_{i2} - y_{i1} \right) + k_2 \sum_{i=1}^{N} d_{ij} y_{i1} \right] \right\}
\end{aligned}
$$

$$+\sum_{i=1}^{N}e_{i1}\left\{-\left(\gamma_{i1}z_{i1}-\alpha_1x_{i1}-\beta_1y_{i1}\right)-a_2\left(\gamma_{i2}z_{i2}-\alpha_2x_{i2}-\beta_2y_{i2}\right)-\gamma_{i1}\left[a_4\left(z_{i2}-z_{i1}\right)+k_3\sum_{i=1}^{N}h_{ij}z_{i1}\right]+\alpha_1\left(-x_{i2}-x_{i3}+k_1\sum_{i=1}^{N}g_{ij}x_{i1}\right)+\beta_1\left[a_2\left(y_{i2}-y_{i1}\right)+k_2\sum_{i=1}^{N}d_{ij}y_{i1}\right]\right\}$$

$$+\sum_{i=1}^{N}e_{i2}\left\{\gamma_{i2}\left(\left(c_4-a_4\right)z_{i1}+c_4z_{i2}-z_{i1}z_{i3}+k_3\sum_{i=1}^{N}h_{ij}z_{i2}\right)-\alpha_2\left(x_{i1}+a_1x_{i2}+k_1\sum_{i=1}^{N}g_{ij}x_{i2}\right)-\beta_2\left(c_2y_{i1}-y_{i1}y_{i3}+k_2\sum_{i=1}^{N}d_{ij}y_{i2}\right)\right\}$$

$$+\sum_{i=1}^{N}e_{i2}\left\{-\left(\gamma_{i2}z_{i2}-\alpha_2x_{i2}-\beta_2y_{i2}\right)+a_2\left(\gamma_1z_{i1}-\alpha_1x_{i1}-\beta_1y_{i1}\right)+a_3\left(\gamma_{i3}z_{i3}-\alpha_3x_{i3}-\beta_3y_{i3}\right)-\gamma_{i2}\left(\left(c_4-a_4\right)z_{i1}+c_4z_{i2}-z_{i1}z_{i3}+k_3\sum_{i=1}^{N}h_{ij}z_{i2}\right)+\alpha_2\left(x_{i1}+a_1x_{i2}+k_1\sum_{i=1}^{N}g_{ij}x_{i2}\right)+\beta_2\left(c_2y_{i1}-y_{i1}y_{i3}+k_2\sum_{i=1}^{N}d_{ij}y_{i2}\right)\right\}$$

$$+\sum_{i=1}^{N}e_{i3}\left\{\gamma_{i3}\left[z_{i1}z_{i2}-b_4z_{i3}+k_3\sum_{i=1}^{N}h_{ij}z_{i3}\right]-\alpha_3\left[b_1+x_{i3}\left(x_{i2}-c_1\right)+k_1\sum_{i=1}^{N}g_{ij}x_{i3}\right]-\beta_3\left(y_{i1}y_{i2}-b_2y_{i3}+k_2\sum_{i=1}^{N}d_{ij}y_{i3}\right)\right\}+\sum_{i=1}^{N}e_{i3}\left\{-\left(\gamma_{i3}z_{i3}-\alpha_3x_{i3}-\beta_3y_{i3}\right)-a_3\left(\gamma_{i2}z_{i2}-\alpha_2x_{i2}-\beta_2y_{i2}\right)-\gamma_{i3}\left[z_{i1}z_{i2}-b_4z_{i3}+k_3\sum_{i=1}^{N}h_{ij}z_{i3}\right]+\alpha_3\left[b_1+x_{i3}\left(x_{i2}-c_1\right)+k_1\sum_{i=1}^{N}g_{ij}x_{i3}\right]+\beta_3\left(y_{i1}y_{i2}-b_2y_{i3}+k_2\sum_{i=1}^{N}d_{ij}y_{i3}\right)\right\}$$

$$=\sum_{i=1}^{N}e_{i1}\left(-e_{i1}-a_2e_{i2}\right)+\sum_{i=1}^{N}e_{i2}\left(-e_{i2}+a_2e_{i1}+a_3e_{i3}\right)+\sum_{i=1}^{N}e_{i3}\left(-e_{i3}-a_3e_{i2}\right)=-\sum_{i=1}^{N}e_{i1}^2-\sum_{i=1}^{N}e_{i2}^2-\sum_{i=1}^{N}e_{i3}^2. \tag{14}$$

□

In G_1 and G_2, since $\dot{V}<0$ as $t\longrightarrow\infty$, it is confirmed through experimental proof that the network-combination synchronization between the drive systems (1) and (2) and the response system (3) is obtained and asymptotical stability is proved on the basis of the Lyapunov Theorem. The results are further illustrated through numerical experiments below. By Fourth-order Runge-Kutta method and with the time step equivalent to 0.001, $\alpha_1=\alpha_2=\alpha_3=\beta_1=\beta_2=\beta_3=\gamma_{i1}=\gamma_{i2}=\gamma_{i3}=1$ is presumed with the original states of the N drive systems, the response system being $(x_{i1}, x_{i2}, x_{i3})=([-5,5],[-5,5],[-5,5])$, $(y_{i1}, y_{i2}, y_3)=([-6,6],[-6,6],[-2,5])$ and $(z_{i1}, z_{i2}, z_{i3})=([-5,5],[-5,5],[0,3])$, respectively, at random by numerical simulation and the coupling strength being $k_1=0.0003$, $k_2=k_3=0.0001$. The validity of the theoretical analysis is verified by numerical simulation results from the two perspectives of full and partial control.

In both cases, suppose the Rössler small-world network has 100 nodes, each with the capability of sending a chaotic signal, while the drive Lü scale-free network can do the same. The corresponding driving signals interacting with each other constitute 100 combined driving signals and there are also 100 response signals in one community network ($h_{ii}=-99$, $h_{ij}=h_{ji}=1$) with the former half Lorenz response signals in the first community and the latter half Chen response signals in the second community. The combined driving signals from two different drive networks can then be loaded to the first and second fifty nodes in two parts of one community network, respectively. The small-world topology with $k=4$ and $p=0.8$ and the scale-free topology with $m_0=m=3$ have been considered.

Remark 3. There has been extensive research into the classic small-world, scale-free, and globally coupled networks, while the study of network-combination synchronization leaves much room to be explored. The advantage of combining those three networks lies not only in the different dynamical structures and scaling matrices displayed by the corresponding nodes of three networks, but also in the different network features between the drive target and the response network. Adjustable synchronous node number and multiple network characteristics are the novelties of the proposed theory.

In Figures 2–4, multiple red curves and blue curves are the state evolution of the first fifty nodes from the first community and the remaining fifty nodes from the second community respectively. As can be seen from the illustrations, the community network has two different sets of curve types, showing entirely different evolution patterns. In Figures 5–7, there are three coexisting states of system $z_{i1}(t)$, $z_{i2}(t)$, and

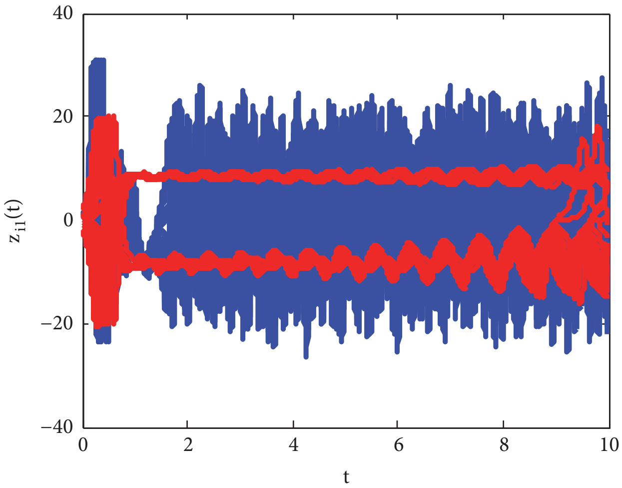

FIGURE 2: (Color online) Evolution patterns of community network $z_{i1}(t)$ with t.

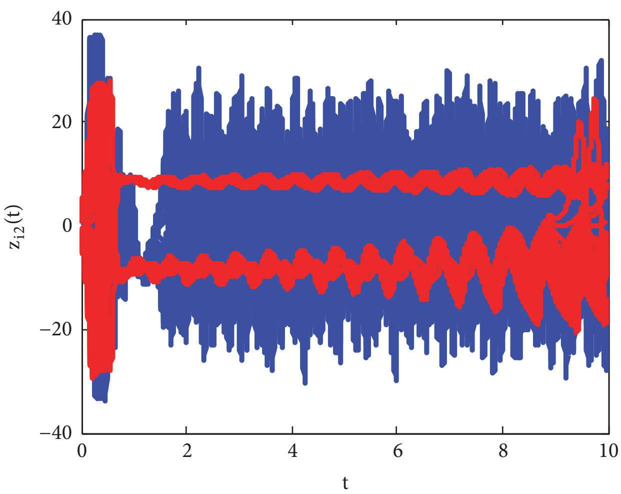

FIGURE 3: (Color online) Evolution patterns of community network $z_{i2}(t)$ with t.

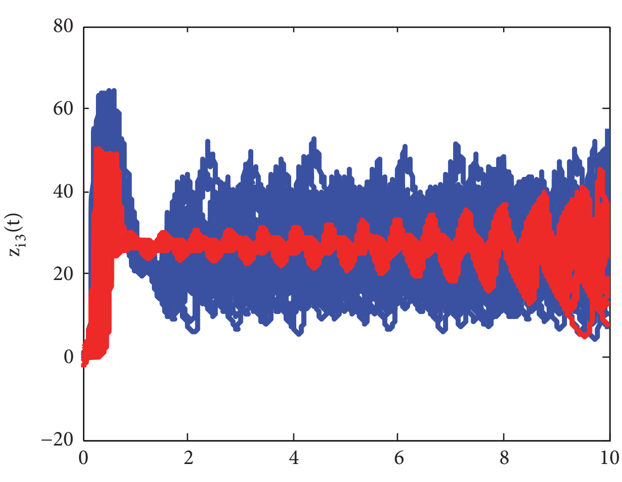

FIGURE 4: (Color online) Evolution patterns of community network $z_{i3}(t)$ with t.

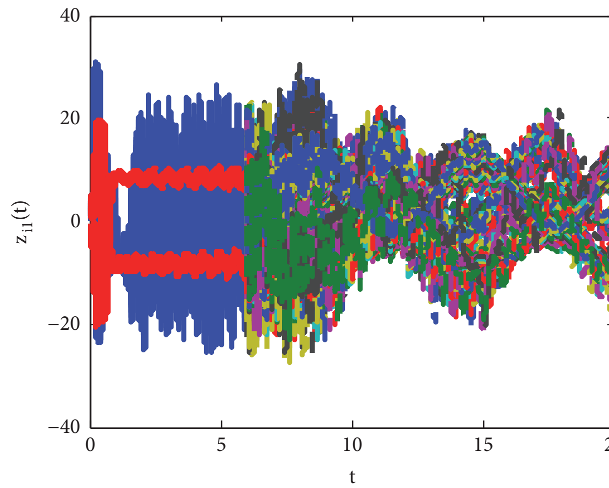

FIGURE 5: (Color online) Evolution curves of state variables $z_{i1}(t)$ after adding control.

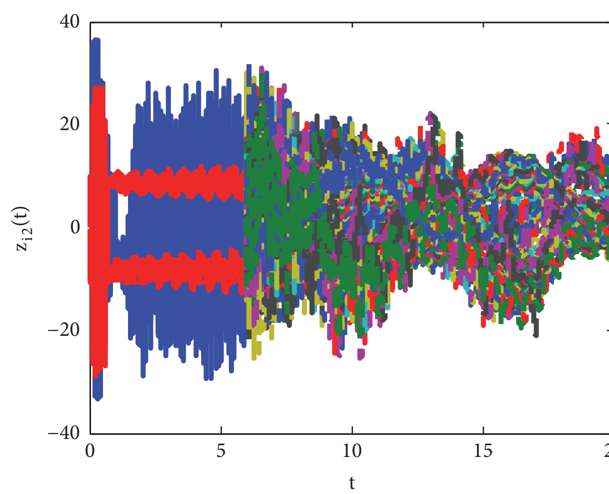

FIGURE 6: (Color online) Evolution curves of state variables $z_{i2}(t)$ after adding control.

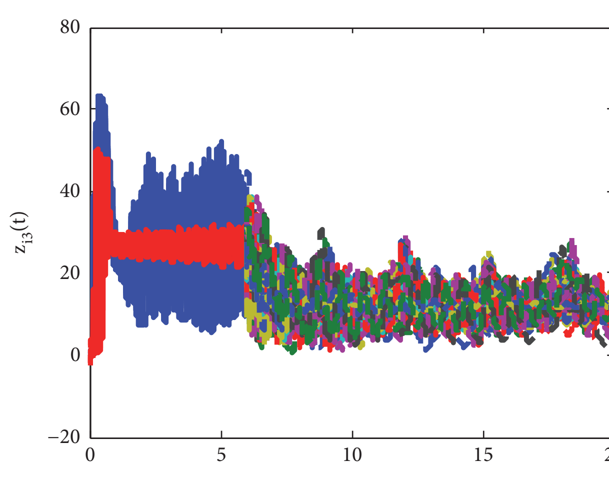

FIGURE 7: (Color online) Evolution curves of state variables $z_{i3}(t)$ after adding control.

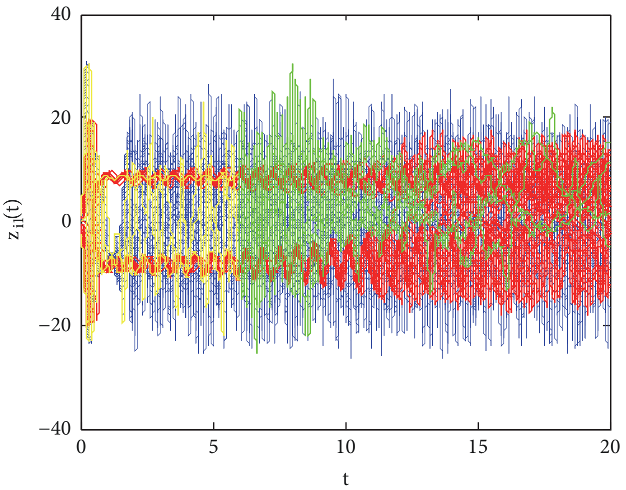

FIGURE 8: (Color online) Less node control community network $z_{i1}(t)$ with t.

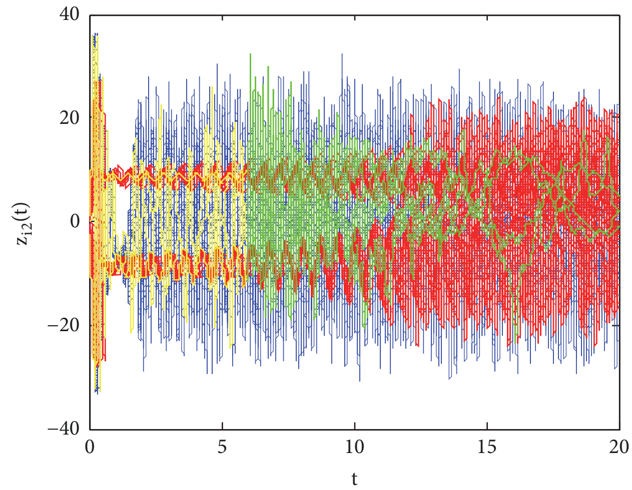

FIGURE 9: (Color online) Less node control community network $z_{i2}(t)$ with t.

$z_{i3}(t)$ with t. Before adding control, 100 state variables of response nodes in the community networks express two kinds of curve patterns of red and blue curve evolution, while after adding control at $t = 6$, the red and blue curves change into colorful curve evolution constituted by the combined signals from $Ax_i(t) + By_i(t)$, verifying the realization of network-combination synchronization between 100 state variables of corresponding nodes. Thus, multisignals from two drive networks and one response network are combined to present three coexisting states.

Remark 4. The previous one-to-one and two-to-one drive-response synchronization models belong to full control, which is not a necessity for synchronizing nodes to combined drive signals in network-combination sometimes. Instead, partial control strategy design is more practical. As one of the two types of partial control, less node control is used when a network has many nodes and there is no need to add controllers to all of them except the crucial ones; more node control is applied when the information of more nodes should be retained during the process of chaotic encryption.

In less node control type, four nodes from two response communities are selected to be controlled. Four controllers ($\gamma_{1,2,\ldots,48} = 0$, $\gamma_{51,52,\ldots,98} = 0$, $\gamma_{49,50,99,100} = 1$) are added to nodes 49 and 50 in the first community and nodes 99 and 100 in the second community. As can be seen from Figures 8–10, red and blue curves represent the set of 48 curves from the response nodes in the first and the second communities without control. Four yellow curves indicating controlled nodes evolve to green driving states after adding control at $t = 6$. After $t = 6$, with time evolution, two sets of red and blue curves show the response states of two communities without control, in spite of which four controlled response states synchronize to the driving states of four corresponding nodes.

In more node control type, two nodes ($\gamma_{1,2,\ldots,49} = 1$, $\gamma_{51,52,\ldots,99} = 1$, $\gamma_{50,100} = 0$) from two response communities are

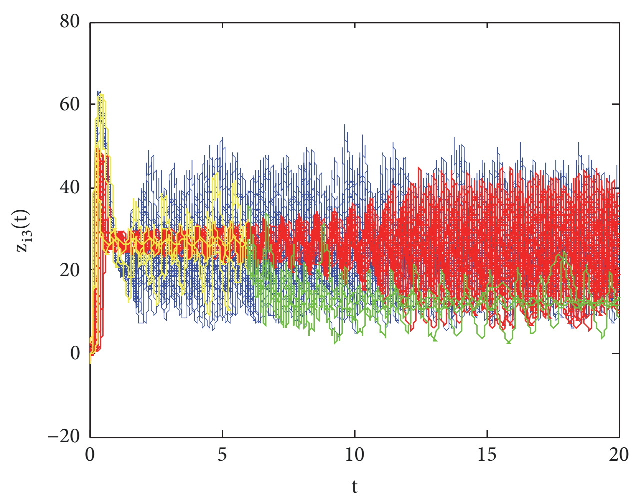

FIGURE 10: (Color online) Less node control community network $z_{i3}(t)$ with t.

selected without control. Only two controllers are not added to node 50 in the first community and node 100 in the second community. As can be seen from Figures 11–13, red curves and blue curves represent the error curves between two response communities and two combined driving signals, but a red and a blue error curve alone in G_1 and G_2 will not reduce to zero gradually due to the property of more node control.

In a word, there are less response nodes synchronizing to the states of drive signals, leaving the rest evolving along self-state in less node control type, while there are more response nodes synchronizing to the states of drive signals in more node control type. The two designs show the characteristics of diversity and selectivity in network-combination synchronization.

Remark 5. Memristors have great application value in a lot of actual nonlinear areas. Autonomous memristor chaotic systems and hybrid memristor chaotic system with multichaotic

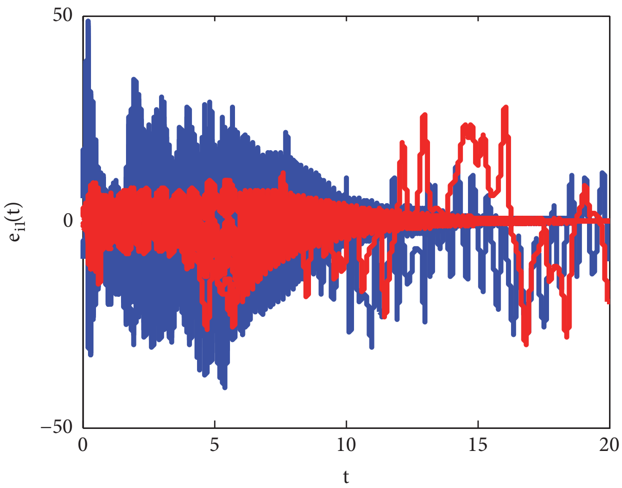

FIGURE 11: (Color online) More node control community network $e_{i1}(t)$ with t.

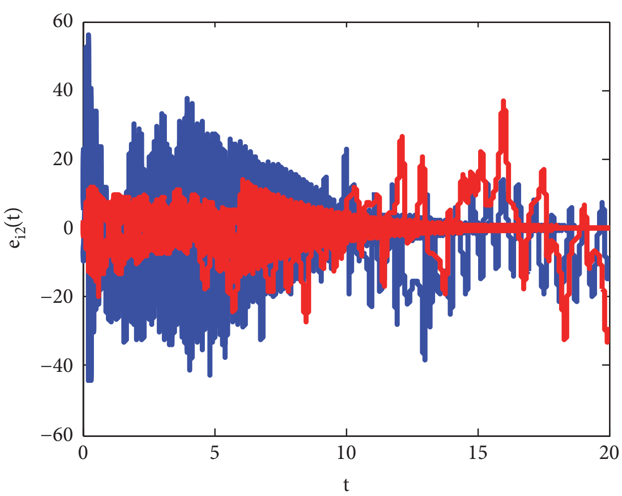

FIGURE 12: (Color online) More node control community network $e_{i2}(t)$ with t.

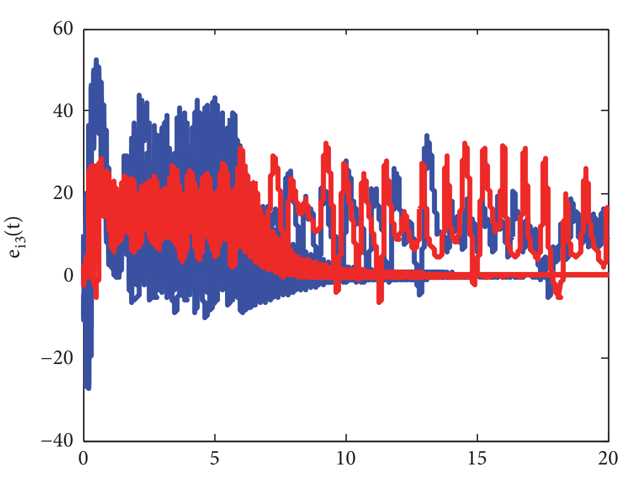

FIGURE 13: (Color online) More node control community network $e_{i3}(t)$ with t.

characteristics, in particular, represent the latest develop trend—the three-dimensional to five-dimensional memristor chaotic systems given in [30] can be chosen as the model of research into nodes of various dimensional networks in the control design process, while the six-dimensional model of hybrid memristor chaotic system, with higher dimensional variables and more parameters, is safer and steadier than the system in [31] and can be used to enhance the security of information exchange during the delivery process. The control of high-dimensional memristor chaotic systems is our next research direction.

4. Conclusion and Perspectives

A novel type of synchronization, called network-combination synchronization, combining three inhomogeneous kinds of classic networks is presented in this paper. Based on the Lyapunov Stability Theorem, some controllers with different settings are proposed for the three combined networks. The simulations of full and partial control are given to show the selectivity and diversity of the design. The network-combination synchronization can meet the demands of realistic network synchronization with high-capacity, multistructures, and multisignal connectivity. The partial combination in consistency with intelligent control is designed in a time and cost efficient way. Furthermore, the multifunctional characteristics of the design are likely to be applied extensively in secure communication and artificial intelligence.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This research was supported by the National Natural Science Foundation of China (Grants 11502139, 11502141, 11601317, 11526135, and 11701359) and the Shanghai Universities Young Teacher Training Scheme (Grants ZZsdl15116).

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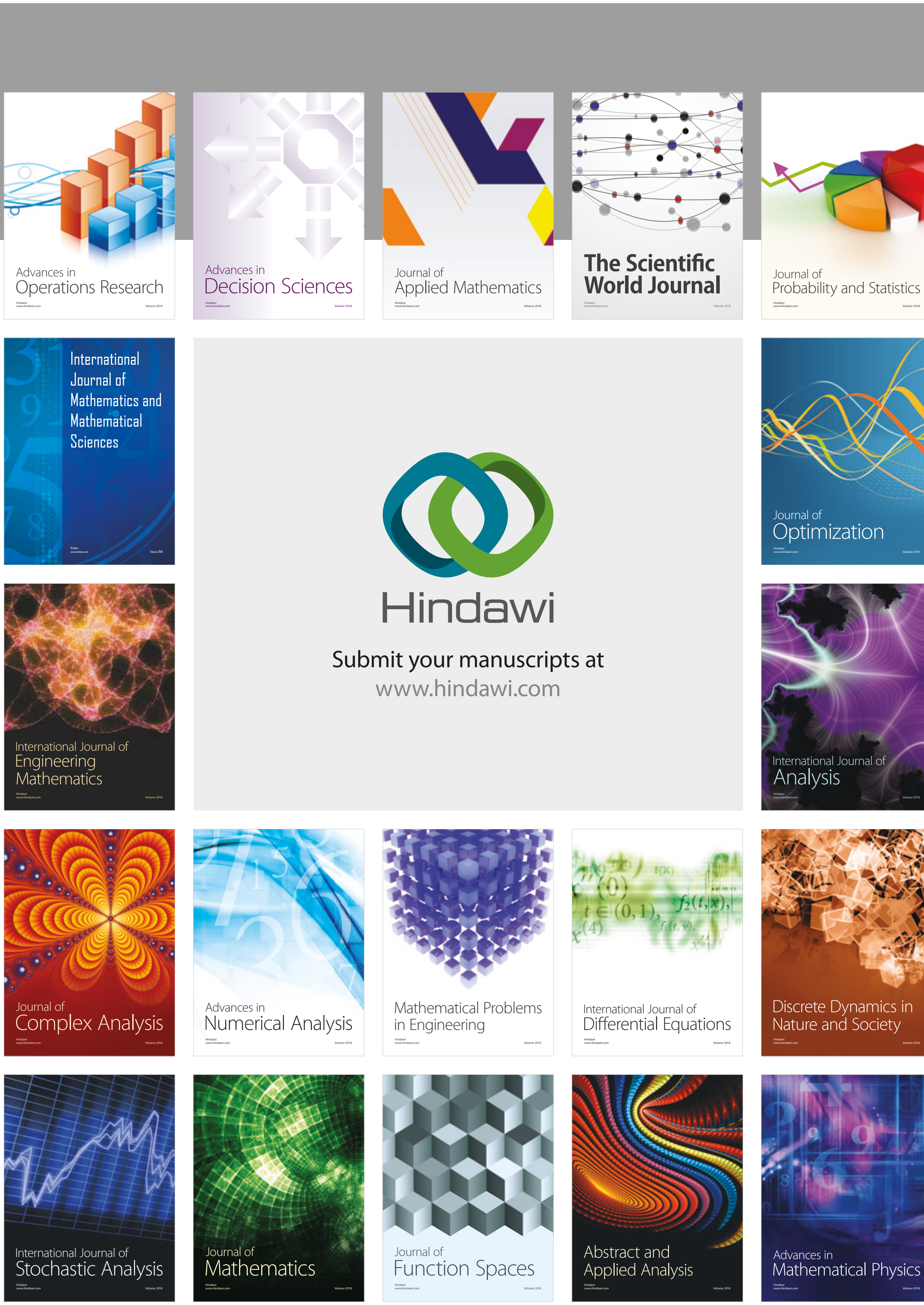
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