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Research Article

Application of Laplace Transform on Fractional Kinetic Equation Pertaining to the Generalized Galué Type Struve Function

Haile Habenom, D. L. Suthar, and Melaku Gebeyehu

Department of Mathematics, Wollo University, Dessie, P.O. Box 1145, Amhara Region, Ethiopia

Correspondence should be addressed to D. L. Suthar; dlsuthar@gmail.com

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In this paper, we establish extensive form of the fractional kinetic equation involving generalized Galué type Struve function using the technique of Laplace transforms. The results are expressed in terms of Mittag-Leffler function. Further, numerical values of the results and their graphical interpretation are interpreted to study the behaviour of these solutions. The results obtained here are quite general in nature and capable of yielding a very large number of known and (presumably) new results.

1. Introduction and Preliminaries

In the field of applied science, the significance of fractional differential equations has gained more attention not only in mathematical direction but also in mathematical physics, dynamical systems, control systems, and engineering, to generate the mathematical model of numerous physical phenomena. Particularly, the kinetic equations define the continuity of motion of substance and are the elementary equations of mathematical physics and natural science. The extension and generality of fractional kinetic equations and various fractional operators with special functions were found (Agarwal et al. [1], Amsalu and Suthar [2], Baleanu et al. [3, 4], Chaurasia and Pandey [5], Choi and Agarwal [6, 7], Zaslavsky [8], Gupta and Parihar [9], Gupta and Sharma [10], Haubold and Mathai [11], Kumar et al. [12], Nisar et al. [13], Saxena et al. [14–16], Saichev and Zaslavsky [17], Suthar et al. [18], and Tariboon et al. [19]). In view of the effectiveness and a great significance of the kinetic equation in some astrophysical problems the authors develop a further generalized form of the fractional kinetic equation involving generalized Galué type Struve function.

Recently, generalized form of Struve function so-called as generalized Galué type Struve function (GTSF) is defined by Nisar et al. [20], following as

$$ {}_{\alpha}w_{p,\beta,\gamma,\xi}^{\lambda,\mu}(z) = \sum_{k=0}^{\infty} \frac{(-\gamma)^k}{\Gamma(\lambda k+\mu)\,\Gamma(\alpha k + p/\xi + (\beta+2)/2)} \left(\frac{z}{2}\right)^{2k+p+1}, \quad (1) $$

where $\lambda > 0, \xi > 0, \alpha \in \mathbb{N}, p, \beta, \gamma \in \mathbb{C}$, and μ is an arbitrary parameter.

For the description of the Struve function and its more overview, the interested reader may refer to many papers (Bhow-Mick [21, 22], Kanth [23], Nisar et al. [24], Singh [25], and Suthar et al. [26]).

Special Cases. We have a number of special functions, which can be expressed as follows:

$$ {}_{1}w_{p,\beta,\gamma,1}^{1,3/2}(z) = H_{p,\beta,\gamma}(z), \quad (2) $$

where $H_{p,\beta,\gamma}(z)$ is generalized Struve function, which is defined by Orhan and Yagmur [27].

$$ {}_{1}w_{p,-1,1,1}^{1,3/2}(z) = H_{p}(z), \quad (3) $$

where $H_p(z)$ is Struve function of order p, which is defined by Nisar et al. [20].

$$ {}_{q}w^{1,1}_{2\nu+2\lambda,-1,1,1}(z) = \frac{(z/2)^{\nu}\Gamma(\lambda+n+1)}{\Gamma(n+1)} J^{q}_{\nu,\lambda}(z), \quad (4) $$

where $J^{q}_{\nu,\lambda}(z)$ is Bessel Maitland function (see [28]).

$$ {}_{1}w^{1,1}_{\upsilon-1,2,1,1}(z) = J_{\nu}(z), \quad (5) $$

where $J_\nu(z)$ is Bessel function of first kind (see [29]).

$$ {}_{q}w^{1,1}_{p-1,1,-1,1}(z) = {}_{q}I_{p}(z), \quad (6) $$

where ${}_{q}I_p(z)$ is the Galué type generalization of modified Bessel function [30].

Now, we recall the fractional differential equation between rate of change of reaction $N = N(t)$, the destruction rate $d = d(N)$, and the production rate $p = p(N)$ given by Haubold and Mathai [11] as follows:

$$ \frac{d(N)}{dt} = -d(N_t) + p(N_t), \quad (7) $$

where N_t is the function denoted by $N_t(t^*) = N(t-t^*)$, $t^* > 0$. Neglecting the inhomogeneity in the quantity $N(t)$, the special case of (7) is

$$ \frac{dN_i}{dt} = -c_i N_i(t), \quad (8) $$

with the initial condition $N_i(t = 0) = N_0$ which is the number of density of species "i" at time $t = 0$.

The solution of equation (8) is expressed as

$$ N_i(t) = N_0 e^{-c_i t}. \quad (9) $$

Alternatively, we can use

$$ N(t) - N_0 = c\, {}_{0}D^{-1}_{t} N(t), \quad (10) $$

having in mind that ${}_{0}D^{-1}_{t}$ is the standard fractional integral operator. The fractional generalization of the standard kinetic equation (10) defined by Haubold and Mathai [11] has the form

$$ N(t) - N_0 = c^{\nu}\, {}_{0}D^{-\nu}_{t} N(t), \quad (11) $$

where ${}_{0}D^{-\nu}_{t}$ is the most common Riemann-Liouville (R-L) fractional integral operator. Further details of R-L are in the studies by Oldham and Spanier [31] and Miller and Ross [32] and it is defined as

$$ {}_{0}D^{-\nu}_{t} f(x) = \frac{1}{\Gamma(\nu)} \int (t-u)^{\nu-1} f(u)\,du, \quad (x > 0, \Re(\nu) > 0). \quad (12) $$

Haubold and Mathai [11] have given the solution of (11) in the form

$$ N(t) = N_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(\nu k+1)} (ct)^{\nu k}. \quad (13) $$

Further, Saxena and Kalla [14] considered the following fractional kinetic equation:

$$ N(t) - N_0 f(t) = -c^{\nu} \left({}_{0}D^{-\nu}_{t} N \right)(t), \quad (\Re(\nu) > 0), \quad (14) $$

where $N(t)$ denotes the number density of a given species at time t, $N_0 = N(0)$ which is the number density of that species at time $t = 0$, c that is a constant and $f \in L(0,\infty)$.

Also we recall the Laplace transform of $f(t)$ defined by Sneddon [33] as

$$ F(s) = L\{f(t); s\} = \int_0^{\infty} e^{-st} f(t)\,dt. \quad (15) $$

For $\alpha, \beta \in \mathbb{C}; \Re(\alpha) > 0; \Re(\beta) > 0$ the two-parameter Mittag-Leffler function is defined by [34] as

$$ E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+\beta)} \quad (16) $$

The aim of this paper is to develop a new and further generalized solution of the fractional kinetic equation involving generalized Galué type Struve function using the technique of Laplace transform. The manifold generality of the generalized Galué type Struve function is discussed in terms of the solution of the above fractional kinetic equation. Furthermore, the results gained here are quite capable of yielding a very huge number of known and (presumably) new results.

2. Solution of Generalized Fractional Kinetic Equations

In this section, we solve the fractional kinetic equation associated with generalized Galué type Struve function using the method of Laplace transforms.

Theorem 1. *If $\alpha > 0, d > 0, \nu > 0$, $\alpha \in \mathbb{N}, p, \beta, \gamma \in \mathbb{C}, \lambda > 0, \xi > 0$, and μ is an arbitrary parameter, then the solution of the equation*

$$ N(t) - N_0\, {}_{\alpha}w^{\lambda,\mu}_{p,\beta,\gamma,\xi}(t) = -d^{\nu}\, {}_{0}D^{-\nu}_{t} N(t) \quad (17) $$

is given by the formula

$$ N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-\gamma)^r \Gamma(2r+p+2)}{\Gamma(\lambda r+\mu)\Gamma(\alpha r + p/\xi + (\beta+2)/2)} \cdot \left(\frac{t}{2}\right)^{2r+p+1} \mathrm{E}_{\nu,2r+p+2}(-d^{\nu} t^{\nu}). \quad (18) $$

Proof. The Laplace transform of Riemann-Liouville fractional integral operator is given by [35, 36] as follows:

$$ L\{{}_{0}D^{-\nu}_{t} f(t); s\} = s^{-\nu} F(s) \quad (19) $$

where $F(s)$ is defined in (15). Now, applying the Laplace transform on both sides of (17) and using (1) and (19) lead to

$$ L[N(t); s] = N_0 L\left[{}_{\alpha}w^{\lambda,\mu}_{p,\beta,\gamma,\xi}(t); s\right] - d^{\nu} L\left[{}_{0}D^{-\nu}_{t} N(t); s\right]. \quad (20) $$

$$N(s) = N_0 \int_0^\infty e^{-st} \sum_{r=0}^{\infty} \frac{(-\gamma)^r}{\Gamma(\lambda r+\mu)\Gamma(\alpha r + p/\xi + (\beta+2)/2)} \left(\frac{t}{2}\right)^{2r+p+1} dt - d^\nu s^{-\nu} N(s), \quad (21)$$

$$N(s) + d^\nu s^{-\nu} N(s) = N_0 \sum_{r=0}^{\infty} \frac{(-\gamma)^r}{\Gamma(\lambda r+\mu)\Gamma(\alpha r + p/\xi + (\beta+2)/2)} \left(\frac{1}{2}\right)^{2r+p+1} \times \int_0^\infty e^{-st} t^{2r+p+1} dt, \quad (22)$$

$$N(s)\{1 + d^\nu s^{-\nu}\} = N_0 \sum_{r=0}^{\infty} \frac{(-\gamma)^r}{\Gamma(\lambda r+\mu)\Gamma(\alpha r + p/\xi + (\beta+2)/2)} \left(\frac{1}{2}\right)^{2r+p+1} \times \frac{\Gamma(2r+p+2)}{s^{(2r+p+2)}}, \quad (23)$$

$$N(s) = N_0 \sum_{r=0}^{\infty} \frac{(-\gamma)^r \Gamma(2r+p+2)}{\Gamma(\lambda r+\mu)\Gamma(\alpha r + p/\xi + (\beta+2)/2)} \left(\frac{1}{2}\right)^{2r+p+1} \times s^{-(2r+p+2)} \sum_{k=0}^{\infty} (1)_k \cdot \frac{[-(s/d)^{-\nu}]^k}{k!}, \quad (24)$$

Taking inverse Laplace transform on both sides of (24) and using $L^{-1}\{s^{-\nu}\} = t^{\nu-1}/\Gamma(\nu)$ for $\Re(\nu) > 0$, we have

$$L^{-1}(N(s)) = N_0 \sum_{r=0}^{\infty} \frac{(-\gamma)^r \Gamma(2r+p+2)}{\Gamma(\lambda r+\mu)\Gamma(\alpha r + p/\xi + (\beta+2)/2)} \left(\frac{1}{2}\right)^{2r+p+1} \times L^{-1}\left\{\sum_{k=0}^{\infty} (-1)^k d^{\nu k} s^{-(2r+p+2+\nu k)}\right\}, \quad (25)$$

and

$$N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-\gamma)^r \Gamma(2r+p+2)}{\Gamma(\lambda r+\mu)\Gamma(\alpha r + p/\xi + (\beta+2)/2)} \left(\frac{1}{2}\right)^{2r+p+1} \times \left\{\sum_{k=o}^{\infty} (-1)^k d^{\nu k} \frac{t^{(2r+p+1+\nu k)}}{\Gamma(\nu k + 2r + p + 2)}\right\}, \quad (26)$$

Interpreting the above result in (26) in the view of (16), we get the required result (18). □

Theorem 2. *If $d > 0, \nu > 0$, $\alpha \in \mathbb{N}, p, \beta, \gamma \in \mathbb{C}, \lambda > 0, \xi > 0$, and μ is an arbitrary parameter, then the solution of the equation*

$$N(t) - N_0\, {}_\alpha w_{p,\beta,\gamma,\xi}^{\lambda,\mu}(d^\nu t^\nu) = -d^\nu\, {}_0D_t^{-\nu} N(t) \quad (27)$$

is given by the formula

$$N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-\gamma)^r \Gamma((2r+p+1)\nu+1)}{\Gamma(\lambda r+\mu)\Gamma(\alpha r + p/\xi + (\beta+2)/2)} \left(\frac{d^\nu t^\nu}{2}\right)^{2r+p+1} \mathrm{E}_{\nu,(2r+p+1)\nu+1}(-d^\nu t^\nu). \quad (28)$$

Proof. Detail proof of Theorem 2 is parallel to that of Theorem 1; therefore we omit the details. □

Theorem 3. *If $a > 0, d > 0, \nu > 0$, $\alpha \in \mathbb{N}, p, \beta, \gamma \in \mathbb{C}, \lambda > 0, \xi > 0$, $a \neq d$, and μ is an arbitrary parameter, then the solution of the equation*

$$N(t) - N_0\, {}_\alpha w_{p,\beta,\gamma,\xi}^{\lambda,\mu}(d^\nu t^\nu) = -a^\nu\, {}_0D_t^{-\nu} N(t) \quad (29)$$

is given by the formula

$$N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-\gamma)^r \Gamma((2r+p+1)\nu+1)}{\Gamma(\lambda r+\mu)\Gamma(\alpha r + p/\xi + (\beta+2)/2)} \left(\frac{d^\nu t^\nu}{2}\right)^{2r+p+1} \mathrm{E}_{\nu,(2r+p+1)\nu+1}(-a^\nu t^\nu), \quad (30)$$

Proof. Detail proof of Theorem 3 is parallel to that of Theorem 1; therefore we omit the details. □

3. Special Cases

(i) Setting $\alpha = \lambda = \xi = 1$ and $\mu = 3/2$ the generalized Galué type Struve function is reduced into the following form:

$$ {}_1w_{p,\beta,\gamma,1}^{1,3/2}(z) = \sum_{k=0}^{\infty} \frac{(-\gamma)^k}{\Gamma(k+3/2)\Gamma(k+p+(\beta+2)/2)} \left(\frac{z}{2}\right)^{2k+p+1} = H_{p,\beta,\gamma}(z). \quad (31)$$

The formula (31) is obtained by suitable settings in Theorem 1-Theorem 3 that are used for the Corollaries 4–6, respectively.

Corollary 4. *If $d > 0, \nu > 0$, $p, \beta, \gamma \in \mathbb{C}$ then the solution of the equation*

$$N(t) - N_0 H_{p,\beta,\gamma}(t) = -d^\nu\, {}_0D_t^{-\nu} N(t) \quad (32)$$

is given by the following formula:

$$N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-\gamma)^r \Gamma(2r+p+2)}{\Gamma(r+(3/2))\Gamma(r+p+(\beta+2)/2)} \left(\frac{t}{2}\right)^{2r+p+1} \mathrm{E}_{\nu,2r+p+2}(-d^\nu t^\nu). \quad (33)$$

Corollary 5. *If $d > 0, \nu > 0$, $p, \beta, \gamma \in \mathbb{C}$ then the solution of the equation*

$$N(t) - N_0 H_{p,\beta,\gamma}(d^\nu t^\nu) = -d^\nu\, {}_0D_t^{-\nu} N(t) \quad (34)$$

is given by the following formula:

$$N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-\gamma)^r \Gamma((2r+p+1)\nu+1)}{\Gamma(r+(3/2))\Gamma(r+p+(\beta+2)/2)} \left(\frac{d^\nu t^\nu}{2}\right)^{2r+p+1} \mathrm{E}_{\nu,(2r+p+1)\nu+1}(-d^\nu t^\nu). \quad (35)$$

Corollary 6. *If $d > 0, \nu > 0$, $p, \beta, \gamma \in \mathbb{C}$, $a \neq d$, then solution of the equation*

$$N(t) - N_0 H_{p,\beta,\gamma}(d^\nu t^\nu) = -a^\nu\, {}_0D_t^{-\nu} N(t) \quad (36)$$

is given by the formula

$$N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-\gamma)^r \Gamma((2r+p+1)\nu+1)}{\Gamma(r+(3/2))\Gamma(r+p+(\beta+2)/2)} \left(\frac{d^\nu t^\nu}{2}\right)^{2r+p+1} \mathrm{E}_{\nu,(2r+p+1)\nu+1}(-a^\nu t^\nu). \tag{37}$$

(ii) On setting $\alpha = \lambda = \xi = \gamma = 1, \beta = -1$ and $\mu = 3/2$ the generalized Galué type Struve function is reduced into Struve function of the following form:

$$\begin{aligned} {}_1w^{1,3/2}_{p,-1,1,1}(z) &= \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(k+3/2)\Gamma(k+p+1/2)} \left(\frac{z}{2}\right)^{2k+p+1} \\ &= H_p(z). \end{aligned} \tag{38}$$

The formula (38) is obtained by suitable settings in Theorem 1 to Theorem 3 that are used for the Corollaries 7–9, respectively.

Corollary 7. *If $d > 0, \nu > 0$, $p \in \mathbb{C}$ then the solution of the equation*

$$N(t) - N_0 H_p(t) = -d^\nu\, {}_0D_t^{-\nu} N(t) \tag{39}$$

is given by the following formula:

$$\begin{aligned} N(t) &= N_0 \sum_{r=0}^{\infty} \frac{(-1)^r \Gamma(2r+p+2)}{\Gamma(r+(3/2))\Gamma(r+p+(1/2))} \left(\frac{t}{2}\right)^{2r+p+1} \\ &\quad \cdot \mathrm{E}_{\nu,2r+p+2}(-d^\nu t^\nu). \end{aligned} \tag{40}$$

Corollary 8. *If $d > 0, \nu > 0$, $p \in \mathbb{C}$ then the solution of the equation*

$$N(t) - N_0 H_p(d^\nu t^\nu) = -d^\nu\, {}_0D_t^{-\nu} N(t) \tag{41}$$

is given by the following formula:

$$N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-1)^r \Gamma((2r+p+1)\nu+1)}{\Gamma(r+(3/2))\Gamma(r+p+(1/2))} \left(\frac{d^\nu t^\nu}{2}\right)^{2r+p+1} \mathrm{E}_{\nu,(2r+p+1)\nu+1}(-d^\nu t^\nu). \tag{42}$$

Corollary 9. *If $d > 0, \nu > 0$, $p \in \mathbb{C}$, $a \neq d$, then the solution of the equation*

$$N(t) - N_0 H_p(d^\nu t^\nu) = -a^\nu\, {}_0D_t^{-\nu} N(t) \tag{43}$$

is given by the formula

$$N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-1)^r \Gamma((2r+p+1)\nu+1)}{\Gamma(r+(3/2))\Gamma(r+p+(1/2))} \left(\frac{d^\nu t^\nu}{2}\right)^{2r+p+1} \mathrm{E}_{\nu,(2r+p+1)\nu+1}(-a^\nu t^\nu). \tag{44}$$

(iii) On setting $\alpha = \lambda = \xi = \gamma = \mu = 1, \beta = 2$ and $p = l-1$ the generalized Galué type Struve function is reduced into Bessel function of first kind as follows:

$${}_1w^{1,1}_{l-1,2,1,1}(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\Gamma(k+l+1)} \left(\frac{z}{2}\right)^{2k+l} = J_l(z). \tag{45}$$

The formula (45) is obtained by suitable settings in Theorem 1 to Theorem 3 that are used for the Corollaries 10–12, respectively.

Corollary 10. *If $d > 0, \nu > 0, l, t \in \mathbb{C}$, $\Re(l) > -1$ then for the solution of the equation*

$$N(t) - N_0 J_l(t) = -d^\nu\, {}_0D_t^{-\nu} N(t) \tag{46}$$

there holds the formula

$$\begin{aligned} N(t) &= N_0 \sum_{r=0}^{\infty} \frac{(-1)^r \Gamma(2r+l+1)}{r!\Gamma(r+l+1)} \left(\frac{t}{2}\right)^{2r+l} \\ &\quad \cdot \mathrm{E}_{\nu,2r+l+1}(-d^\nu t^\nu). \end{aligned} \tag{47}$$

Corollary 11. *If $d > 0, \nu > 0, l, t \in \mathbb{C}$, $\Re(l) > -1$, then for the solution of the equation*

$$N(t) - N_0 J_l(d^\nu t^\nu) = -d^\nu\, {}_0D_t^{-\nu} N(t) \tag{48}$$

there holds the formula

$$\begin{aligned} N(t) &= N_0 \sum_{r=0}^{\infty} \frac{(-1)^r \Gamma((2r+l)\nu+1)}{\Gamma(r+1)\Gamma(r+l+1)} \left(\frac{d^\nu t^\nu}{2}\right)^{2r+l} \\ &\quad \cdot \mathrm{E}_{\nu,(2r+l)\nu+1}(-d^\nu t^\nu). \end{aligned} \tag{49}$$

Corollary 12. *If $d > 0, \nu > 0, l, t \in \mathbb{C}$, $\Re(l) > -1$, $a \neq d$, then the solution of the equation*

$$N(t) - N_0 J_\nu(d^\nu t^\nu) = -a^\nu\, {}_0D_t^{-\nu} N(t) \tag{50}$$

is given by the formula

$$\begin{aligned} N(t) &= N_0 \sum_{r=0}^{\infty} \frac{(-1)^r \Gamma((2r+l)\nu+1)}{\Gamma(r+1)\Gamma(r+l+1)} \left(\frac{d^\nu t^\nu}{2}\right)^{2r+l} \\ &\quad \cdot \mathrm{E}_{\nu,(2r+l)\nu+1}(-a^\nu t^\nu), \end{aligned} \tag{51}$$

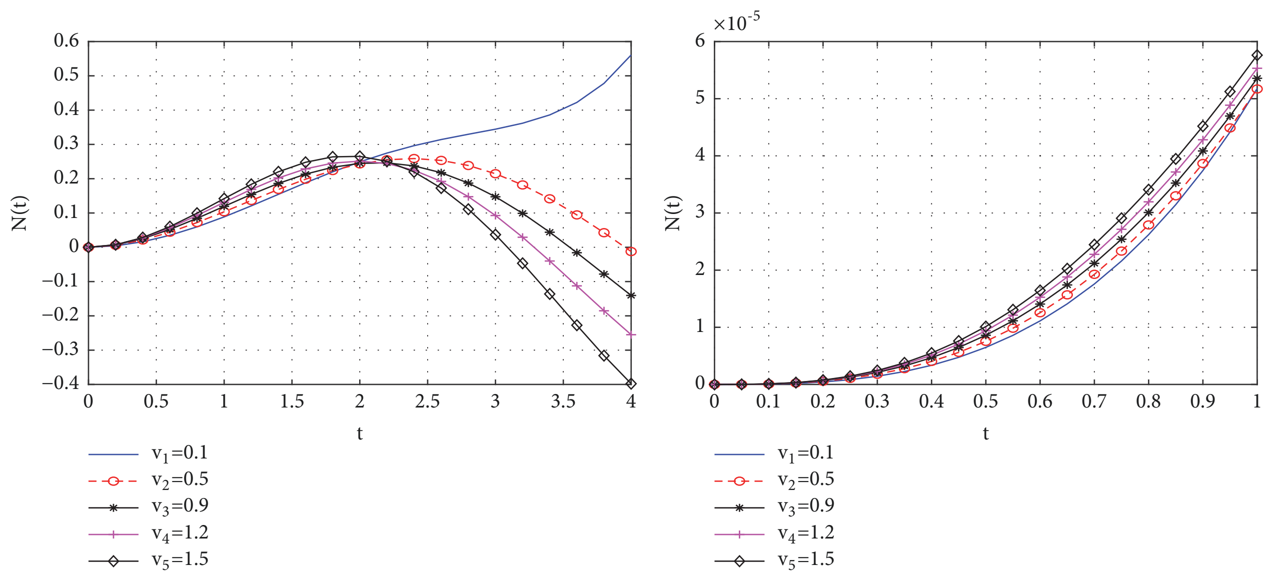
Remark. The special cases for Theorem 1 to Theorem 3 can be developed on similar lines to the above, but we do not state them here due to lack of space.

4. Graphical Interpretation

Here, we are going to illustrate a tabular and graphical expression of the results in Theorem 1 to Theorem 3 with different and suitable assignments of the parameters there. Figure 1(a) is the graphical solution of Theorem 1 using the

Table 1: The numerical results of Theorem 1 for the parameters listed in Figure 1(a).

t	v=0.1	v=0.5	v=0.9	v=1.2	v=1.5
0	0	0	0	0	0
0.2	0.019	0.0441	0.0184	0.0068	0.0022
0.4	0.018	0.0679	0.0553	0.0334	0.0172
0.6	0.0175	0.0811	0.0974	0.08	0.055
0.8	0.0184	0.0867	0.1361	0.1408	0.1209
1	0.0222	0.0866	0.1648	0.2053	0.2122
1.2	0.0319	0.082	0.1786	0.2599	0.3163
1.4	0.0523	0.0738	0.174	0.2888	0.4083
1.6	0.0897	0.0627	0.149	0.2765	0.4516
1.8	0.1533	0.0493	0.1028	0.2091	0.4016
2	0.2549	0.0339	0.0361	0.0768	0.2122
2.2	0.4094	0.0171	-0.0494	-0.124	-0.1524
2.4	0.6357	-0.0008	-0.1509	-0.3887	-0.7033
2.6	0.9564	-0.0196	-0.2644	-0.7039	-1.4121
2.8	1.3989	-0.0391	-0.3856	-1.0465	-2.1988
3	1.9954	-0.0589	-0.5093	-1.3848	-2.9279
3.2	2.7837	-0.079	-0.63	-1.6802	-3.4181
3.4	3.8072	-0.099	-0.7422	-1.8905	-3.4683
3.6	5.1159	-0.119	-0.8404	-1.9738	-2.9004
3.8	6.7663	-0.1387	-0.9194	-1.8937	-1.6143
4	8.8224	-0.158	-0.9745	-1.6238	0.3559



(a) The solution of Theorem 1 for t=0:0.2:4

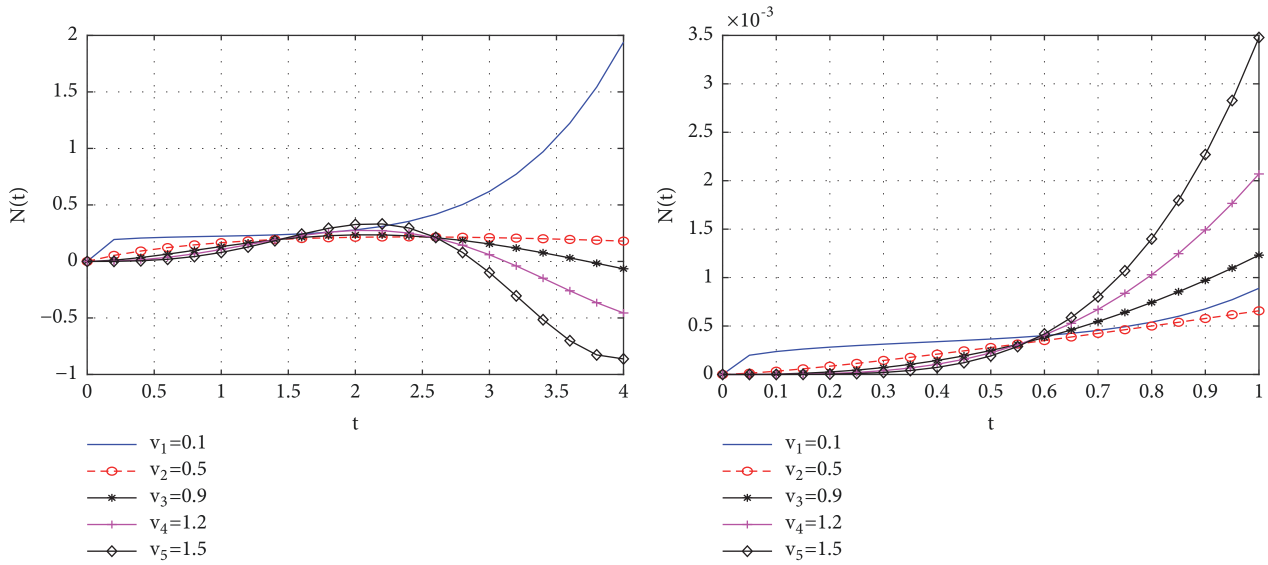
(b) The solution of Theorem 1 for t=0:0.05:1

Figure 1: Solution of fractional kinetic equation (18).

assignment on the parameters $N_0 = 1; c = 1; q = 1; \lambda = 1; \mu = 1; m = 1; \eta = 1; b = 1; d = 1$. The numerical results of Theorem 1 for the parameters listed in Figure 1(a) are given in Table 1. And also Figure 1(b) is based on assignment for the parameters $N_0 = 10; c = 1; q = 2; \lambda = 7; \mu = 7; m = 4; \eta = 7; b = 7; d = 3$.

Figure 2(a) is the graphical solution of Theorem 2 using the assignment on the parameters $N_0 = 1, c = 1, q = 1, \lambda = 1, \mu = 1, m = 1, \eta = 1, b = 1, d = 1$. And also Figure 2(b) is based on assignment for the parameters $N_0 = 10, c = 1, q = 2, \lambda = 7, \mu = 7, m = 4, \eta = 7, b = 7, d = 3$.

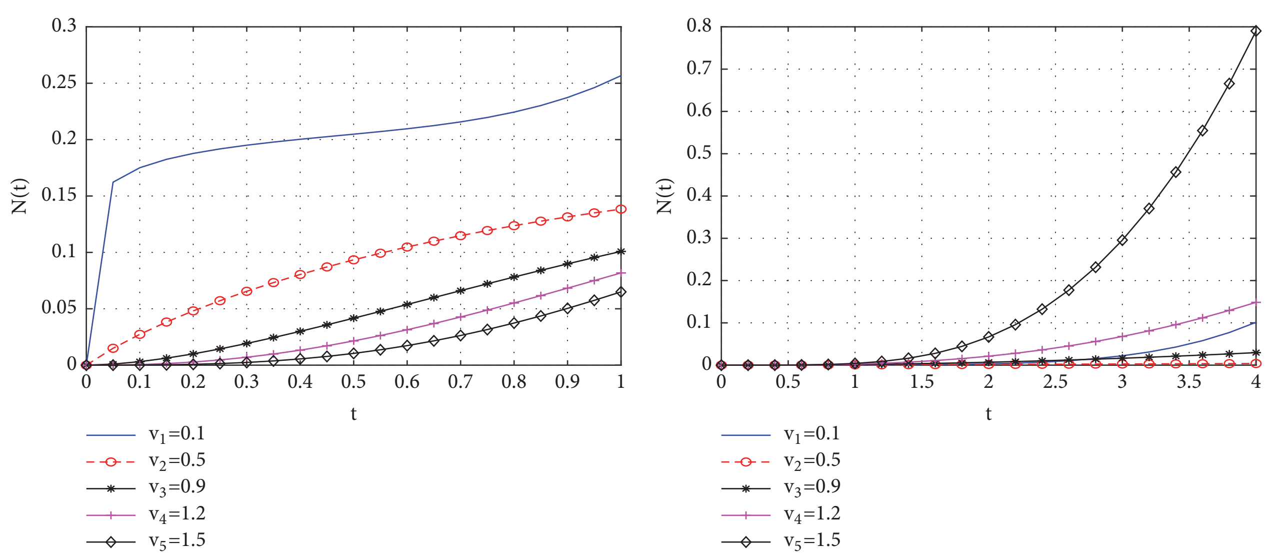
Figure 3(a) is the graphical solution of Theorem 3 using the assignment on the parameters $N_0 = 1, c = 1, q = 1, \lambda = 1, \mu = 1, m = 1, \eta = 1, b = 1, a = 2, d = 1$. And also Figure 3(b) is based on assignment for the parameters



(a) The solution of Theorem 2 for t=0:0.2:4

(b) The solution of Theorem 2 for t=0:0.05:1

FIGURE 2: Solution of fractional kinetic equation (28).



(a) The solution of Theorem 3 for t=0:0.05:1

(b) The solution of Theorem 3 for t=0:0.2:4

FIGURE 3: Solution of fractional kinetic equation (30).

$N_0 = 10, c = 1, q = 2, \lambda = 7, \mu = 7, m = 4, \eta = 7, b = 7, a = 2, d = 3$.

5. Concluding Remarks

In this paper, we derived a new fractional generalization of the kinetic equation on Theorem 1 to Theorem 3 and their related corollaries. It is not difficult to obtain several further analogous fractional kinetic equations and their solutions as shown by main results. For various other special cases we refer to [12, 13, 37, 38] and we left results for the interested readers. Also, regarding the graphical and tabular expressions in Section 3, it is easy to observe and conclude that $N(t)$ has both positive and negative results for different values of the parameters at different time intervals.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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