



Research Article

On the Analytical and Numerical Solutions in the Quantum Magnetoplasmas: The Atangana Conformable Derivative $(1 + 3)$ -ZK Equation with Power-Law Nonlinearity

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In this research paper, our work is connected with one of the most popular models in quantum magnetoplasma applications. The computational wave and numerical solutions of the Atangana conformable derivative $(1 + 3)$ -Zakharov-Kuznetsov (ZK) equation with power-law nonlinearity are investigated via the modified Khater method and septic-B-spline scheme. This model is formulated and derived by employing the well-known reductive perturbation method. Applying the modified Khater (mK) method, septic B-spline scheme to the $(1 + 3)$ -ZK equation with power-law nonlinearity after harnessing suitable wave transformation gives plentiful unprecedented ion-solitary wave solutions. Stability property is checked for our results to show their applicability for applying in the model's applications. The result solutions are constructed along with their 2D, 3D, and contour graphical configurations for clarity and exactitude.

1. Introduction

In the existence of a magnetized e-p-i plasma [1], the ZK equation is one of the widely common methods to characterize the ion-acoustic solitary waves. The magnetized load-varying dusty plasma is the best location to look for alternate placed dust ion acoustic waves of nonthermal electrons with a vortex-like spread of velocity [2]. In a comprehensive computational analysis, the ZK method was used to spread the dust-acoustic waves in a magnetized dusty plasma [3] and to excite the electrostatic ion-acoustic lone wave in two dimensions of negative ion magnetoplasmas of superthermal electrons [4]. This plasma comprises of nonthermal ions and

negatively charged mobile dust crystals, and q-distributed temperature electrons of distinct nonextensivity power [5]. The ZK equation's mathematical formula found by the well-known reductive disruption process [6] is given by

$$\mathcal{D}_t^\alpha \mathfrak{B} + \mathfrak{Q} \mathfrak{B} \mathfrak{B}_z + \mathfrak{B}_{zzz} + \mathfrak{Q} (\mathfrak{B}_{xxz} + \mathfrak{B}_{yyz}) = 0, \quad (1)$$

where $\mathcal{D}_t^\alpha = d^\alpha / dt^\alpha$, $0 < \alpha < 1$, $\mathfrak{B} = \mathfrak{B}(x, y, z, t)$, $\mathfrak{Q} = 2$, $\mathfrak{Q} = 1 - (\mathfrak{H}_e^2/8)$, $\mathfrak{H}_e = (\mathcal{H} \sqrt{\omega_{ci} \omega_{ce}})/2 \mathcal{H}_B \mathcal{T}_{Fe}$, $\mathcal{H} = 2\pi$, $\omega_{ci} = e \mathcal{B}_0 / 2 \mathcal{H}_B \mathcal{T}_{Fe}$, and $\omega_{ce} = e \mathcal{B}_0 / m_e c$. Additionally, \mathcal{H} is Planck's constant; $\sqrt{\omega_{ci} \omega_{ce}}$ is the lower-hybrid resonance frequency; $\omega_{ci} = e \mathcal{B}_0 / \mathcal{M}_i c$, $\omega_{ce} = e \mathcal{B}_0 / \mathcal{M}_e c$ are the ion

(electron) gyrofrequency; \mathcal{M}_i is the ion mass; and c is the speed of light in vacuum.

Solving this kind of models has attracted many researchers in various areas, chemical physics [7], geochemistry [8], plasma physics [8], fluid mechanics [9], optical fiber [10], solid-state physics [11], and so on [12–15]. Consequently, constructing the exact solutions of these mathematical models is an indispensable tool for detecting novel properties of them that can be used in their various applications. However, finding the exact solutions of them are not easy to process but is also considered a hard and complex process where there is no unified computational or numerical technique that is able to be applied to all nonlinear evolution (NLE) equations. Almost all computational and numerical techniques depend on an auxiliary equation that is considered a pivot tool in these techniques where all obtained solutions via these schemes are special cases of its general solutions [16–24].

For the fractional models, many analytical and numerical methods with various fractional operators have been derived such as the exponential expansion method, Khater method, Kudryashov method, simplest equation method, (Ψ'/Ψ) -expansion method, Riccati expansion method, first integral method, tanh method, and the functional variable method [25–34].

This paper studies the analytical and numerical solutions of the Atangana conformable derivative (1 + 3)-ZK equation with power-law nonlinearity that is given by [35–38].

$$\mathcal{D}_t^\alpha \mathfrak{B} + a \mathfrak{B}^n \mathfrak{B}_x + b (\mathfrak{B}_{xxx} + \mathfrak{B}_{yyx} + \mathfrak{B}_{zxx}) = 0, \quad (2)$$

where a, b , respectively, represent the nonlinearity and dispersion real valued constants. Also, \mathfrak{B}_t is the evolution term while n represents the power law nonlinearity parameter. Using the following wave transformation [39, 40] $[\mathfrak{B} = \mathfrak{B}(x, y, z, t) = \mathfrak{P}(\mathfrak{F}), \mathfrak{F} = x + y + z + (\lambda/\alpha)(t + (1/\Gamma(\alpha)))^\alpha]$ on Equation (1) where λ is an arbitrary constant yields

$$\lambda \mathfrak{P}' + \frac{a}{n+1} \mathfrak{P}^n \mathfrak{P}' + b (3 \mathfrak{P}'') = 0. \quad (3)$$

Integrating Equation (3) once with zero constant of the integration leads to

$$\lambda \mathfrak{P} + \frac{a}{n+1} \mathfrak{P}^{n+1} + 3b \mathfrak{P}'' = 0. \quad (4)$$

Through the balancing principle, the terms \mathfrak{P}^{n+1} and \mathfrak{P}'' force that $m = 2/n$. Thus, we employ another transformation $\mathfrak{P} = \mathfrak{U}^{2/m}$ on Equation (1) gives

$$\lambda \mathfrak{U}^2 + \frac{a}{n+1} \mathfrak{U}^4 + \frac{3b(4-2n)}{n^2} \mathfrak{U}'^2 + \frac{6b}{n} \mathfrak{U} \mathfrak{U}'' = 0. \quad (5)$$

Balancing between the terms of Equation (5) leads to $m = 1$.

The outline of this research paper is given as follows. Section 2 employs the mK method and septic B-spline scheme to get the abundant explicit wave and numerical solutions of the

Atangana conformable derivative (1 + 3)-ZK equation with power-law nonlinearity. Section 3 investigates the stability of the results solutions. Section 4 shows and discusses the obtained results in our research paper. Section 5 gives the graphical demonstration of some of our solutions. Section 6 explains the conclusion of our study.

2. Implementation

In this section, we employ three recent analytical schemes to find the explicit wave solutions of the Atangana conformable derivative (1 + 3)-ZK equation with power-law nonlinearity.

2.1. Ion-Acoustic Solitary Waves Solutions. This section gives a transitory elucidation of the mK method. We now explore a nontrivial solution for Equation (5) in the form

$$\begin{aligned} \mathfrak{U} &= \sum_{i=1}^m a_i \mathcal{K}^{i\mathfrak{F}(\mathfrak{F})} + \sum_{i=1}^m b_i \mathcal{K}^{-i\mathfrak{F}(\mathfrak{F})} + a_0 \\ &= a_1 \mathcal{K}^{\mathfrak{F}(\mathfrak{F})} + a_0 + b_1 \mathcal{K}^{-\mathfrak{F}(\mathfrak{F})}, \end{aligned} \quad (6)$$

where a_0, a_1 , and b_1 are arbitrary constants while $\mathfrak{F}(\mathfrak{F})$ is a function that satisfies the next ODE

$$\mathfrak{F}'(\mathfrak{F}) = \frac{u \mathcal{K}^{-\mathfrak{F}(\mathfrak{F})} + \rho \mathcal{K}^{\mathfrak{F}(\mathfrak{F})} + \delta}{\ln(\mathcal{K})}. \quad (7)$$

where u, ρ , and δ are arbitrary constants. Exchanging the values of $\mathfrak{U}, \mathfrak{U}''$ with Equation (6) along (7) and aggregation of all terms with the same power of $\mathcal{K}^{j\mathfrak{F}(\mathfrak{F})}$, ($j = -4, -3, \dots, 3, 4$) then equating the gathering terms with zero lead to a system of equations. Solving this system yields

Family I

$$\begin{aligned} a_0 &\longrightarrow \frac{a_1 \sqrt{\delta^2 - 4\rho u} + a_1 \delta}{2\rho}, \\ b_1 &\longrightarrow 0, \\ \lambda &\longrightarrow \frac{1}{4}(-3)(b\delta^2 - 4b\rho u), \\ a &\longrightarrow -\frac{9b\rho^2}{4a_1^2}, \\ n &\longrightarrow -4. \end{aligned} \quad (8)$$

Family II

$$\begin{aligned} a_0 &\longrightarrow \frac{b_1 \sqrt{\delta^2 - 4\rho u} + b_1 \delta}{2u}, \\ a_1 &\longrightarrow 0, \\ \lambda &\longrightarrow \frac{1}{4}(-3)(b\delta^2 - 4b\rho u), \\ a &\longrightarrow -\frac{9bu^2}{4b_1^2}, \\ n &\longrightarrow -4. \end{aligned} \quad (9)$$

Family III

$$\begin{aligned}
 a_1 &\longrightarrow \frac{a_0 \rho}{\delta}, \\
 b_1 &\longrightarrow \frac{a_0 u}{\delta}, \\
 \lambda &\longrightarrow \frac{1}{4}(-3)(b\delta^2 - 4b\rho u), \\
 a &\longrightarrow -\frac{9b\delta^2}{4a_0^2}, \\
 n &\longrightarrow -4.
 \end{aligned}
 \tag{10}$$

Family VI

$$\begin{aligned}
 a_0 &\longrightarrow \frac{b_1 \delta}{2u}, \\
 a_1 &\longrightarrow 0, \\
 \lambda &\longrightarrow \frac{3}{2}(b\delta^2 - 4b\rho u), \\
 a &\longrightarrow -\frac{18bu^2}{b_1^2}, \\
 n &\longrightarrow 2.
 \end{aligned}
 \tag{12}$$

Family IV

$$\begin{aligned}
 a_0 &\longrightarrow \frac{a_1 \delta}{2\rho}, \\
 b_1 &\longrightarrow 0, \\
 \lambda &\longrightarrow \frac{3}{2}(b\delta^2 - 4b\rho u), \\
 a &\longrightarrow -\frac{18b\rho^2}{a_1^2}, \\
 n &\longrightarrow 2.
 \end{aligned}
 \tag{11}$$

Thus, using the above families leads to the new exact solitary wave solutions to the Atangana conformable derivative (1 + 3)-ZK equation with power-law nonlinearity in the next formulas.

For $\delta^2 - 4\rho u < 0, \rho \neq 0$, we get

$$\begin{aligned}
 \mathfrak{B}_{I,1}(x, t) &= \frac{\sqrt{2}}{\sqrt{\left(a_1 \left(\sqrt{4\rho u - \delta^2} \tan \left(\left(\sqrt{4\rho u - \delta^2} (4\alpha \mathcal{H} - 3b\phi(\delta^2 - 4\rho u))\right)/8\alpha\right) + \sqrt{\delta^2 - 4\rho u}\right)/\rho}}, \\
 \mathfrak{B}_{I,2}(x, t) &= \frac{\sqrt{2}}{\sqrt{\left(a_1 \left(\sqrt{4\rho u - \delta^2} \cot \left(\left(\sqrt{4\rho u - \delta^2} (4\alpha \mathcal{H} - 3b\phi(\delta^2 - 4\rho u))\right)/8\alpha\right) + \sqrt{\delta^2 - 4\rho u}\right)/\rho}}, \\
 \mathfrak{B}_{II,1}(x, t) &= \frac{\sqrt{2}}{\sqrt{b_1 \left(\left(\left(\sqrt{\delta^2 - 4\rho u} + \delta\right)/u\right) - \left(4\rho/\left(\delta - \sqrt{4\rho u - \delta^2} \tan \left(\left(\sqrt{4\rho u - \delta^2} (4\alpha \mathcal{H} - 3b\phi(\delta^2 - 4\rho u))\right)/8\alpha\right)\right)\right)}}, \\
 \mathfrak{B}_{II,2}(x, t) &= \frac{\sqrt{2}}{\sqrt{b_1 \left(\left(\left(\sqrt{\delta^2 - 4\rho u} + \delta\right)/u\right) - \left(4\rho/\left(\delta - \sqrt{4\rho u - \delta^2} \cot \left(\left(\sqrt{4\rho u - \delta^2} (4\alpha \mathcal{H} - 3b\phi(\delta^2 - 4\rho u))\right)/8\alpha\right)\right)\right)}}, \\
 \mathfrak{B}_{III,1}(x, t) &= \frac{1}{\sqrt{\left(a_0(\delta^2 - 4\rho u)\right)/\left(\delta \left(-\sqrt{4\rho u - \delta^2} \sin \left(\sqrt{4\rho u - \delta^2} (\mathcal{H} - (3b\phi(\delta^2 - 4\rho u)/4\alpha))\right) + \delta \cos \left(\sqrt{4\rho u - \delta^2} (\mathcal{H} - (3b\phi(\delta^2 - 4\rho u)/4\alpha))\right) + \delta\right)}}, \\
 \mathfrak{B}_{III,2}(x, t) &= \frac{1}{\sqrt{\left(a_0(\delta^2 - 4\rho u)\right)/\left(\delta \left(\sqrt{4\rho u - \delta^2} \sin \left(\sqrt{4\rho u - \delta^2} (\mathcal{H} - (3b\phi(\delta^2 - 4\rho u)/4\alpha))\right) + \delta \cos \left(\sqrt{4\rho u - \delta^2} (\mathcal{H} - (3b\phi(\delta^2 - 4\rho u)/4\alpha))\right) - \delta\right)}}, \\
 \mathfrak{B}_{IV,1}(x, t) &= \frac{a_1 \sqrt{4\rho u - \delta^2} \tan \left(\left(\sqrt{4\rho u - \delta^2} (3b\phi(\delta^2 - 4\rho u) + 2\alpha \mathcal{H})\right)/4\alpha\right)}{2\rho}, \\
 \mathfrak{B}_{IV,2}(x, t) &= \frac{a_1 \sqrt{4\rho u - \delta^2} \cot \left(\left(\sqrt{4\rho u - \delta^2} (3b\phi(\delta^2 - 4\rho u) + 2\alpha \mathcal{H})\right)/4\alpha\right)}{2\rho}, \\
 \mathfrak{B}_{V,1}(x, t) &= \frac{1}{2} b_1 \left(\frac{\delta}{u} - \frac{4\rho}{\delta - \sqrt{4\rho u - \delta^2} \tan \left(\left(\sqrt{4\rho u - \delta^2} (3b\phi(\delta^2 - 4\rho u) + 2\alpha \mathcal{H})\right)/4\alpha\right)} \right), \\
 \mathfrak{B}_{V,2}(x, t) &= \frac{1}{2} b_1 \left(\frac{\delta}{u} - \frac{4\rho}{\delta - \sqrt{4\rho u - \delta^2} \cot \left(\left(\sqrt{4\rho u - \delta^2} (3b\phi(\delta^2 - 4\rho u) + 2\alpha \mathcal{H})\right)/4\alpha\right)} \right).
 \end{aligned}
 \tag{13}$$

For $\rho\delta^2 - 4\rho u < 0, \neq 0$, we get

$$\begin{aligned}
 \mathfrak{B}_{I,3}(x, t) &= \frac{\sqrt{2}}{\sqrt{-(a_1\sqrt{\delta^2 - 4\rho u}(\tanh((\sqrt{\delta^2 - 4\rho u}(4\alpha\mathcal{R} - 3b\phi(\delta^2 - 4\rho u)))/8\alpha) - 1)))/\rho}, \\
 \mathfrak{B}_{I,4}(x, t) &= \frac{\sqrt{2}}{\sqrt{-(a_1\sqrt{\delta^2 - 4\rho u}(\coth((\sqrt{\delta^2 - 4\rho u}(4\alpha\mathcal{R} - 3b\phi(\delta^2 - 4\rho u)))/8\alpha) - 1)))/\rho}, \\
 \mathfrak{B}_{II,3}(x, t) &= \frac{\sqrt{2}}{\sqrt{b_1\left(\left(\left(\sqrt{\delta^2 - 4\rho u} + \delta\right)/u\right) - \left(4\rho/\left(\sqrt{\delta^2 - 4\rho u} \tanh\left(\left(\sqrt{\delta^2 - 4\rho u}(4\alpha\mathcal{R} - 3b\phi(\delta^2 - 4\rho u))\right)/8\alpha\right) + \delta\right)\right)\right)}, \\
 \mathfrak{B}_{II,4}(x, t) &= \frac{\sqrt{2}}{\sqrt{b_1\left(\left(\left(\sqrt{\delta^2 - 4\rho u} + \delta\right)/u\right) - \left(4\rho/\left(\sqrt{\delta^2 - 4\rho u} \coth\left(\left(\sqrt{\delta^2 - 4\rho u}(4\alpha\mathcal{R} - 3b\phi(\delta^2 - 4\rho u))\right)/8\alpha\right) + \delta\right)\right)\right)}, \\
 \mathfrak{B}_{III,3}(x, t) &= \frac{1}{\sqrt{(a_0(\delta^2 - 4\rho u))/\left(\delta\left(\sqrt{\delta^2 - 4\rho u} \sinh\left(\sqrt{\delta^2 - 4\rho u}(\mathcal{R} - (3b\phi(\delta^2 - 4\rho u)/4\alpha))\right) + \delta \cosh\left(\sqrt{\delta^2 - 4\rho u}(\mathcal{R} - (3b\phi(\delta^2 - 4\rho u)/4\alpha))\right) + \delta\right)\right)}, \\
 \mathfrak{B}_{III,4}(x, t) &= \frac{1}{\sqrt{-(a_0(\delta^2 - 4\rho u))/\left(\delta\left(\sqrt{\delta^2 - 4\rho u} \sinh\left(\sqrt{\delta^2 - 4\rho u}(\mathcal{R} - (3b\phi(\delta^2 - 4\rho u)/4\alpha))\right) + \delta \cosh\left(\sqrt{\delta^2 - 4\rho u}(\mathcal{R} - (3b\phi(\delta^2 - 4\rho u)/4\alpha))\right) - \delta\right)\right)}, \\
 \mathfrak{B}_{IV,3}(x, t) &= -\frac{a_1\sqrt{\delta^2 - 4\rho u} \tanh\left(\left(\sqrt{\delta^2 - 4\rho u}(3b\phi(\delta^2 - 4\rho u) + 2\alpha\mathcal{R})\right)/4\alpha\right)}{2\rho}, \\
 \mathfrak{B}_{IV,4}(x, t) &= -\frac{a_1\sqrt{\delta^2 - 4\rho u} \coth\left(\left(\sqrt{\delta^2 - 4\rho u}(3b\phi(\delta^2 - 4\rho u) + 2\alpha\mathcal{R})\right)/4\alpha\right)}{2\rho}, \\
 \mathfrak{B}_{V,3}(x, t) &= \frac{1}{2}b_1\left(\frac{\delta}{u} - \frac{4\rho}{\sqrt{\delta^2 - 4\rho u} \tanh\left(\left(\sqrt{\delta^2 - 4\rho u}(3b\phi(\delta^2 - 4\rho u) + 2\alpha\mathcal{R})\right)/4\alpha\right) + \delta}\right), \\
 \mathfrak{B}_{V,4}(x, t) &= \frac{1}{2}b_1\left(\frac{\delta}{u} - \frac{4\rho}{\sqrt{\delta^2 - 4\rho u} \coth\left(\left(\sqrt{\delta^2 - 4\rho u}(3b\phi(\delta^2 - 4\rho u) + 2\alpha\mathcal{R})\right)/4\alpha\right) + \delta}\right).
 \end{aligned} \tag{14}$$

For $\rho u > 0, u \neq 0, \rho \neq 0, \delta = 0$, we get

$$\begin{aligned}
 \mathfrak{B}_{I,5}(x, t) &= \frac{1}{\sqrt{(a_1(\sqrt{\rho u} \tan(\sqrt{\rho u}((3b\rho u\phi)/\alpha + \mathcal{R})) + \sqrt{\rho(-u)})))/\rho}, \\
 \mathfrak{B}_{I,6}(x, t) &= \frac{1}{\sqrt{(a_1(\sqrt{\rho(-u)} - \sqrt{\rho u} \cot(\sqrt{\rho u}((3b\rho u\phi)/\alpha + \mathcal{R}))))/\rho}, \\
 \mathfrak{B}_{II,5}(x, t) &= \frac{1}{\sqrt{(b_1(\sqrt{\rho u} \cot(\sqrt{\rho u}((3b\rho u\phi)/\alpha + \mathcal{R})) + \sqrt{\rho(-u)})))/u}, \\
 \mathfrak{B}_{II,6}(x, t) &= \frac{1}{\sqrt{(b_1(\sqrt{\rho(-u)} - \sqrt{\rho u} \tan(\sqrt{\rho u}((3b\rho u\phi)/\alpha + \mathcal{R}))))/u}, \\
 \mathfrak{B}_{IV,5}(x, t) &= \frac{a_1 u \tan(\sqrt{\rho u}(\mathcal{R} - ((6b\rho u\phi)/\alpha)))}{\sqrt{\rho u}}, \\
 \mathfrak{B}_{IV,6}(x, t) &= -\frac{a_1 u \cot(\sqrt{\rho u}(\mathcal{R} - ((6b\rho u\phi)/\alpha)))}{\sqrt{\rho u}}, \\
 \mathfrak{B}_{V,5}(x, t) &= \frac{b_1 \rho \cot(\sqrt{\rho u}(\mathcal{R} - ((6b\rho u\phi)/\alpha)))}{\sqrt{\rho u}}, \\
 \mathfrak{B}_{V,6}(x, t) &= -\frac{b_1 \rho \tan(\sqrt{\rho u}(\mathcal{R} - ((6b\rho u\phi)/\alpha)))}{\sqrt{\rho u}}.
 \end{aligned} \tag{15}$$

For $\rho u < 0, u \neq 0, \rho \neq 0, \delta = 0$, we get

$$\mathfrak{B}_{I,7}(x, t) = \frac{1}{\sqrt{\left(a_1 u \left(\tanh \left(\sqrt{\rho(-u)}((3b\rho u\phi/\alpha) + \mathcal{H}) - 1\right)\right) / \sqrt{\rho(-u)}\right)}}, \quad (16)$$

$$\mathfrak{B}_{I,8}(x, t) = \frac{1}{\sqrt{\left(a_1 u \left(\coth \left(\sqrt{\rho(-u)}((3b\rho u\phi/\alpha) + \mathcal{H}) - 1\right)\right) / \sqrt{\rho(-u)}\right)}}, \quad (17)$$

$$\mathfrak{B}_{II,7}(x, t) = \frac{1}{\sqrt{\left(b_1 \sqrt{\rho(-u)} \left(\coth \left(\sqrt{\rho(-u)}((3b\rho u\phi/\alpha) + \mathcal{H}) + 1\right)\right) / u}\right)}, \quad (18)$$

$$\mathfrak{B}_{II,8}(x, t) = \frac{1}{\sqrt{\left(b_1 \sqrt{\rho(-u)} \left(\tanh \left(\sqrt{\rho(-u)}((3b\rho u\phi/\alpha) + \mathcal{H}) + 1\right)\right) / u}\right)}, \quad (19)$$

$$\mathfrak{B}_{IV,7}(x, t) = \frac{a_1 u \tanh \left(\sqrt{\rho(-u)}(\mathcal{H} - (6b\rho u\phi)/\alpha)\right)}{\sqrt{\rho(-u)}}, \quad (20)$$

$$\mathfrak{B}_{IV,8}(x, t) = \frac{a_1 u \coth \left(\sqrt{\rho(-u)}(\mathcal{H} - (6b\rho u\phi)/\alpha)\right)}{\sqrt{\rho(-u)}}, \quad (21)$$

$$\mathfrak{B}_{V,7}(x, t) = \frac{b_1 \sqrt{\rho} \cot \left(\sqrt{\rho} \sqrt{u}(\mathcal{H} - (6b\rho u\phi)/\alpha)\right)}{\sqrt{u}}, \quad (22)$$

$$\mathfrak{B}_{V,8}(x, t) = \frac{b_1 \sqrt{\rho(-u)} \tanh \left(\sqrt{\rho(-u)}(\mathcal{H} - \mathcal{H} - (6b\rho u\phi)/\alpha)\right)}{u}. \quad (23)$$

For $\delta = u/2 = \kappa, \rho = 0$, we get

$$\begin{aligned} \mathfrak{B}_{II,9}(x, t) &= \frac{2}{\sqrt{b_1 \left((4 / (e^{\kappa \mathcal{H}} - ((3b\kappa^3 \phi)/4\alpha) - 2)) + (\kappa / \sqrt{\kappa^2}) + 1 \right)}}, \\ \mathfrak{B}_{III,9}(x, t) &= \frac{1}{\sqrt{a_0 (2 / (e^{\kappa \mathcal{H}} - ((3b\kappa^3 \phi)/4\alpha) - 2) + 1)}}, \\ \mathfrak{B}_{VI,9}(x, t) &= \frac{1}{4} b_1 \left(\frac{4}{e^{((3b\kappa^3 \phi)/2\alpha) + \kappa \mathcal{H}} - 2} + 1 \right). \end{aligned} \quad (24)$$

For $\delta = \rho = \kappa, u = 0$, we get

$$\begin{aligned} \mathfrak{B}_{I,10}(x, t) &= \frac{\sqrt{2}}{\sqrt{a_1 \left((\kappa / \sqrt{\kappa^2}) - \coth \left((\kappa \mathcal{H} / 2) - ((3b\kappa^3 \phi) / 8\alpha) \right) \right)}}, \\ \mathfrak{B}_{III,10}(x, t) &= \frac{1}{\sqrt{-a_0 / e^{\kappa \mathcal{H}} - ((3b\kappa^3 \phi) / 4\alpha) - 1}}, \\ \mathfrak{B}_{IV,10}(x, t) &= -\frac{1}{2} a_1 \coth \left(\frac{3b\kappa^3 \phi}{4\alpha} + \frac{\kappa \mathcal{H}}{2} \right). \end{aligned} \quad (25)$$

For $u = 0, \delta \neq 0, \rho \neq 0$, we get

$$\begin{aligned} \mathfrak{B}_{I,11}(x, t) &= \frac{\sqrt{2}}{\sqrt{\left(a_1 \left(\delta \left(-4 / \left(\rho e^{\delta \mathcal{H}} - ((3b\delta^3 \phi) / 4\alpha) - 2 \right) - 1 \right) + \sqrt{\delta^2} \right) / \rho}\right)}, \\ \mathfrak{B}_{III,10}(x, t) &= \frac{1}{\sqrt{2} \sqrt{-a_0 / \left(\rho e^{(\delta \mathcal{H}) - ((3b\delta \wedge 3 \phi) / 4\alpha)} - 2 \right)}}, \\ \mathfrak{B}_{IV,10}(x, t) &= \frac{a_1 \delta (4 / (2 - \rho e^{((3b\delta \wedge 3 \phi) / 2\alpha + \delta \mathcal{H})}) - 1)}{2\rho}. \end{aligned} \quad (26)$$

For $\delta = \rho = 0, u \neq 0$, we get

$$\begin{aligned} \mathfrak{B}_{II,10}(x, t) &= \frac{1}{\sqrt{b_1 / \lambda t u + x u}}, \\ \mathfrak{B}_{V,10}(x, t) &= \frac{b_1}{\lambda t u + x u}. \end{aligned} \quad (27)$$

For $\delta = 0, u = \rho$, we get

$$\begin{aligned} \mathfrak{B}_{I,11}(x, t) &= \frac{1}{\sqrt{a_1 \left(\tan \left((3bu^3 \phi / \alpha) + C + \mathcal{H} u \right) + (\sqrt{-u} / \sqrt{u}) \right)}}, \\ \mathfrak{B}_{II,11}(x, t) &= \frac{1}{\sqrt{b_1 \left(\cot \left((3bu^3 \phi / \alpha) + C + \mathcal{H} u \right) + (\sqrt{-u} / \sqrt{u}) \right)}}, \\ \mathfrak{B}_{IV,11}(x, t) &= \frac{1}{\sqrt{b_1 \left(\cot \left((3bu^3 \phi / \alpha) + C + \mathcal{H} u \right) + (\sqrt{-u} / \sqrt{u}) \right)}}, \\ \mathfrak{B}_{V,11}(x, t) &= a_1 \tan \left(-\frac{6bu^3 \phi}{\alpha} + C + \mathcal{H} u \right), \\ \mathfrak{B}_{VI,11}(x, t) &= b_1 \cot \left(-\frac{6bu^3 \phi}{\alpha} + C + \mathcal{H} u \right). \end{aligned} \quad (28)$$

For $\rho = 0, \delta \neq 0, u \neq 0$, we get

$$\begin{aligned} \mathfrak{B}_{II,12}(x, t) &= \frac{1}{2} b_1 \left(\frac{2\delta}{\delta e^{\delta \mathcal{H} - (3b\delta^3 \phi/4\alpha)} - u} + \frac{\sqrt{\delta^2 + \delta}}{u} \right), \\ \mathfrak{B}_{III,12}(x, t) &= a_0 \left(\frac{u}{\delta e^{\delta \mathcal{H} - (3b\delta^3 \phi/4\alpha)} - u} + 1 \right), \\ \mathfrak{B}_{VI,12}(x, t) &= \frac{1}{2} b_1 \delta \left(\frac{2}{\delta e^{(3b\delta^3 \phi/2\alpha) + \delta \mathcal{H}} - u} + \frac{1}{u} \right). \end{aligned} \tag{29}$$

where $\mathcal{H} = x + y + z, \phi = (t + (1/\Gamma(\alpha)))^\alpha$.

2.2. Numerical Solutions. Here, we use three different analytical solutions Equations (16), (19) and (20) to evaluate

the numerical solutions of the Atangana conformable derivative (1 + 3)-ZK equation with power-law nonlinearity. Employing the septic spline technique to Equation (5) with the following conditions $a_1 = 3, a_0 = -6, a = -5/4, b = 5, b_1 = 0, \delta = 0, \lambda = -60, n = -4, \rho = -1, u = 4 \& a_0 = -25, a_1 = 0, a = -9/20, b_1 = 5, b = 2, \delta = 0, \lambda = -150, n = -4, \rho = 25, u = -1 \& a_1 = 3, a_0 = 0, a = -4, b = 2, b_1 = 0, \delta = 0, \lambda = 108, n = 2, \rho = -1$, and $u = 9$ gives its numerical solutions in the next form

$$\mathfrak{B}(\mathfrak{F}) = \sum_{\mathfrak{Z}=-1}^{\mathfrak{M}+1} \mathfrak{C}_{\mathfrak{M}} \mathfrak{E}_{\mathfrak{M}}, \tag{30}$$

where $c_{\mathfrak{M}}, \mathfrak{E}_{\mathfrak{M}}$ follow the next conditions, respectively:

$\mathfrak{Q} \mathfrak{B}(\mathfrak{F}) = \mathcal{F}(\mathfrak{F}_{\mathfrak{M}}, \mathfrak{B}(\mathfrak{F}_{\mathfrak{M}}))$ where $(\mathfrak{M} = 0, 1, \dots, n)$

$$\mathfrak{E}_{\mathfrak{Z}}(\mathfrak{F}) = \frac{1}{\mathfrak{J}^5} \begin{cases} (\mathfrak{F} - \mathfrak{F}_{\mathfrak{Z}-4})^7, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{Z}-4}, \mathfrak{F}_{\mathfrak{Z}-3}], \\ (\mathfrak{F} - \mathfrak{F}_{\mathfrak{Z}-4})^7 - 8(\mathfrak{F} - \mathfrak{F}_{\mathfrak{Z}-3})^7, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{Z}-3}, \mathfrak{F}_{\mathfrak{Z}-2}], \\ (\mathfrak{F} - \mathfrak{F}_{\mathfrak{Z}-4})^7 - 8(\mathfrak{F} - \mathfrak{F}_{\mathfrak{Z}-3})^7 + 28(\mathfrak{F} - \mathfrak{F}_{\mathfrak{Z}-2})^7, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{Z}-2}, \mathfrak{F}_{\mathfrak{Z}-1}], \\ (\mathfrak{F} - \mathfrak{F}_{\mathfrak{Z}-4})^7 - 8(\mathfrak{F} - \mathfrak{F}_{\mathfrak{Z}-3})^7 + 28(\mathfrak{F} - \mathfrak{F}_{\mathfrak{Z}-2})^7 + 56(\mathfrak{F} - \mathfrak{F}_{\mathfrak{Z}-1})^7, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{Z}-1}, \mathfrak{F}_{\mathfrak{Z}}], \\ (\mathfrak{F}_{\mathfrak{Z}+4} - \mathfrak{F})^7 - 8(\mathfrak{F}_{\mathfrak{Z}+3} - \mathfrak{F})^7 + 28(\mathfrak{F}_{\mathfrak{Z}+2} - \mathfrak{F})^7 + 56(\mathfrak{F}_{\mathfrak{Z}+1} - \mathfrak{F})^7, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{Z}}, \mathfrak{F}_{\mathfrak{Z}+1}], \\ (\mathfrak{F}_{\mathfrak{Z}+4} - \mathfrak{F})^7 - 8(\mathfrak{F}_{\mathfrak{Z}+3} - \mathfrak{F})^7 + 28(\mathfrak{F}_{\mathfrak{Z}+2} - \mathfrak{F})^7, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{Z}+1}, \mathfrak{F}_{\mathfrak{Z}+2}], \\ (\mathfrak{F}_{\mathfrak{Z}+4} - \mathfrak{F})^7 - 8(\mathfrak{F}_{\mathfrak{Z}+3} - \mathfrak{F})^7, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{Z}+2}, \mathfrak{F}_{\mathfrak{Z}+3}], \\ (\mathfrak{F}_{\mathfrak{Z}+4} - \mathfrak{F})^7, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{Z}+3}, \mathfrak{F}_{\mathfrak{Z}+4}], \\ 0, & \text{otherwise.} \end{cases} \tag{31}$$

For $\mathfrak{Z} \in [-3, \mathfrak{M} + 3]$, we get

$$\begin{aligned} \mathfrak{B}_{\mathfrak{Z}}(\mathfrak{F}) &= \mathfrak{C}_{\mathfrak{Z}-3} + 120 \mathfrak{C}_{\mathfrak{Z}-2} + 1191 \mathfrak{C}_{\mathfrak{Z}-1} \\ &+ 2416 \mathfrak{C}_{\mathfrak{Z}} + 1191 \mathfrak{C}_{\mathfrak{Z}+1} + 120 \mathfrak{C}_{\mathfrak{Z}+2} + \mathfrak{C}_{\mathfrak{Z}+3}. \end{aligned} \tag{32}$$

Substituting Equation (32) into Equation (5) gives $(\mathfrak{M} + 7)$ of equations. Resolving this system leads to the following values of exact, numerical, and absolute values or error.

3. Stability Characteristics

In this section, the stability property has been tested of the obtained results based on the Hamiltonian system characteristics. This system imposes a single condition to ensure the

stability of the solution. This condition is given by

$$\left. \frac{\partial \mathcal{M}}{\partial \lambda} \right|_{\lambda=\mathfrak{F}} > 0, \tag{33}$$

where $\mathcal{M} = (1/2) \int_{-\mathfrak{C}}^{\mathfrak{C}} \mathfrak{B}^2 d\mathfrak{F}$ where \mathfrak{C} is an arbitrary constants, λ is the frequency, and \mathfrak{F} is an arbitrary constant.

Applying the stability check of Equation (20) with the following values of the parameters $a_1 = 3, a_0 = 0, a = -4, b = 2, b_1 = 0, \delta = 0, \lambda = 108, n = 2, \rho = -1$, and $u = 9$, leads to

$$\left. \frac{\partial \mathcal{M}}{\partial \lambda} \right|_{\lambda=6} = -2.3447910280083306 \times 10^{-13} < 0. \tag{34}$$

Consequently, this solution is not stable and applying the

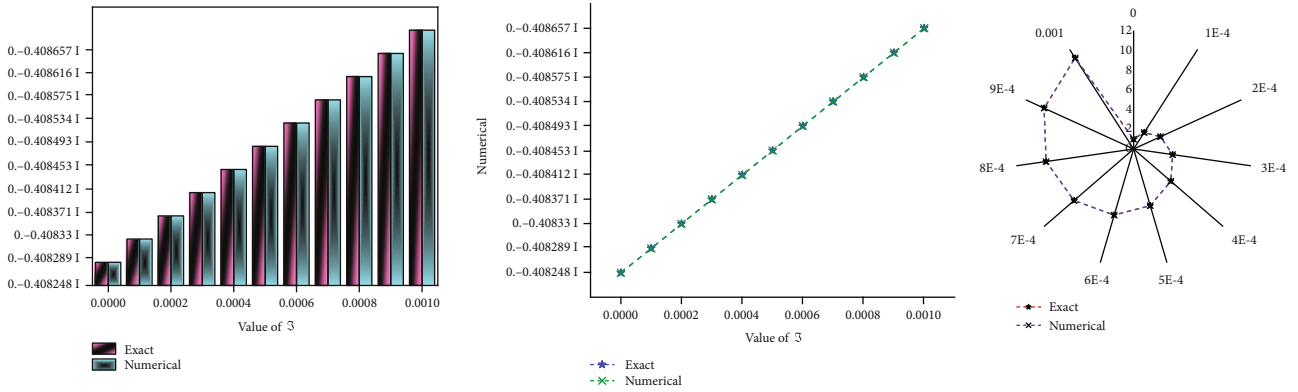


FIGURE 1: Exact, and numerical solutions based on the obtained analytical solution Equation (16) and septic B-spline scheme.

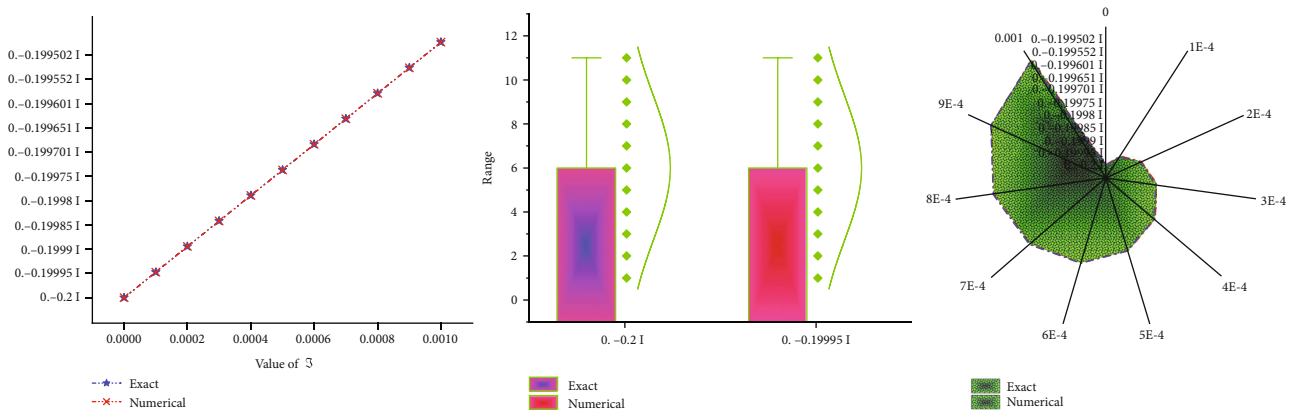


FIGURE 2: Exact and numerical solutions based on the obtained analytical solution Equation (19) and septic B-spline scheme.

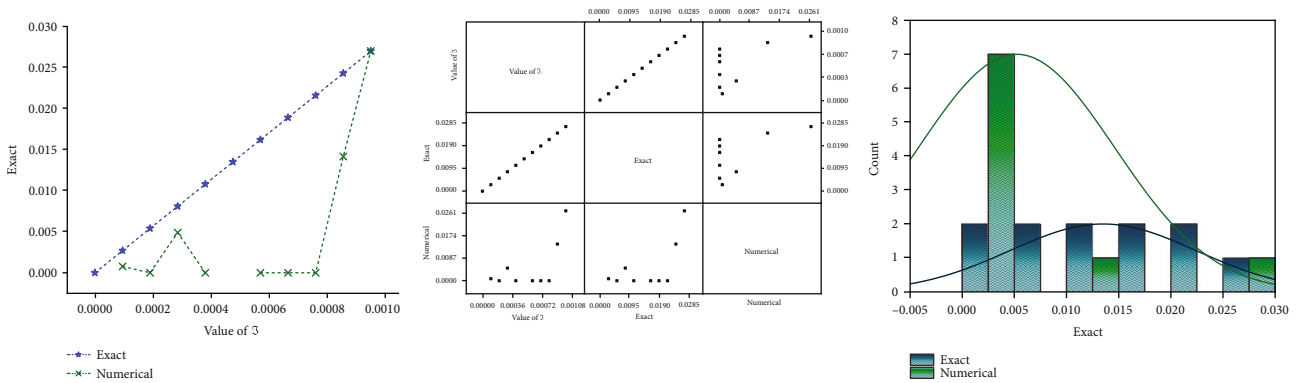


FIGURE 3: Exact and numerical solutions based on the obtained analytical solution Equation (20) and septic B-spline scheme.

same steps to other obtained solutions investigates their stability property.

4. Result and Discussion

Here, we discuss our obtained solutions of the Atangana conformable derivative (1 + 3)-ZK equation with power-law nonlinearity that have been obtained through one of the most recent computational schemes in nonlinear evolution equation field (the mK method) via two main axes which are a compar-

ison between our obtained computational solutions and other previous obtained solutions, while the second axis of this discussion is studying our exact and numerical solutions.

(i) Computational solutions

- (1) Applying the modified Khater method to the Atangana conformable derivative (1 + 3)-ZK equation with power-law nonlinearity has obtained sixty distinct traveling wave solutions

- (2) The difference between our obtained solutions and that have been obtained in [41] by Aminikhah et al. who had used the functional variable method; however, they have just found three solutions and accurate in their and our solutions, we can figure out the complete difference between these solutions that thing makes our solutions are novel

(ii) Numerical solutions

- (1) Applying the septic B-spline scheme to the Atangana conformable derivative (1 + 3)-ZK equation with power-law nonlinearity by using three of our obtained solutions in evaluating the initial and boundary conditions that give the ability of employing the septic B-spline scheme to the fractional model

5. Figure and Table Interpretation

This section illustrates our explained Figures 1–3 and Tables 1–3 with the abovementioned values of the parameters.

- (i) Figure 1 and Table 1 show the value of the exact and numerical solutions and absolute error of Equation (5) with Equation (16) in three distinct types of sketches to explain the convergence between the two types of solutions
- (ii) Figure 2 and Table 2 show the value of exact and numerical solutions and absolute error of Equation (5) with Equation (19) in three distinct types of sketches to illustrate the closer between the two types of solutions
- (iii) Figure 3 and Table 3 explain the value of exact, numerical solutions and absolute error of Equation (5) with Equation (20) in three distinct types of sketches to show the matching between the two types of solutions

6. Conclusion

This paper has succeeded in the implementation of the mK method and septic B-spline scheme to the Atangana conformable derivative (1 + 3)-ZK equation with power-law nonlinearity. Sixty distinct novel computational solutions have been obtained. Three of these solutions have been used to evaluate the initial and boundary conditions that have allowed the application of the numerical scheme. Calculating the absolute value of error between the exact and numerical is the aim of our study. Moreover, the stability of our obtained solutions has been illustrated based on the Hamiltonian system characteristics. The effectiveness and power of our two used schemes have been verified, and all obtained solutions have been also verified by putting them back in the original equation via Mathematica 12 software.

TABLE 1: Exact and numerical value of the Atangana conformable derivative (1 + 3)-ZK equation with power-law nonlinearity through the obtained analytical solutions via the modified Khater method Equation (16) and septic B-spline scheme.

Value of \mathfrak{F}	Exact	Numerical	Absolute error
0	0. -0.408248 I	0. -0.0000110311 I	0.408237
0.0001	0. -0.408289 I	0. -8.37587 $\times 10^{-6}$ I	0.408281
0.0002	0. -0.40833 I	0. -5.50221 $\times 10^{-6}$ I	0.408324
0.0003	0. -0.408371 I	0. -2.53611 $\times 10^{-6}$ I	0.408368
0.0004	0. -0.408412 I	0. -1.13958 $\times 10^{-6}$ I	0.408411
0.0005	0. -0.408453 I	0. +1.58999 $\times 10^{-7}$ I	0.408453
0.0006	0. -0.408493 I	0. -1.01297 $\times 10^{-6}$ I	0.408492
0.0007	0. -0.408534 I	0. -2.41857 $\times 10^{-6}$ I	0.408532
0.0008	0. -0.408575 I	0. -5.26488 $\times 10^{-6}$ I	0.40857
0.0009	0. -0.408616 I	0. -7.96518 $\times 10^{-6}$ I	0.408608
0.001	0. -0.408657 I	0. -0.0000103138 I	0.408647

TABLE 2: Exact and numerical value of the Atangana conformable derivative (1 + 3)-ZK equation with power-law nonlinearity through the obtained analytical solutions via the modified Khater method Equation (19) and septic B-spline scheme.

Value of \mathfrak{F}	Exact	Numerical	Absolute error
0	0. -0.2 I	0. -0.0000794823 I	0.199921
0.0001	0. -0.19995 I	0. -0.0000595992 I	0.19989
0.0002	0. -0.1999 I	0. -0.0000399605 I	0.19986
0.0003	0. -0.19985 I	0. -0.0000214 I	0.199829
0.0004	0. -0.1998 I	0. -8.76169 $\times 10^{-6}$ I	0.199792
0.0005	0. -0.19975 I	0. +1.47924 $\times 10^{-6}$ I	0.199752
0.0006	0. -0.199701 I	0. -7.5007 $\times 10^{-6}$ I	0.199693
0.0007	0. -0.199651 I	0. -0.0000199929 I	0.199631
0.0008	0. -0.199601 I	0. -0.0000376969 I	0.199564
0.0009	0. -0.199552 I	0. -0.000056177 I	0.199495
0.001	0. -0.199502 I	0. -0.0000736582 I	0.199428

TABLE 3: Exact and numerical value of the Atangana conformable derivative (1 + 3)-ZK equation with power-law nonlinearity through the obtained analytical solutions via the modified Khater method Equation (20) and septic B-spline scheme.

Value of \mathfrak{F}	Exact	Numerical	Absolute error
0	0	3.46945 $\times 10^{-17}$	3.46945 $\times 10^{-17}$
0.0001	0.0027	0.000779236	0.00192076
0.0002	0.0054	-8.67362E-18	0.0054
0.0003	0.0081	0.00489544	0.00320455
0.0004	0.0108	0	0.0108
0.0005	0.0135	6.50521E-19	0.0135
0.0006	0.0162	0	0.0162
0.0007	0.0189	-4.33681E-19	0.0189
0.0008	0.0216	-8.67362E-19	0.0216
0.0009	0.0242999	0.0141565	0.0101435
0.001	0.0269999	0.0269999	3.46945 $\times 10^{-18}$

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this article.

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