Research Article

Fractal Ion Acoustic Waves of the Space-Time Fractional Three Dimensional KP Equation

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Methods known as fractional subequation and sine-Gordon expansion (FSGE) are employed to acquire new exact solutions of some fractional partial differential equations emerging in plasma physics. Fractional operators are employed in the sense of conformable derivatives (CD). New exact solutions are constructed in terms of hyperbolic, rational, and trigonometric functions. Computational results indicate the power of the method.

1. Introduction

Nonlinear propagation of electrostatic excitations in electron-positron ion plasmas and nonthermal distribution of electrons is an important research area in astrophysical and space plasmas [1–6].

Many important phenomena such as the effective behavior of the ionized matter, magnetic field near the surfaces of the sun and stars, emission mechanisms of pulsars, the origin of cosmic rays and radio sources, dynamics of magnetosphere, and propagation of electromagnetic radiation through the upper atmosphere required the study of plasma physics. Equations such as Korteweg de Vries (KdV), Burgers, KdV-Burgers, and Kadomtsev-Petviashvili (KP) were highly used models in the description of plasma systems.

We study the physical phenomena for space-time fractional KP equation with the aid of fractional calculus and examine the resulting solutions in detail. The fractional calculus [7–13] has a wide range of applications and is deeply rooted in the field of probability, mathematical physics, differential equations, and so on. Very recently, fractional differential equations have got a lot of consideration as they define many complex phenomena in various fields. Several fractional-order models play very important roles in different areas including physics, engineering, mechanics and dynamical systems, signal and image processing, control theory, biology, and materials [14–18].

The paper is summarized as follows. Definitions and properties of conformable derivatives are discussed. In Section 2, a discussion about the two algorithms method,

namely, fractional subequation method and sine-Gordon expansion method for solving FPDEs arising in plasma physics are given. In Section 3, two schemes are employed for some new exact solutions for the FKPE. We presented a graphical description of some of the solutions with a fixed value of fractal order α in a brief conclusion at the end of the article.

Definition 1. Let $\psi : (0, \infty) \longrightarrow R$. Some definitions, useful properties, and a theorem about conformable derivatives are given as follows:

$$\begin{aligned} U_\alpha(\psi)(t) &= \lim_{\varepsilon \rightarrow 0} \frac{\psi(t + \varepsilon t^{1-\alpha}) - \psi(t)}{\varepsilon}, \quad t > 0, \quad 0 < \alpha < 1, \\ U_\alpha(b\psi + ch) &= BU_\alpha(\psi) + CU_\alpha(h), \quad B, C \in R, \\ U_\alpha t^\lambda &= \lambda t^{\lambda-\alpha}, \quad \lambda \in R, \\ U_\alpha(\psi h) &= \psi U_\alpha(h) + h U_\alpha(\psi), \\ U_\alpha\left(\frac{\psi}{h}\right) &= \frac{h U_\alpha(\psi) - \psi U_\alpha(h)}{h^2}. \end{aligned} \quad (1)$$

If ψ is differentiable, then $U_\alpha(\psi)(t) = t^{1-\alpha}(d\psi/dh)$.

Theorem 2. Let $\psi : (0, \infty) \longrightarrow R$ be a differentiable function. Then,

$$U_\alpha(\psi * h) = t^{1-\alpha} h'(t) \psi'(h(t)). \quad (2)$$

2. Solution Method

2.1. Extended Fractional Subequation Method. For a given nonlinear FPDE as

$$\chi(u_1 u_k, D_t^\alpha u_1 D_t^\alpha u_k, D_{x_1}^\alpha u_1 D_{x_1}^\alpha u_k) = 0, \quad 0 < \alpha < 1. \quad (3)$$

in which χ is a polynomial of u . Using wave transformation as

$$u_i(t, x_1, x_2, \dots, x_n) = U_i(\xi), \quad \xi = ct + \dots + k_n x_n, \quad (4)$$

Eq. (3) reads

$$\phi(U_1 \dots U_k, c^\alpha D_\xi^\alpha U_1 \dots c^\alpha D_\xi^\alpha U_k, k_1^\alpha D_\xi^\alpha U_1 \dots k_n^\alpha D_\xi^\alpha U_k) = 0, \quad 0 < \alpha < 1. \quad (5)$$

Thus,

$$U_i(\xi) = \sum_{i=0}^M a_i \left(\frac{D_\xi^\alpha w(\xi)}{w(\xi)} \right)^i, \quad (6)$$

where $w = w(\xi)$ satisfies

$$Aw(\xi) D_\xi^{2\alpha} w(\xi) - Bw(\xi) D_\xi^\alpha w(\xi) - C[D_\xi^\alpha w(\xi)]^2 - Ew(\xi)^2 = 0, \quad (7)$$

where $D_\xi^\alpha w(\xi)$ is a RL fractional operator of order α . To solve Eq. (7), assume $w(\xi) = z(\eta)$, with the fractional complex transformation, then

$$Az((\eta))z'(\eta) - Bz(\eta)z'(\eta) - C[z'(\eta)]^2 - Ez^2(\eta) = 0. \quad (8)$$

Since $D_\xi^\alpha w(\xi) = D_\eta^\alpha z(\eta) = z'(\eta)$. The general solutions Eq. (7) is as follows: where $\nu_1 = B^2 + 4E(A - C)$, $\nu_2 = E(A - C)$, L_1 , and L_2 are arbitrary constants and $\eta = \xi^\alpha / \Gamma(1 + \alpha)$. Inserting Eq. (6) into (5) knowing Eq. (7), collecting the same order terms ($D_\xi^\alpha w(\xi)/w(\xi)$), then equating it to zero, k and c are obtained. As long as the solutions are obtained with the general expression $[D_\xi^\alpha w(\xi)/w(\xi)]$, admits several solutions of Eq. (3).

Family 1. As long as $B \neq 0$, $\nu_1 > 0$, admits to

$$\begin{aligned} \left[\frac{D_\xi^\alpha w(\xi)}{w(\xi)} \right] &= \frac{B}{2(A - C)} \\ &+ \frac{\sqrt{\nu_1}}{2(A - C)} \left[\frac{L_1 \sin h(\sqrt{\nu_1} \eta / 2(A - C)) + L_2 \cos h(\sqrt{\nu_1} \eta / 2(A - C))}{L_1 \cos h(\sqrt{\nu_1} \eta / 2(A - C)) + L_2 \sin h(\sqrt{\nu_1} \eta / 2(A - C))} \right]. \end{aligned} \quad (9)$$

Family 2. Limiting case $B \neq 0$, $\nu_1 < 0$ gains

$$\begin{aligned} \left[\frac{D_\xi^\alpha w(\xi)}{w(\xi)} \right] &= \frac{B}{2(A - C)} \\ &+ \frac{\sqrt{-\nu_1}}{2(A - C)} \left[\frac{-L_1 \sin(\sqrt{-\nu_1} \eta / 2(A - C)) + L_2 \cos(\sqrt{-\nu_1} \eta / 2(A - C))}{L_1 \cos(\sqrt{-\nu_1} \eta / 2(A - C)) + L_2 \sin(\sqrt{-\nu_1} \eta / 2(A - C))} \right]. \end{aligned} \quad (10)$$

Family 3. For $B \neq 0$, $\nu_1 = 0$,

$$\left[\frac{D_\xi^\alpha w(\xi)}{w(\xi)} \right] = \frac{B}{2(A - C)} + \frac{L_2}{L_1 + L_2 \eta}. \quad (11)$$

Family 4. When $B = 0$, $\nu_2 > 0$,

$$\left[\frac{D_\xi^\alpha w(\xi)}{w(\xi)} \right] = \frac{\sqrt{\nu_2}}{2(A - C)} \left[\frac{L_1 \sin h(\sqrt{\nu_2} \eta / 2(A - C)) + L_2 \cos h(\sqrt{\nu_2} \eta / 2(A - C))}{L_1 \cos h(\sqrt{\nu_2} \eta / 2(A - C)) + L_2 \sin h(\sqrt{\nu_2} \eta / 2(A - C))} \right]. \quad (12)$$

Family 5. When $B = 0$, $\nu_2 < 0$, then

$$\left[\frac{D_\xi^\alpha w(\xi)}{w(\xi)} \right] = \frac{\sqrt{-\nu_2}}{2(A - C)} \left[\frac{-L_1 \sin(\sqrt{-\nu_2} \eta / 2(A - C)) + L_2 \cos(\sqrt{-\nu_2} \eta / 2(A - C))}{L_1 \cos(\sqrt{-\nu_2} \eta / 2(A - C)) + L_2 \sin(\sqrt{-\nu_2} \eta / 2(A - C))} \right], \quad (13)$$

2.2. Analysis of the Fractional Sine-Gordon Expansion (FSGE) Method. Let us first consider the fractional sine-Gordon equation as

$$V_{xx} - D_t^{2\alpha} V = m^2 \sin(V), \quad (14)$$

m is constant.

By using the transformation $V = V(\xi)$, $\xi = a(x - v(t^\alpha/\alpha))$. Then Eq. (14) yields

$$\frac{d^2(V/2)}{d\xi^2} = \frac{m^2}{a^2(1-v^2)} \sin^2(V/2) + C, \quad (15)$$

where C is an integration constant to be zero. Setting $\chi(\xi) = V(\xi)/2$, $b^2 = m^2/a^2(1-v^2)$. Then Eq. (15) reads

$$\frac{d(\chi w(\xi))}{d\xi} = b \sin(\chi), \quad (16)$$

Setting $b = 1$, we have

$$\sin(\chi(\xi)) = \frac{2de^\xi}{d^2e^{2\xi} + 1} = \sec h(\xi), \quad \cos(\chi(\xi)) = \frac{d^2e^{2\xi} - 1}{d^2e^{2\xi} + 1} = \tan h(\xi), \quad (17)$$

$$\sin(\chi(\xi)) = i \csc h(\xi), \quad \cos(\chi(\xi)) = \cot h(\xi), \quad d = 1. \quad (18)$$

In view of this method, we assume the trail solutions by

$$V(\xi) = \sum_{j=1}^N \tan h^{j-1}(\xi) [B_j \sec h(\xi) + A_j \csc h(\xi)] + A_0, \quad (19)$$

$$V(\xi) = \sum_{j=1}^N \cot h^{j-1}(\xi) [jB_j \csc h(\xi) + A_j \cot h(\xi)] + A_0. \quad (20)$$

Making use of Eq. (18), then Eq. (19) can be rewritten as follows

$$V(\xi) = \sum_{j=1}^N \cos^{j-1}(\xi) [B_j \sin(\xi) + A_j \cos(\xi)] + A_0, \quad (21)$$

where N can be obtained by balancing principle. Inserting Eq. (21) into (15) and the collecting the same power of $\cos^i(\xi)$ si $n^j(\xi)$, admitting the system of algebraic equation, by solving them by Maple, the coefficient values A_j, B_j, v can be determined. Inserting these values into Eq. (19), the exact solutions of Eq. (14) are determined.

3. New Applications

In this part of our research, we apply a novel computational approach mentioned above to illustrate the advantages for finding analytical solutions of (3 + 1)-dimension space-time FKPE which is as follows

$$\begin{aligned} & D_x^\alpha D_t^\alpha u(x, y, z, t) + d D_x^\alpha [u(x, y, z, t) D_x^\alpha u(x, y, z, t)] \\ & + v D_y^{\alpha\alpha\alpha} u(x, y, z, t) + \beta D_z^{\alpha\alpha} u(x, y, z, t) \\ & + v D_x^{\alpha\alpha} u(x, y, z, t) = 0, \end{aligned} \quad (22)$$

where $u(\xi) = U(x, y, z, t)$ is the field function, v, δ, d , and $\beta \in \mathbb{R}$. Let $u(x, y, z, t) = U(\xi)$, where $\xi = kx + ct + ly + mz + \xi_0$, k, l, c, m, ξ_0 then

$$\begin{aligned} D_x^\alpha U &= D_x^\alpha U(\xi) = (D_\xi^\alpha U) \left(\xi_x' \right)^\alpha = k^\alpha D_\xi^\alpha U, \\ D_t^\alpha U &= D_t^\alpha U(\xi) = (D_\xi^\alpha U) \left(\xi_t' \right)^\alpha = c^\alpha D_\xi^\alpha U. \end{aligned} \quad (23)$$

Then, Eq. (22) reduces to

$$\begin{aligned} & c^\alpha k^\alpha D_\xi^{\alpha\alpha} U(\xi) + dk^{2\alpha} D_\xi^\alpha [U(\xi) D_\xi^\alpha U(\xi)] + vk^{4\alpha} D_\xi^{\alpha\alpha\alpha\alpha} U(\xi) \\ & + \delta l^{2\alpha} D_\xi^{\alpha\alpha} U(\xi) + m^{2\alpha} \beta D_\xi^{\alpha\alpha} U(\xi) = 0. \end{aligned} \quad (24)$$

Now, we assume the solution of Eq. (24) as

$$U(\xi) = \sum_{i=0}^M a_i \left(\frac{D_\xi^\alpha w(\xi)}{w(\xi)} \right)^i, \quad (25)$$

where $w = w(\xi)$. Using the proposed algorithm for Eq. (24), we have $M = 2$.

Then,

$$U(\xi) = a_0 + a_1 \left(\frac{D_\xi^\alpha w(\xi)}{w(\xi)} \right) + a_2 \left(\frac{D_\xi^\alpha w(\xi)}{w(\xi)} \right)^2. \quad (26)$$

Inserting (26) into (24) and collecting the terms with a similar degree of $(D_\xi^\alpha w(\xi)/w(\xi))$, equating it to zero, we have two values of a_i , ($i = 0, 1, 2..$), k, c, l , and m

$$a_0 = \frac{-(k^{-2\alpha} A^2 m^{2\alpha} \beta + k^{-2\alpha} A^2 \delta l^{2\alpha} + k^{-2\alpha} A^2 c^\alpha k^\alpha - 8k^{2\alpha} vEA + k^{2\alpha} vB^2 + 8k^{2\alpha} vEC)}{dA^2}, \quad (27)$$

$$\begin{aligned} a_1 &= \frac{12(-C + A)vk^{4\alpha}B}{dk^{2\alpha}A^2}, \\ a_2 &= \frac{-12(C^2 - 2AC + A^2)vk^{4\alpha}}{dk^{2\alpha}A^2}, \\ c^\alpha &= c^\alpha, k^\alpha = k^\alpha. \end{aligned} \quad (28)$$

From Eqs. (28) and (26), we gain

$$\begin{aligned} U(\xi) &= \frac{-(k^{-2\alpha} A^2 m^{2\alpha} \beta + k^{-2\alpha} A^2 \delta l^{2\alpha} + k^{-2\alpha} A^2 c^\alpha k^\alpha - 8k^{2\alpha} vEA + k^{2\alpha} vB^2 + 8k^{2\alpha} vEC)}{dA^2} \\ &+ \left[\frac{12(-C + A)vk^{4\alpha}B}{dk^{2\alpha}A^2} \right] \left(\frac{D_\xi^\alpha w(\xi)}{w(\xi)} \right) + \left[\frac{-12(C^2 - 2AC + A^2)vk^{4\alpha}}{dk^{2\alpha}A^2} \right] \left(\frac{D_\xi^\alpha w(\xi)}{w(\xi)} \right)^2. \end{aligned} \quad (29)$$

In view of Family 1–5 in (26), we obtain the following

Family 6. When $B \neq 0$, $\nu_1 > 0$

$$\begin{aligned}
 U_1(\xi) = & \frac{-(k^{-2\alpha} A^2 m^{2\alpha} \beta + k^{-2\alpha} A^2 \delta l^{2\alpha} + k^{-2\alpha} A^2 c^\alpha k^\alpha - 8k^{2\alpha} \nu EA + k^{2\alpha} \nu B^2 + 8k^{2\alpha} \nu EC)}{dA^2} \\
 & + \left[\frac{12(-C+A)\nu k^{4\alpha} B}{dk^{2\alpha} A^2} \right] \left[\frac{B}{2(A-C)} + \frac{\sqrt{\nu_1}}{2(A-C)} \left[\frac{L_1 \sin h(\sqrt{\nu_1} \eta/2(A-C)) + L_2 \cos h(\sqrt{\nu_1} \eta/2(A-C))}{L_1 \cos h(\sqrt{\nu_1} \eta/2(A-C)) + L_2 \sin h(\sqrt{\nu_1} \eta/2(A-C))} \right] \right] \\
 & + \left[\frac{-12(C^2 - 2AC + A^2)\nu k^{4\alpha}}{dk^{2\alpha} A^2} \right] \left[\frac{B}{2(A-C)} + \frac{\sqrt{\nu_1}}{2(A-C)} \left[\frac{L_1 \sin h(\sqrt{\nu_1} \eta/2(A-C)) + L_2 \cos h(\sqrt{\nu_1} \eta/2(A-C))}{L_1 \cos h(\sqrt{\nu_1} \eta/2(A-C)) + L_2 \sin h(\sqrt{\nu_1} \eta/2(A-C))} \right] \right]^2.
 \end{aligned} \tag{30}$$

Family 7. When $B \neq 0$, $\nu_1 < 0$

$$\begin{aligned}
 U_2(\xi) = & \frac{-(k^{-2\alpha} A^2 m^{2\alpha} \beta + k^{-2\alpha} A^2 \delta l^{2\alpha} + k^{-2\alpha} A^2 c^\alpha k^\alpha - 8k^{2\alpha} \nu EA + k^{2\alpha} \nu B^2 + 8k^{2\alpha} \nu EC)}{dA^2} \\
 & + \left[\frac{12(-C+A)\nu k^{4\alpha} B}{dk^{2\alpha} A^2} \right] \left[\frac{B}{2(A-C)} + \frac{\sqrt{-\nu_1}}{2(A-C)} \left[\frac{-L_1 \sin(\sqrt{-\nu_1} \eta/2(A-C)) + L_2 \cos(\sqrt{-\nu_1} \eta/2(A-C))}{L_1 \cos(\sqrt{-\nu_1} \eta/2(A-C)) + L_2 \sin(\sqrt{-\nu_1} \eta/2(A-C))} \right] \right] \\
 & + \left[\frac{-12(C^2 - 2AC + A^2)\nu k^{4\alpha}}{dk^{2\alpha} A^2} \right] \left[\frac{B}{2(A-C)} + \frac{\sqrt{-\nu_1}}{2(A-C)} \left[\frac{-L_1 \sin(\sqrt{-\nu_1} \eta/2(A-C)) + L_2 \cos(\sqrt{-\nu_1} \eta/2(A-C))}{L_1 \cos(\sqrt{-\nu_1} \eta/2(A-C)) + L_2 \sin(\sqrt{-\nu_1} \eta/2(A-C))} \right] \right]^2.
 \end{aligned} \tag{31}$$

Family 8. When $B \neq 0$, $\nu_1 = 0$

$$\begin{aligned}
 U_3(\xi) = & \frac{-(k^{-2\alpha} A^2 m^{2\alpha} \beta + k^{-2\alpha} A^2 \delta l^{2\alpha} + k^{-2\alpha} A^2 c^\alpha k^\alpha - 8k^{2\alpha} \nu EA + k^{2\alpha} \nu B^2 + 8k^{2\alpha} \nu EC)}{dA^2} \\
 & + \left[\frac{12(-C+A)\nu k^{4\alpha} B}{dk^{2\alpha} A^2} \right] \left[\frac{B}{2(A-C)} + \frac{L_2}{L_1 + L_2 \eta} \right] + \left[\frac{-12(C^2 - 2AC + A^2)\nu k^{4\alpha}}{dk^{2\alpha} A^2} \right] \left[\frac{B}{2(A-C)} + \frac{L_2}{L_1 + L_2 \eta} \right]^2.
 \end{aligned} \tag{32}$$

Family 9. For $B = 0$, $\nu_2 > 0$

$$\begin{aligned}
 U_4(\xi) = & \frac{-(k^{-2\alpha} A^2 m^{2\alpha} \beta + k^{-2\alpha} A^2 \delta l^{2\alpha} + k^{-2\alpha} A^2 c^\alpha k^\alpha - 8k^{2\alpha} \nu EA + 8k^{2\alpha} \nu EC)}{dA^2} \\
 & + \left[\frac{-12(C^2 - 2AC + A^2)\nu k^{4\alpha}}{dk^{2\alpha} A^2} \right] \left[\frac{\sqrt{\nu_2}}{2(A-C)} \left[\frac{L_1 \sin h(\sqrt{\nu_2} \eta/2(A-C)) + L_2 \cos h(\sqrt{\nu_2} \eta/2(A-C))}{L_1 \cos h(\sqrt{\nu_2} \eta/2(A-C)) + L_2 \sin h(\sqrt{\nu_2} \eta/2(A-C))} \right] \right]^2.
 \end{aligned} \tag{33}$$

Family 10. In case of $B = 0$, $v_2 < 0$

$$U_5(\xi) = \frac{-(k^{-2\alpha}A^2m^{2\alpha}\beta + k^{-2\alpha}A^2\delta l^{2\alpha} + k^{-2\alpha}A^2c^\alpha k^\alpha - 8k^{2\alpha}vEA + 8k^{2\alpha}vEC)}{dA^2} + \left[\frac{-12(C^2 - 2AC + A^2)vk^{4\alpha}}{dk^{2\alpha}A^2} \right] \left[\frac{\sqrt{-v_2}}{2(A-C)} \left[\frac{-L_1 \sin(\sqrt{-\Delta_2}\eta/2(A-C)) + L_2 \cos(\sqrt{-v_2}\eta/2(A-C))}{L_1 \cos(\sqrt{-v_2}\eta/2(A-C)) + L_2 \sin(\sqrt{-v_2}\eta/2(A-C))} \right] \right]^2, \quad (34)$$

where $v_1 = 4E(A-C)$, $v_2 = E(A-C)$, $\eta = \xi^\alpha/\Gamma(\alpha+1)$, and $\xi = kx + ct + mz + ly + \xi_0$. It is clearly seen that the solutions depend on α , and when $\alpha = 1$, we have the solutions that are obtained for normal derivative. The results introduce free parameters. Hence, five solutions are essential in handling initial and boundary problems. To solve the reduced Eq. (24) by the sine-Gordon expansion (FSGE) method, assume the solution of Eq. (24) as

$$U(\xi) = A_0 + B_1 \sin(\xi) + A_1 \cos(\xi) + A_2 \cos^2(\xi) + B_2 \cos(\xi) \sin(\xi). \quad (35)$$

Inserting Eq. (35) into (24) and the collecting the same power of $\cos^i(\chi(xi))\sin^j(\chi(xi))$, admitting the system of algebraic equation, by solving them by Maple, admits to

Set 1.

$$k^\alpha = k^\alpha, \quad c^\alpha = c^\alpha, \quad A_1 = 0, \quad B_1 = 0, \quad B_2 = \pm \frac{6ik^{2\alpha}v}{d}, \quad A_2 = -\frac{6vk^{4\alpha}}{dk^{2\alpha}}, \\ A_0 = \frac{(5k^{2\alpha}v - k^{-2\alpha}\delta L^{2\alpha} - k^{-2\alpha}c^\alpha k^\alpha - k^{-2\alpha}m^{2\alpha}\beta)}{d}, \quad (36)$$

Set 2.

$$k^\alpha = k^\alpha, \quad c^\alpha = c^\alpha, \quad A_1 = 0, \quad B_1 = 0, \quad B_2 = 0, \quad A_2 = \frac{-12vk^{4\alpha}}{dk^{2\alpha}}, \\ A_0 = \frac{8k^{2\alpha}v - k^{-2\alpha}c^\alpha k^\alpha - k^{-2\alpha}\delta L^{2\alpha} - k^{-2\alpha}m^{2\alpha}\beta}{d}. \quad (37)$$

Inserting Set 1 into (35), we obtain the exact solution of Eq. (22) as

$$u_1(\xi) = \left[\frac{(5k^{2\alpha}v - k^{-2\alpha}\delta L^{2\alpha} - k^{-2\alpha}c^\alpha k^\alpha - k^{-2\alpha}m^{2\alpha}\beta)}{d} \right] - \left[\frac{6vk^{4\alpha}}{dk^{2\alpha}} \right] \tan h^2(\xi) \pm \left[\frac{6ik^{2\alpha}v}{d} \right] \tan h(\xi) \operatorname{sech}(\xi), \quad (38)$$

$$u_2(\xi) = \left[\frac{(5k^{2\alpha}v - k^{-2\alpha}\delta L^{2\alpha} - k^{-2\alpha}c^\alpha k^\alpha - k^{-2\alpha}m^{2\alpha}\beta)}{d} \right] - \left[\frac{6vk^{4\alpha}}{dk^{2\alpha}} \right] \cot h^2(\xi) \pm \left[\frac{6ik^{2\alpha}v}{d} \right] i \coth(\xi) \csc h(\xi), \quad (39)$$

where $\xi = kx + ct + ly + mz + \xi_0$. Knowing Set 2 and Eq. (35), we gain the exact solution of Eq. (22) as follows:

$$u_3(\xi) = \left[\frac{(8k^{2\alpha}v - k^{-2\alpha}c^\alpha k^\alpha - k^{-2\alpha}\delta L^{2\alpha} - k^{-2\alpha}m^{2\alpha}\beta)}{d} \right] - \left[\frac{12vk^{4\alpha}}{d(k^{2\alpha})} \right] \tan h^2(\chi(\xi)), \quad (40)$$

$$u_4(\xi) = \left[\frac{(8k^{2\alpha}v - k^{-2\alpha}c^\alpha k^\alpha - k^{-2\alpha}\delta L^{2\alpha} - k^{-2\alpha}m^{2\alpha}\beta)}{d} \right] - \left[\frac{12vk^{4\alpha}}{dk^{2\alpha}} \right] \cot h^2(\chi(\xi)), \quad (41)$$

where $\xi = kx + ct + ly + mz + \xi_0$. It is to be noted that, the 3D graph represent the obtained solutions with fixed $y = z = 1$ of Eqs. (38) and (40) are shown graphically (see Figures 1–7) for fixed parameter with a different choice of fractal order α .

4. Concluding Remarks

In this article, the extended fractional subequation method and sine-Gordon expansion (FSGE) method have been proposed for finding exact solutions of fractional partial differential equations (FPDEs) in the sense of conformable derivative. This paper studies (3 + 1)-dimensions space-time FKPE which appears in plasma physics in the sense of conformable derivatives via two algorithms, namely, the extended fractional subequation method and FSGE method to obtain sets of exact solutions. Using suitable wave transform, the equations are reduced to some ODEs. Then, the admissible solutions are substituted into the resultant ODE. Equating the coefficients of $(D_\xi^\alpha w(\xi)/w(\xi))$ in extended

FIGURE 1: The 3D graph represents of Eq. (38) via sine-Gordon expansion method under the fixed values $m = 0.05, l = 1; k = 0.5, \nu = 1, y = z = 1, c = 0.05, d = 1, \beta = 1, \delta = 1, L = 0.5, 10 < x < 10, 10 < t < 10$ with fractal order $\alpha = 1$.

FIGURE 2: The 3D graph represents of Eq. (38) via sine-Gordon expansion method with a fixed values $m = 0.05, l = 1; k = 0.5, \nu = 1, y = z = 1, c = 0.05, d = 1, \beta = 1, \delta = 1, L = 0.5, 10 < x < 10, 10 < t < 10$ with fractal order $\alpha = 0.55$

FIGURE 3: The 3D graph represents of Eq. (38) via sine-Gordon expansion method with a fixed values $m = 0.05, l = 1; k = 0.5, \nu = 1, y = z = 1, c = 0.05, d = 1, \beta = 1, \delta = 1, L = 0.5, 10 < x < 10, 10 < t < 10$ with fractal order $\alpha = 0.85$

FIGURE 4: The 3D graph represents of Eq. (38) via sine-Gordon expansion method under the values $m = 0.05, l = 1; k = 0.5, \nu = 1, y = z = 1, c = 0.05, d = 1, \beta = 1, \delta = 1, L = 0.5, 10 < x < 10, 10 < t < 10$ with fractal order $\alpha = 0.45$

FIGURE 5: The 3D graph represents of the wave solutions of Eq. (40) via sine-Gordon expansion method with a fixed values $m = 0.05, l = 1; k = 0.5, \nu = 1, y = z = 1, c = 0.05, d = 1, \beta = 1, \delta = 1, L = 0.5, 10 < x < 10, 10 < t < 10$ with fractal order $\alpha = 1$

FIGURE 6: The 3D graph represents of Eq. (40) via sine-Gordon expansion method with a fixed values $m = 0.05, l = 1; k = 0.5, \nu = 1, y = z = 1, c = 0.05, d = 1, \beta = 1, \delta = 1, L = 0.5, 10 < x < 10, 10 < t < 10$ with fractal order $\alpha = 0.35$

FIGURE 7: The 3D graph represents of Eq. (40) via sine-Gordon expansion method with a fixed values $m = 0.05, l = 1; k = 0.5, \nu = 1, y = z = 1, c = 0.05, d = 1, \beta = 1, \delta = 1, L = 0.5, 10 < x < 10, 10 < t < 10$ with fractal order $\alpha = 0.55$

fractional subequation method and cosine and sine functions and their multiplications in FSGE method to zero leads to some algebraic system of equations. Solving this system gives the relations among the parameters. Some 3-D solution graphs are presented in some finite domains to comprehend the effects of α .

The presence of parameters makes our results useful for the IBVP with fractional order. For $\alpha = 1$, our solutions go back to that previously obtained solution. The performance

of these approaches shows the ability for applying on various space-time fractional nonlinear equations in nonlinear science.

Data Availability

No any data availability

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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