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Research Article

Positive Solutions Depending on Parameters for a Nonlinear Fractional System with p-Laplacian Operators

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This paper considers a system of fractional differential equations involving p-Laplacian operators and two parameters

$$\begin{cases} D_{0^+}^{\alpha_1}(\varphi_{p_1}(D_{0^+}^{\beta_1}u(t))) + \lambda f(t,u(t),v(t)) = 0, 0 < t < 1, \\ D_{0^+}^{\alpha_2}(\varphi_{p_2}(D_{0^+}^{\beta_2}v(t))) + \mu g(t,u(t),v(t)) = 0, 0 < t < 1, \\ u(0) = u(1) = u'(0) = u'(1) = 0, D_{0^+}^{\beta_1}u(0) = 0, D_{0^+}^{\beta_1}u(1) = b_1 D_{0^+}^{\beta_1}u(\eta_1), \\ v(0) = v(1) = v'(0) = v'(1) = 0, D_{0^+}^{\beta_2}v(0) = 0, D_{0^+}^{\beta_2}v(1) = b_2 D_{0^+}^{\beta_2}v(\eta_2), \end{cases}$$

where $\alpha_i \in (1,2]$, $\beta_i \in (3,4]$, $D_{0^+}^{\alpha_i}$ and $D_{0^+}^{\beta_i}$ are the standard Riemann-Liouville derivatives, $\varphi_{p_i}(s) = |s|^{p_i-2}s, p_i > 1, \varphi_{p_i}^{-1} = \varphi_{q_i}$, $(1/p_i) + (1/q_i) = 1, \eta_i \in (0,1), b_i \in (0, \eta_i^{(1-\alpha_i)/(p_i-1)})$, $i = 1, 2$, and f, $g \in C([0,1] \times [0,+\infty) \times [0,+\infty), [0,+\infty))$ and λ and μ are two positive parameters. We obtain the existence and uniqueness of positive solutions depending on parameters for the system by utilizing a recent fixed point theorem. Furthermore, an example is present to illustrate our main result.

1. Introduction

During the past several decades, many fractional problems with differential equations have been paid much attention, see [1–10] for example. Also, much attention has been focused on the existence of positive solutions for such equations, see [3–29] and the references therein. As we know, the p-Laplacian operator has very a important position in theoretical research and engineering applications. In 1945, to discuss turbulent flow in a porous medium, a basic mechanical problem, Leibenson [30] introduced a differential equation with a p-Laplacian operator:

$$\left(\varphi_p\left(u'(t)\right)\right)' = f(t, u(t)). \tag{1}$$

Since then, there are many papers investigating differential equations with p-Laplacian operators. Recently, the study of fractional equations with a p-Laplacian operator has also gained plenty of attention, see [19, 20, 31–40] for instance. In [35], the authors studied a fractional equation with a p-Laplacian operator:

$$\begin{cases} -D_{0^+}^{\alpha}\left(\varphi_p\left(D_{0^+}^{\beta}u(t)\right)\right) = f(t,u(t)), 0 < t < 1, \\ u(0) = u(1) = u'(0) = u'(1) = 0, D_{0^+}^{\beta}u(0) = 0, D_{0^+}^{\beta}u(1) = bD_{0^+}^{\beta}u(\eta), \end{cases} \tag{2}$$

where $\alpha \in (1,2]$, $\beta \in (3,4]$, $D_{0^+}^{\alpha}$ and $D_{0^+}^{\beta}$ denote the Riemann-Liouville derivatives, $b \in (0, \eta^{(1-\alpha)/(p-1)})$, and $f \in C([0,1] \times [0,+\infty), [0,+\infty))$. Based on Schauder's fixed point theorem and by using the upper-lower solution method, they obtained the existence and uniqueness of solutions.

Recently, fractional differential systems have been also studied by many people because of their great application value, see [5, 20, 23–30]. So, the results on fractional systems with p-Laplacian operator are many, see [11, 41–45]. For example, Rodica [41] discussed a fractional differential system:

$$\begin{cases} D_{0^+}^{\alpha_1}\left(\varphi_{r_1}\left(D_{0^+}^{\beta_1}u(t)\right)\right)+\lambda f(t,u(t),v(t))=0, 0<t<1, \\ D_{0^+}^{\alpha_2}\left(\varphi_{r_2}\left(D_{0^+}^{\beta_2}v(t)\right)\right)+\mu g(t,u(t),v(t))=0, 0<t<1, \\ u^{(j)}(0)=0, j=0,\cdots,n-2\,; D_{0^+}^{\beta_1}u(0)=0, D_{0^+}^{p_1}u(1)=\sum_{i=1}^{N}a_iD_{0^+}^{q_1}u(\xi_i), \\ v^{(j)}(0)=0, j=0,\cdots,m-2\,; D_{0^+}^{\beta_2}v(0)=0, D_{0^+}^{p_2}v(1)=\sum_{i=1}^{M}b_iD_{0^+}^{q_2}v(\eta_i), \end{cases} \tag{3}$$

where $\alpha_1,\alpha_2\in(0,1]$, $\beta_1\in(n-1,n]$, $\beta_2\in(m-1,m]$, $n,m\in\mathbb{N}$, $n,m\geq 3$, $p_1,p_2,q_1,q_2\in\mathbb{R}$, $p_1\in[1,n-2]$, $p_2\in[1,m-2]$, $q_1\in[0,p_1]$, $q_2\in[0,p_2]$, $\xi_i,a_i\in\mathbb{R}$ for all $i=1,\cdots,N(N\in\mathbb{N})$, $0<\xi_1<\cdots<\xi_N\leq 1$, $\eta_i,b_i\in\mathbb{R}$ for all $i=1,\cdots,M(M\in\mathbb{N})$, $0<\eta_1<\cdots<\eta_M\leq 1$, $r_1,r_2>1$, $\lambda,\mu>0$, and $f,g\in C([0,1]\times[0,\infty)\times[0,\infty),[0,\infty))$. The existence of solutions was obtained via Guo-Krasnosel'skii's fixed point theorem.

From literature, we see that most results are the existence of solutions, but the uniqueness is scarce. Inspired by [34], we discuss the following system of fractional differential equations with p-Laplacian operators:

$$\begin{cases} D_{0^+}^{\alpha_1}\left(\varphi_{p_1}\left(D_{0^+}^{\beta_1}u(t)\right)\right)+\lambda f(t,u(t),v(t))=0, 0<t<1, \\ D_{0^+}^{\alpha_2}\left(\varphi_{p_2}\left(D_{0^+}^{\beta_2}v(t)\right)\right)+\mu g(t,u(t),v(t))=0, 0<t<1, \\ u(0)=u(1)=u'(0)=u'(1)=0, D_{0^+}^{\beta_1}u(0)=0, D_{0^+}^{\beta_1}u(1)=b_1D_{0^+}^{\beta_1}u(\eta_1), \\ v(0)=v(1)=v'(0)=v'(1)=0, D_{0^+}^{\beta_2}v(0)=0, D_{0^+}^{\beta_2}v(1)=b_2D_{0^+}^{\beta_2}v(\eta_2), \end{cases} \tag{4}$$

where $\alpha_i\in(1,2]$, $\beta_i\in(3,4]$, $D_{0^+}^{\alpha_i}$ and $D_{0^+}^{\beta_i}$ denote the standard Riemann-Liouville derivatives, $\varphi_{p_i}(s)=|s|^{p_i-2}s, p_i>1$, $\varphi_{p_i}^{-1}=\varphi_{q_i}$, $(1/p_i)+(1/q_i)=1, \eta_i\in(0,1), b_i\in(0,\eta_i^{(1-\alpha_i)/(p_i-1)})$, $i=1,2$ and $f,g\in C([0,1]\times[0,+\infty)\times[0,+\infty),[0,+\infty))$, and λ and μ are two positive parameters. It should be pointed out, in [45], that Hao et al. investigated the existence of solutions for system (4) without considering the uniqueness. They used Guo-Krasnosel'skii's fixed point theorem to get some existence results for positive solutions under different values of λ and μ. In this paper, based upon a recent fixed point theorem, we aim to present the existence and uniqueness of positive solutions for system (4) depending on fixed positive constants λ and μ. Our results can tell us that the unique positive solution exists in a product set and can be approximated by giving an iterative sequence for any initial point in the product set. Therefore, our result is an extension and improvement of the previous works. At the end, an example is given to illustrate the result.

2. Preliminaries

Lemma 1 (see [45]). *Assume $\alpha_1\in(1,2]$, $\beta_1\in(3,4]$, $p_1>1$, $(1/p_1)+(1/q_1)=1$, $\eta_1\in(0,1)$, $b_1\in(0,\eta_1^{(1-\alpha_1)/(p_1-1)})$. If $y\in C[0,1]$, then the unique solution of the following problem:*

$$\begin{cases} D_{0^+}^{\alpha_1}\left(\varphi_{p_1}\left(D_{0^+}^{\beta_1}u(t)\right)\right)+y(t)=0, 0<t<1, \\ u(0)=u(1)=u'(0)=u'(1)=0, D_{0^+}^{\beta_1}u(0)=0, D_{0^+}^{\beta_1}u(1)=b_1D_{0^+}^{\beta_1}u(\eta_1), \end{cases} \tag{5}$$

is

$$u(t)=\int_0^1 G_1(t,s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)y(\tau)d\tau\right)ds, \tag{6}$$

where

$$G_1(t,s)=\frac{1}{\Gamma(\beta_1)}\begin{cases} t^{\beta_1-2}(1-s)^{\beta_1-2}[(s-t)+(\beta_1-2)(1-t)s], 0\leq t\leq s\leq 1, \\ t^{\beta_1-2}(1-s)^{\beta_1-2}[(s-t)+(\beta_1-2)(1-t)s]+(t-s)^{\beta_1-1}, 0\leq s\leq t\leq 1, \end{cases} \tag{7}$$

$$H_1(t,s)=h_1(t,s)+\frac{b_1^{p_1-1}t^{\alpha_1-1}}{1-b_1^{p_1-1}\eta_1^{\alpha_1-1}}h_1(\eta_1,s), \tag{8}$$

$$h_1(t,s)=\frac{1}{\Gamma(\alpha_1)}\begin{cases} [t(1-s)]^{\alpha_1-1}, 0\leq t\leq s\leq 1, \\ [t(1-s)]^{\alpha_1-1}-(t-s)^{\alpha_1-1}, 0\leq s\leq t\leq 1. \end{cases} \tag{9}$$

For convenience, we can easily give the following Lemma by using Lemma 1.

Lemma 2. *Let $\alpha_2\in(1,2]$, $\beta_2\in(3,4]$, $p_2>1$, $(1/p_2)+(1/q_2)=1$, $\eta_2\in(0,1)$, $b_2\in(0,\eta_2^{(1-\alpha_2)/(p_2-1)})$. If $y\in C[0,1]$, then*

$$\begin{cases} D_{0^+}^{\alpha_2}\left(\varphi_{p_2}\left(D_{0^+}^{\beta_2}v(t)\right)\right)+y(t)=0, 0<t<1, \\ v(0)=v(1)=v'(0)=v'(1)=0, D_{0^+}^{\beta_2}v(0)=0, D_{0^+}^{\beta_2}v(1)=b_2D_{0^+}^{\beta_2}v(\eta_2), \end{cases} \tag{10}$$

has a unique solution

$$v(t)=\int_0^1 G_2(t,s)\varphi_{q_2}\left(\int_0^1 H_2(s,\tau)y(\tau)d\tau\right)ds, \tag{11}$$

where

$$G_2(t,s)=\frac{1}{\Gamma(\beta_2)}\begin{cases} t^{\beta_2-2}(1-s)^{\beta_2-2}[(s-t)+(\beta_2-2)(1-t)s], 0\leq t\leq s\leq 1, \\ t^{\beta_2-2}(1-s)^{\beta_2-2}[(s-t)+(\beta_2-2)(1-t)s]+(t-s)^{\beta_2-1}, 0\leq s\leq t\leq 1, \end{cases} \tag{12}$$

$$H_2(t,s)=h_2(t,s)+\frac{b_2^{p_2-1}t^{\alpha_2-1}}{1-b_2^{p_2-1}\eta_2^{\alpha_2-1}}h_2(\eta_2,s), \tag{13}$$

$$h_2(t,s)=\frac{1}{\Gamma(\alpha_2)}\begin{cases}[t(1-s)]^{\alpha_2-1}, 0\le t\le s\le 1,\\ [t(1-s)]^{\alpha_2-1}-(t-s)^{\alpha_2-1}, 0\le s\le t\le 1.\end{cases} \tag{14}$$

By Lemmas 4 and 5 in [45], the following conclusion is clear.

Lemma 3. *The functions $G_i(t,s)$, $i=1,2$ defined by (7) and (12) have several properties:*

(i) $G_i(t,s)$ is continuous on $[0,1]\times[0,1]$ and $G_i(t,s)>0$ for $(t,s)\in(0,1)\times(0,1)$

(ii) $(\beta_i-2)k_i(t)l_i(s)\le\Gamma(\beta_i)G_i(t,s)\le M_il_i(s)$, $(t,s)\in[0,1]\times[0,1]$

(iii) $(\beta_i-2)k_i(t)l_i(s)\le\Gamma(\beta_i)G_i(t,s)\le M_ik_i(s)$, $(t,s)\in[0,1]\times[0,1]$, where

$$k_i(t)=t^{\beta_i-2}(1-t)^2, l_i(s)=s^2(1-s)^{\beta_i-2}, M_i=\max\{\beta_i-1,(\beta_i-2)^2\} \tag{15}$$

Suppose that $(X,\|\cdot\|)$ is a real Banach space with a partial order induced by a cone $P\subset X$. For any $x,y\in X$, the notation $x\sim y$ denotes that there exist $\lambda>0$ and $\mu>0$ such that $\lambda x\le y\le\mu x$. For $h>\theta$ (i.e., $h\ge\theta$ and $h\ne\theta$), define a set $P_h=\{x\in X\mid x\sim h\}$. Evidently, $P_h\subset P$. For $h_1,h_2\in P$ with $h_1,h_2\ne\theta$. Suppose $h=(h_1,h_2)$, then $h\in\bar{P}:=P\times P$. If P is normal, then $\bar{P}=(P,P)$ is normal.

Lemma 4 (see [46, 47]). $\bar{P}_h=\{(x,y): x\in P_{h_1}, y\in P_{h_2}\}=P_{h_1}\times P_{h_2}$.

Lemma 5 (see [47]). *Let P be a normal cone in a Banach space X and $h=(h_1,h_2)\in P\times P$ with $h_1,h_2\ne\theta$. Operators $A,B:P\times P\longrightarrow P$ are increasing and satisfy the following:*

(M_1) There exist $\varphi_1,\varphi_2:(0,1)\longrightarrow(0,1)$ such that

$$A(rx,ry)\ge\varphi_1(r)A(x,y), B(rx,ry)\ge\varphi_2(r)B(x,y), x,y\in P \tag{16}$$

where $\varphi_i(r)>r$, $r\in(0,1)$, $i=1,2$;

(M_2) There is $(e_1,e_2)\in\bar{P}_h$ such that $A(e_1,e_2)\in P_{h_1}$, $B(e_1,e_2)\in P_{h_2}$.

Then,

(a) $A:P_{h_1}\times P_{h_2}\longrightarrow P_{h_1}$, $B:P_{h_1}\times P_{h_2}\longrightarrow P_{h_2}$, and exist $x_1,y_1\in P_{h_1}$, $x_2,y_2\in P_{h_2}$, $\gamma\in(0,1)$ such that $\gamma(y_1,y_2)\le(x_1,x_2)\le(y_1,y_2)$ and $x_1\le A(x_1,x_2)\le y_1$, $x_2\le B(x_1,x_2)\le y_2$

(b) for any given $\lambda,\mu>0$, the equation $(x,y)=(\lambda A(x,y),\mu B(x,y))$ has a unique solution $(x^*_{\lambda,\mu},y^*_{\lambda,\mu})$ in $\bar{P}_h$. Moreover, take any fixed point $(x_0,y_0)\in\bar{P}_h$, let

$$(x_n,y_n)=(\lambda A(x_{n-1},y_{n-1}),\mu B(x_{n-1},y_{n-1})), n=1,2,\cdots \tag{17}$$

then $\|x_n-x^*_{\lambda,\mu}\|\longrightarrow 0$, $\|y_n-y^*_{\lambda,\mu}\|\longrightarrow 0$, as $n\longrightarrow\infty$.

3. Positive Solutions Depending on Parameters

Let $X=C[0,1]$, a Banach space with the norm $\|u\|=\sup\{|u(t)|:t\in[0,1]\}$. We study (4) in the product space $X\times X$. For $(u,v)\in X\times X$, let $\|(u,v)\|=\max\{\|u\|,\|v\|\}$. Then, $(X\times X,\|(\cdot,\cdot)\|)$ is a Banach space. Let $\bar{P}=\{(u,v)\in X\times X\mid u(t)\ge 0, v(t)\ge 0, t\in[0,1]\}$, $P=\{u\in X\mid u(t)\ge 0, t\in[0,1]\}$, then $\bar{P}\subset X\times X$ is a cone and $\bar{P}=P\times P$ is normal, and the space $X\times X$ has a partial order:

$$(u_1,v_1)\le(u_2,v_2)\Leftrightarrow u_1(t)\le u_2(t), v_1(t)\le v_2(t), t\in[0,1]. \tag{18}$$

Lemma 6. *Let $f(t,u,v)$, $g(t,u,v)$ be continuous. By using Lemmas 1 and 2 and some results in [45], $(u,v)\in P\times P$ is a positive solution of (4) if and only if $(u,v)\in P\times P$ is a solution of the following equations:*

$$\begin{cases}u(t)=\displaystyle\int_0^1 G_1(t,s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)\lambda f(\tau,u(\tau),v(\tau))d\tau\right)ds,\\ v(t)=\displaystyle\int_0^1 G_2(t,s)\varphi_{q_2}\left(\int_0^1 H_2(s,\tau)\mu g(\tau,u(\tau),v(\tau))d\tau\right)ds.\end{cases} \tag{19}$$

Theorem 7. *Let $\alpha_i\in(1,2]$, $\beta_i\in(3,4]$, $h_1(t)=t^{\beta_1-2}(1-t)^2$, $h_2(t)=t^{\beta_2-2}(1-t)^2$, $t\in[0,1]$. Assume that*

(H_1) $f,g\in C([0,1]\times[0,+\infty)\times[0,+\infty),[0,+\infty))$ and $f(t,0,0)\not\equiv 0$, $g(t,0,0)\not\equiv 0$, $t\in[0,1]$

(H_2) f,g are increasing with respect to the second, third variables, i.e., $f(t,u_1,v_1)\le f(t,u_2,v_2)$, $g(t,u_1,v_1)\le g(t,u_2,v_2)$ for $t\in[0,1]$, $0\le u_1\le u_2$, $0\le v_1\le v_2$

(H_3) for $r\in(0,1)$, there is $\psi_i(r)$: $(0,1)\longrightarrow(0,1)$, $i=1,2$, such that $\psi_i(r)>r^{1/(q_i-1)}$ and

$$f(t,ru,rv)\ge\psi_1(r)f(t,u,v), g(t,ru,rv)\ge\psi_2(r)g(t,u,v), \tag{20}$$

for $t\in[0,1]$, $u,v\in[0,+\infty)$

Then

(a) there are $u_1,v_1\in P_{h_1}$, $u_2,v_2\in P_{h_2}$, $\gamma\in(0,1)$ such that $\gamma(v_1,v_2)\le(u_1,u_2)\le(v_1,v_2)$ and

$$u_1(t) \le \int_0^1 G_1(t,s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,u_1(\tau),u_2(\tau))d\tau\right)ds \le v_1(t),\ t\in[0,1],$$
$$u_2(t) \le \int_0^1 G_2(t,s)\varphi_{q_2}\left(\int_0^1 H_2(s,\tau)g(\tau,u_1(\tau),u_2(\tau))d\tau\right)ds \le v_2(t),\ t\in[0,1], \tag{21}$$

where G_i, $i=1,2$, are the Green functions in Lemmas 1 and 2

(b) *System (4) has a unique positive solution $(x^*_{\lambda,\mu}, y^*_{\lambda,\mu})$ depending on $\lambda,\mu>0$ in $\bar{P}_h$, where $h(t)=(t^{\beta_1-2}(1-t)^2, t^{\beta_2-2}(1-t)^2)$, $t\in[0,1]$*

(c) *Take any initial point $(u_0,v_0)\in\bar{P}_h$, let*

$$u_{n+1}(t) = \lambda^{q_1-1}\int_0^1 G_1(t,s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,u_n(\tau),v_n(\tau))d\tau\right)ds,\ n=1,2,\cdots,$$
$$v_{n+1}(t) = \mu^{q_2-1}\int_0^1 G_2(t,s)\varphi_{q_2}\left(\int_0^1 H_2(s,\tau)g(\tau,u_n(\tau),v_n(\tau))d\tau\right)ds,\ n=1,2,\cdots, \tag{22}$$

then $u_n(t)\longrightarrow x^*_{\lambda,\mu}(t)$, $v_n(t)\longrightarrow y^*_{\lambda,\mu}(t)$ as $n\longrightarrow\infty$

Proof. We consider three operators $A,B : P\times P\longrightarrow X$ and $T : P\times P\longrightarrow X\times X$ defined by

$$A(u,v)(t) = \int_0^1 G_1(t,s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,u(\tau),v(\tau))d\tau\right)ds,$$
$$B(u,v)(t) = \int_0^1 G_2(t,s)\varphi_{q_2}\left(\int_0^1 H_2(s,\tau)g(\tau,u(\tau),v(\tau))d\tau\right)ds,$$
$$T(u,v)(t) = \left(\tilde{\lambda}A(u,v)(t), \tilde{\mu}B(u,v)(t)\right), \tag{23}$$

where $\tilde{\lambda} \coloneqq \lambda^{q_1-1}$, $\tilde{\mu} \coloneqq \mu^{q_2-1}$, G_i, H_i, and $i=1,2$ are defined by (7) and (12). From Lemma 3 and (H_1), it is clear that $A,B : \bar{P}\longrightarrow P$ and $T : \bar{P}\longrightarrow\bar{P}$. From our above discussion, we can easily claim that $(u,v)\in\bar{P}$ is a solution of system (4) if and only if $(u,v)\in\bar{P}$ is a fixed point of operator T. Next, we only need to prove that all assumptions of Lemma 5 are satisfied for operators A,B.

We first show that A,B are increasing. To do this, for $u_i, v_i\in P$, $i=1,2$, with $u_1\le u_2$, $v_1\le v_2$, one has $u_1(t)\le u_2(t)$, $v_1(t)\le v_2(t)$, $t\in[0,1]$ and by (H_2) and Lemma 3,

$$\begin{aligned}A(u_1,v_1)(t) &= \int_0^1 G_1(t,s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,u_1(\tau),v_1(\tau))d\tau\right)ds\\ &\le\le\int_0^1 G_1(t,s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,u_2(\tau),v_2(\tau))d\tau\right)ds\\ &= A(u_2,v_2)(t),\end{aligned}$$

$$\begin{aligned}B(u_1,v_1)(t) &= \int_0^1 G_2(t,s)\varphi_{q_2}\left(\int_0^1 H_2(s,\tau)g(\tau,u_1(\tau),v_1(\tau))d\tau\right)ds\\ &\le\int_0^1 G_2(t,s)\varphi_{q_1}\left(\int_0^1 H_2(s,\tau)g(\tau,u_2(\tau),v_2(\tau))d\tau\right)ds\\ &= B(u_2,v_2)(t).\end{aligned} \tag{24}$$

That is, $A(u_1,v_1)\le A(u_2,v_2)$ and $B(u_1,v_1)\le B(u_2,v_2)$.

Second, we indicate that A,B satisfy condition (M_1) of Lemma 5. Let $\Psi_1(r)=\varphi_{q_1}(\psi_1(r))$, $\Psi_2(r)=\varphi_{q_2}(\psi_2(r))$. Then, for $r\in(0,1)$, by (H_2), we have

$$\Psi_1(r) = (\psi_1(r))^{q_1-1} > \left(r^{1/(q_1-1)}\right)^{q_1-1} = r. \tag{25}$$

Similarly, $\Psi_2(r)>r$. For $r\in(0,1)$ and $u,v\in P$, by (H_3), we obtain

$$\begin{aligned}A(ru,rv)(t) &= \int_0^1 G_1(t,s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,ru(\tau),rv(\tau))d\tau\right)ds\\ &\ge\int_0^1 G_1(t,s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)\psi_1(r)f(\tau,u(\tau),v(\tau))d\tau\right)ds\\ &= \varphi_{q_1}(\psi_1(r))\int_0^1 G_1(t,s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,u(\tau),v(\tau))d\tau\right)ds\\ &= \Psi_1(r)A(u,v)(t),\end{aligned}$$

$$\begin{aligned}B(ru,rv)(t) &= \int_0^1 G_2(t,s)\varphi_{q_2}\left(\int_0^1 H_2(s,\tau)g(\tau,ru(\tau),rv(\tau))d\tau\right)ds\\ &\ge\int_0^1 G_2(t,s)\varphi_{q_2}\left(\int_0^1 H_2(s,\tau)\psi_2(r)g(\tau,u(\tau),v(\tau))d\tau\right)ds\\ &= \varphi_{q_2}(\psi_2(r))\int_0^1 G_2(t,s)\varphi_{q_2}\left(\int_0^1 H_2(s,\tau)g(\tau,u(\tau),v(\tau))d\tau\right)ds\\ &= \Psi_2(r)B(u,v)(t).\end{aligned} \tag{26}$$

That is, $A(ru,rv)\ge\Psi_1(r)A(u,v)$, $B(ru,rv)\ge\Psi_2(r)B(u,v)$ for $r\in(0,1)$, $u,v\in P$.

Set $h=(h_1,h_2)$, where $h_1(t)=t^{\beta_1-2}(1-t)^2$, $h_2(t)=t^{\beta_2-2}(1-t)^2$, $t\in[0,1]$. Then, $(h_1,h_2)\in\bar{P}_h$. Now, we prove that $A(h_1,h_2)\in P_{h_1}$, $B(h_1,h_2)\in P_{h_2}$. In view of (H_2) and Lemma 3, for $t\in[0,1]$, we have

$$\begin{aligned}A(h_1,h_2)(t) &= \int_0^1 G_1(t,s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,h_1(\tau),h_2(\tau))d\tau\right)ds\\ &\ge\int_0^1\frac{(\beta_1-2)k_1(t)l_1(s)}{\Gamma(\beta_1)}\varphi_{q_1}\\ &\quad\cdot\left(\int_0^1 H_1(s,\tau)f\left(\tau,\tau^{\beta_1-2}(1-\tau)^2,\tau^{\beta_2-2}(1-\tau)^2\right)d\tau\right)ds\\ &\ge\frac{(\beta_1-2)k_1(t)}{\Gamma(\beta_1)}\int_0^1 l_1(s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,0,0)d\tau\right)ds\\ &= \frac{(\beta_1-2)}{\Gamma(\beta_1)}h_1(t)\int_0^1 l_1(s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,0,0)d\tau\right)ds,\end{aligned}$$

$$\begin{aligned}
A(h_1,h_2)(t) &= \int_0^1 G_1(t,s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,h_1(\tau),h_2(\tau))d\tau\right)ds \\
&\leq \int_0^1 \frac{M_1k_1(t)}{\Gamma(\beta_1)}\varphi_{q_1} \\
&\quad \cdot\left(\int_0^1 H_1(s,\tau)f\left(\tau,\tau^{\beta_1-2}(1-\tau)^2,\tau^{\beta_2-2}(1-\tau)^2\right)d\tau\right)ds \\
&\leq \frac{M_1k_1(t)}{\Gamma(\beta_1)}\int_0^1 \varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,1,1)d\tau\right)ds \\
&= \frac{M_1}{\Gamma(\beta_1)}h_1(t)\int_0^1 \varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,1,1)d\tau\right)ds.
\end{aligned} \tag{27}$$

Noting that (H_1) and (H_2) guarantee that $f(t,1,1)\geq f(t,0,0)\geq 0$ and $f(t,0,0)\not\equiv 0$, $t\in[0,1]$. Because $l_1(s)=s^2(1-s)^{\beta_1-2}\leq 1$, $s\in[0,1]$, then

$$\begin{aligned}
&\int_0^1 \varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,1,1)d\tau\right)ds \\
&\quad\geq \int_0^1 l_1(s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,0,0)d\tau\right)ds>0.
\end{aligned} \tag{28}$$

So, we have

$$\begin{aligned}
m_1 &:= \frac{(\beta_1-2)}{\Gamma(\beta_1)}\int_0^1 l_1(s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,0,0)d\tau\right)ds>0, \\
m_2 &:= \frac{M_1}{\Gamma(\beta_1)}\int_0^1 \varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,1,1)d\tau\right)ds>0.
\end{aligned} \tag{29}$$

By the definition of M_1, it is clear that $M_1\geq\beta_1-2$; then, we have $m_1\leq m_2$, so $m_1h_1(t)\leq A(h_1,h_2)(t)\leq m_2h_1(t)$, $t\in[0,1]$, that is, $A(h_1,h_2)\in P_{h_1}$. Similarly, from Lemma 3 and $(H_1)-(H_2)$, we get $B(h_1,h_2)\in P_{h_2}$.

Now, by Lemma 5, we obtain the following conclusions:

(1) We can find $u_1,v_1\in P_{h_1}$, $u_2,v_2\in P_{h_2}$, $\gamma\in(0,1)$ such that $\gamma(v_1,v_2)\leq(u_1,u_2)\leq(v_1,v_2)$ and $u_1\leq A(u_1,u_2)\leq v_1$, $u_2\leq B(u_1,u_2)\leq v_2$, that is,

$$\begin{aligned}
u_1(t) &\leq \int_0^1 G_1(t,s)\varphi_{q_1}\left(\int_0^1 H_1(s,\tau)f(\tau,u_1(\tau),u_2(\tau))d\tau\right)ds \\
&\leq v_1(t),\, t\in[0,1], \\
u_2(t) &\leq \int_0^1 G_2(t,s)\varphi_{q_2}\left(\int_0^1 H_2(s,\tau)g(\tau,u_1(\tau),u_2(\tau))d\tau\right)ds \\
&\leq v_2(t),\, t\in[0,1]
\end{aligned} \tag{30}$$

(2) The operator equation $(u,v)=(\tilde{\lambda}A(u,v),\tilde{\mu}B(u,v))$ has a unique solution $(u^*_{\lambda,\mu},v^*_{\lambda,\mu})$ depending on λ, $\mu>0$ in $\bar{P}_h$, where $\tilde{\lambda}=\lambda^{q_1-1}$, $\tilde{\mu}=\mu^{q_2-1}$. That is, $(u^*,v^*)=T(u^*,v^*)$. So, system (4) has a unique positive solution $(u^*_{\lambda,\mu},v^*_{\lambda,\mu})$ in $\bar{P}_h$

(3) Taking any initial point $(u_0,v_0)\in\bar{P}_h$, let

$$\begin{aligned}
u_{n+1}(t) &= \lambda^{q_1-1}\int_0^1 G_1(t,s)\varphi_{q_1} \\
&\quad\cdot\left(\int_0^1 H_1(s,\tau)f(\tau,u_n(\tau),v_n(\tau))d\tau\right)ds,\, n=1,2,\cdots, \\
v_{n+1}(t) &= \mu^{q_2-1}\int_0^1 G_2(t,s)\varphi_{q_2} \\
&\quad\cdot\left(\int_0^1 H_2(s,\tau)g(\tau,u_n(\tau),v_n(\tau))d\tau\right)ds,\, n=1,2,\cdots,
\end{aligned} \tag{31}$$

then $u_n(t)\longrightarrow u^*_{\lambda,\mu}(t)$, $v_n(t)\longrightarrow v^*_{\lambda,\mu}(t)$ as $n\longrightarrow\infty$

Taking $\lambda=\mu=1$, we have the following conclusion.

Corollary 8. *Let $\alpha_i\in(1,2]$, $\beta_i\in(3,4]$, $h_1(t)=t^{\beta_1-2}(1-t)^2$, and $h_2(t)=t^{\beta_2-2}(1-t)^2$, $t\in[0,1]$. Assume that (H_1), (H_2), and (H_3) hold. Then, the following system:*

$$\begin{cases}
D^{\alpha_1}_{0^+}\left(\varphi_{p_1}\left(D^{\beta_1}_{0^+}u(t)\right)\right)+f(t,u(t),v(t))=0,\, 0<t<1, \\
D^{\alpha_2}_{0^+}\left(\varphi_{p_2}\left(D^{\beta_2}_{0^+}v(t)\right)\right)+g(t,u(t),v(t))=0,\, 0<t<1, \\
u(0)=u(1)=u'(0)=u'(1)=0,\, D^{\beta_1}_{0^+}u(0)=0,\, D^{\beta_1}_{0^+}u(1)=b_1D^{\beta_1}_{0^+}u(\eta_1), \\
v(0)=v(1)=v'(0)=v'(1)=0,\, D^{\beta_2}_{0^+}v(0)=0,\, D^{\beta_2}_{0^+}v(1)=b_2D^{\beta_2}_{0^+}v(\eta_2),
\end{cases} \tag{32}$$

has a unique positive solution (x^*,y^*) in $\bar{P}_h$. In addition, take any given point $(u_0,v_0)\in\bar{P}_h$, construct

$$\begin{aligned}
u_{n+1}(t) &= \int_0^1 G_1(t,s)\varphi_{q_1} \\
&\quad\cdot\left(\int_0^1 H_1(s,\tau)f(\tau,u_n(\tau),v_n(\tau))d\tau\right)ds,\, n=1,2,\cdots, \\
v_{n+1}(t) &= \int_0^1 G_2(t,s)\varphi_{q_2} \\
&\quad\cdot\left(\int_0^1 H_2(s,\tau)g(\tau,u_n(\tau),v_n(\tau))d\tau\right)ds,\, n=1,2,\cdots,
\end{aligned} \tag{33}$$

then $u_n(t)\longrightarrow x^*(t)$, $v_n(t)\longrightarrow y^*(t)$ as $n\longrightarrow\infty$.

4. An Example

Considering the following system:

$$\begin{cases} D_{0^+}^{3/2}\left(\varphi_{3/2}\left(D_{0^+}^{7/2}u(t)\right)\right)+\lambda\left(u^{1/3}+v^{1/3}+2t^2\right)=0, 0<t<1, \\ D_{0^+}^{3/2}\left(\varphi_{3/2}\left(D_{0^+}^{7/2}v(t)\right)\right)+\mu\left(u^{1/4}+v^{1/4}+3t^3\right)=0, 0<t<1, \\ u(0)=u(1)=u'(0)=u'(1)=0, D_{0^+}^{7/2}u(0)=0, D_{0^+}^{7/2}u(1)=b_1 D_{0^+}^{7/2}u(1/2), \\ v(0)=v(1)=v'(0)=v'(1)=0, D_{0^+}^{7/2}v(0)=0, D_{0^+}^{7/2}v(1)=b_2 D_{0^+}^{7/2}v(1/3), \end{cases} \tag{34}$$

where $\alpha_1=\alpha_2=3/2$, $\beta_1=\beta_2=7/2$, $\lambda,\mu>0$, $\eta_1=1/2$, $\eta_2=1/3$, $b_1=1/2$, $b_2=1/3$, $p_1=p_2=3/2$, $q_1=q_2=3$ and

$$f(t,u,v)=u^{1/3}+v^{1/3}+2t^2, g(t,u,v)=u^{1/4}+v^{1/4}+3t^3. \tag{35}$$

Obviously, $f,g\in C([0,1])\times[0,+\infty)\times[0,+\infty),[0,+\infty))$ and $f(t,0,0)=2t^2\not\equiv 0$, $g(t,0,0)=3t^3\not\equiv 0$.

Note that $x^{1/3}$ and $x^{1/4}$ are increasing in $[0,+\infty)$, it implies that $f(t,u,v)$, $g(t,u,v)$ are increasing with respect to the second and third variables for $t\in[0,1]$. Moreover, set $\psi_1(r)=r^{1/3}$, $\psi_2(r)=r^{1/4}$, $r\in(0,1)$. Then, $\psi_1(r),\psi_2(r)\in(0,1)$, $\psi_1(r)=r^{1/3}>r^{1/2}=r^{1/(q_1-1)}$, $\psi_2(r)=r^{1/4}>r^{1/2}=r^{1/(q_2-1)}$, and

$$\begin{aligned} f(t,ru,rv)&=r^{1/3}\left(u^{1/3}+v^{1/3}\right)+2t^2\geq r^{1/3}\left(u^{1/3}+v^{1/3}\right)\\ &\quad+r^{1/3}2t^2=r^{1/3}f(t,u,v)=\psi_1(r)f(t,u,v),\\ g(t,ru,rv)&=r^{1/4}\left(u^{1/4}+v^{1/4}\right)+3t^3\geq r^{1/4}\left(u^{1/4}+v^{1/4}\right)\\ &\quad+r^{1/4}3t^3=r^{1/4}g(t,u,v)=\psi_2(r)g(t,u,v), \end{aligned} \tag{36}$$

for $t\in[0,1]$, $u,v\in[0,+\infty)$. Hence, all conditions of Theorem 7 are satisfied. Then, Theorem 7 shows that system (34) has a unique positive solution $(u^*_{\lambda,\mu},v^*_{\lambda,\mu})$ in $\bar{P}_h$, where $h(t)=(t^{3/2}(1-t)^2,t^{3/2}(1-t)^2)$, $t\in[0,1]$, and taking any given point $(u_0,v_0)\in\bar{P}_h$, let

$$\begin{aligned} u_{n+1}(t)&=\lambda^2\int_0^1 G_1(t,s)\left[\int_0^1 H_1(s,\tau)\left(u_n(\tau)^{1/3}+v_n(\tau)^{1/3}\right.\right.\\ &\quad\left.\left.+2\tau^2\right)d\tau\right]^2 ds, n=1,2,\cdots,\\ v_{n+1}(t)&=\mu^2\int_0^1 G_2(t,s)\left[\int_0^1 H_2(s,\tau)\left(u_n(\tau)^{1/4}+v_n(\tau)^{1/4}\right.\right.\\ &\quad\left.\left.+3\tau^3\right)d\tau\right]^2 ds, n=1,2,\cdots, \end{aligned} \tag{37}$$

then $u_n(t)\longrightarrow u^*_{\lambda,\mu}(t)$, $v_n(t)\longrightarrow v^*_{\lambda,\mu}(t)$ as $n\longrightarrow\infty$, where

$$\begin{aligned} G_1(t,s)&=G_2(t,s)=\frac{1}{\Gamma(7/2)}\begin{cases} t^{3/2}(1-s)^{3/2}[(s-t)+3/2(1-t)s], 0\leq t\leq s\leq 1,\\ t^{3/2}(1-s)^{3/2}[(s-t)+3/2(1-t)s]+(t-s)^{5/2}, 0\leq s\leq t\leq 1,\end{cases}\\ H_1(t,s)&=h_1(t,s)+\sqrt{2}t^{1/2}h_1\left(\frac{1}{2},s\right),\\ h_1(t,s)&=\frac{1}{\Gamma(3/2)}\begin{cases}[t(1-s)]^{1/2}, 0\leq t\leq s\leq 1,\\ [t(1-s)]^{1/2}-(t-s)^{1/2}, 0\leq s\leq t\leq 1,\end{cases}\\ H_2(t,s)&=h_2(t,s)+\frac{\sqrt{3}}{2t^{1/2}h_1}\left(\frac{1}{3},s\right),\\ h_2(\mathrm{t},s)&=\frac{1}{\Gamma(3/2)}\begin{cases}[t(1-s)]^{1/2}, 0\leq t\leq s\leq 1,\\ [t(1-s)]^{1/2}-(t-s)^{1/2}, 0\leq s\leq t\leq 1.\end{cases} \end{aligned} \tag{38}$$

Data Availability

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

The authors declare that the study was realized in collaboration with the same responsibility. All authors read and approved the final manuscript.

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