Research Article

Solitary Wave Solutions to the Multidimensional Landau–Lifshitz Equation

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In this paper, we study the different types of new soliton solutions to the Landau–Lifshitz equation with the aid of the auxiliary equation method. Then, we get some special soliton solutions for this equation. Without the Gilbert damping term, we present a travelling wave solution with a finite energy in the initial time. The parameters of the soliton envelope are obtained as a function of the dependent model coefficients.

1. Introduction

Nonlinear partial differential equations have different types of equations; one of them is the Landau–Lifshitz equation that is relevant to the classical and quantum mechanics. Nonlinear evolution equations (NEEs) which describe many physical phenomena are often illustrated by nonlinear partial differential equations. So, the exact solutions of NLPDE are explored in detail in order to understand the physical structure of natural phenomena that are described by such equations. A variety of powerful methods have been used to study the nonlinear evolution equations, for the analytic and numerical solutions. Some of these methods, the Riccati Equation method [1], Hirota's bilinear operators [2], exponential rational function method [3], the Jacobi elliptic function expansion [4], the homogeneous balance method [5], the tanh-function expansion [6], first integral method [7, 8], the subequation method [9], the exp-function method [10], the Backlund transformation, and similarity reduction [11–29], are used to obtain the exact solutions of NLPDE.

In physics, the Landau–Lifshitz–Gilbert equation, named for Lev Landau, Evgeny Lifshitz, and T. L. Gilbert, is a name used for a differential equation describing the precessional motion of magnetization M in a solid. It is a modification

by Gilbert of the original equation of LLG equation (see [30]) can be written down as

$$\frac{\partial}{\partial t} S = \alpha S \Delta S - \beta S \Delta (S \Delta S), \quad (1)$$

here, $S = (S_1(t, \vec{x}), S_2(t, \vec{x}), S_3(t, \vec{x})) \in S^2 \hookrightarrow \mathbb{R}^3$, $\alpha \geq 0$, $\alpha^2 + \beta^2 = 1$, Δ denotes the cross product. The term multiplying with α represents the exchange interaction, while the β term denotes the Gilbert damping term. Especially, two extreme cases of (1) ($\beta = 0$ and $\alpha = 0$, respectively) include as special cases the well-known Schrödinger map equation and harmonic map heat flow, respectively. The well-posedness problem of the LL(G) equation are intensively studied in mathematics, to list a few, in 1986, of the weak solution of the LL(G) equation. Under the small initial value, the global existence of the solution in different spaces [31–33] was proved. The first progress on the existence of partially regular solutions to the LLG equation was found [30, 33–36]. Even for the small initial data, the exact form of the solution is still unknown. On the other hand, whether the LLG equation admits a global solution will develop a finite time singularity from the large initial data is an open equation.

2. Auxiliary Equation Method

Let us consider a typical nonlinear PDE for $q = q(x, t)$, given by

$$U(q_x, q_t, \dots) = 0. \quad (2)$$

Under the wave transformations of $q(x, t) = Q(\xi)$ and $\xi = \sigma x - lt$, Equation (2) becomes an ordinary differential equation given by

$$U(Q, \sigma Q', -lQ', \dots) = 0. \quad (3)$$

We assume that the solution $\phi(\xi)$ of the nonlinear Equation (18) can be presented as

$$Q(\xi) = \sum_{i=0}^M A_i Y^i(\xi), \quad (4)$$

in which $A_i (i = 0, \dots, n)$ are all real constants to be determined, the balancing number M is a positive integer which can be determined by balancing the highest order derivative terms with the highest power nonlinear terms in Equation (2) and $Y^i(\xi)$ expresses the solutions of the following auxiliary ordinary differential equation:

$$\left(\frac{dY}{d\xi}\right)^2 = s_1 Y^2(\xi) + s_2 Y^3(\xi) + s_3 Y^4(\xi), \quad (5)$$

where s_1, s_2 , and s_3 are real parameters. Equations (6), (7), (8), (9), (10), (11), (12), (13), (14), (15), (16), (17), (18), and (19) with $\Delta = s_2^2 - 4s_1s_3$ give the following solutions:

Case 1. For $s_1 > 0$,

$$\frac{-s_1 s_2 \operatorname{sech}^2\left(\sqrt{(s_1/2)}\xi\right)}{s_2^2 - s_1 s_3 \left(1 + \xi \tanh\left(\sqrt{(s_1/2)}\xi\right)\right)}. \quad (6)$$

Case 2. For $s_1 > 0$,

$$\frac{-s_1 s_2 \csc^2\left(\sqrt{(s_1/2)}\xi\right)}{s_2^2 - s_1 s_3 \left(1 + \xi \tanh\left(\sqrt{(s_1/2)}\xi\right)\right)}. \quad (7)$$

Case 3. For $s_1 > 0, \Delta > 0$,

$$\frac{2s_1 \operatorname{sech}\left(\sqrt{s_1}\xi\right)}{\varepsilon\sqrt{\Delta} - s_2 \operatorname{sech}\left(\sqrt{s_1}\xi\right)}. \quad (8)$$

Case 4. For $s_1 < 0, \Delta > 0$,

$$\frac{2s_1 \sec\left(\sqrt{-s_1}\xi\right)}{\varepsilon\sqrt{\Delta} - s_2 \sec\left(\sqrt{-s_1}\xi\right)}. \quad (9)$$

Case 5. For $s_1 > 0, \Delta < 0$,

$$\frac{2s_1 \operatorname{csch}\left(\sqrt{s_1}\xi\right)}{\varepsilon\sqrt{-\Delta} - s_2 \operatorname{csch}\left(\sqrt{s_1}\xi\right)}. \quad (10)$$

Case 6. For $s_1 < 0, \Delta > 0$,

$$\frac{2s_1 \csc\left(\sqrt{-s_1}\xi\right)}{\varepsilon\sqrt{\Delta} - s_2 \csc\left(\sqrt{-s_1}\xi\right)}. \quad (11)$$

Case 7. For $s_1 > 0, s_3 > 0$

$$\frac{-s_1 \operatorname{sech}^2\left(\left(\sqrt{s_1}/2\right)\xi\right)}{s_2 + 2\xi\sqrt{s_1 s_3} \tanh\left(\left(\sqrt{s_1}/2\right)\xi\right)}. \quad (12)$$

Case 8. For $s_1 < 0, s_3 > 0$,

$$\frac{-s_1 \sec^2\left(\left(\sqrt{-s_1}/2\right)\xi\right)}{s_2 + 2\xi\sqrt{-s_1 s_3} \tan\left(\left(\sqrt{-s_1}/2\right)\xi\right)}. \quad (13)$$

Case 9. For $s_1 > 0, s_3 > 0$

$$\frac{s_1 \operatorname{csch}^2\left(\left(\sqrt{s_1}/2\right)\xi\right)}{s_2 + 2\xi\sqrt{s_1 s_3} \coth\left(\left(\sqrt{s_1}/2\right)\xi\right)}. \quad (14)$$

Case 10. For $s_1 < 0, s_3 > 0$,

$$\frac{-s_1 \csc^2\left(\left(\sqrt{-s_1}/2\right)\xi\right)}{s_2 + 2\xi\sqrt{-s_1 s_3} \cot\left(\left(\sqrt{-s_1}/2\right)\xi\right)}. \quad (15)$$

Case 11. For $s_1 > 0, \Delta = 0$,

$$-\frac{s_1}{s_2} \left(1 + \xi \tanh\left(\frac{\sqrt{s_1}}{2}\xi\right)\right). \quad (16)$$

Case 12. For $s_1 > 0, \Delta = 0$,

$$-\frac{s_1}{s_2} \left(1 + \xi \coth\left(\frac{\sqrt{s_1}}{2}\xi\right)\right). \quad (17)$$

Case 13. For $s_1 > 0$,

$$\frac{4s_1 e^{\varepsilon\sqrt{s_1}\xi}}{(e^{\varepsilon\sqrt{s_1}\xi} - s_2)^2 - 4s_1 s_2}. \quad (18)$$

Case 14. For $s_1 > 0, s_2 = 0$,

$$\frac{\pm 4s_1 \varepsilon e^{\varepsilon\sqrt{s_1}\xi}}{1 - 4s_1 s_3 e^{2\varepsilon\sqrt{s_1}\xi}}. \quad (19)$$

Stage 2: substituting Equations (4) and (5) into Equation (3) and collecting all terms with the same order of Y^j together, we convert the left-hand side of Equation (3) into a polynomial in Y^j . Setting each coefficient of each polynomial to zero, we derive a set of algebraic equations for A_0 ,

A_1 , and A_2 . By solving these algebraic equations, we obtain several cases of variables solutions [15, 37]

Stage 3: by substituting the obtained solutions in stage 2 into Equation (4) along with general solutions of Equations (6), (7), (8), (9), (10), (11), (12), (13), (14), (15), (16), (17), (18), and (19), it finally generates new exact solutions for the nonlinear PDE (1)

3. Travelling Wave Solutions of the LL Equation via Aem

In this section, we construct a travelling wave solution without the Gilbert term [30]. Under the condition of the auxiliary equation method, we consider the following wave transformations:

$$W_{c,w}(t, \bar{r}) = e^{-iwt} \phi(\bar{r} - ct) e^{i\psi(\bar{r} - ct)}, \quad (20)$$

where c and w are constants undetermined.

Here, we assume $-c^2 + 4w > 0$. Substitute (5) into (1) [30], the separate real and the virtual parts, respectively, as

$$\begin{aligned} & \phi(\xi) \left(w - 2(\xi)^2 + c\psi'(\xi) - \psi'(\xi)^2 \right) \\ & + \phi(\xi)^3 \left(w + c\psi'(\xi) + \psi'(\xi)^2 \right) + \phi(\xi)^2 \phi''(\xi) = 0, \end{aligned} \quad (21)$$

$$\begin{aligned} & \phi'(\xi) \left(-c + 2\psi'(\xi) \right) - \phi(\xi)^2 \phi'(\xi) \left(c + 2\psi'(\xi) \right) \\ & + \phi(\xi) \psi''(\xi) + \phi(\xi)^3 \psi''(\xi) = 0, \end{aligned} \quad (22)$$

where $\xi = \bar{r} - ct$. (21) and (22) are the nonlinear constant coefficients of ordinary differential equation system with the variable ξ . According to (22), we can obtain a relationship between ψ and ϕ :

$$\psi'(\xi) = \frac{(1 + \phi(\xi)^2)(-c + 2C_1 + 2C_1(\xi)^2)}{2\phi(\xi)^2}, \quad (23)$$

where C_1 is the arbitrary constant. If we set $C_1=0$, we have

$$\psi'(\xi) = -\frac{c(1 + \phi(\xi)^2)}{2\phi(\xi)^2}. \quad (24)$$

Substituting (24) into (21), we get

$$\begin{aligned} & c^2 + 3c^2 \phi(\xi)^2 + (c^2 - 4w) \phi(\xi)^6 + \phi(\xi)^2 (3c^2 - 4w + 8\phi'(\xi)^2) \\ & - 4\phi(\xi)^3 (1 + \phi(\xi)^2) \phi''(\xi) = 0. \end{aligned} \quad (25)$$

4. Results

By the auxiliary equation method, the solution of (25) is assumed as

$$\phi(\xi) = A_1 Y(\xi) + A_0. \quad (26)$$

From (5), we have

$$\begin{aligned} & (Y')^2 = s_1 Y^2(\xi) + s_2 Y^3(\xi) + s_3 Y^4(\xi), \\ & Y'' = \frac{1}{2} (s_1 Y(\xi) + 3s_2 Y^2(\xi) + 4s_3 Y^3(\xi)), \end{aligned} \quad (27)$$

where A_1 and A_0 are constants. Substituting (26) into (25) along with (27) and comparing the coefficients of alike powers of $Y(\xi)$ provides an algebraic system of equations, and solving this set of algebraic equations by used of the Maple, we obtain several case solutions. For example is as follows.

Set 1:

$$A_1 = -\frac{20}{3} s_3 \left(\frac{3s_1 - 12s_2}{240s_3 - 4w} \right),$$

$$A_0 = \frac{3s_1 - 12s_2}{240s_3 - 4w},$$

$$s_2 = 0,$$

$$c = \frac{1}{30} \left(\sqrt{10 \sqrt{25s_3(3s_1)^2 + (-3120w - 10800s_3)s_3(3s_1) + 144(w + 90s_3)^2 + (150s_1 - 10800)s_3 + 480w}} \right). \quad (28)$$

FIGURE 1: Graphical behavior for $\phi_1(\bar{r}, t)$, for $\bar{r}, t = -\pi.. \pi, s_1 = -1, s_3 = 1$, and $w = 1$.

From set 1 and Eq. (26), we have

$$\phi(\xi) = -\frac{20}{3}s_3 \left(\frac{3s_1}{240s_3 - 4w} \right) \Upsilon(\xi) + \frac{3s_1}{240s_3 - 4w}. \quad (29)$$

So, solutions of Equation (5) are obtained in (6), (7), (8), (9), (10), (11), (12), (13), (14), (15), (16), (17), (18), and (19), we have final solutions of Equation (1) and Equation (25) as follows:

$$\begin{aligned} \phi_1(\bar{r}, t) = & -\frac{20}{3}s_3 \left(\frac{3s_1}{240s_3 - 4w} \right) \\ & \times \frac{-s_1 \csc^2 \left(\left(\frac{\sqrt{-s_1}}{2} \right) (\bar{r} - ct) \right)}{2\varepsilon \sqrt{-s_1 s_3} \cot \left(\left(\frac{\sqrt{-s_1}}{2} \right) (\bar{r} - ct) \right)} + \frac{3s_1 - 12s_2}{240s_3 - 4w}. \end{aligned} \quad (30)$$

In Figure 1, the graphical behavior of solutions ϕ_1 for $s_1 = -1, s_3 = 1$, and $w = 1$ in different values of $\bar{r}$ and t is illustrated.

$$\begin{aligned} \phi_2(\bar{r}, t) = & -\frac{20}{3}s_3 \left(\frac{3s_1}{240s_3 - 4w} \right) \times \frac{2s_1 \sec \left(\left(\frac{\sqrt{-s_1}}{2} \right) (\bar{r} - ct) \right)}{\varepsilon \sqrt{-4s_1 s_3}} \\ & + \frac{3s_1 - 12s_2}{240s_3 - 4w}. \end{aligned} \quad (31)$$

FIGURE 2: Graphical behavior for $\phi_2(\bar{r}, t)$, for $\bar{r}, t = -\pi.. \pi, s_1 = -1, s_3 = 1$, and $w = 1$.

FIGURE 3: Graphical behavior for $\phi_3(\bar{r}, t)$, for $\bar{r}, t = -\pi.. \pi, s_1 = 1, s_3 = 1$, and $w = 1$.

In Figure 2, the graphical behavior of solutions ϕ_2 for $s_1 = -1, s_3 = 1$, and $w = 1$ in different values of $\bar{r}$ and t is illustrated.

$$\begin{aligned} \phi_3(\bar{r}, t) = & -\frac{20}{3}s_3 \left(\frac{3s_1}{240s_3 - 4w} \right) \\ & \times \frac{s_1 \csc^2 \left(\left(\frac{\sqrt{s_1}}{2} \right) (\bar{r} - ct) \right)}{2\varepsilon \sqrt{s_1 s_3} \coth \left(\left(\frac{\sqrt{s_1}}{2} \right) (\bar{r} - ct) \right)} + \frac{3s_1}{240s_3 - 4w}. \end{aligned} \quad (32)$$

FIGURE 4: Graphical behavior for $\phi_6(\bar{r}, t)$, for $\bar{r}, t = -\pi.. \pi$, $s_1 = 1$, and $s_2 = 1$.

FIGURE 5: Graphical behavior for $\phi_7(\bar{r}, t)$, for $\bar{r}, t = -\pi.. \pi$, $s_1 = 1$, and $s_2 = 1$.

In Figure 3, the graphical behavior of solutions ϕ_3 for $s_1 = 1$, $s_3 = 1$, and $w = 1$ in different values of $\bar{r}$ and t is illustrated.

$$\begin{aligned}\phi_4(\bar{r}, t) &= -\frac{20}{3}s_3 \left(\frac{3s_1}{240s_3 - 4w} \right) \times \frac{4s_1 e^{\varepsilon\sqrt{s_1}(\bar{r}-ct)}}{(e^{\varepsilon\sqrt{s_1}(\bar{r}-ct)})^2} + \frac{3s_1}{240s_3 - 4w}, \\ \phi_5(\bar{r}, t) &= -\frac{20}{3}s_3 \left(\frac{3s_1}{240s_3 - 4w} \right) \times \frac{\pm 4s_1 \varepsilon e^{\varepsilon\sqrt{s_1}(\bar{r}-ct)}}{1 - 4s_1 s_3 e^{2\varepsilon\sqrt{s_1}(\bar{r}-ct)}} + \frac{3s_1}{240s_3 - 4w},\end{aligned}\quad (33)$$

$$W_{c,w}(t, \bar{r}) = e^{-i\omega t} \phi(\bar{r} - ct) e^{i\psi(\bar{r}-ct)},$$

$$\begin{aligned}\psi'(\xi) &= -\frac{c(1 + \phi(\xi)^2)}{2\phi(\xi)^2} \text{ so } \psi(\xi) = -\int \frac{c(1 + \phi(\xi)^2)}{2\phi(\xi)^2} d\xi, \\ c &= \frac{1}{30} \left(\sqrt{10\sqrt{25s_3(3s_1)^2 + (-3120w - 10800s_3)s_3(3s_1) + 144(w + 90s_3)^2 + (150s_1 - 10800)s_3 + 480w}} \right).\end{aligned}\quad (34)$$

Set 2:

$$\begin{aligned}A_1 &= \frac{-s_1^2(s_1^2 + 225s_2^2)(-90s_2 + s_1)}{12w(2s_1^4 - 75s_1^3s_2 + 118125s_2^4)}, \\ A_0 &= -\frac{1}{15} \frac{s_1}{s_2}, \\ s_3 &= 0, \\ c &= \sqrt{\left(\frac{4}{15} \frac{s_1}{s_2} + 4 \right) w / \left(6 - \frac{1}{15} \frac{s_1}{s_2} \right)}.\end{aligned}\quad (35)$$

From set 2 and Eq. (26), we have

$$\phi(\xi) = \frac{-s_1^2(s_1^2 + 225s_2^2)(-90s_2 + s_1)}{12w(2s_1^4 - 75s_1^3s_2 + 118125s_2^4)} \Upsilon(\xi) - \frac{1}{15} \frac{s_1}{s_2}. \quad (36)$$

So, solutions of Equation (5) are obtained in (6), (7), (8), (9), (10), (11), (12), (13), (14), (15), (16), (17), (18), and (19), we have final solutions of Equation (1) and Equation (25) as follows:

$$\phi_6(\bar{r}, t) = \frac{-s_1^2(s_1^2 + 225s_2^2)(-90s_2 + s_1)}{12w(2s_1^4 - 75s_1^3s_2 + 118125s_2^4)} \times \frac{-s_1s_2 \operatorname{sech}^2(\sqrt{s_1/2}(\bar{r} - ct))}{s_2^2} - \frac{1}{15} \frac{s_1}{s_2}. \quad (37)$$

In Figure 4, the graphical behavior of solutions ϕ_6 for $s_1 = 1, s_2 = 1$ in different values of $\bar{r}$ and t is illustrated.

$$\phi_7(\bar{r}, t) = \frac{-s_1^2(s_1^2 + 225s_2^2)(-90s_2 + s_1)}{12w(2s_1^4 - 75s_1^3s_2 + 118125s_2^4)} \times \frac{-s_1s_2 \operatorname{csch}^2(\sqrt{s_1/2}(\bar{r} - ct))}{s_2^2} - \frac{1}{15} \frac{s_1}{s_2}. \quad (38)$$

In Figure 5, the graphical behavior of solutions ϕ_7 for $s_1 = 1, s_3 = 1$ in different values of $\bar{r}$ and t is illustrated.

$$\phi_8(\bar{r}, t) = \frac{-s_1^2(s_1^2 + 225s_2^2)(-90s_2 + s_1)}{12w(2s_1^4 - 75s_1^3s_2 + 118125s_2^4)} \times \frac{2s_1 \operatorname{sech}(\sqrt{s_1}(\bar{r} - ct))}{\varepsilon s_2 - s_2 \operatorname{sech}(\sqrt{s_1}(\bar{r} - ct))} - \frac{1}{15} \frac{s_1}{s_2},$$

$$\phi_9(\bar{r}, t) = \frac{-s_1^2(s_1^2 + 225s_2^2)(-90s_2 + s_1)}{12w(2s_1^4 - 75s_1^3s_2 + 118125s_2^4)} \times \frac{4s_1 e^{\varepsilon\sqrt{s_1}(\bar{r}-ct)}}{(e^{\varepsilon\sqrt{s_1}(\bar{r}-ct)} - s_2)^2 - 4s_1s_2} - \frac{1}{15} \frac{s_1}{s_2}, \quad (39)$$

$$W_{c,w}(t, \bar{r}) = e^{-i\omega t} \phi(\bar{r} - ct) e^{i\psi(\bar{r}-ct)},$$

$$\psi'(\xi) = -\frac{c(1 + \phi(\xi)^2)}{2\phi(\xi)^2} \text{ so } \psi(\xi) = -\int \frac{c(1 + \phi(\xi)^2)}{2\phi(\xi)^2} d\xi,$$

$$c = \sqrt{\left(\frac{4}{15} \frac{s_1}{s_2} + 4\right) w / \left(6 - \frac{1}{15} \frac{s_1}{s_2}\right)}. \quad (40)$$

For more convenience and understanding, the graphical behavior of the answers is considered (see Figures 1–5).

5. Conclusions

This paper derived new optical soliton solutions of the Landau–Lifshitz equation, which describe the propagation of ultrashort pulses in nonlinear optical fibers by using the auxiliary equation method. We boldly say that the work here is valuable and may be beneficial for studying in other nonlinear science. The exact solutions obtained from the model equations provide important insight into the dynamics of solitary waves. The solutions obtained in this paper have not been reported in the old research.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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