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Research Article

Mixed Rational Lump-Solitary Wave Solutions to an Extended (2+1)-Dimensional KdV Equation

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Based on the bilinear method, rational lump and mixed lump-solitary wave solutions to an extended (2+1)-dimensional KdV equation are constructed through the different assumptions of the auxiliary function in the trilinear form. It is found that the rational lump decays algebraically in all directions in the space plane and its amplitude possesses one maximum and two minima. One kind of the mixed solution describes the interaction between one lump and one line solitary wave, which exhibits fission and fusion phenomena under the different parameters. The other kind of the mixed solution shows one lump interacting with two paralleled line solitary waves, in which the evolution of the lump gives rise to a two-dimensional rogue wave. This shows that these three interesting phenomena exist in the corresponding physical model.

1. Introduction

The study of integrable nonlinear systems has become a hot topic in wave propagations and mathematical physics. Integrable systems approximately describe the evolution of various waves in many physical settings, including shallow-water waves with weakly nonlinear restoring forces, pulse propagation in optical fibers and wave guides, long internal waves in a density-stratified ocean, and ion acoustic waves in plasma [1–16]. In the higher-dimensional extensions of integrable nonlinear wave equation, the (2+1)-dimensional KdV equation or the asymmetrical Nizhnik-Novikov-Veselov (ANNV) equation [17]

$$u_t + u_{xxx} + 3\left(u\partial_y^{-1}u_x\right)_x = 0 \tag{1}$$

was firstly proposed by Boiti et al. in the sense of the weak Lax pair. This model arose in the incompressible fluid and was shown to possess an infinite number of conservation laws, multiple soliton solutions, and other integrability properties [17]. By introducing two terms $\partial_y^{-1}u_{xx}$ and $u_y = \partial_y^{-1}u_{yy}$ into equation (1), a generalized (2+1)-dimensional KdV equation with arbitrary constant coefficients has been recently developed [18]

$$u_t + v_{xxx} + \alpha\left(u\partial_y^{-1}u_x\right)_x + \beta\left(\partial_y^{-1}u_{xx}\right) + \gamma\left(\partial_y^{-1}u_{yy}\right) = 0, \tag{2}$$

which describes the ion-acoustic waves in plasmas, shallow water waves in oceans, and pulse waves in large arteries. Here, α, β, and γ are real constants. Equation (2) reduces to the ANNV equation (1) when $\alpha = 3$ and $\beta = \gamma = 0$ and becomes the classical KdV equation when $y = x$. The (2+1)-dimensional equation (2) was investigated through the Painlevé test, and its multiple-soliton solutions were derived via the simplified Hirota algorithm [18]. More recently, a lot of rational lump solutions, hybrid solutions consisting of lump waves and kink waves, loop-like kink breather solutions, and the lump interacting with the line soliton solutions have been constructed via the Hirota bilinear method [19–46]. Although these results can also be derived by Darboux transformation [47–50], modified extended mapping method [1], and direct algebraic method [2], the bilinear method is still a powerful tool for solving integrable systems. It is worth mentioning that Seadawy et al. obtained some new exact solutions of many integrable systems by using various methods, such as extend simple equation method and the exp $(\phi(\xi))$ expansion method [1–16]. For the ANNV equation (1), the lump solutions, mixed lump-stripe solutions, and periodic lump solutions were presented in [19].

Recently, researches about trilinear form have become a hot topic. Trilinear form is an extension of Hirota's bilinear form [51]. A group of scholars who work on integrable systems have found that some new analytic solutions of nonlinear PDEs can be obtained through trilinear differential equations [52, 53]. Hence, we aim to construct the rational lump and the lump-solitary wave solutions to the extended (2+1)-dimensional KdV equation (2) through the trilinear form.

The rest of the paper is organized as follows. In Section 2, we firstly transform the extended (2+1)-dimensional KdV equation (2) to the trilinear form through the certain variable transformation and construct the exact rational lump solution. The mixed solution composed of one lump and one line solitary wave is derived in Section 3. Section 4 devotes to studying the interaction solution consisting of one lump and two line solitary waves, which can be viewed as a two-dimensional rogue wave excited from the line soliton pair.

2. Lump Solution

Through the dependent variable transformation $u=(6/\alpha)(\ln f)_{xy}$, the extended (2+1)-dimensional KdV equation (2) is transformed to the following trilinear form:

$$\begin{aligned}&[f, B_1 f\cdot f]_y+\beta\left[f, D_x^2 f\cdot f\right]_x-3\left[f_{xx}, D_x D_y f\cdot f\right]_x\\&\quad+3\left[f_{xx}, D_x^2 f\cdot f\right]_y=0,\end{aligned}\tag{3}$$

where

$$\begin{aligned}B_1 f\cdot f&=D_x D_t f\cdot f+D_x^4 f\cdot f+\gamma D_x D_y f\cdot f,\\ [a,b]_x&=a_x b-\frac{1}{2}ab_x,\\ [a,b]_y&=a_y b-\frac{1}{2}ab_y,\end{aligned}\tag{4}$$

and the Hirota bilinear operators D_x, D_y, and D_t are defined by [54]

$$\begin{aligned}D_x^n D_y^m D_t^l(a\cdot b)&=\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^n\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial y'}\right)^m\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^l\\&\quad\cdot a(x,y,t)bx',y',t'|_{x'=x,y'=y,t'=t}.\end{aligned}\tag{5}$$

In order to find the rational lump solution for equation (2), we set the auxiliary variable f in equation (3) as the following form:

$$\begin{aligned}f&=g^2+h^2+a_9,\\ g&=a_1x+a_2y+a_3t+a_4,\\ h&=a_5x+a_6y+a_7t+a_8,\end{aligned}\tag{6}$$

with

$$\begin{vmatrix}a_1 & a_2\\ a_5 & a_6\end{vmatrix}\neq 0,\tag{7}$$

where $a_i(i=1,2,\cdots,9)$ are real parameters and will be determined. Based on symbolic computation, we substitute the assumption equation (6) into the trilinear equation (3) and then collect the coefficients of the independent variables x, y, and t. Consequently, one has a set of algebraic equations with respect to the parameters $a_i(i=1,2,\cdots,9)$. To solve these equations, one can get the parameter relations as follows:

$$\begin{aligned}a_3&=-\frac{\beta\left[a_2\left(a_1^2-a_5^2\right)+2a_1a_5a_6\right]}{a_2^2+a_6^2}-\gamma a_2,\\ a_7&=-\frac{\beta\left[a_6\left(a_5^2-a_1^2\right)+2a_1a_2a_5\right]}{a_2^2+a_6^2}-\gamma a_6,\end{aligned}\tag{8}$$

$$a_9=-\frac{3\left(a_1^2+a_5^2\right)\left(a_2^2+a_6^2\right)\left(a_1a_2+a_5a_6\right)}{\beta\left(a_1a_6-a_2a_5\right)^2}.\tag{9}$$

This in turn gives rise to the rational lump solution as

$$u=\frac{12\left(a_1a_2+a_5a_6\right)}{\alpha\left(g^2+h^2+a_9\right)}-\frac{24\left(a_1g+a_5h\right)\left(a_2g+a_6h\right)}{\alpha\left(g^2+h^2+a_9\right)^2},\tag{10}$$

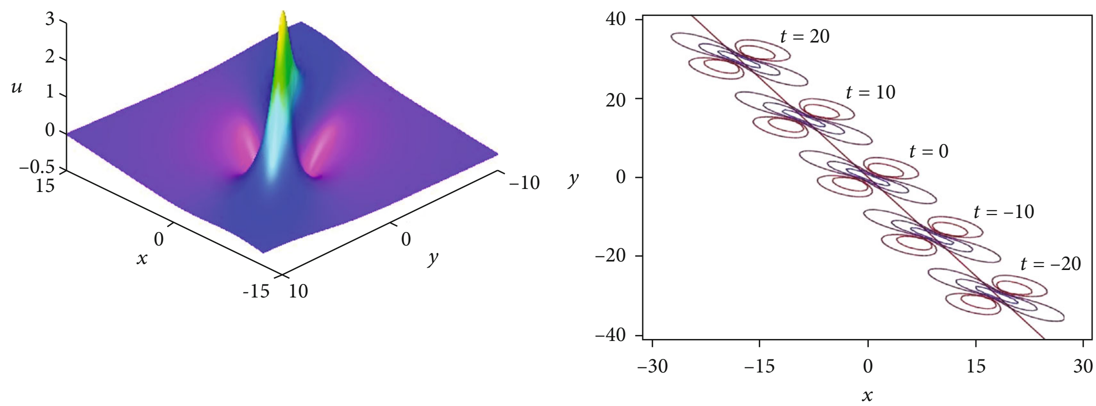
where the functions g and h are given in equation (6) with the parameters' conditions (8) and (9). To guarantee that the function f is well defined and the solution u in equation (10) decays in all directions in the x, y plane, these parameters are restricted by three conditions: $a_2^2+a_6^2\neq 0$ and $-((3(a_1^2+a_5^2)(a_1a_2+a_5a_6))/\beta)>0$.

For the local analysis, we find that the rational lump solution u in equation (10) possesses the maximum amplitude $-((4\beta(a_1a_6-a_2a_5)^2)/(\alpha(a_1^2+a_5^2)(a_2^2+a_6^2)))$, which is centered at the point

$$\left(\frac{a_2a_8-a_4a_6+\left(a_2a_7-a_3a_6\right)t}{a_1a_6-a_2a_5},\frac{a_4a_5-a_1a_8+\left(a_3a_5-a_1a_7\right)t}{a_1a_6-a_2a_5}\right).\tag{11}$$

It is concluded that this rational lump moves along the route line l_0: $y=((a_3a_5-a_1a_7)/(a_2a_7-a_3a_6))x+((a_3a_8-a_4a_7)/(a_2a_7-a_3a_6))$ and with the velocities $V_x=(a_2a_7-a_3a_6)/(a_1a_6-a_2a_5)$ and $V_y=(a_3a_5-a_1a_7)/(a_1a_6-a_2a_5)$, respectively.

The specific lump's structure and its moving path are illustrated in Figure 1. With the given parameter's values,



(a) (b)

Figure 1: The rational lump solution equation (10) with $a_1 = a_5 = \alpha = -\beta = \gamma = 1$, $a_2 = -1/5$, $a_6 = 2$, and $a_4 = a_8 = 0$: (a) the three-dimensional plot at $t = 0$; (b) the moving path with the contour curves of lumps at the different times.

Figure 1(a) shows the three-dimensional plot of the rational lump equation (10) at time $t = 0$, and Figure 1(b) corresponds to the contour curves at the different time, which exhibits the moving track of the lump. The direct calculation indicates that the illustrated lump in Figure 1 moves along the straight line $l_0 : y = -(151/90)x$, its velocities are $V_x = -(90/101)$, $V_y = 151/101$, and its maximum amplitude is 242/101.

3. The Mixed Solution Composed of One Rational Lump and One Line Solitary Wave

To construct the mixed solution that is composed of one rational lump and one line solitary wave, we use the following assumption of the function f:

$$\begin{aligned} f &= g^2 + h^2 + a_9 + ke^{\eta}, \\ g &= a_1 x + a_2 y + a_3 t + a_4, \\ h &= a_5 x + a_6 y + a_7 t + a_8, \\ \eta &= k_1 x + k_2 y + k_3 t, \end{aligned} \tag{12}$$

where $a_i (i = 1, 2, \cdots, 9)$, k, and $k_i (i = 1, 2, 3)$ are real parameters and will be determined. Here, the rational and the exponential functions are responsible for the rational lump and the line solitary wave, respectively. Similar to the case of the purely rational lump, one needs to collect the coefficients of x, y, and t and exponential functions after substituting equation (12) into equation (3). Then, we have a set of algebraic equations with respect to the parameters $a_i (i = 1, 2, \cdots, 9)$, k, and $k_i (i = 1, 2, 3)$, which gives the parameters' relations as follows:

$$\begin{aligned} a_3 &= -\frac{\beta\left[a_2\left(a_1^2 - a_5^2\right) + 2a_1 a_5 a_6\right]}{a_2^2 + a_6^2} - \gamma a_2, \\ a_7 &= -\frac{\beta\left[a_6\left(a_5^2 - a_1^2\right) + 2a_1 a_2 a_5\right]}{a_2^2 + a_6^2} - \gamma a_6, \end{aligned} \tag{13}$$

$$\begin{aligned} a_9 &= -\frac{3\left(a_1^2 + a_5^2\right)\left(a_2^2 + a_6^2\right)\left(a_1 a_2 + a_5 a_6\right)}{\beta\left(a_1 a_6 - a_2 a_5\right)^2}, \\ k_3 &= -k_1^3 - \beta\frac{k_1^2}{k_2} - \gamma k_2, \end{aligned} \tag{14}$$

$$\begin{aligned} k_1 &= \frac{a_1 a_2 + a_5 a_6}{a_2^2 + a_6^2} k_2 - \frac{3\left(a_1^2 + a_5^2\right)}{2\beta\left(a_2^2 + a_6^2\right)} k_2^3, \\ k_2 &= \frac{\delta_1}{3}\left[\frac{6\beta\left(a_1 a_2 + a_5 a_6\right)}{a_1^2 + a_5^2} + 6\sigma_2\beta\left(\frac{a_2^2 + a_6^2}{a_1^2 + a_5^2}\right)^{1/2}\right]^{1/2}, \end{aligned} \tag{15}$$

with $\sigma_i^2 = 1$ for $i = 1, 2$. This in turn leads to the mixed solution composed of one lump and one line solitary wave as

$$\begin{aligned} u = {} & \frac{6\left(2a_1 a_2 + 2a_5 a_6 + kk_1 k_2 e^{\eta}\right)}{\alpha\left(g^2 + h^2 + a_9 + ke^{\eta}\right)} \\ & - \frac{6\left(2a_1 g + 2a_5 h + kk_1 e^{\eta}\right)\left(2a_2 g + 2a_6 h + kk_2 e^{\eta}\right)}{\alpha\left(g^2 + h^2 + a_9 + ke^{\eta}\right)^2}, \end{aligned} \tag{16}$$

where

$$a_1 a_6 - a_2 a_5 \neq 0, 2_2 + a_6^2 \neq 0, -\frac{3\left(a_1^2 + a_5^2\right)\left(a_1 a_2 + a_5 a_6\right)}{\beta} > 0, k > 0. \tag{17}$$

Here, g, h, and η are defined by equation (12) with the parameters' relations (13), (14), and (15). The restricted conditions in equation (16) are to be able to form a lump wave and guarantee the regularity of the function f.

In the interaction processes between one lump and one line solitary wave, fission and fusion phenomena [55, 56] will appear under the different parameters. If we set x and y as constants, the structure of the mixed solution equation (16) can be explained as follows. When the coefficient of the time $k_3 > 0$, the exponential term is dominant and only the line solitary wave exists for $t > 0$, while the

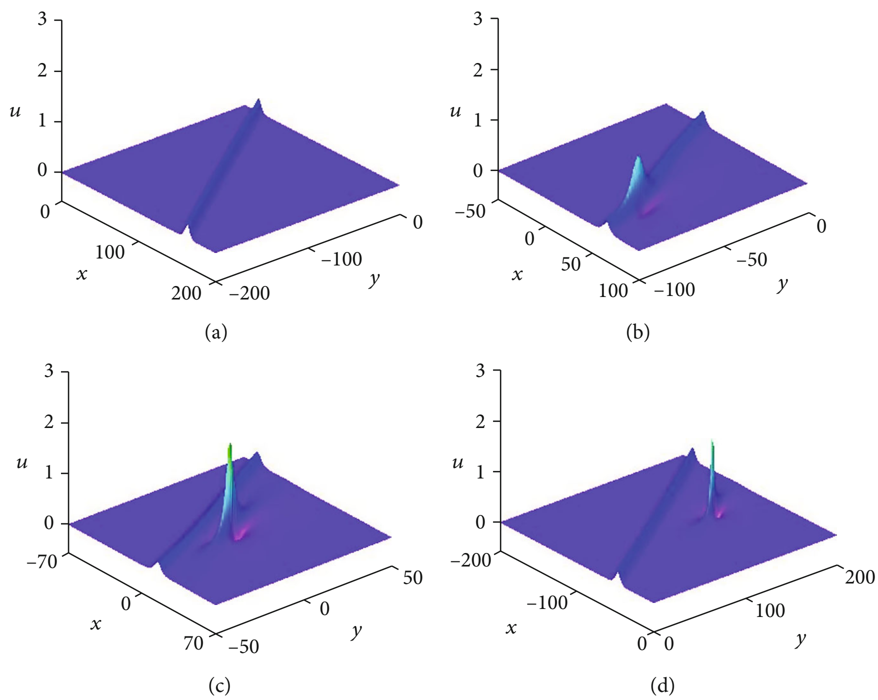


Figure 2: The three-dimensional plots of the mixed solution equation (16) with $a_1 = 1/20$, $a_5 = 2$, $k = 1/10$, $a_2 = a_6 = \alpha = -\beta = \gamma = -\sigma_1 = -\sigma_2 = 1$, and $a_4 = a_8 = 0$: (a) $t = -100$; (b) $t = -25$; (c) $t = 0$; (d) $t = 50$.

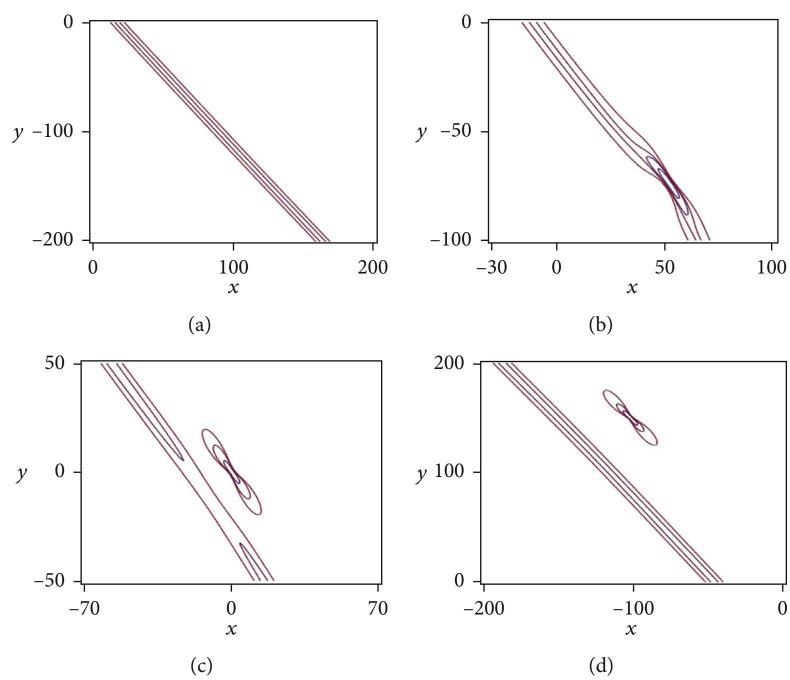


Figure 3: The corresponding contour plots of the mixed solution equation (16) with $a_1 = 1/20$, $a_5 = 2$, $k = 1/10$, $a_2 = a_6 = \alpha = -\beta = \gamma = -\sigma_1 = -\sigma_2 = 1$, and $a_4 = a_8 = 0$: (a) $t = -100$; (b) $t = -25$; (c) $t = 0$; (d) $t = 50$.

rational term is dominant and the rational lump emerges for $t < 0$. Thus, such interaction processes correspond to a fission phenomenon. On the contrary, the negative coefficient of the time k_3 gives rise to a fusion phenomenon. To illustrate this type of the mixed solution, we exhibit the fission phenomenon through the three-dimensional plots in Figure 2 and the corresponding contour plots in Figure 3. It can be seen clearly that only one line solitary wave exists firstly and then one rational lump arises gradually. Although the integrable system studied in this paper is not the same system as those in Refs. [55, 56], they have similar fusion and fission phenomena.

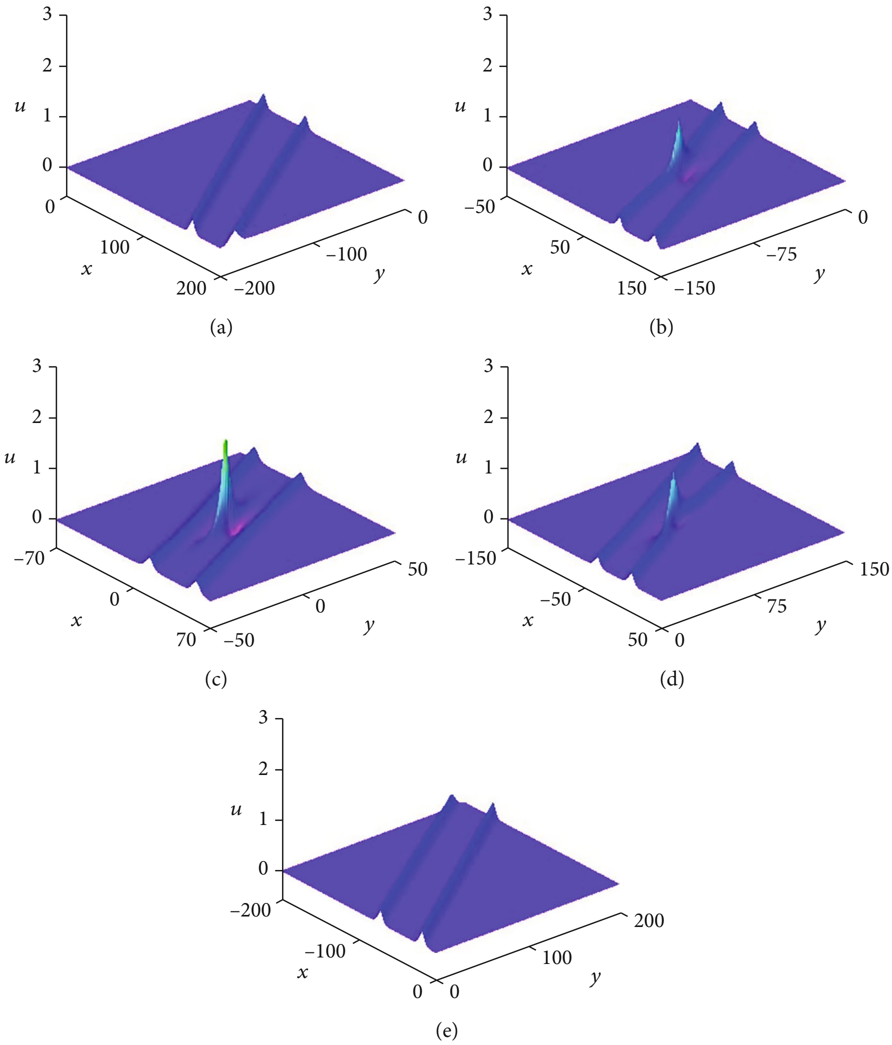


Figure 4: The three-dimensional plots of the mixed solution equation (23) with $a_1 = 1/20$, $a_5 = 2$, $k = l = 1/10$, $a_2 = a_6 = \alpha = -\beta = \gamma = -\sigma_1 = -\sigma_2 = 1$, and $a_4 = a_8 = 0$: (a) $t = -100$; (b) $t = -25$; (c) $t = 0$; (d) $t = 25$; (e) $t = 100$.

4. The Mixed Solution Composed of One Rational Lump and Two Line Solitary Waves

In this section, we seek to construct the mixed solution composed of one rational lump and two line solitary waves. This type of interaction solution will describe fission and fusion phenomena simultaneously. According to the last section, we need to assume the function f as the following form:

$$
\begin{aligned}
f &= g^2 + h^2 + a_9 + ke^{\eta} + le^{-\eta},\\
g &= a_1 x + a_2 y + a_3 t + a_4,\\
h &= a_5 x + a_6 y + a_7 t + a_8,\\
\eta &= k_1 x + k_2 y + k_3 t,
\end{aligned} \tag{18}
$$

where $a_i (i = 1, 2, \cdots, 9)$, $k, l,$ and $k_i (i = 1, 2, 3)$ are real parameters and will be determined. Here, the rational and the exponential terms support the rational lump and the line soliton pair, respectively. Proceeding as before, we have the parameters' relations as follows:

$$
\begin{aligned}
a_3 &= -\frac{\beta\left[a_2\left(a_1^2 - a_5^2\right) + 2a_1 a_5 a_6\right]}{a_2^2 + a_6^2} - \gamma a_2,\\
a_7 &= -\frac{\beta\left[a_6\left(a_5^2 - a_1^2\right) + 2a_1 a_2 a_5\right]}{a_2^2 + a_6^2} - \gamma a_6,
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
a_9 = &-\frac{3\left(a_1^2 + a_5^2\right)\left(a_2^2 + a_6^2\right)\left(a_1 a_2 + a_5 a_6\right)}{\beta\left(a_1 a_6 - a_2 a_5\right)^2}\\
&+ \frac{8\beta kl\left[\beta\left(a_1 a_6 - a_2 a_5\right)^2 + 3k_2^2\left(a_1^2 + a_5^2\right)\left(a_1 a_2 + a_5 a_6\right)\right]}{3\left(a_1 a_6 - a_2 a_5\right)^2\left[3k_2^2\left(a_1^2 + a_5^2\right) - 2\beta\left(a_1 a_2 + a_5 a_6\right)\right]},
\end{aligned} \tag{20}
$$

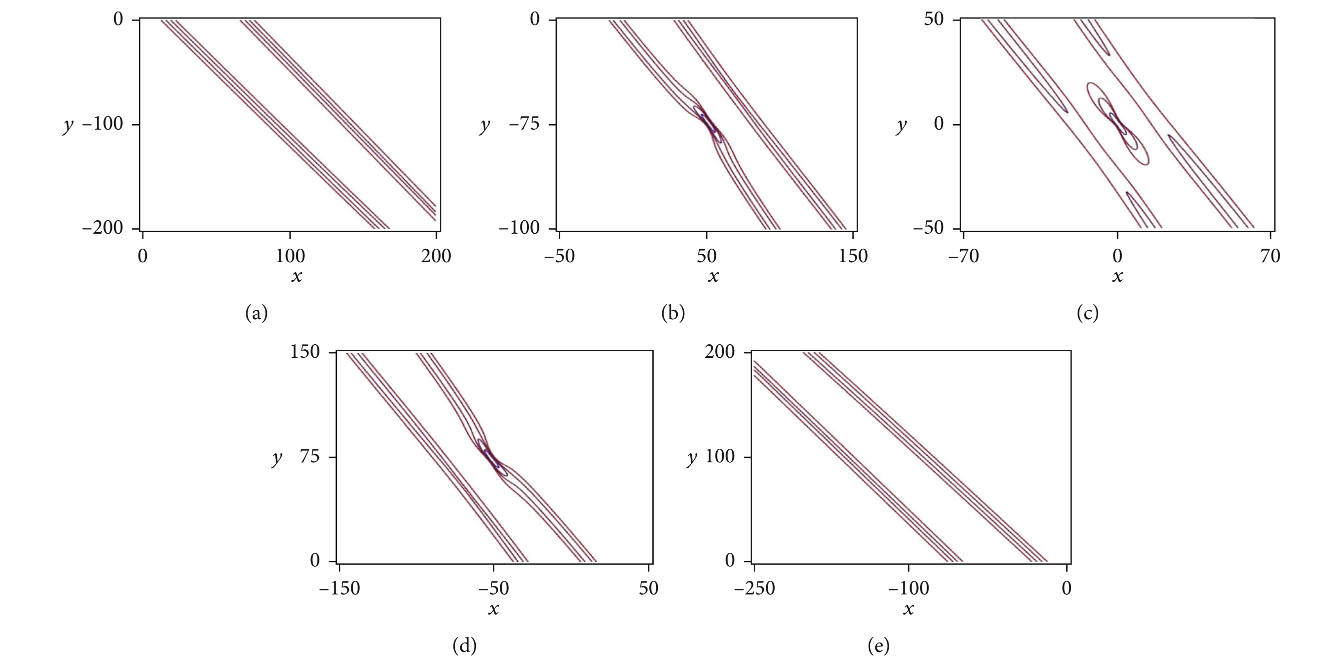


Figure 5: The corresponding contour plots of the mixed solution equation (23) with $a_1 = 1/20$, $a_5 = 2$, $k = l = 1/10$, $a_2 = a_6 = \alpha = -\beta = \gamma = -\sigma_1 = -\sigma_2 = 1$, and $a_4 = a_8 = 0$: (a) $t = -100$; (b) $t = -25$; (c) $t = 0$; (d) $t = 25$; (e) $t = 100$.

$$k_3 = -k_1^3 - \beta \frac{k_1^2}{k_2} - \gamma k_2,$$
$$k_1 = \frac{a_1 a_2 + a_5 a_6}{a_2^2 + a_6^2} k_2 - \frac{3\left(a_1^2 + a_5^2\right)}{2\beta\left(a_2^2 + a_6^2\right)} k_2^3, \tag{21}$$

$$k_2 = \frac{\delta_1}{3}\left[\frac{6\beta(a_1 a_2 + a_5 a_6)}{a_1^2 + a_5^2} + 6\sigma_2 \beta \left(\frac{a_2^2 + a_6^2}{a_1^2 + a_5^2}\right)^{1/2}\right]^{1/2}, \tag{22}$$

with $\sigma_i^2 = 1$ for $i = 1, 2$. This in turn gives the mixed solution composed of one lump and two line solitary waves as

$$u = \frac{6\left[2a_1 a_2 + 2a_5 a_6 + k_1 k_2 (k e^{\eta} + l e^{-\eta})\right]}{\alpha\left(g^2 + h^2 + a_9 + k e^{\eta} + l e^{-\eta}\right)} - \frac{6\left[2a_1 g + 2a_5 h + k_1 (k e^{\eta} - l e^{-\eta})\right]\left[2a_2 g + 2a_6 h + k_2 (k e^{\eta} - l e^{-\eta})\right]}{\alpha\left(g^2 + h^2 + a_9 + k e^{\eta} + l e^{-\eta}\right)^2}, \tag{23}$$

with

$$a_1 a_6 - a_2 a_5 \neq 0,\ a_2^2 + a_6^2 \neq 0, \quad a_9 > 0, k > 0, l > 0. \tag{24}$$

Here, g, h, and η are defined by equation (18) with the parameters' relations (19), (20), (21), and (22). In this interaction processes described by equation (23), both fission and fusion phenomena will occur under the certain parameters' values. Thus, it can be realized that the lump is only observed on a certain region or during a specific time period. More precisely, by setting x and y as the fixed constants in the mixed solution equation (18), one can give the simple analysis:

$$\lim_{t \longrightarrow \pm\infty} \frac{g^2}{h^2} = \frac{a_3^2}{a_7^2},$$
$$\lim_{t \longrightarrow \pm\infty} \frac{g^2}{k e^{\eta} + l e^{-\eta}} = 0, \tag{25}$$
$$\lim_{t \longrightarrow \pm\infty} \frac{h^2}{k e^{\eta} + l e^{-\eta}} = 0,$$
$$(k, l > 0).$$

It implies that only two line solitary waves exist when the time approaches to infinity, and the lump emerges and reaches its maximum amplitude when the time approaches to zero. Hence, the evolution of the lump coincides with the characters of rogue wave: short-lived occurrence and large amplitude. The rational lump is identified as a two-dimensional rogue wave originating in the line soliton pair. The three-dimensional plots and corresponding contour plots for this type of the mixed solution at different times are shown in Figures 4 and 5, respectively. It can be observed that in the evolution process the lump acts as a rogue wave but the line soliton pair remains the same shape. The whole interaction means that a two-dimensional rogue wave is excited from two paralleled line solitary waves.

5. Conclusions

In this paper, we have constructed rational lump and mixed lump-solitary wave solutions of the extended (2+1)-dimensional KdV equation by using the bilinear method. Under the appropriate variable transformation, the extended (2+1)-dimensional KdV equation is firstly changed into the trilinear form. Then, three groups of exact solutions are derived by assuming the auxiliary function as the quadratic and exponential functions. The first kind of solution is given by the purely rational form, it possesses one maximum and two minima, and its peak decays algebraically in all directions in the space plane. Figure 1 shows these characteristics of a lump wave intuitively and clearly. The second kind of solution is expressed by the mixed rational-exponential function, which exhibits fission and fusion phenomena between one lump and one line solitary wave. Equation (16) gives specific mathematical expressions for the second type of solution, and Figures 2 and 3 elaborate on these interesting fission and fusion phenomena. The last one contains one lump and two line solitary waves; these local waves' interaction shown in Figure 5 is able to describe a two-dimensional rogue wave excited from the line soliton pair. Because the extended (2+1)-dimensional KdV equation describes the ion-acoustic waves in plasmas, shallow water waves in oceans, and pulse waves in large arteries, we believe that there are fission and fusion phenomena in corresponding physical models.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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