Research Article

Application of the Multiple Exp-Function, Cross-Kink, Periodic-Kink, Solitary Wave Methods, and Stability Analysis for the CDG Equation

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In this article, the exact wave structures are discussed to the Caudrey-Dodd-Gibbon equation with the assistance of Maple based on the Hirota bilinear form. It is investigated that the equation exhibits the trigonometric, hyperbolic, and exponential function solutions. We first construct a combination of the general exponential function, periodic function, and hyperbolic function in order to derive the general periodic-kink solution for this equation. Then, the more periodic wave solutions are presented with more arbitrary autocephalous parameters, in which the periodic-kink solution localized in all directions in space. Furthermore, the modulation instability is employed to discuss the stability of the available solutions, and the special theorem is also introduced. Moreover, the constraint conditions are also reported which validate the existence of solutions. Furthermore, 2-dimensional graphs are presented for the physical movement of the earned solutions under the appropriate selection of the parameters for stability analysis. The concluded results are helpful for the understanding of the investigation of nonlinear waves and are also vital for numerical and experimental verification in engineering sciences and nonlinear physics.

1. Introduction

It is known that these exact solutions of nonlinear evolution equations (NLEEs), especially the soliton solutions [1–3], can be given by using a variety of different methods [4, 5], such as Jacobi elliptic function expansion method [6], inverse scattering transformation (IST) [7, 8], Darboux transformation (DT) [9], extended generalized DT [10], Lax pair (LP) [11], Lie symmetry analysis [12], Hirota bilinear method [13], and others [14, 15]. The Hirota bilinear method is an efficient tool to construct exact solutions of NLEEs, and there exists plenty of completely integrable equations which are studied in this way. For instance, generalized bilinear equations [16], the lump-type solutions in a homogenous-dispersive medium [17], the (2 + 1)-dimensional KdV equation [18], the (3 + 1)-D potential-YTSF equation [19], the generalized

BKP equation [20], the (3 + 1)-dimensional BKP-Boussinesq equation [21], the (3 + 1) dimensional generalized KP-Boussinesq equation [22], the (2 + 1)-dimensional integrable Boussinesq model [23], and the (2 + 1)-dimensional Breaking Soliton equation [24, 25]. More recently, lump waves and rogue waves have attracted a growing amount of attention, and many theoretical and experimental studies of lump waves are mentioned [26–33]. A novel method for finding the special rogue waves with predictability of NLEEs is proposed by using the Hirota bilinear method by powerful researchers in Refs. [34, 35], in which some results are very helpful for us to study some physical phenomena in engineering.

The Caudrey-Dodd-Gibbon equation introduced by Aiyer et al. [36] who describes the inelastic interactions between the solitary waves under strong physical contexts

in certain integrable or nonintegrable systems and has been investigated the related dynamic behavior [37], which reads

$$\Phi_t + \Phi_{xxxxx} + 30\Phi\Phi_{xxx} + 30\Phi_x\Phi_{xx} + 180\Phi^2\Phi_x = 0. \quad (1)$$

In 2006, the tanh solutions of the equation [38] and, in 2008, the multiple-soliton solutions utilizing the Hirota bilinear method combined with the simplified Hereman method [39] for the above equation are derived by Wazwaz. Also, the physical comprehension of Equation (1) was demonstrated by plenty of scholars who investigated its solitary type solutions and given in Refs. [40] and [41]. The homotopy perturbation method has been utilized to find solutions for the aforementioned equation [42–44]. Based on the obtained transformation of integrating Equation (1), we get to the following nonlinear PDE [45]:

$$\Lambda_t + 60\Lambda_x^3 + 30\Lambda_x\Lambda_{xxx} + \Lambda_{xxxxx} = 0. \quad (2)$$

According to [46], the Hirota bilinear form of the CDG equation reads

$$\Lambda = 2(\ln \Gamma)_x, \quad (\Phi = \Lambda_x) \quad (3)$$

and, also by applying the dependent variable transformation, turns into the Hirota bilinear form

$$(D_{xt} + D_x^6)\Gamma \cdot \Gamma = 0, \quad (4)$$

where D is a bilinear operator. By deeming the D -operator defined with the aid of the functions Γ_1 and Γ_2 , we get to the following relation:

$$D_x^{\pi_1} D_t^{\pi_2} (\Gamma_1 \cdot \Gamma_2) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)^{\pi_1} \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right)^{\pi_2} \Gamma_1(x, t) \cdot \Gamma_2(x', t') \Big|_{x=x', t=t'}. \quad (5)$$

With the help of the transformation Equation (3), the general periodic-kink solutions of Equation (1) can be given. We get to the bilinear form the of Γ as

$$\Gamma\Gamma_{xt} - \Gamma_x\Gamma_t + \Gamma\Gamma_{xxxxx} - 6\Gamma_x\Gamma_{xxxx} + 15\Gamma_{xx}\Gamma_{xxx} - 10\Gamma_{xxx}^2 = 0. \quad (6)$$

Moreover, the stability analysis and the more general periodic-wave solutions and special rogue waves with predictability are investigated in our paper, which have never been studied. Various types of studies were investigated by capable authors in which some of them can be mentioned, for example, the Caudrey–Dodd–Gibbon equation [47], the pZK equation using Lie point symmetries [48], group-invariant solutions of the (3 + 1)-dimensional generalized KP equation [49], optimal system and dynamics of solitons for a higher-dimensional Fokas equation [50], dynamics of solitons for (2 + 1)-dimensional NNV equations [51], the combined MCBS-nMCBS equation [52], Lie symmetry reductions for (2 + 1)-dimensional Pavlov equation [53], Schrödinger-Hirota equation with variable coefficients [54],

the (2 + 1)-dimension paraxial wave equation [55], the fractional Drinfeld-Sokolov-Wilson equation [56], the (3 + 1)-dimensional extended Jimbo-Miwa equations [57], and a high-order partial differential equation with fractional derivatives [58]. In the valuable work, the capable authors studied the periodic wave solutions and stability analysis for the KP-BBM equation [59] and breather and periodic wave solution for generalized Bogoyavlensky-Konopelchenko equation [60] with the aid of Hirota operator.

To make this paper more self-contained, a combination of general exponential function, periodic function, and hyperbolic function of the (3 + 1)-dimensional CDG equation is constructed with the help of a bilinear operator, which is crucial to obtain the periodic-wave solution of Equation (1). Based on the Hirota bilinear form Equation (6), the general periodic-wave solution is derived in Section 2 and the novel periodic solutions which can be arisen with twenty one classes. In Section 4, we shall investigate the stability analysis to obtain the modulation stability spectrum of this equation. The final section will be reserved for the conclusions and the discussions.

2. Multiple Exp-Function Method

In this section, according to [61–63] so that it can be further employed to the nonlinear partial differential equation (NLPDE) in order to furnish its exact solutions, it can be presented as:

Step 1. The following is the NLPDE:

$$\mathcal{P}(x, y, t, \Psi, \Psi_x, \Psi_y, \Psi_z, \Psi_t, \Psi_{xx}, \Psi_{tt}, \dots) = 0. \quad (7)$$

We commence a sequence of new variables $\xi_i = \xi_i(x, t)$, $1 \leq i \leq n$, by solvable PDEs, for example, the linear ones,

$$\xi_{i,x} = \alpha_i \xi_i, \quad \xi_{i,t} = \delta_i \xi_i, \quad 1 \leq i \leq n, \quad (8)$$

where α_i , $1 \leq i \leq n$, is the angular wave number and δ_i , $1 \leq i \leq n$, is the wave frequency. It should be pointed that this is frequently the initiating step for constructing the exact solutions to NLPDEs, and moreover, solving such linear equations redounds to the exponential function solutions:

$$\xi_i = \omega_i e^{\theta_i}, \quad \theta_i = \alpha_i x - \delta_i t, \quad 1 \leq i \leq n, \quad (9)$$

in which ω_i , $1 \leq i \leq n$, are undetermined constants.

Step 2. Determine the solution of Equation (7) as the following form in terms of the new variables ξ_i , $1 \leq i \leq n$:

$$\Psi(x, t) = \frac{\Delta(\xi_1, \xi_2, \dots, \xi_n)}{\Omega(\xi_1, \xi_2, \dots, \xi_n)}, \quad \Delta = \sum_{r,s=1}^n \sum_{i,j=0}^M \Delta_{rs,ij} \xi_r^i \xi_s^j, \quad \Omega = \sum_{r,s=1}^n \sum_{i,j=0}^N \Omega_{rs,ij} \xi_r^i \xi_s^j, \quad (10)$$

in which $\Delta_{rs,ij}$ and $\Omega_{rs,ij}$ are the amounts to be settled. Appending Equation (10) into Equation (7) and ordering the numerator of the rational function to zero, we can achieve

a series of nonlinear algebraic equations about the variables $\alpha_i, \delta_i, \Delta_{rs,ij}$ and $\Omega_{rs,ij}$. Solving the solutions for these nonlinear algebraic equations and putting these solutions into Equation (10), the multiple soliton solutions to Equation (7) can be obtained in the below form as

$$\Psi(x, t) = \frac{\Delta(\omega_1 e^{\alpha_1 x - \delta_1 t}, \dots, \omega_n e^{\alpha_n x - \delta_n t})}{\Omega(\omega_1 e^{\alpha_1 x - \delta_1 t}, \dots, \omega_n e^{\alpha_n x - \delta_n t})}, \quad (11)$$

in which $\Omega \neq 0$, and also, we have

$$\begin{aligned} \Delta_t &= \sum_{i=1}^n \Delta_{\xi_i} \xi_{i,t}, \Omega_t = \sum_{i=1}^n \Omega_{\xi_i} \xi_{i,t}, \Delta_x = \sum_{i=1}^n \Delta_{\xi_i} \xi_{i,x}, \Omega_x = \sum_{i=1}^n \Omega_{\xi_i} \xi_{i,x}, \\ \Psi_t &= \frac{\Omega \sum_{i=1}^n \Delta_{\xi_i} \xi_{i,t} - \Delta \sum_{i=1}^n \Omega_{\xi_i} \xi_{i,t}}{\Omega^2}, \Psi_x = \frac{\Omega \sum_{i=1}^n \Delta_{\xi_i} \xi_{i,x} - \Delta \sum_{i=1}^n \Omega_{\xi_i} \xi_{i,x}}{\Omega^2}. \end{aligned} \quad (12)$$

3. Multiple Soliton Solutions for the CDG Equation

3.1. Set I: One-Wave Solution. We start up with one-wave function based on the explanation in Step 2 in the previous section, we deem that Equation (1) has the below form of one-wave solution as

$$\Psi(x, t) = \frac{\eta_1}{\eta_2}, \quad \eta_1 = \sigma_1 + \sigma_2 e^{\alpha_1 x - \delta_1 t}, \eta_2 = 1 + \rho_1 + \rho_2 e^{\alpha_1 x - \delta_1 t}, \quad (13)$$

in which ρ_1, ρ_2, σ_1 , and σ_2 are the unfound constants. Plugging (13) into Equation (1), we get to the following cases:

Case 1.

$$\begin{aligned} \alpha_1 &= \alpha_1, \beta_1 = \beta_1, \rho_1 = \frac{\rho_2 \sigma_1 - \sigma_2}{\sigma_2}, \rho_2 = \rho_2, \\ \sigma_1 &= \sigma_1, \sigma_2 = \sigma_2, \delta_1 = \alpha_1^5. \end{aligned} \quad (14)$$

Case 2.

$$\begin{aligned} \alpha_1 &= \alpha_1, \beta_1 = \beta_1, \rho_1 = -1, \rho_2 = \rho_2, \\ \sigma_1 &= \sigma_1, \sigma_2 = \sigma_2, \delta_1 = \alpha_1^5. \end{aligned} \quad (15)$$

Case 3.

$$\begin{aligned} \alpha_1 &= \alpha_1, \beta_1 = \beta_1, \rho_1 = \rho_1, \rho_2 = \frac{\sigma_2(1 + \rho_1)}{\sigma_1}, \\ \sigma_1 &= \sigma_1, \sigma_2 = \sigma_2, \delta_1 = \delta_1. \end{aligned} \quad (16)$$

For example, the resulting one-wave solution for Cases 1 to 3 will be read, respectively, as

$$\begin{aligned} \Psi_1(x, t) &= \frac{\sigma_1 + \sigma_2 e^{-t\alpha_1^5 + x\alpha_1}}{1 + ((\rho_2 \sigma_1 - \sigma_2)/\sigma_2) + \rho_2 e^{-t\alpha_1^5 + x\alpha_1}}, \\ \Psi_2(x, t) &= \frac{\sigma_1 + \sigma_2 e^{-t\alpha_1^5 + x\alpha_1}}{\rho_2 e^{-t\alpha_1^5 + x\alpha_1}}, \\ \Psi_3(x, t) &= \frac{\sigma_1 + \sigma_2 e^{-t\delta_1 + x\alpha_1}}{1 + \rho_1 + ((\sigma_2(1 + \rho_1)e^{-t\delta_1 + x\alpha_1})/\sigma_1)}. \end{aligned} \quad (17)$$

3.2. Set II: Two-Wave Solutions. We start up with two-wave functions based on the explanations in Step 2 in the previous section; we deem that Equation (1) has the bellow form of two-wave solutions as

$$\Psi(x, t) = \frac{\eta_1}{\eta_2}, \quad (18)$$

$$\eta_1 = \rho_0 + \rho_1 e^{\alpha_1 x - \delta_1 t} + \rho_2 e^{\alpha_2 x - \delta_2 t} + \rho_1 \rho_2 \rho_{12} e^{(\alpha_1 + \alpha_2)x - (\delta_1 + \delta_2)t}, \quad (19)$$

$$\eta_2 = 1 + \sigma_1 e^{\alpha_1 x - \delta_1 t} + \sigma_2 e^{\alpha_2 x - \delta_2 t} + \sigma_1 \sigma_2 \sigma_{12} e^{(\alpha_1 + \alpha_2)x - (\delta_1 + \delta_2)t}. \quad (20)$$

Plugging (18) along with (19) into Equation (1), we gain the following cases:

Case 1.

$$\alpha_1 = 0, \alpha_2 = \alpha_2, \delta_1 = \delta_1, \delta_2 = \delta_1, \rho_0 = \frac{\rho_2}{\sigma_2}, \quad (21)$$

$$\rho_1 = 0, \rho_2 = \rho_2, \rho_{12} = \rho_{12}, \sigma_1 = \sigma_1, \sigma_2 = \sigma_2, \sigma_{12} = 1.$$

Case 2.

$$\begin{aligned} \alpha_1 &= 0, \alpha_2 = \alpha_2, \delta_1 = \delta_1, \delta_2 = \delta_2, \rho_0 = \frac{1}{\rho_{12}}, \\ \rho_1 &= \rho_1, \rho_2 = \frac{\sigma_2}{\rho_{12}}, \rho_{12} = \rho_{12}, \sigma_1 = 0, \sigma_2 = \sigma_2, \sigma_{12} = \sigma_{12}. \end{aligned} \quad (22)$$

Case 3.

$$\begin{aligned} \alpha_1 &= 0, \alpha_2 = \alpha_2, \delta_1 = \delta_1, \delta_2 = \alpha_2^5, \rho_0 = \frac{1}{\rho_{12}}, \\ \rho_1 &= \rho_1, \rho_2 = \rho_2, \rho_{12} = \rho_{12}, \sigma_1 = \sigma_1, \sigma_2 = 0, \sigma_{12} = \sigma_{12}. \end{aligned} \quad (23)$$

Case 4.

$$\begin{aligned} \alpha_1 &= 0, \alpha_2 = \alpha_2, \delta_1 = \delta_1, \delta_2 = \delta_2, \rho_0 = \frac{\rho_2}{\sigma_2}, \\ \rho_1 &= \rho_1, \rho_2 = \rho_2, \rho_{12} = \frac{\sigma_2}{\rho_2}, \sigma_1 = \sigma_1, \sigma_2 = \sigma_2, \sigma_{12} = 1. \end{aligned} \quad (24)$$

Case 5.

$$\begin{aligned}\alpha_1 &= \alpha_1, \alpha_2 = \alpha_2, \delta_1 = \alpha_1^5, \delta_2 = \alpha_2^5, \\ \rho_0 &= \frac{\alpha_1^4 - 3\alpha_1^3\alpha_2 + 4\alpha_1^2\alpha_2^2 - 3\alpha_1\alpha_2^3 + \alpha_2^4}{\rho_{12}(\alpha_1^2 + \alpha_1\alpha_2 + \alpha_2^2)(\alpha_1 + \alpha_2)^2}, \\ \rho_1 &= \rho_1, \rho_2 = \rho_2, \rho_{12} = \rho_{12}, \sigma_1 = 0, \sigma_2 = 0, \sigma_{12} = \sigma_{12}.\end{aligned}\quad (25)$$

Case 6.

$$\begin{aligned}\alpha_1 &= \alpha_1, \alpha_2 = \alpha_2, \delta_1 = \alpha_1^5, \delta_2 = \delta_2, \rho_0 = \frac{1}{\rho_{12}}, \\ \rho_1 &= \rho_1, \rho_2 = \frac{\sigma_2}{\rho_{12}}, \rho_{12} = \rho_{12}, \\ \sigma_1 &= 0, \sigma_2 = \sigma_2, \sigma_{12} = \sigma_{12}.\end{aligned}\quad (26)$$

Case 7.

$$\begin{aligned}\alpha_1 &= \alpha_1, \alpha_2 = \alpha_2, \delta_1 = \delta_1, \delta_2 = \delta_2, \rho_0 = \frac{\rho_2}{\sigma_2}, \\ \rho_1 &= \frac{\rho_2\sigma_1}{\sigma_2}, \rho_2 = \rho_2, \rho_{12} = \frac{\sigma_2\sigma_{12}}{\rho_2}, \sigma_1 = 0, \sigma_2 = \sigma_2, \sigma_{12} = \sigma_{12}.\end{aligned}\quad (27)$$

For example, the resulting two-wave solution for Cases 1–7 will be read, respectively, as

$$\begin{aligned}\Psi_1(x, t) &= \frac{(\rho_2/\sigma_2) + \rho_2 e^{-t\delta_2 + x\alpha_2}}{1 + \sigma_1 e^{-t\delta_1} + \sigma_2 e^{-t\delta_2 + x\alpha_2} + \sigma_1 \sigma_2 e^{-t\delta_1 - t\delta_2 + x\alpha_2}}, \\ \Psi_2(x, t) &= \frac{\rho_{12}^{-1} + \rho_1 e^{-t\delta_1} + ((\sigma_2 e^{-t\delta_2 + x\alpha_2})/\rho_{12}) + \rho_1 \sigma_2 e^{-t\delta_1 - t\delta_2 + x\alpha_2}}{1 + \sigma_2 e^{-t\delta_2 + x\alpha_2}}, \\ \Psi_3(x, t) &= \frac{\rho_{12}^{-1} + \rho_1 e^{-t\delta_1} + \rho_2 e^{-t\delta_2 + x\alpha_2} + \rho_1 \rho_2 \rho_{12} e^{-t\delta_1 - t\delta_2 + x\alpha_2}}{1 + \sigma_1 e^{-t\delta_1}}, \\ \Psi_4(x, t) &= \frac{(\rho_2/\sigma_2) + \rho_1 e^{-t\delta_1} + \rho_2 e^{-t\delta_2 + x\alpha_2} + \rho_1 \sigma_2 e^{-t\delta_1 - t\delta_2 + x\alpha_2}}{1 + \sigma_1 e^{-t\delta_1} + \sigma_2 e^{-t\delta_2 + x\alpha_2} + \sigma_1 \sigma_2 e^{-t\delta_1 - t\delta_2 + x\alpha_2}}, \\ \Psi_5(x, t) &= \frac{\alpha_1^4 - 3\alpha_1^3\alpha_2 + 4\alpha_1^2\alpha_2^2 - 3\alpha_1\alpha_2^3 + \alpha_2^4}{\rho_{12}(\alpha_1^2 + \alpha_1\alpha_2 + \alpha_2^2)(\alpha_1 + \alpha_2)^2} \\ &\quad + \rho_1 e^{-t\alpha_1^5 + x\alpha_1} + \rho_2 e^{-t\alpha_2^5 + x\alpha_2} + \rho_1 \rho_2 \rho_{12} e^{-t\alpha_1^5 - t\alpha_2^5 + x\alpha_1 + x\alpha_2}, \\ \Psi_6(x, t) &= \frac{\rho_{12}^{-1} + \rho_1 e^{-t\alpha_1^5 + x\alpha_1} + ((\sigma_2 e^{-t\delta_2 + x\alpha_2})/\rho_{12}) + \rho_1 \sigma_2 e^{-t\alpha_1^5 - t\delta_2 + x\alpha_1 + x\alpha_2}}{1 + \sigma_2 e^{-t\delta_2 + x\alpha_2}}, \\ \Psi_7(x, t) &= \frac{(\rho_2/\sigma_2) + ((\rho_2 \sigma_1 e^{-t\delta_1 + x\alpha_1})/\sigma_2) + \rho_2 e^{-t\delta_2 + x\alpha_2} + \rho_2 \sigma_1 \sigma_{12} e^{-t\delta_1 - t\delta_2 + x\alpha_1 + x\alpha_2}}{1 + \sigma_1 e^{-t\delta_1 + x\alpha_1} + \sigma_2 e^{-t\delta_2 + x\alpha_2} + \sigma_1 \sigma_2 \sigma_{12} e^{-t\delta_1 - t\delta_2 + x\alpha_1 + x\alpha_2}}.\end{aligned}\quad (28)$$

3.3. Set III: Three-Wave Solutions. We start up with three-wave functions according to the given explanations in Step 2 in the previous section, we deem that Equation (1) has the below form of three-wave solutions as

$$\Psi(x, t) = \frac{df(\eta_1)/dx}{\eta_1}, \quad (29)$$

$$\begin{aligned}\eta_1 &= 1 + \rho_1 e^{\Lambda_1} + \rho_2 e^{\Lambda_2} + \rho_3 e^{\Lambda_3} \\ &\quad + \rho_1 \rho_2 \rho_{12} e^{\Lambda_1 + \Lambda_2} \\ &\quad + \rho_1 \rho_3 \rho_{13} e^{\Lambda_1 + \Lambda_3} + \rho_2 \rho_3 \rho_{23} e^{\Lambda_2 + \Lambda_3} \\ &\quad + \rho_1 \rho_2 \rho_3 \rho_{12} \rho_{13} \rho_{23} e^{\Lambda_1 + \Lambda_2 + \Lambda_3},\end{aligned}\quad (30)$$

in which $\Lambda_i = \alpha_i x - \delta_i t$, $i = 1, 2, 3$. Appending (29) along with (30) into Equation (1), we obtain the following case:

$$\begin{aligned}\alpha_i &= \alpha_i, \delta_i = \alpha_i^5, \quad i = 1, 2, 3, \\ \rho_{ij} &= \rho_{ij}, \quad i, j = 1, 2, 3, i \neq j.\end{aligned}\quad (31)$$

Therefore, three-wave solution will be as

$$\begin{aligned}\Psi_1(x, t) &= \rho_1 \alpha_1 e^{-t\alpha_1^5 + x\alpha_1} + \rho_2 \alpha_2 e^{-t\alpha_2^5 + x\alpha_2} + \rho_3 \alpha_3 e^{-t\alpha_3^5 + x\alpha_3} \\ &\quad + \rho_1 \rho_2 \rho_{12} (\alpha_1 + \alpha_2) e^{-t\alpha_1^5 - t\alpha_2^5 + x\alpha_1 + x\alpha_2} \\ &\quad + \rho_1 \rho_3 \rho_{13} (\alpha_1 + \alpha_3) e^{-t\alpha_1^5 - t\alpha_3^5 + x\alpha_1 + x\alpha_3} \\ &\quad + \rho_2 \rho_3 \rho_{23} (\alpha_2 + \alpha_3) e^{-t\alpha_2^5 - t\alpha_3^5 + x\alpha_2 + x\alpha_3} \\ &\quad + \rho_1 \rho_2 \rho_3 \rho_{12} \rho_{13} \rho_{23} (\alpha_1 + \alpha_2 + \alpha_3) e^{-t\alpha_1^5 - t\alpha_2^5 - t\alpha_3^5 + x\alpha_1 + x\alpha_2 + x\alpha_3} \\ &\quad \cdot \left(1 + \rho_1 e^{-t\alpha_1^5 + x\alpha_1} + \rho_2 e^{-t\alpha_2^5 + x\alpha_2} + \rho_3 e^{-t\alpha_3^5 + x\alpha_3} \right. \\ &\quad + \rho_1 \rho_2 \rho_{12} e^{-t\alpha_1^5 - t\alpha_2^5 + x\alpha_1 + x\alpha_2} + \rho_1 \rho_3 \rho_{13} e^{-t\alpha_1^5 - t\alpha_3^5 + x\alpha_1 + x\alpha_3} \\ &\quad + \rho_2 \rho_3 \rho_{23} e^{-t\alpha_2^5 - t\alpha_3^5 + x\alpha_2 + x\alpha_3} \\ &\quad \left. + \rho_1 \rho_2 \rho_3 \rho_{12} \rho_{13} \rho_{23} e^{-t\alpha_1^5 - t\alpha_2^5 - t\alpha_3^5 + x\alpha_1 + x\alpha_2 + x\alpha_3} \right).\end{aligned}\quad (32)$$

3.4. Cross-Kink Solutions. Here, we will consider the cross-kink wave solution with selecting the below function which for Equation (1) has been taken as

$$\begin{aligned}f &= \exp(\tau_1) + \theta_{10} \exp(-\tau_1) + \sinh(\tau_2) \\ &\quad + \sin(\tau_3) + \theta_{11}, \tau_1 = \theta_1 x + \theta_2 t \\ &\quad + \theta_3, \tau_2 = \theta_4 x + \theta_5 t \\ &\quad + \theta_6, \tau_3 = \theta_7 x + \theta_8 t + \theta_9,\end{aligned}\quad (33)$$

$$\Psi(x, t) = \ln(f)_{xx}, \quad (34)$$

where $\theta_i, i = 1, \dots, 11$, are undetermined amounts which should be detected. Appending (34) into Equation (1) and afterwards collecting the coefficients, we obtain the following consequences:

Case 1.

$$\begin{aligned}\theta_1 &= \theta_4 = i\theta_7, \theta_2 = \theta_5 = -16i\theta_7^5, \\ \theta_8 &= -16\theta_7^5, \theta_{11} = 0, i^2 = -1.\end{aligned}\quad (35)$$

Substituting (35) into (33) and (34), we achieve a cross-kink wave solution of Equation (1) as follows:

$$f = e^{-16i\theta_7^5 t + i\theta_7 x + \theta_3} + \theta_{10} e^{16i\theta_7^5 t - i\theta_7 x - \theta_3} - \sinh(16i\theta_7^5 t - i\theta_7 x - \theta_6) - \sin(16\theta_7^5 t - \theta_7 x - \theta_9),$$

$$\Psi_1 = \frac{-\theta_7^2 e^{-16i\theta_7^5 t + i\theta_7 x + \theta_3} - \theta_{10} \theta_7^2 e^{16i\theta_7^5 t - i\theta_7 x - \theta_3} + \sinh(16i\theta_7^5 t - i\theta_7 x - \theta_6) \theta_7^2 + \sin(16\theta_7^5 t - \theta_7 x - \theta_9) \theta_7^2}{e^{-16i\theta_7^5 t + i\theta_7 x + \theta_3} + \theta_{10} e^{16i\theta_7^5 t - i\theta_7 x - \theta_3} - \sinh(16i\theta_7^5 t - i\theta_7 x - \theta_6) - \sin(16\theta_7^5 t - \theta_7 x - \theta_9)}$$

$$- \frac{\left(i\theta_7 e^{-16i\theta_7^5 t + i\theta_7 x + \theta_3} - i\theta_{10} \theta_7 e^{16i\theta_7^5 t - i\theta_7 x - \theta_3} + i \cosh(16i\theta_7^5 t - i\theta_7 x - \theta_6) \theta_7 + \cos(16\theta_7^5 t - \theta_7 x - \theta_9) \theta_7\right)^2}{\left(e^{-16i\theta_7^5 t + i\theta_7 x + \theta_3} + \theta_{10} e^{16i\theta_7^5 t - i\theta_7 x - \theta_3} - \sinh(16i\theta_7^5 t - i\theta_7 x - \theta_6) - \sin(16\theta_7^5 t - \theta_7 x - \theta_9)\right)^2}. \quad (36)$$

Case 2.

$$\theta_1 = \sqrt{3}\theta_7 = i\sqrt{3}\theta_4, \theta_2 = -\sqrt{3}\theta_8 = 16\sqrt{3}i\theta_4^5, \quad (37)$$

$$\theta_5 = -16\theta_4^5, \theta_{10} = 0, \theta_{11} = 0, i^2 = -1.$$

Substituting (37) into (33) and (34), we achieve a cross-kink wave solution of Equation (1) which can be written as follows:

$$f = e^{16\sqrt{3}\theta_4^5 it + \sqrt{3}i\theta_4 x + \theta_3} - \sinh(16t\theta_4^5 - x\theta_4 - \theta_6) - \sin(16\theta_4^5 it - i\theta_4 x - \theta_9),$$

$$\Psi_1 = \frac{3i^2\theta_4^2 e^{16\sqrt{3}\theta_4^5 it + \sqrt{3}i\theta_4 x + \theta_3} - \sinh(16t\theta_4^5 - x\theta_4 - \theta_6) \theta_4^2 + \sin(16\theta_4^5 it - i\theta_4 x - \theta_9) i^2 \theta_4^2}{e^{16\sqrt{3}\theta_4^5 it + \sqrt{3}i\theta_4 x + \theta_3} - \sinh(16t\theta_4^5 - x\theta_4 - \theta_6) - \sin(16\theta_4^5 it - i\theta_4 x - \theta_9)}$$

$$- \frac{\left(\sqrt{3}i\theta_4 e^{16\sqrt{3}\theta_4^5 it + \sqrt{3}i\theta_4 x + \theta_3} + \cosh(16t\theta_4^5 - x\theta_4 - \theta_6) \theta_4 + \cos(16\theta_4^5 it - i\theta_4 x - \theta_9) i\theta_4\right)^2}{\left(e^{16\sqrt{3}\theta_4^5 it + \sqrt{3}i\theta_4 x + \theta_3} - \sinh(16t\theta_4^5 - x\theta_4 - \theta_6) - \sin(16\theta_4^5 it - i\theta_4 x - \theta_9)\right)^2}. \quad (38)$$

3.5. Periodic-Kink Wave Solutions. Here, we will consider the periodic-kink wave solution with selecting the below function which for Equation (1) has been taken as

$$f = \exp(\tau_1) + \theta_{10} \exp(-\tau_1) + \cosh(\tau_2) + \cos(\tau_3)$$

$$+ \theta_{11}, \tau_1 = \theta_1 x + \theta_2 t + \theta_3, \tau_2 = \theta_4 x + \theta_5 t \quad (39)$$

$$+ \theta_6, \tau_3 = \theta_7 x + \theta_8 t + \theta_9,$$

$$\Psi(x, t) = \ln(f)_{xx}, \quad (40)$$

where $\theta_i, i = 1, \dots, 11$, are undetermined amounts which should be detected. Appending (40) into Equation (1) and

afterwards collecting the coefficients, we obtain the following consequences:

Case 1.

$$\theta_1 = \theta_4 = i\theta_7, \theta_2 = \theta_5 = -16i\theta_7^5, \theta_8 = -16\theta_7^5, \theta_{11} = 0, i^2 = -1. \quad (41)$$

Substituting (41) into (39) and (40), we achieve a periodic-kink wave solution of Equation (1) which can be written as follows:

$$f = e^{-16i\theta_7^5 t + i\theta_7 x + \theta_3} + \theta_{10} e^{16i\theta_7^5 t - i\theta_7 x - \theta_3} + \cosh(-16i\theta_7^5 t + i\theta_7 x + \theta_6) + \cos(-16\theta_7^5 t + \theta_7 x + \theta_9),$$

$$\Psi_2 = \frac{i^2\theta_7^2 e^{-16i\theta_7^5 t + i\theta_7 x + \theta_3} + \theta_{10} i^2\theta_7^2 e^{16i\theta_7^5 t - i\theta_7 x - \theta_3} + \cosh(16i\theta_7^5 t - i\theta_7 x - \theta_6) i^2\theta_7^2 - \cos(16t\theta_7^5 - x\theta_7 - \theta_9) \theta_7^2}{e^{-16i\theta_7^5 t + i\theta_7 x + \theta_3} + \theta_{10} e^{16i\theta_7^5 t - i\theta_7 x - \theta_3} + \cosh(16i\theta_7^5 t - i\theta_7 x - \theta_6) + \cos(16t\theta_7^5 - x\theta_7 - \theta_9)}$$

$$- \frac{\left(i\theta_7 e^{-16i\theta_7^5 t + i\theta_7 x + \theta_3} - \theta_{10} i\theta_7 e^{16i\theta_7^5 t - i\theta_7 x - \theta_3} - \sinh(16i\theta_7^5 t - i\theta_7 x - \theta_6) i\theta_7 + \sin(16t\theta_7^5 - x\theta_7 - \theta_9) \theta_7\right)^2}{\left(e^{-16i\theta_7^5 t + i\theta_7 x + \theta_3} + \theta_{10} e^{16i\theta_7^5 t - i\theta_7 x - \theta_3} + \cosh(16i\theta_7^5 t - i\theta_7 x - \theta_6) + \cos(16t\theta_7^5 - x\theta_7 - \theta_9)\right)^2}. \quad (42)$$

Case 2.

Substituting (43) into (39) and (40), we achieve a periodic-kink wave solution of Equation (1) as follows:

$$\begin{aligned}\theta_1 &= \sqrt{3}\theta_7 = i\sqrt{3}\theta_4, \theta_2 = -\sqrt{3}\theta_8 = 16\sqrt{3}i\theta_4^5, \\ \theta_5 &= -16\theta_4^5, \theta_{10} = 0, \theta_{11} = 0, i^2 = -1.\end{aligned}\quad (43)$$

$$\begin{aligned}f &= e^{16\sqrt{3}\theta_4^5 it + \sqrt{3}i\theta_4 x + \theta_3} + \sinh(-16t\theta_4^5 + x\theta_4 + \theta_6) + \sin(-16\theta_4^5 it + i\theta_4 x + \theta_9), \\ \Psi_2 &= \frac{3i^2\theta_4^2 e^{16\sqrt{3}\theta_4^5 it + \sqrt{3}i\theta_4 x + \theta_3} + \cosh(16t\theta_4^5 - x\theta_4 - \theta_6)\theta_4^2 - \cos(16\theta_4^5 it - i\theta_4 x - \theta_9)i^2\theta_4^2}{e^{16\sqrt{3}\theta_4^5 it + \sqrt{3}i\theta_4 x + \theta_3} + \cosh(16t\theta_4^5 - x\theta_4 - \theta_6) + \cos(16\theta_4^5 it - i\theta_4 x - \theta_9)} \\ &\quad - \frac{\left(\sqrt{3}i\theta_4 e^{16\sqrt{3}\theta_4^5 it + \sqrt{3}i\theta_4 x + \theta_3} - \sinh(16t\theta_4^5 - x\theta_4 - \theta_6)\theta_4 + \sin(16\theta_4^5 it - i\theta_4 x - \theta_9)i\theta_4\right)^2}{\left(e^{16\sqrt{3}\theta_4^5 it + \sqrt{3}i\theta_4 x + \theta_3} + \cosh(16t\theta_4^5 - x\theta_4 - \theta_6) + \cos(16\theta_4^5 it - i\theta_4 x - \theta_9)\right)^2}.\end{aligned}\quad (44)$$

3.6. Periodic Type Wave Solutions-I. Here, we will consider the periodic type wave solution with selecting the below function which for Equation (1) has been taken as

$$\begin{aligned}f &= \theta_{10} \exp(\tau_1) + \theta_{11} \exp(-\tau_2) + \theta_{12} \cos(\tau_3), \tau_1 = \theta_1 x \\ &\quad + \theta_2 t + \theta_3, \tau_2 = \theta_4 x + \theta_5 t + \theta_6, \tau_3 = \theta_7 x + \theta_8 t + \theta_9,\end{aligned}\quad (45)$$

$$\Psi(x, t) = \ln(f)_{xx}, \quad (46)$$

where $\theta_i, i = 1, \dots, 11$, are undetermined amounts which should be detected. Appending (46) into Equation (1) and afterwards collecting the coefficients, we obtain the following consequences:

Case 1.

$$\theta_1 = \sqrt{3}\theta_7, \theta_2 = 16\sqrt{3}\theta_7^5, \theta_8 = -16\theta_7^5, \theta_{11} = 0. \quad (47)$$

Substituting (47) into (45) and (46), a periodic type wave solution of Equation (1) can be obtained as follows:

$$\begin{aligned}f &= \theta_{10} e^{16\theta_7^5 \sqrt{3}t + \sqrt{3}\theta_7 x + \theta_3} + \theta_{12} \cos(16t\theta_7^5 - x\theta_7 - \theta_9), \\ \Psi_1 &= \frac{3\theta_{10}\theta_7^2 e^{16\theta_7^5 \sqrt{3}t + \sqrt{3}\theta_7 x + \theta_3} - \theta_{12} \cos(16t\theta_7^5 - x\theta_7 - \theta_9)\theta_7^2}{\theta_{10} e^{16\theta_7^5 \sqrt{3}t + \sqrt{3}\theta_7 x + \theta_3} + \theta_{12} \cos(16t\theta_7^5 - x\theta_7 - \theta_9)} \\ &\quad - \frac{\left(\theta_{10} \sqrt{3}\theta_7 e^{16\theta_7^5 \sqrt{3}t + \sqrt{3}\theta_7 x + \theta_3} + \theta_{12} \sin(16t\theta_7^5 - x\theta_7 - \theta_9)\theta_7\right)^2}{\left(\theta_{10} e^{16\theta_7^5 \sqrt{3}t + \sqrt{3}\theta_7 x + \theta_3} + \theta_{12} \cos(16t\theta_7^5 - x\theta_7 - \theta_9)\right)^2}.\end{aligned}\quad (48)$$

Case 2.

$$\theta_1 = -\theta_4 = -\sqrt{3}\theta_7, \theta_2 = -\theta_5 = -16\sqrt{3}\theta_7^5, \theta_8 = -16\theta_7^5. \quad (49)$$

Substituting (49) into (45) and (46), a periodic type wave solution of Equation (1) can be obtained as follows:

$$\begin{aligned}f &= \theta_{10} e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x + \theta_3} + \theta_{11} e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x - \theta_6} + \theta_{12} \cos(16t\theta_7^5 - x\theta_7 - \theta_9), \\ \Psi_2 &= \frac{3\theta_{10}\theta_7^2 e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x + \theta_3} + 3\theta_{11}\theta_7^2 e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x - \theta_6} - \theta_{12} \cos(16t\theta_7^5 - x\theta_7 - \theta_9)\theta_7^2}{\theta_{10} e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x + \theta_3} + \theta_{11} e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x - \theta_6} + \theta_{12} \cos(16t\theta_7^5 - x\theta_7 - \theta_9)} \\ &\quad - \frac{\left(-\theta_{10} \sqrt{3}\theta_7 e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x + \theta_3} - \theta_{11} \sqrt{3}\theta_7 e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x - \theta_6} + \theta_{12} \sin(16t\theta_7^5 - x\theta_7 - \theta_9)\theta_7\right)^2}{\left(\theta_{10} e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x + \theta_3} + \theta_{11} e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x - \theta_6} + \theta_{12} \cos(16t\theta_7^5 - x\theta_7 - \theta_9)\right)^2}.\end{aligned}\quad (50)$$

3.7. Periodic Type Wave Solutions-II. Here, we will consider the periodic wave solution with selecting the below function which for Equation (1) has been taken as

$$\begin{aligned}f &= \theta_{10} \exp(\tau_1) + \theta_{11} \exp(-\tau_2) + \theta_{12} \sin(\tau_3), \tau_1 = \theta_1 x \\ &\quad + \theta_2 t + \theta_3, \tau_2 = \theta_4 x + \theta_5 t + \theta_6, \tau_3 = \theta_7 x + \theta_8 t + \theta_9,\end{aligned}\quad (51)$$

$$\Psi(x, t) = \ln(f)_{xx}, \quad (52)$$

where $\theta_i, i = 1, \dots, 11$, are undetermined amounts which should be detected. Appending (52) into Equation (1) and afterwards collecting the coefficients, we obtain the following consequences:

Case 1.

$$\theta_1 = \sqrt{3}\theta_7, \theta_2 = 16\sqrt{3}\theta_7^5, \theta_8 = -16\theta_7^5, \theta_{11} = 0. \quad (53)$$

Substituting (53) into (51) and (52), a periodic type wave solution of Equation (1) can be obtained as follows:

$$\begin{aligned} f &= \theta_{10} e^{16\theta_7^5 \sqrt{3}t + \sqrt{3}\theta_7 x + \theta_3} + \theta_{12} \sin(-16t\theta_7^5 + x\theta_7 + \theta_9), \\ \Psi_1 &= \frac{3\theta_{11}\theta_7^2 e^{16\theta_7^5 \sqrt{3}t + \sqrt{3}\theta_7 x + \theta_3} + \theta_{12} \sin(16t\theta_7^5 - x\theta_7 - \theta_9)\theta_7^2}{\theta_{11} e^{16\theta_7^5 \sqrt{3}t + \sqrt{3}\theta_7 x + \theta_3} - \theta_{12} \sin(16t\theta_7^5 - x\theta_7 - \theta_9)} \\ &\quad - \frac{(\theta_{11} \sqrt{3}\theta_7 e^{16\theta_7^5 \sqrt{3}t + \sqrt{3}\theta_7 x + \theta_3} + \theta_{12} \cos(16t\theta_7^5 - x\theta_7 - \theta_9)\theta_7)^2}{(\theta_{11} e^{16\theta_7^5 \sqrt{3}t + \sqrt{3}\theta_7 x + \theta_3} - \theta_{12} \sin(16t\theta_7^5 - x\theta_7 - \theta_9))^2}. \end{aligned} \quad (54)$$

Case 2.

$$\theta_1 = -\theta_4 = -\sqrt{3}\theta_7, \theta_2 = -\theta_5 = -16\sqrt{3}\theta_7^5, \theta_8 = -16\theta_7^5. \quad (55)$$

Substituting (55) into (51) and (52), a periodic type wave solution of Equation (1) can be obtained as follows:

$$\begin{aligned} f &= \theta_{10} e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x + \theta_3} + \theta_{11} e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x - \theta_6} - \theta_{12} \sin(16t\theta_7^5 - x\theta_7 - \theta_9), \\ \Psi_2 &= \frac{3\theta_{10}\theta_7^2 e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x + \theta_3} + 3\theta_{11}\theta_7^2 e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x - \theta_6} + \theta_{12} \sin(16t\theta_7^5 - x\theta_7 - \theta_9)\theta_7^2}{\theta_{10} e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x + \theta_3} + \theta_{11} e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x - \theta_6} - \theta_{12} \sin(16t\theta_7^5 - x\theta_7 - \theta_9)} \\ &\quad - \frac{(-\theta_{10} \sqrt{3}\theta_7 e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x + \theta_3} - \theta_{11} \sqrt{3}\theta_7 e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x - \theta_6} + \theta_{12} \cos(16t\theta_7^5 - x\theta_7 - \theta_9)\theta_7)^2}{(\theta_{10} e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x + \theta_3} + \theta_{11} e^{-16\theta_7^5 \sqrt{3}t - \sqrt{3}\theta_7 x - \theta_6} - \theta_{12} \sin(16t\theta_7^5 - x\theta_7 - \theta_9))^2}. \end{aligned} \quad (56)$$

3.8. Solitary Wave Solutions. Here, we will consider the solitary wave solution with selecting the below function which for Equation (1) has been taken as

$$\begin{aligned} f &= \theta_7 + \theta_8 \exp(\tau_1) + \theta_9 \exp(-\tau_2), \tau_1 = \theta_1 x \\ &\quad + \theta_2 t + \theta_3, \tau_2 = \theta_4 x + \theta_5 t + \theta_6, \end{aligned} \quad (57)$$

$$\Psi(x, t) = \ln(f)_{xx}, \quad (58)$$

where $\theta_i, i = 1, \dots, 9$, are undetermined amounts which should be detected. Appending (58) into Equation (1) and afterwards collecting the coefficients, we obtain the below consequences:

Case 1.

$$\begin{aligned} \theta_2 &= -\theta_1^5 - 5\theta_1^4\theta_4 - 10\theta_1^3\theta_4^2 - 10\theta_1^2\theta_4^3 \\ &\quad - 5\theta_1\theta_4^4 - \theta_4^5 - \theta_5, \theta_7 = 0. \end{aligned} \quad (59)$$

Substituting (59) into (57) and (58), a solitary wave solution of Equation (1) can be written as follows:

$$\begin{aligned} f &= \theta_8 e^{(-\theta_1^5 - 5\theta_1^4\theta_4 - 10\theta_1^3\theta_4^2 - 10\theta_1^2\theta_4^3 - 5\theta_1\theta_4^4 - \theta_4^5 - \theta_5)t + \theta_1 x + \theta_3} \\ &\quad + \theta_9 e^{-t\theta_5 - x\theta_4 - \theta_6}, \end{aligned} \quad (60)$$

$$\begin{aligned} \Psi_1 &= \frac{\theta_8 \theta_1^2 e^{(-\theta_1^5 - 5\theta_1^4\theta_4 - 10\theta_1^3\theta_4^2 - 10\theta_1^2\theta_4^3 - 5\theta_1\theta_4^4 - \theta_4^5 - \theta_5)t + \theta_1 x + \theta_3} + \theta_9 \theta_4^2 e^{-t\theta_5 - x\theta_4 - \theta_6}}{\theta_8 e^{(-\theta_1^5 - 5\theta_1^4\theta_4 - 10\theta_1^3\theta_4^2 - 10\theta_1^2\theta_4^3 - 5\theta_1\theta_4^4 - \theta_4^5 - \theta_5)t + \theta_1 x + \theta_3} + \theta_9 e^{-t\theta_5 - x\theta_4 - \theta_6}} \\ &\quad - \frac{(\theta_8 \theta_1 e^{(-\theta_1^5 - 5\theta_1^4\theta_4 - 10\theta_1^3\theta_4^2 - 10\theta_1^2\theta_4^3 - 5\theta_1\theta_4^4 - \theta_4^5 - \theta_5)t + \theta_1 x + \theta_3} - \theta_9 \theta_4 e^{-t\theta_5 - x\theta_4 - \theta_6})^2}{(\theta_8 e^{(-\theta_1^5 - 5\theta_1^4\theta_4 - 10\theta_1^3\theta_4^2 - 10\theta_1^2\theta_4^3 - 5\theta_1\theta_4^4 - \theta_4^5 - \theta_5)t + \theta_1 x + \theta_3} + \theta_9 e^{-t\theta_5 - x\theta_4 - \theta_6})^2}. \end{aligned} \quad (61)$$

By using suitable values of parameters, the analytical treatment of periodic wave solution is presented in Figure 1 including 3D plot and 2D plot with three points of time including $t = 0$, $t = 0.02$, and $t = 0.04$.

Case 2.

$$\theta_1 = -\theta_4, \theta_2 = \theta_4^5, \theta_5 = -\theta_4^5. \quad (62)$$

Substituting (62) into (57) and (58), a solitary wave solution of Equation (1) can be obtained as follows:

$$\begin{aligned} f &= \theta_7 + \theta_8 e^{t\theta_4^5 - x\theta_4 + \theta_3} + \theta_9 e^{t\theta_4^5 - x\theta_4 - \theta_6}, \\ \Psi_2 &= \frac{\theta_4^2 (\theta_8 e^{t\theta_4^5 - x\theta_4 + \theta_3} + \theta_9 e^{t\theta_4^5 - x\theta_4 - \theta_6})\theta_7}{(\theta_7 + \theta_8 e^{t\theta_4^5 - x\theta_4 + \theta_3} + \theta_9 e^{t\theta_4^5 - x\theta_4 - \theta_6})^2}. \end{aligned} \quad (63)$$

FIGURE 1: The solitary wave (61) at $\theta_1 = 1, \theta_4 = 2, \theta_3 = 2, \theta_5 = .5, \theta_6 = 3, \theta_8 = 1.3, \theta_9 = 2.4$.

FIGURE 2: The solitary wave (63) at $\theta_4 = 2, \theta_3 = 2, \theta_6 = 3, \theta_7 = 1.3, \theta_8 = 2, \theta_9 = 2.4$.

By using suitable values of parameters, the analytical treatment of periodic wave solution is presented in Figure 2 including 3D plot and 2D plot with three points of time including $t = 0, t = 0.02$, and $t = 0.04$.

Case 3.

$$\theta_1 = -\frac{1}{2}(-1 \pm \sqrt{3}i)\theta_4, \theta_2 = \frac{1}{2}(1 \pm \sqrt{3}i)\theta_4^5, \theta_5 = -\theta_4^5. \quad (64)$$

Substituting (64) into (57) and (58), we achieve a solitary wave solution of Equation (1) as follows:

$$\begin{aligned} f &= \theta_7 + \theta_8 e^{(1/2)t\theta_4^5(i\sqrt{3}+1) - (1/2)x\theta_4(i\sqrt{3}-1) + \theta_3} + \theta_9 e^{t\theta_4^5 - x\theta_4 - \theta_6}, \\ \Psi_3 &= \frac{(1/4)\theta_8\theta_4^2(i\sqrt{3}-1)^2 e^{(1/2)t\theta_4^5(i\sqrt{3}+1) - (1/2)x\theta_4(i\sqrt{3}-1) + \theta_3} + \theta_9\theta_4^2 e^{t\theta_4^5 - x\theta_4 - \theta_6}}{\theta_7 + \theta_8 e^{(1/2)t\theta_4^5(i\sqrt{3}+1) - (1/2)x\theta_4(i\sqrt{3}-1) + \theta_3} + \theta_9 e^{t\theta_4^5 - x\theta_4 - \theta_6}} \\ &\quad - \frac{\left(- (1/2)\theta_8\theta_4(i\sqrt{3}-1) e^{(1/2)t\theta_4^5(i\sqrt{3}+1) - (1/2)x\theta_4(i\sqrt{3}-1) + \theta_3} - \theta_9\theta_4 e^{t\theta_4^5 - x\theta_4 - \theta_6}\right)^2}{\left(\theta_7 + \theta_8 e^{(1/2)t\theta_4^5(i\sqrt{3}+1) - (1/2)x\theta_4(i\sqrt{3}-1) + \theta_3} + \theta_9 e^{t\theta_4^5 - x\theta_4 - \theta_6}\right)^2}. \end{aligned} \quad (65)$$

4. Stability Analysis of CDG Equation

According to [59], in order to analyze the propagation characteristics of the rogue wave in detail, we choose the linear stability analysis for the CDG equation via the following

function along with appropriate parameters:

$$\Phi(x, t) = \theta + \delta \Omega(x, t), \quad (66)$$

where the relation constant θ is a steady state solution of Equation (66). Appending (66) into Equation (1), one can obtain

$$\begin{aligned} & \delta \frac{\partial}{\partial t} \Omega(x, t) + \delta \frac{\partial^5}{\partial x^5} \Omega(x, t) + 30\delta \left(\frac{\partial^3}{\partial x^3} \Omega(x, t) \right) \theta \\ & + 30\delta^2 \left(\frac{\partial^3}{\partial x^3} \Omega(x, t) \right) \Omega(x, t) + 30\delta^2 \left(\frac{\partial}{\partial x} \Omega(x, t) \right) \\ & \cdot \frac{\partial^2}{\partial x^2} \Omega(x, t) + 180\delta \left(\frac{\partial}{\partial x} \Omega(x, t) \right) \theta^2 + 360\delta^2 \left(\frac{\partial}{\partial x} \Omega(x, t) \right) \\ & \cdot \theta \Omega(x, t) + 180\delta^3 \left(\frac{\partial}{\partial x} \Omega(x, t) \right) (\Omega(x, t))^2 = 0. \end{aligned} \quad (67)$$

By linearization of Equation (67), we get

$$\begin{aligned} & \delta \frac{\partial}{\partial t} \Omega(x, t) + \delta \frac{\partial^5}{\partial x^5} \Omega(x, t) + 30\delta \left(\frac{\partial^3}{\partial x^3} \Omega(x, t) \right) \theta \\ & + 180\delta \left(\frac{\partial}{\partial x} \Omega(x, t) \right) \theta^2 = 0. \end{aligned} \quad (68)$$

Theorem 1. Presume that the solution of Equation (68) has the following form:

$$\Omega(x, t) = \rho_1 e^{i(\alpha x + \beta t)}, \quad (69)$$

where α, β are the normalized wave numbers, by putting (69) into Equation (68), then by solving for β , we can achieve the following form

$$\beta(\alpha) = -\alpha^5 + 30\alpha^3\theta - 180\alpha\theta^2. \quad (70)$$

Proof. By appending the equality (69) in the linear PDE (68), we obtain

$$\begin{aligned} & \delta \frac{\partial}{\partial t} \Omega(x, t) + \delta \frac{\partial^5}{\partial x^5} \Omega(x, t) + 30\delta \left(\frac{\partial^3}{\partial x^3} \Omega(x, t) \right) \theta \\ & + 180\delta \left(\frac{\partial}{\partial x} \Omega(x, t) \right) \theta^2 = i e^{i(\alpha x + \beta t)} \\ & \cdot \delta \rho_1 (\alpha^5 - 30\alpha^3\theta + 180\alpha\theta^2 + \beta) = 0. \end{aligned} \quad (71)$$

By solving and simplifying, we can find the value of $\beta(\alpha)$ as follows:

$$\beta(\alpha) = -\alpha^5 + 30\alpha^3\theta - 180\alpha\theta^2. \quad (72)$$

After that, we get to the needed solution. Hence, the proof of the theorem is complete.

FIGURE 3: The behavior of stability analysis $\beta(\alpha)$ and wave number α with the disparate amounts $\theta = 1, \theta = 2, \theta = 3$.

FIGURE 4: The behavior of stability analysis $\beta(\alpha)$ and wave number α with the disparate amounts $\theta = -5, \theta = -3, \theta = -1$.

In Figures 3–5, it can be seen that when the sign of $\beta(\alpha)$ is positive for all amounts of α , then any superposition of solutions of the form $e^{i(\alpha x + \beta t)}$ will come to ascent, while the sign of $\beta(\alpha)$ is negative for all amounts of α , then any superposition of solutions of the form $e^{i(\alpha x + \beta t)}$ will come to decay and the steady condition is stable. After that, in Figures 3 and 4, it can be observed that if the $\beta(\alpha)$ is positive or negative for some amounts of α , then with increasing time some components of a superposition will become descent, and the steady condition is stable. Finally, in Figure 5, it can be perceived that when the sign of $\beta(\alpha)$ is positive for all amounts of α , then any superposition of solutions of the form $e^{i(\alpha x + \beta t)}$ will come to ascent, while the sign of $\beta(\alpha)$ is negative for all amounts of α , then any superposition of solutions of the form $e^{i(\alpha x + \beta t)}$ will come to decay and the steady condition is stable.

FIGURE 5: The behavior of stability analysis $\beta(\alpha)$ and wave number α with the disparate amounts $\theta = -1, \theta = 0, \theta = 1$.

5. Conclusion

In this work, the multiple exp-function, cross-kink, periodic-kink, and solitary wave methods with predictability of the $(1+1)$ -dimensional CDG equation are investigated with more arbitrary autocephalous parameters. It is not hard to see that the general periodic-kink solution is an algebraically wave solution, and we noticed that some obtained solutions are singular periodic solitary wave solution which is periodic wave or periodic-kink, or solitary wave solutions in $x-t$ direction. Also, the other presented solution is a breather type of two-solitary wave solution which contains a periodic wave and two solitary waves, whose amplitude periodically oscillates with the evolution of time. Moreover, the kink and periodic solutions were analyzed and investigated. In addition, the periodic-kink waves appeared when the periodic solution cut by a stripe soliton before or after a special time. Meanwhile, the modulation instability was applied to discuss the stability of earned solutions. Finally, we show some graphs to explain these solutions.

Data Availability

The datasets supporting the conclusions of this article are included within the article and its additional file.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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